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Matriculation Arithmetic

WITH

NUMEROUS EXAMPLES AND SIMPLE GRAPHS

FOR THE USE OF SCHOOLS AND COLLEGES

BY

GAURI SANKAR DE, 

PROFESSOR OF MATHEMATICS, SCOTTISH CHURCH'S COLLEGE AND
HONORARY FELLOW OF THE CALCUTTA UNIVERSITY

TWELFTH EDITION.

(Thoroughly Revised and Improved)

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PREFACE.

IN this book will be found all that is required in Arithmetic of the students of our Indian Universities. It will be useful to students who may afterwards have to serve in Mercantile offices. Any one, who intends to learn Arithmetic thoroughly, will find in it a safe and complete guide. It differs from the existing treatises in the greater prominence given to the *Unitary Method* and *Arithmetical Equations*. The Unitary Method (called *Subhankar's* method in this country) is practically more useful than the method of Rule of Three. The solution of a problem by the Unitary method gives a greater insight into it than the method of Rule of Three, the use of which in most cases is merely mechanical. The Arithmetical Equations require only certain axioms which are common to all branches of Mathematics.

The Examples in this book are more numerous and of greater variety in the belief that a sound knowledge of the analytical part of Mathematics requires a sound knowledge of Arithmetic, and this can be effected only by the student being drilled with home exercises of at least four sums of Arithmetic every day from the 8th to the 3d class. The collection of examples in this book is sufficiently large to obviate the necessity of buying another book of Arithmetic. Typical examples of every variety have been worked out, and no pains have been spared to make them really instructive.

One third of the more important examples in each set should be worked out in the class and the remaining two-thirds may be given as home exercises. The more difficult examples in each set and the Miscellaneous Examples may advantageously be left for a revisional course. The Oral Examples should not be neglected.

Typographical errors are likely to have crept in in this the first edition. I shall, therefore, feel highly obliged if any one using this book would be good enough to point them out either to me or to the publishers.

In conclusion, I have to thank many friends who have assisted me in the verification of the Answers of the examples of this book, and especially Babu Chunilal Sil, late principal Mathematical Teacher of the General Assembly's Institution and author of several mathematical works, who has materially helped me in the preparation of this work and without whose help it would perhaps not have been possible for me to complete it.

38/2, NIMMONY MITTFR'S STREET, } GAURI SANKAR DE.
Calcutta the 15th December, 1897 }

PREFACE TO THE SECOND EDITION

I AM very grateful to the Heads of Institutions, and the reading public for the very cordial reception given to this book, the first edition of which has been sold off in the very brief space of two months

I also take this opportunity of acknowledging the help given me by several of my friends in pointing out errors, verifying answers of examples, and making valuable suggestions

In this edition only slight alterations have been made here and there, and errors corrected. About 200 of the less important miscellaneous examples have been omitted from the latter part to reduce the size of the book

38/2, NILMONY MITTER'S STREET, } GAURI SANKAR DE
20th April, 1898

PREFACE TO THE THIRD EDITION

IN this edition the book has been thoroughly revised and only slight alterations and additions have been made in certain places. Almost all the examples have been worked anew in the course of preparing the Key to this book which has been out about a month ago. I hope that few errors are left in this edition

I have to tender my thanks to my friends and correspondents who have pointed out errors and communicated suggestions for the improvement of the book. Any communication for the improvement of the book will be thankfully received

38/2, NILMONY MITTER'S STREET, } GAURI SANKAR DE
The 29th December, 1898

PREFACE TO THE SEVENTH EDITION

IN this edition the book has been thoroughly revised and several alterations and additions have been made. Many unimportant articles and examples have been omitted to reduce the bulk of the book and at the same time great pains have been taken to ensure accuracy in the examples and the answers

38/2, NILMONY MITTER'S STREET, } GAURI SANKAR DE
The 2nd February, 1905

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MATRICULATION ARITHMETIC.

CHAPTER I

Definitions, Names of Numbers, Notation and Numeration

I DEFINITIONS AND PRELIMINARY NOTIONS

1 Anything that is capable of increase or diminution is called a **magnitude**

2 A magnitude may be *whole and undivided*, as the length of a stick, a period of time, or it may consist of *separate and distinct* parts, as a heap of pebbles, a herd of oxen, a pack of dogs

3 When a magnitude is whole and undivided, we select some well marked magnitude of that kind which we call its **unit**, and by counting this unit a sufficient number of times, we make up the given magnitude, but if the magnitude be made up of distinct objects, we select an object of that kind as our **unit**, and see how many of these units are to be taken to make up the given magnitude

4 Hence, a **unit**, or as it is generally called **unity**, is the representation of a thing considered in its *individual* capacity, without regard to the *parts* of which it may be made up, and it is the *Base* or *Element* of all our computations

Thus, each of the terms, *a man, a house, a pound, &c* denotes one individual of its kind, being the same as *one man, one house, one pound, &c* respectively, and these are the bases or elements by means of which *several men, several houses, several pounds, &c*, may be computed

5 A magnitude represented as made up of one or more of its unit, is called a **quantity**, and the result of the comparison of the given magnitude with its unit respecting how many times it contains its unit is called **number**

Thus, the length of a stick, a heap of pebbles are *magnitudes*, ten yards, a hundred pebbles are *quantities*, ten and a hundred are *numbers*

6 Hence, **number** signifies *one or more* units, or denotes one or more *distinct* objects of the same kind

Thus, *one man, two houses, three pounds, &c* which are represented by the numbers, *one, two, three, &c* denote one or more individuals of the same kind

7 Numbers thus viewed or considered are termed **whole numbers** or **integers**, and the *unit* is considered as the *first* or *least* integer

8 The **measure** or **numerical value** of any quantity is the *number* of times the quantity contains the unit

Thus, when a foot is used as the unit of length, and we speak of a rod as four feet long, the number *four* represents the measure of the stick

9 Hence the measure of a quantity represents its *relative* magnitude, but the measure and the unit together indicate its *absolute* magnitude.

10 Numbers are either **abstract** or **concrete**

A *concrete* or *applicate* number is, a number of objects or units of any kind, an *abstract* number is a number considered separately and without any relation to objects

Thus, *five* apples, *ten* pounds, *four* men are *concrete* numbers, *five*, *ten*, *four* are *abstract* numbers

11 Hence, an *abstract* number is a number in its literal sense, giving the idea of times or repetitions, but a *concrete* number is simply a quantity

12 **Arithmetic** is the Science of numbers. It investigates their properties, and points out methods of calculations by means of them

II NAMES OF NUMBERS

13 The *Symbol* or *Representation* of unit or unity is 1, but instead of other numbers being expressed by assemblages or multitudes of units *placed* together, which would soon become embarrassing, other characters or symbols have been invented, by means of which every number however great may be expressed, again, instead of a different symbol being adopted for every different number, which would soon become equally inconvenient, *all* numbers are expressed by means of the following *ten* symbols, or as they are usually termed **figures**, and sometimes **digits**, which have their names respectively annexed —

1,	2,	3,	4,	5,	6,	7,	8,	9,	0
one,	two,	three,	four,	five,	six,	seven,	eight,	nine	zero

the first *nine* of which are all defined by their names thus, one and one is **two**, two and one is **three**, three and one is **four**, four and one is **five**, and so on, and the last which is variously denominated **zero**, **cipher**, or **nought**, when standing by itself has no signification or at most, denotes the absence of number, and is to be regarded merely as an *auxiliary* digit for the purposes hereafter to be explained. These nine digits are called *simple numbers*, and *units of the first order*. Their names are perfectly arbitrary

14 The next number is *nine* and *one*, and we give it the name *ten*. Ten forms a single *unit of the second order*, and by counting by *ten*, as we before counted by *one*, we have

one ten, two ten, three-ten, four-ten nine ten ,

or more briefly, remembering that "ty" is equivalent to ten, and treating ten as a simple number, we say

ten, twenty, thirty, forty, . . . ninety

The names of the nine numbers between ten and twenty, are eleven, twelve, thirteen, fourteen, fifteen, . . . nineteen

The names of the nine numbers between twenty and thirty, thirty and forty, as also the nine numbers that follow ninety are formed by placing in order the names of the first nine numbers after twenty, thirty, . . . , ninety. Thus we get at last ninety-nine

15 The number which follows ninety-nine is ninety-nine and one, or *ten tens*, and is called a *hundred*. It is a single *unit of the third order*, and by counting by *hundreds* as we counted by simple units, we have

one-hundred, two hundred, three hundred, nine-hundred

The names of the ninety-nine numbers between one hundred and two hundred, two hundred and three hundred, . . . , as also the ninety-nine numbers that follow nine hundred, are formed by placing in order the names of the first ninety-nine numbers after one hundred, two hundred, . . . , nine hundred. Thus we get at last nine hundred and ninety-nine

16 The number which follows nine hundred and ninety-nine is nine hundred ninety-nine and one or *ten hundred*, and is called a *thousand*. It is a single *unit of the fourth order* - Proceeding as before, we get *ten-thousand* as forming a single *unit of the fifth order*, and *ten ten thousands* or a *hundred thousand* as a single *unit of the sixth order*, but there being no independent names for these units we call a *thousand* as a *second principal unit*, and count by units, tens and hundreds of thousands

The names of the nine hundred and ninety nine numbers between one thousand and two thousand, two thousand and three thousand, . . . , as also the nine hundred and ninety-nine numbers that follow hundreds of thousand are formed by placing in order the names of the first nine hundred and ninety-nine numbers after one thousand, two thousand, . . . , hundreds of thousands

17 The next number is a *thousand thousands*, and forms a single *unit of the seventh order*. It has an independent name and is called a *million*. Considering a million as a *third principal unit*, we count by units, tens, hundreds, thousands, ten-thousands, and hundred-thousands of millions

18 Lastly, we come to a *million millions*, which is called a *billion*, and forms a single *unit of the thirteenth order*. Proceeding

in this way, we get a *million billions*, which is called a *trillion*, a *million trillions*, which is called a *quadrillion*, and so on

The periods which follow the above in succession are *quintillion*, *sextillion*, *septillion*, *octillion*, &c

In France and some of the United States of America a *thousand millions* is called a *billion*, a *thousand billions* a *trillion*, and so on, hence a billion in England is a trillion in America, &c

19 From what has been said above, it appears that we practically employ not more than *thirteen* independent words — *one, two, three, four, five, six, seven, eight, nine, ten, hundred, thousand, million*, and that *ten* units of any order always make one unit of the next higher order

III NOTATION

20 Notation is the method of expressing by certain symbols or characters, any proposed number expressed in words

21 Beginners have already learnt from Art 13, how to express the numbers from one to nine by *one* figure, the following Article will teach them to express numbers from ten to ninety-nine by the use of *two* figures

22 When a figure is placed on the *right* of the same or any other figure it has by *universal agreement* the effect of increasing the value of the last mentioned figure *tenfold*, at the same time that it retains its own value

Thus, beginning with the auxiliary digit 0, we have the following numbers and their representations —

10 ten	29 twenty nine	48 forty eight
11 eleven	30 thirty	49 forty nine
12 twelve	31 thirty one	50 fifty
13 thirteen	32 thirty two	51 fifty-one
14 fourteen	33 thirty three	52 fifty-two
15 fifteen	34 thirty-four	53 fifty three
16 sixteen	35 thirty five	54 fifty four
17 seventeen	36 thirty six	55 fifty-five
18 eighteen	37 thirty seven	56 fifty six
19 nineteen	38 thirty eight	57 fifty seven
20 twenty	39 thirty-nine	58 fifty-eight
21 twenty-one	40 forty	59 fifty nine
22 twenty two	41 forty one	60 sixty
23 twenty three	42 forty-two	61 sixty one
24 twenty four	43 forty three	62 sixty two
25 twenty five	44 forty four	63 sixty three
26 twenty six	45 forty five	64 sixty four
27 twenty seven	46 forty six	65 sixty five
28 twenty eight	47 forty seven	66 sixty six

67 sixty-seven	78 seventy-eight	89 eighty-nine
68 sixty-eight	79 seventy nine	90 ninety
69 sixty nine	80 eighty	91 ninety one
70 seventy	81 eighty-one	92 ninety two
71 seventy-one	82 eighty-two	93 ninety-three
72 seventy-two	83 eighty three	94 ninety-four
73 seventy-three	84 eighty four	95 ninety-five
74 seventy-four	85 eighty five	96 ninety-six
75 seventy five	86 eighty six	97 ninety seven
76 seventy-six	87 eighty seven	98 ninety eight
77 seventy-seven	88 eighty-eight	99 ninety nine

which is the largest number that can be expressed by two digits

23 The use of two, either the *same* or *different* figures, will not enable us to go beyond this number, but a repetition of the contrivance in the last Article, will by means of *more* figures supply the defect

Thus, supposing the effect of any figure's being placed on the right of symbols formed as above, to be to increase all their values *tenfold*, we shall have

100 one hundred	200 two hundred
101 one hundred and one	201 two hundred and one
102 one hundred and two	202 two hundred and two
&c	&c

so likewise of succeeding numbers, thus, we have

345 three hundred and forty-five	750 seven hundred and fifty
586 five hundred and eighty-six	946 nine hundred and forty six

and again 999 will be *nine hundred and ninety nine*, which is the largest number capable of being expressed by *three* figures

Here, the *first* figure on the right hand is said to occupy the *units place*, the *second*, the place of *tens*, and the *third*, that of *hundreds*

Of the auxiliary digit 0, the sole use is in the effect specified in the last two Articles, and all figures to the *right* of it will therefore be unaffected by it

24 In estimating numerical magnitudes, we proceed in order from *hundreds*, to *thousands*, *tens of thousands*, and *hundreds of thousands*, *millions*, *tens of millions*, and *hundreds of millions*, in precisely the same manner as we have done above from *units* to *tens*, and from *tens* to *hundreds*

25 Agreeably to the principle of Art 22, it is *assumed* that 'any figure placed on the right of one or more figures, has the effect of increasing every one of them tenfold without altering its own value', and this enables us to express with facility any number whatever

Thus, 1000 will represent one thousand
 5493 will represent five thousand, four hundred and ninety-three
 23456 will represent twenty three thousand, four hundred and fifty six
 729054 will represent seven hundred twenty-nine thousand and fifty four
 1803205 will represent one million, eight hundred three thousand, two hundred and five
 32754081 will represent thirty two million, seven hundred fifty-four thousand and eighty one
 473025004 will represent four hundred seventy three million, twenty five thousand and four

26 If the first three figures beginning from the right hand be denominated so many *units*, tens of *units* and hundreds of *units*, it follows that the next three figures taken in the same way will be *thousands*, tens of *thousands*, and hundreds of *thousands*, the next three in order will be *millions*, tens of *millions*, and hundreds of *millions*, and so on

Whence, to express in figures any number proposed, we have only to consider in which of these divisions each part of it ought to be found, observing that *three* figures from the right must be taken to make each division *complete*, before we proceed to the next Thus,

Ex 1 Express by means of figures *Thirty-five thousand, eight hundred and nineteen*

Here, eight hundred and nineteen belongs to the *first* division on the right, and is written 819 also, thirty five thousand must be found in the *second* division from the right, and is 35 whence the proposed number will be expressed by 35,819

Ex 2 Write down in figures the number *Five million, twenty five thousand, six hundred and seven*

In this case, the *first* division on the right will be 607, the *second* will be 025, the digit 0 being affixed to the left of the others without altering their values, to make up the required number of *three*, and the third is 5, so that the expression required will be 5,025,607

Ex 3 Express by figures the following number *Five hundred and seventy million, two hundred six thousand and fifty-four*

Here, the *first* division is 054, the 0 altering only the values of the figures in the *subsequent* divisions, the *second* division is 206, and the *third* is 570, whence the number proposed is correctly expressed by 570,206,054

27 This method of notation can never present any difficulty, provided it be carefully remembered that every division of figures, as we proceed from the right hand towards the left must be *completed* as far as it is possible, and by a little practice, we shall be enabled to write down any number by beginning at the *left hand*

Ex 1 To write down *Six hundred and thirteen million five hundred and twenty seven*, we observe that the division of *millions* will be 613, that of *thousands* will be 000, and that of *units* 527, so that the number is expressed by 613,000,527

Ex 2 To represent *Ten thousand million* by figures, for the *fourth* division we have 10, and for each of the *third, second* and *first* 000, so that the representation required is 10,000,000,000

Examples I

Represent the following numbers in figures —

1 Forty-three, seventy-nine, sixty-five, eighty four, fifty-eight, ninety seven, sixty, eighty-seven

2 Four hundred and forty-nine, five hundred and ninety-eight, seven hundred and four, four hundred and five, two hundred and thirty five, nine hundred and fifty eight, seven hundred and twenty-five, eight hundred and thirty-five

3 Four thousand, seven thousand, eight hundred and four, eighty nine thousand and sixty three, fifty-three thousand, two hundred and twenty three, eight thousand and forty-six, six hundred three thousand, two hundred and forty, five hundred thousand, five hundred and five, nine hundred nine thousand and nine

4 Three hundred forty one thousand three hundred and twenty-three, two hundred thousand and seventy five, seven hundred seven thousand and seventy, five hundred thousand, eighty thousand and eight, four hundred two thousand and seven hundred

5 Nine million, forty-three thousand, six hundred and two, seven million, eight hundred fifty nine thousand, six hundred and thirty-two, three million, forty thousand and twenty, one million, four hundred and three thousand, five million, five hundred thousand six hundred and seventy-six, eleven million and five, one million, three hundred, seventy-eight thousand, two hundred and sixty-seven, one million, ten thousand and one

6 Forty-five million, three hundred eighty seven thousand and twenty five, ninety two million, five hundred sixty eight thousand, nine hundred and eighty five, eleven million five hundred sixty-five thousand, four hundred and thirty-seven, forty million, forty thousand and five, ninety-six million, ninety six thousand and ninety six

7 Three hundred forty-nine million, four thousand and sixty-five, one hundred million, thirteen thousand and one, nine hundred nine million, nine thousand and ninety nine, eight hundred forty-two million, two hundred forty-six thousand, four hundred and eighty-four, three million, four hundred fifty two thousand, one hundred and sixty-one, four hundred ninety-four million

8 Ninety-nine million, ninety-nine thousand and ninety-nine, one hundred eleven million, six hundred fifty thousand and fifty,

six hundred forty million, sixty-four thousand and six hundred, five hundred million, seven hundred three thousand and two, six hundred nine million, one thousand, two hundred and eight

9 Two thousand, eight hundred four million, two hundred fifty two thousand and ninety seven, twelve thousand, thirty-six million, fifty four thousand and seventy nine, four thousand million, nine hundred thousand and five, six thousand three hundred four million, five hundred six thousand, five hundred and six, forty thousand, two hundred eighty million, five hundred thirty thousand, two hundred and fifty-nine

10 Four hundred thousand million and ten thousand, eight hundred thirty-six thousand, five hundred and seventy-three million, two hundred forty-four thousand and six, nine hundred thousand, nine hundred million, nine hundred thousand and nine, six hundred thousand, sixty million, six thousand and six

11 Nine thousand four hundred five million four thousand, five hundred and fifty, four hundred thirteen thousand, seven hundred twenty three million, nine thousand and four, five thousand, and eight hundred eight million, sixty-eight thousand and eighty

12 Eight billion, two hundred seven thousand and five, three billion, four thousand, seven hundred two million, one hundred sixty-four thousand, seven hundred and twenty two, one billion, three hundred thousand and five

13 Ninety-nine billion, ninety million, ninety nine thousand, nine hundred and nine, one hundred billion, one hundred ninety six thousand, four hundred million, ten thousand and nine

14 Six hundred fifty-four thousand, three hundred twenty-three billion, four thousand, twenty-one million, fifty thousand, three hundred and one, forty-seven thousand, five hundred twenty-six billion, eight hundred seventy thousand, seven hundred forty-four million, one hundred three thousand, two hundred and eighty four

15 Nine trillion, four billion, six hundred forty million, three hundred and sixty five

16 Write in figures the least number of six digits and the greatest number of eight digits. How many numbers are represented by three digits?

17 Write down in figures all the numbers between eighty-seven and ninety-three, between six hundred and eleven to six hundred and twenty, and between nine hundred and forty seven to nine hundred and seventy

18 When told to write five million, five hundred five thousand, five hundred and five in figures, one boy wrote 550555, and another wrote 50550505, what mistakes did they commit?

28 It will be observed, from what has been said, that each of the nine figures or digits, 1, 2, 3, 4, 5, 6, 7, 8, 9, has a simple

absolute or **intrinsic** value of its own, whereas the auxiliary digit 0 has no such value, and on this account the former are termed **significant** figures, in contradistinction to the last. It will moreover have occurred to the reader, that every one of these significant digits, in addition to its **simple** value, which is fixed and certain, possesses also a **local** or **accidental** value dependent upon the situation in which it is placed.

Thus, in the expression of the number, *Four thousand three hundred and twenty one*, which will be 4321, the 1 in the first place on the right hand, retains its *simple* value, the second figure 2, in its situation denotes two *tens* or *twenty*, the third is three *hundreds*, and the fourth is four *thousands*, so that the *local* values of 2, 3 and 4 here, are respectively, *ten* times, a *hundred* times and a *thousand* times, as great as their *simple* values, and it is the circumstance of assigning to each of the significant figures a **local** as well as a **simple** value, which confers upon the system, the immense powers it possesses.

29 The characters 1, 2, 3, 4, 5, 6, 7, 8, 9, 0 and the mode of representing numbers by their combinations were first invented by the Hindus. The word *Digit* (denoting a *Finger*) usually applied to these characters, seems the point out the means originally employed in estimating numerical magnitudes, the number 10, which is called the *Base* or *Radix* of the system, and by which the *local values* of the *digits* are regulated being that of the *Fingers* of both hands. Thus came the name **Decimal System** of Notation. The system was borrowed from the Hindus by the Arabs, who introduced it into Europe about the 11th century. Hence the Europeans call it the *Arabic Notation*. The Notation appears to be as *complete* and *convenient* as can well be imagined, and in its present state may certainly be regarded as one of the greatest and most successful efforts of human ingenuity ever exhibited to the world.

IV NUMERATION

30 **Numeration** is the art of reading or estimating the value of a number expressed by figures, and is therefore the *reverse* of Notation.

31 From the circumstance of every figure possessing a *local* as well as a *simple* value, it follows that the value of each figure must be estimated by the place which it occupies, hence, a figure standing by itself expresses so many *units*, a figure in the second place from the right, denotes so many *tens*, a figure in the third place, so many *hundreds*, and so on. consequently, if we suppose any numerical expression to be divided into **periods**, or portions each consisting of three figures as far as they go, the figures of the **period** on the right will be **units**, and tens and hundreds of **units**, those of the next will be units, tens and hundreds of **thousands**, those of the third will be units, tens and hundreds of **millions**, and so on.

34 In reference to what was said in Art 32, it may be proper to observe that the method of proceeding differs from that adopted by the *French* and some other *European* Arithmeticians, who adhere throughout to divisions of *three* figures, according to the principle of Art 31, and after the division of *millions*, proceed directly to that of *billions*, tens of *billions*, and hundreds of *billions* then to *trillions*, tens of *trillions*, and hundreds of *trillions*, and so on and this method certainly possesses some advantages in point of simplicity, but as numbers of these magnitudes are not of very frequent occurrence, it has not been thought necessary to depart from the *Notation* and *Nomenclature* established in England

Examples II

Write down in words the following numbers —

- 1 17, 24, 35, 46, 27 48, 59, 76, 84, 95, 66, 75, 89.
- 2 217, 319, 583, 695, 725, 308, 406, 846, 932, 725
- 3 3406, 5260, 4236, 3298, 5678, 2405, 9286
- 4 43201, 87054, 34002, 49803, 58030, 76503
- 5 903756, 903284, 827109, 319420, 243065, 123456
- 6 2714325, 8047328, 4010010, 8004640, 1234007
- 7 12870045, 20084216, 79030284, 43002005
- 8 321408653, 408076032, 314159265, 123456789
- 9 571268405, 3179040601, 319680209078
- 10 1234567654321, 5020040003060, 4302500764009
- 11 200900600002, 43287000006321, 64000002646002
- 12 319080259417, 236045978213478
- 13 1327875430029, 5432176989007

14 Write the largest and the smallest numbers possible with the symbols 5, 4, 9, 2, 7

15 Give the local value of each of the significant digits in the following numbers —

95, 64, 575, 8297, 40276, 3205, 478296, 40302605, 50003029, 70300006, 9786002030

16 Express in words the greatest number of five figures and the least number of seven figures

17 Write down *all* the numbers that can be formed by the digits 2, 3, 4, taken all together

V THE HINDU METHOD OF NUMERATION

35 The following is the Indian Numeration Table in common use —

१८	hundred, of crores चार	१८	tens of lacs निम्न	१८	tens of thousands अथ	१८	hundreds शत
१८	tens of crores अक्ष	१८	lacs लक्ष	१८	thousands सहस्र	१८	tens दश
१८	crores कोटि	१८		१८		१८	units ए
4		7		2		3	
5		5		0		1	
6						4	

The above number is read thus — Four hundred and fifty six crores, seventy five lacs, twenty eight thousand, three hundred and sixty-four

The Hindu names of places of figures are as follow — *eka*, *dasha*, *shata*, *sahasra*, *ajuta*, *laksha* (lac), *myuta*, *koti* (crore), *arabuda*, *brinda*, *lharva*, *mkharva*, *santia*, *sagara*, *padma*, *padma*, *nava*, *mohapadma*, *kshum*, *atshuhum*, *dhuli*, *mohadhuli*, *antya*, *paradha*

Examples III

Write down in words the following numbers according to the Indian Numeration —

- 1 19237 , 60081 , 49027 , 167208 , 200753 , 830005
- 2 7090709 , 8001025 , 3905086 , 24050008 , 4001745
- 3 40217815 , 4030024340 , 4780230016 23456000
- 4 123456789 , 6450300000 , 760242900
- 5 4500002430 , 8000785000 , 4020504008

Express in figures —

6 Four lacs, fifteen thousand, two hundred and eight , fifty six lacs, four thousand and twenty nine , eight hundred forty three lacs, seventy four thousand, two hundred and nine , eight lacs and five , seventy five lacs , thirty lacs, seven hundred and eight

7 Two crores, fifteen lacs and four , thirty seven crores, seven lacs, four thousand and twelve , one hundred and forty five crores, nineteen lacs, and seven , five thousand and ninety nine crores, four lacs, five thousand, six hundred and seven

8 Eighty crores, thirty lacs, one thousand and eleven, four thousand two hundred and ninety-five crores, fourteen lacs, and eighty-five, seventy five thousand four hundred and ten crores, four teen lacs, nine thousand and nine

9 How many lacs are there in twenty millions? How many thousands are in ten lacs? How many millions in forty crores?

10 Read according to the Indian numeration the number—four hundred five million, seventy-five thousand, nine hundred and four

11 Express a *billion* in Indian, and a *akshuhini* in English Notation

12 A boy was told to write nine crores, five lacs four thousand, seven hundred and fifty-six and he wrote 905407056 Find out his mistakes

VI THE ROMAN SYSTEM OF NOTATION

36 A different system of Notation was in use among the Romans, long before the introduction of the Arabic Notation into Europe by the Moors in Spain

In this system the characters chiefly used are I, V, X, L, C, D and M which denote respectively the numbers 1, 5, 10, 50, 100, 500 and 1000 in the Arabic system. Again when a *bar* or *line* is placed over a character, it increases its value a *thousandfold*

Thus \bar{V} stands for 5000, \bar{C} represents 100000

The following table gives a full view of the method of expressing numbers in the Roman System —

I	1	XV	15	CC	200
II	2	XVI	16	CCC	300
III	3	XVII	17	CD	400
IV	4	XVIII	18	D	500
V	5	XIX	19	DC	600
VI	6	XX	20	DCC	700
VII	7	XXX	30	DCCC	800
VIII	8	XL	40	CM	900
IX	9	L	50	M	1000
X	10	LX	60	MCD	1400
XI	11	LXX	70	MCM	1900
XII	12	LXXX	80	MM	2000
XIII	13	XC	90	MDCCCLXXXVI	1886
XIV	14	C	100	$\bar{D}LX$ DCCCXLIV	560844

Examples V (MENTAL ADDITION)

1 Write down the sums of —

- (1) 2 and 4, 2 and 10, 3 and 5, 4 and 7, 5 and 9, 8 and 7
- (2) 9 and 10, 8 and 8, 7 and 3, 7 and 6, 9 and 1, 5 and 9
- (3) 2 and 9, 0 and 7, 4 and 9, 9 and 7, 4 and 11, 9 and 14
- (4) 7 and 7, 7 and 9, 8 and 10, 9 and 6, 4 and 12, 7 and 13-
- (5) 8 and 2, 8 and 5, 9 and 14, 8 and 13, 7 and 15, 6 and 14
- (6) 10 and 6, 10 and 9, 11 and 5, 13 and 6, 14 and 3
- (7) 4 and 17, 3 and 19, 12 and 12, 13 and 13, 16 and 12
- (8) 8 and 0, 12 and 13, 12 and 15, 11 and 16, 10 and 19
- (9) 15 and 8, 11 and 15, 18 and 12, 16 and 15, 13 and 16
- (10) 18 and 16, 15 and 15, 14 and 14, 16 and 16, 11 and 17
- (11) 10 and 11, 10 and 12, 11 and 13, 11 and 18, 12 and 19
- (12) 17 and 17, 18 and 19, 16 and 18, 19 and 19, 16 and 19

- 2 (1) Add 6 to 28, to 38, to 48, to 58, to 68, to 78, to 88, &c
 (2) Add 8 to 25, to 35, to 45, to 55, to 65, to 75, to 85, &c
 (3) Add 15 to 39, to 49, to 59, to 69, to 79, to 89, to 99 .

3 Add together —

- (1) 12 and 37, 13 and 25, 14 and 84, 14 and 26, 14 and 76
- (2) 19 and 75, 17 and 87, 16 and 56, 18 and 75, 18 and 52
- (3) 26 and 64, 36 and 85, 49 and 24, 39 and 75, 27 and 31
- (4) 39 and 42, 49 and 99, 26 and 37, 75 and 94, 53 and 84
- (5) 16 and 85, 17 and 54, 45 and 33, 64 and 89

4 Count aloud by increments of 7 up to 100, starting at 6, at 9-
 at 13, at 15, at 17, at 19, at 21, at 23, at 25, and at 29

5 Find the sums of —

- (1) 1, 3 and 5, 2, 5 and 3, 3, 9 and 7, 8, 4 and 6, 7, 7 and 7-
- (2) 9, 9 and 2, 7, 3 and 6, 8, 5 and 9, 5, 5 and 9, 7, 5 and 9
- (3) 3, 3, 3 and 3, 4, 6, 1 and 9, 8, 0, 9 and 6, 8, 8, 8 and 8
- (4) 5, 5, 8 and 4, 9, 8, 7 and 6, 4, 7, 2 and 6, 6, 7, 8 and 9
- (5) 4, 0, 3, 5 and 9, 6, 0, 5, 0 and 9, 7, 2, 8, 8 and 5

6 Find the values of —

- (1) $3+4+9+3+3+5$, $3+6+8+5+6+4$, $6+0+4+7+0+5$
- (2) $9+5+7+8+3+4$, $6+9+9+7+7+5$, $5+8+9+7+5+6+3$

7 Ram has 6 books, and his brother 5, how many books have they together?

8 A boy has 8 marbles in one pocket, and 5 in another, how many marbles has he?

9 Bepin has 4 marbles, Gopal 7 and Bejoy 5, how many have they together?

10 In a garden there are 4 mango trees, 6 cocoanut trees, 5 jack trees and 8 plum trees, how many trees are there in all?

11 Shyam paid 3 pice for a loaf, 4 pice for sugar, and 2 pice for butter How much did he pay altogether ?

12 One boy gained 3 prizes, another 2, and another 5 How many prizes did the three boys gain ?

13 Hari has 8 marbles, and Bhuvan 7 more than Hari How many have they both together ?

14 One dovecot has 8 pigeons, another has 10, and a third has 12 How many pigeons have the three dovecots ?

15 A boy paid 4 pice for a pencil, 2 pice for a pen holder, 14 pice for a slate and 7 pice for quills, how many did he pay for the whole ?

16 Ram's age is 4 years, Gopal is 2 years older than Ram, Shyam's age is the sum of the ages of the other two Find the sum of all their ages

17 In a school there are four classes In the first class there are 6 boys, in the second class 7 boys, in the third 2 more than in the first class, in the fourth 5 more than in the second class How many boys are there in the school ?

18 Ram, Hari, and Gopal went to fish Ram caught 9 lobsters, Hari caught none, and Gopal caught 12 How many lobsters did the three boys catch ?

19 Ram has a line 6 feet long, Shyam one 10 feet long, and Bhuvan one 9 feet long If the three lines were joined, how long a line would they make ?

20 Jogin got a prize of 5 rupees, Upendra got 6 rupees more than Jogin, how many rupees did they get altogether ?

21 A farmer has 8 cows, 6 calves, and 5 sheep How many animals has he altogether ?

22 Hari got from his father 9 pice, his two brothers 7 and 8 pice respectively, and his sister 5 pice, how much did the father give in all ?

23 A man's age is 38 years, how old will he be after 12 years ?

24 From a rope are cut off 15 yards and there are 6 yards left, what was the length of the rope ?

25 After giving away 15 rupees, I have 8 rupees still left, how many rupees had I in all ?

26 What number is that from which if I take first 8, and then 5, there will remain 24 ?

27 A man has a son whose age is 10 years, he is older than his son by 26 years, what is his age ?

28 I have 25 nuts in my pocket, and my father gives me 15 more, how many have I in all ?

29 A rupee contains 64 pice, how many pice are there in two rupees ?

30 A woman sold 4 mangoes to *A*, to *B* 5 more than to *A*, to *C* as many as to *A* and *B*, to *D* 9 more than to *B*, had *C* bought as many more mangoes as he did buy, the woman would have sold all her mangoes, how many mangoes had she to sell?

SIMPLE ADDITION

42 The principle usually termed **carrying** in the Rule given below is "*that the tens of any order in a partial sum may be carried as units to the next higher order*," for ten units of any order are equivalent to one unit of the next higher order

43 The following is the Rule for the addition of large numbers —

RULE Place the numbers under one another in such a manner that units may stand under units, tens under tens, hundreds under hundreds, and so on, and draw a line below all the horizontal rows of figures. Then add up the figures in the first vertical row on the right-hand, find the numbers of *tens* and *units* in their sum, and put down the number of *units*, whether it be zero or any of the nine other digits. Carry as many *units* as there are *tens* thus found to the next vertical row and add them up as before, observing the numbers of *tens* and *units* contained in the sum. Place the number of *units* under the row added, and carry the number of *tens* to the next, proceed in the same manner till the last row is added, when put down the numbers both of *tens* and *units*, as there are no more figures of higher denominations. The entire sum thus put down will be the sum of the separate numbers

Ex. Add together 6254, 893, 48 and 5487

Arrange the numbers according to the Rule given above, and proceed to add the columns beginning from the column of units

6254	The sum of 4, 3, 8 and 7 is 22	Place the 2 units under
893	tens	the row of units, and carry on the 2 tens units to the row of
48	The sum of 2, 5, 9, 4 and 8 is 28	Place the 8 tens under
5487	the row of tens, and carry on the 2 hundreds units to the	
12682	row of hundreds	

The sum of 2, 2, 8 and 4 is 16. Put down the 6 hundreds under the row of hundreds, and carry on the 1 thousand units to the row of thousands

The sum of 1, 6 and 5 is 12. Put down the 12 under the row of thousands. Thus the entire sum is 12682

44 A **Proof** is a second operation which serves as a test of the correctness of the first

The *Proofs of Addition* depend on this principle—The sum of several numbers is not affected by the order in which they are added together, thus $4+8=8+4$

45 To ascertain whether the operation is correctly performed, various expedients might be resorted to —first, that of adding the numbers *downwards* instead of *upwards*, which, because the *same* set of numbers cannot have two *different* sums, must give the same result second, that of omitting one of the horizontal rows of figures in a *second* operation, and afterwards adding it to the result of the rest obtained by the Rule third, that of *casting out the nines* from the sum of the digits in the *summands* and the sum of the digits in the amount, if the two results coincide the operation may be *presumed* to be correct (*Casting out the nines* is explained in Art 79)

Examples VI

1 Add together —

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
37	90	57	24	98	68	79	12	87	97
42	45	68	56	55	48	27	56	68	59
<u>23</u>	<u>73</u>	<u>75</u>	<u>35</u>	<u>60</u>	<u>99</u>	<u>94</u>	<u>48</u>	<u>59</u>	<u>68</u>
(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	
85	78	310	704	345	2969	787	347	3214	
92	69	46	450	902	4868	678	238	6786	
<u>99</u>	<u>75</u>	<u>147</u>	<u>979</u>	<u>450</u>	<u>6787</u>	<u>425</u>	<u>410</u>	<u>2345</u>	
(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	
889	654	8888	6748	415	293	814	325	4028	
803	546	5173	5555	278	75	326	748	354	
519	465	7421	7864	614	409	628	493	95	
<u>745</u>	<u>824</u>	<u>7643</u>	<u>5408</u>	<u>932</u>	<u>3</u>	<u>459</u>	<u>869</u>	<u>2076</u>	
(29)	(30)	(31)	(32)	(33)	(34)	(35)	(36)	(37)	
736	9806	785	6045	736	8076	459	9542	17384	
402	1932	8756	4500	405	432	3687	876	12345	
4159	6580	9540	8068	8159	5431	7468	4093	5432	
47	9889	8559	9647	49	458	243	7777	946	
<u>2468</u>	<u>7885</u>	<u>386</u>	<u>9407</u>	<u>7204</u>	<u>9327</u>	<u>5907</u>	<u>4685</u>	<u>24607</u>	
(38)	(39)	(40)	(41)	(42)	(43)	(44)			
71407	15161	96748	33456	67895	796210	894142			
90781	8098	25003	84771	56789	34728	378523			
68943	958	84067	66854	98765	51344	66666			
32605	41978	95674	72984	87956	858521	8524			
<u>72777</u>	<u>78368</u>	<u>98765</u>	<u>99999</u>	<u>78965</u>	<u>936266</u>	<u>981234</u>			
(45)	(46)	(47)	(48)	(49)	(50)				
659873	5678912	2345678	1234567	4893054	9876543				
487	4567891	3192463	2345671	9876543	9999999				
6935	3456789	7283642	8742015	2483109	4602				
70415	2144124	9234925	8888888	7834510	341025				
<u>8796</u>	<u>7878787</u>	<u>8364774</u>	<u>4310943</u>	<u>3469146</u>	<u>46</u>				

(51)	(52)	(53)	(54)	(55)
466779	897654	9466495	768402	27591046
878987	987763	7545478	95320089	5768004
365363	123456	29099	6949	39039587
432598	789099	2988607	84982759	596459
756545	789789	9292929	700897	78534842
<u>487988</u>	<u>437977</u>	<u>7833210</u>	<u>78563412</u>	<u>19827634</u>

2 Find the values of —

- (1) $567 + 90 + 48 + 39 + 4728 + 1000 + 6489 + 327 + 4578$
- (2) $37045 + 6879 + 3724 + 4562 + 82971 + 37256 + 409$
- (3) $5971096 + 7266440 + 5846666 + 5325863 + 5755621 + 5656219 + 2754013 + 4036957$
- (4) $48678 + 53232 + 48214 + 87292 + 93246 + 37527 + 40752 + 53033 + 35002 + 15382 + 1128 + 5404$
- (5) $1541061 + 1891484 + 1817881 + 2265380 + 2323979 + 379153 + 2010958 + 1476985 + 1774013 + 1764304 + 1076539 + 847590$
- (6) $795824 + 1049700 + 1279605 + 593411 + 949908 + 8204 + 208513 + 1250687 + 974983 + 1267694 + 2038505 + 801986 + 608592 + 1007740 + 7292$

3 Find the sum of —

- (1) 774145, 999455, 1016062, 1797223, 1854905, 1681274, 74952, 3467035 and 1226612
- (2) 5971096, 1756856, 2124682, 1964909, 2582060, 2633447, 51027, 2280382 and 1721608
- (3) 36530, 4179, 1899, 52773, 130079, 17801, 15235, 118940, 101665, 35584, 5057 and 12162
- (4) 925682143, 832563297, 4327568, 98526342, 753291424, 643263, 71952875 and 2147397
- (5) 441698853, 37519162, 599678437, 4840, 5128697, 20304009, 679821345, 172564 and 4263721

4 Add together seven hundred and six, twenty-five thousand and eighty-four, nineteen thousand and ninety-nine, seven thousand, four hundred and three, ten thousand, ninety-nine thousand and ninety nine, and eight hundred and eighteen

5 Add together five hundred sixty thousand, two hundred and eighteen, ninety thousand and eighty-five, three hundred six thousand, five hundred and sixty seven, seventeen thousand, eight hundred and nine, seventy eight thousand and eight, twelve thousand and fifty, six hundred twenty thousand, six hundred and twenty-six, and nine thousand and twelve

6 Add together seven hundred seven thousand, four hundred and fifty-nine, ninety-eight thousand and seventy-four, six thousand, eight hundred and seven, five hundred thousand, three hundred and nine, seven thousand, nine hundred and seventy-eight, and nine hundred nine thousand, nine hundred and ninety-nine

7 Add together fifty-five millions, seven hundred thousand and five, seven hundred millions, nine hundred eight thousand, two hundred and five, seventy-six millions, fourteen thousand and fifty-nine, eight hundred seventy seven millions, nine hundred two thousand and forty seven; seven millions, eight hundred four thousand, five hundred and twelve, and five hundred seventy five millions, eight hundred one thousand and ninety-one

8 Add together three hundred nine millions, four hundred seventeen thousand and eighty-seven, six hundred seventy five thousand and forty nine, seven thousand ninety seven millions, eight hundred fourteen thousand, three hundred and five, seventy-nine millions, five hundred four thousand and forty nine, six thousand seventy-eight millions, four hundred thirty nine thousand, six hundred and forty-seven, and seven thousand millions, eight hundred seventy-six thousand, four hundred and twenty-nine

9 Find the amount of five thousand, six hundred and ninety-two, four lacs, thirty-five thousand and eleven, eighty five lacs, four hundred and ninety-nine, forty-three lacs and forty-three, and five hundred and four

10 Find the total of six lacs, six thousand and six, four crores, twenty-five lacs, six hundred and thirty-five, nine hundred and ninety-three crores, seventy five lacs, and seventy-five, eighty-five crores, eighty-five lacs, eighty five thousand and eighty-five, twenty-three crores, five lacs, fifty-eight thousand and eighty nine, and four hundred sixty-three crores, nineteen lacs, four thousand and ninety-five

11 One apple-tree had 816 apples on it, and another had 638, how many apples were on both trees?

12 There are 129 boys, 308 girls, and 60 infants in a school how many children are there altogether in the school?

13 A train contains 63 first-class passengers, 120 second-class and 154 third-class, how many passengers are in the train?

14 A man has been working five days. On Monday he earns 25 annas, on Tuesday 34, on Wednesday 16, on Thursday 38, and on Friday 27, how much does he earn in the five days?

15 In one book there are 525 pages, in another 144, and in another as many as in the other two, how many pages are there in the three books?

16 Figures were used by the Arabs in the year 890 and decimal fractions were invented 574 years later, in what year were they invented?

17 Five mango trees produced as follows the first 657, the second 231 more than the first, the third 892, the fourth 11 more than all the first three, the fifth as many as all the others. How many mangoes were there on all the trees?

18 A gentleman left his property by will, thus to his wife nine thousand and eighty rupees, to each of his two younger sons, five thousand, eight hundred and ninety four rupees, the rest of his property in two equal shares between his three daughters, and eldest son the eldest son's share was fifteen hundred and twenty rupees more than the mother's share, what did the gentleman die worth?

19 Europe contains 3807195 square miles, Asia 17805146, Africa 11647428, America 13542400, and Oceania 3347840, what does this make the extent of the land on the surface of the globe?

20 The number of Mahomedans in the Burdwan division is 957630, in the Presidency division 4063137, in the Rajshahye division 4885165, in the Dacca division 5531869, and in the Chittagong division 2425610, find the total Mahomedan population for Bengal Proper

21 Bought a lot of ground for 675 rupees, erected a house upon the same, at a cost for carpenter's works 2540 rupees, mason's works 637 rupees, printer's works 242 rupees and for grading the lot 293 rupees, what was the cost of the whole?

22 A man bought four chests of oranges. In the first chest there were 589 oranges, in the second 215 more than in the first, in the third 197 more than in the first, in the fourth as many as there were in the first and third. How many oranges did he buy?

23 A man has two thousand and eighty one sovereigns, three thousand and sixty-eight rupees, one thousand, one hundred and eleven dollars, and two hundred and sixty nine half rupees. How many coins has he altogether?

24 Find the sum of six numbers each equal to 7003856

25 A man was born in 1764, in what year was he 83 years old?

26 In a dictionary there are 869 words beginning with the letter A, 742 with the letter B, 1061 with the letter C, and 1154 with the letter D. How many words begin with the letters A, B, C and D?

27 Add together the sum of five numbers each equal to 4597 and the sum of four numbers each equal to 89796

28 January has 31 days, February 28, March 31, April 30, May 31, June 30, July 31, August 31, September 30, October 31, November 30 and December 31. How many days are there in the whole year?

29 From a sum of money I first took away 71407 rupees, and then 90781 rupees and had still 69843 rupees left, what was the sum?

30 The number of soldiers in an army of six regiments are 895, 976, 884, 937, 949 and 982 respectively, the first, third and fifth regiments are respectively joined by 246, 145, and 102 soldiers. Find the whole number of soldiers in the six regiments

II SUBTRACTION

46 **Subtraction** is the method by which we find what number is left when a smaller number is taken from a greater

The greater number is called the **minuend**, the smaller one the **subtrahend**, and the number left the **remainder**

47 The number left is the **difference** between the two given numbers, it is also the **excess** of the greater number over the less, it is also the number which must be added to the less number to make it equal to the greater. Hence **Subtraction** is sometimes called **complementary addition**

48 Like *Addition*, *Subtraction* is of two kinds, **simple** and **compound**

(i) *Simple Subtraction* is one in which the numbers are both *abstract* numbers or both *concrete* numbers of the *same* kind

(ii) *Compound Subtraction* is the method of finding the difference between two *concrete* numbers of the same kind, but of *different* denominations of that kind

49 The operation of *Subtraction*, is indicated or expressed by the sign $-$, which is read **minus**, with the use of the sign $=$

Thus, the *excess* of 7 above 3 will be expressed in the form $7 - 3 = 4$, which is read seven *minus* three *equals* four where the sign $-$ between 7 and 3 denotes the subtraction of the latter from the former and the sign $=$ between 3 and 4 shows the *equality* of the excess to 4

50 To effect the operation of *Subtraction*, it is necessary to *recollect* the difference of every two numbers less than 20. The following Table, called the **Subtraction Table**, should be committed to memory by beginners

1 from	2 from	3 from	4 from	5 from	6 from	7 from	8 from	9 from
1 leave 0	2 leave 0	3 leave 0	4 leave 0	5 leave 0	6 leave 0	7 leave 0	8 leave 0	9 leave 0
2 13	14	15	16	17	18	19	110	111
3 24	25	26	27	28	29	210	211	212
4 35	36	37	38	39	310	311	312	313
5 46	47	48	49	410	411	412	413	414
6 57	58	59	510	511	512	513	514	515
7 68	69	610	611	612	613	614	615	616
8 79	710	711	712	713	714	715	716	717
9 810	811	812	813	814	815	816	817	818
10 911	912	913	914	915	916	917	918	919
11 1012	1013	1014	1015	1016	1017	1018	1019	10110

This Table can easily be extended further, for instance, since 2 from 3 leave 1, 2 from 13, *i.e.*, from $3 + 10$, leave $1 + 10$, or 11, the result being 10 more than in the corresponding case in the Table. Also since 7 from 15 leave 8, 7 from 45, *i.e.*, from $15 + 30$, leave $8 + 30$, or 38, the result leaving 30 more than in the corresponding case in the Table. Also since 9 from 14 leave 5, 9 from 54, *i.e.*, from $14 + 40$

leave $5+40$, or 45, and 9 from 99, i.e., from $19+80$, leave $10+80$ or 90, and so on

Examples VII (MENTAL SUBTRACTION)

- 1 (1) Take 2 from 4, from 7, from 11, from 6, from 12, &c
 (2) Take 3 from 4, from 7, from 6, from 8, from 13, &c
 (3) Take 4 from 6, from 9, from 13, from 15, from 19, &c
 (4) Take 8 from 12, from 15, from 19, from 21, from 25, &c
 (5) Take 9 from 15, from 18, from 20, from 24, from 36, &c
- 2 (1) Subtract 6 from 20, 47, 32, 70, 63, 55, 81, 71 and 99
 (2) Subtract 7 from 18, 22, 49, 33, 84, 51, 94, 88 and 38
 (3) Subtract 5 from 18, 25, 53, 61, 70, 82, 67, 93 and 90
- 3 How many does
 (1) 9 leave from 15, 5 from 14, 7 from 12, 9 from 71, 8 from 21?
 (2) 7 leave from 44, 8 from 38, 9 from 88, 6 from 94, 5 from 47?
- 4 Find the difference between —
 (1) 13 and 18, 3 and 14, 20 and 25, 30 and 45, 15 and 11
 (2) 89 and 47, 46 and 12, 34 and 68, 14 and 31, 14 and 95
- 5 What must be added to 11 to make 15, 7 to make 18, 6 to make 15, 4 to make 11, 9 to make 17, 21 to make 49, 31 to make 44, and 30 to make 82?
- 6 By how much does 13 exceed 7, 17 exceed 8, 19 exceed 8, 26 exceed 14, 29 exceed 13, 69 exceed 26, 95 exceed 32, 98 exceed 36, 82 exceed 64, and 89 exceed 72?
- 7 Count by decrements of 3, 5 and 7, commencing at 100
- 8 How much is 33 less 7, 84 less 5, 49 less 6, 67 less $5+2$, 95 less $4+0+4$, and $67+16$ less $15-4$?
- 9 Take $5+3$ from 11, $7+2$ from 17, 12 from $14+11$, 25 from $48+11$, $9+6$ from $12+5$, $3+8$ from $2+9$, and $1+4$ from $2+7$
- 10 How many times can 5 be taken from 15, 6 from 18, 9 from 27, and 12 from 48?
- 11 A girl has 8 oranges. She gives 2 to her sister. How many has she left?
- 12 Shyam has 6 pice. He pays 1 pice for a top, 2 pice for a whistle, and 2 pice for a kite. How many has he over?
- 13 A boy has 18 pice in his pocket. He loses 7 and spends 4. How many pice has he left?
- 14 If you buy 18 yards of ribbon, and find that you have 3 yards too much, how many yards should you have bought?
- 15 A man planted 25 trees, 8 of them died. How many lived?
- 16 Jadu has 19 apples, and Bhuban has 8. How many has Jadu more than Bhuban?

17 I bought 6 pice worth of apples, and 4 pice worth of pears
What money had I over out of 15 pice?

18 A baker's boy sets out with 21 rolls He leaves 5 in one house, 4 in another, 6 in a third and 5 in a fourth How many rolls has he left?

19 Ram is 19 years old, Gopal is 8 years old How many years is Gopal younger than Ram?

20 A man had 26 sheep, he sold 10, and 6 were stolen How many were left?

SIMPLE SUBTRACTION

15 The following are the Rules for the subtraction of large numbers

(i) When none of the figures of the *Subtrahend* exceeds the corresponding figures of the *Minuend*

RULE Place the less number under the greater, so that units may stand under units tens under tens, hundreds under hundreds, and so on; then draw a line below the lower number Begin at the units place and subtract each figure in the lower line from the corresponding figure in the upper, taken by itself, and put down the remainder below the line just drawn, units under units, tens under tens, hundreds under hundreds, and so on The entire difference or remainder, so put down, will be the difference or remainder of the proposed numbers

Ex 1 Subtract 425 from 1679

Place the smaller number 425 under the greater 1679, and draw a line below it First take 5 from 9 and place the difference 4 under the units' figure below the line drawn, next take 2 from 7 and set down the remainder 5 in the tens' place below the line next take 4 from 6 and put down the difference 2 in the hundreds' place under the line. Lastly bring down 1 since there is nothing below it Thus the remainder is 1254

Ex 2 Subtract 5634 from 9657

As before, put 5634 below 9657 and draw a line
Take 4 from 7, the remainder is 3, 3 from 5 leaves 2 as remainder, 6 from 6 leaves *nothing* or 0 as remainder; lastly 5 from 9 leaves 4 as remainder.
Thus the entire remainder is 4023

(ii) If the units of any order in the *Subtrahend* exceed those of the *Minuend*

In this case we avail ourselves of the following principle, usually termed *borrowing* — "The *Minuend* and *Subtrahend* may be increased by the same number without altering their difference"

Hence we may increase the number of units in any order of the *Minuend* by 10, if we increase that of the next higher order in the *Subtrahend* by 1

RULE Place the numbers as in (1) and draw a line below. Begin at the units' figure, but if the said figure in the lower line exceed that in the upper, increase the upper figure by ten and then subtract the lower figure from the upper figure thus increased. Put down the remainder as in (1), and carry 1 to the next higher figure in the lower line. Proceed with the remaining figures as in (1), observing that whenever ten units have been borrowed, or added to the upper line, one unit must be carried, or added to the next higher denomination in the lower line.

Ex. Subtract 5634 from 7483

$\begin{array}{r} 7483 \\ 5634 \\ \hline 1849 \end{array}$	<p>Since 4 is greater than 3, 3 is made 13 by adding 10 to it, from 13 take 4 and put down the remainder 9. Now add 1 to the next lower figure 3, the sum is 4, which subtracted from 8 leaves 4. Put down 4. Next 6 is greater than 4, so 10 is added to 4, and from the sum 14, subtract 6. The remainder is 8.</p>
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Lastly, add 1 to the next lower figure 5, the sum is 6, which subtracted from 7 leaves the remainder 1. Thus the difference is 1849.

52 In the preceding Example, the same result would be obtained, if we have borrowed ten units of the next denomination from the *Minuend*, as is usual in France. For whether we suppose 1 to be added to the lower line, or subtracted from the upper, the remainder will evidently be the same on both suppositions. In practice, however, the former method is convenient.

53 Subtraction being the reverse of Addition, it follows, that if we add together the remainder and the less of the numbers proposed the sum ought to be equal to the greater, and the operation of subtraction may be presumed to be correct when this is the case. Another method of testing the correctness of the result is this: Cast out the nines from the sum of the digits in the *minuend*, and also from the sum of the digits in the *subtrahend* and the *remainder*, if the two results coincide, the operation may be presumed to be correct.

Examples VIII

1 Perform the following subtractions —

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
59	79	85	70	98	428	526	702	650	912
42	45	69	54	89	274	317	504	56	707
<u>17</u>									
(11)	(12)	(13)	(14)	(15)	(16)	(17)			
7046	7825	4286	9821	8943	6789	5959			
807	4976	3097	6935	4573	697	999			
<u>6239</u>									

(18) <u>23456</u> 4987	(19) <u>56785</u> 39876	(20) <u>76325</u> 59876	(21) <u>62831</u> 48072	(22) <u>708001</u> 39508	(23) <u>542657</u> 214958	(24) <u>203087</u> 76498
(25) <u>6829019</u> 6599341	(26) <u>1531335</u> 1456516	(27) <u>1287657</u> 1000958	(28) <u>78602045</u> 59763567	(29) <u>493827156</u> 246913578	(30) <u>8539410</u> 3438148	
(31) <u>74147863</u> 9701297	(32) <u>370489000</u> 269579235	(33) <u>68539582</u> 45947895	(34) <u>650030042</u> 94090095	(35) <u>13456789</u> 8765432	(36) <u>352100435</u> 79213679	
(37) <u>777722233</u> 38945635	(38) <u>909009099</u> 842248484	(39) <u>453870250</u> 39004065	(40) <u>1000100010</u> 999999999	(41) <u>765007005</u> 400827054		

2 Find the difference between -

- (1) 75011 and 6012, 3095 and 80131, 8019 and 18018
- (2) 110111 and 11012, 916553 and 1683452, 251483 and 77777
- (3) 20470932 and 80476325, 613020303 and 420536075
- (4) 12785462 and 1842567, 92603745298 and 25402487609

3 Find the values of -

- (1) $5124060 - 5083959$, $1056789 - 967899$, $4060124 - 3951035$
- (2) $6284503 - 4995629$, $7014062 - 6985172$, $6001004 - 5480018$
- (3) $1010102 - 956784$, $3601020 - 3598642$, $5490206 - 4301218$
- (4) $500120456 - 499296845$, $4060213697 - 2846545789$

4 What is the excess of 12795 above 8096? How much greater is 2600509050 than 433418175?

5 By how much is 87719808 greater than 68440260?

6 What is the excess of 9497605 above 8688516?

7 By how much is a lac greater than ninety-five thousand, nine hundred and nine, and less than a million?

8 What number must be added to each of the following numbers to make the sum equal to ten millions?—8423458, 457685, 9032401, 7612345, 5040289, 904507 and 9003465

9 What number must be taken from each of the numbers 999999, 425078, 8725900, 6420587 and 428905 to leave 245678?

10 Required the excess of three hundred five million, two hundred and four, above seventy-five thousand, three hundred and eighty six

11 From seven hundred eighteen million, fourteen-thousand and fifty six take ninety-eight million, seven hundred three thousand, six hundred and seventeen

12 Subtract thirteen lacs, four thousand and fifty-six from seventy five crores, two hundred and three

13 Take eleven thousand eleven hundred and eleven from twelve thousand one hundred and twelve

14 A box contains 4074 oranges, 2386 of them were sold How many remained?

15 In 1882 a man was 86 years old In what year was he born?

16 William the Conqueror began to reign in the year 1066, how many years elapsed between that period and the battle of Waterloo, which was fought in 1815?

17 A tea merchant has 4680 maunds of tea He sells 1000 maunds to one customer, 999 to a second, and 354 to a third How many maunds of tea has he left?

18 Jadu has 829 marbles, he gives away 618 and then buys 206 How many has he now?

19 A man was born in 1845, what was his age in 1896?

20 A man was 25 years old at the birth of his son, what is the son's age when the father is 74 years old?

21 A merchant bought a certain quantity of goods for 6246 rupees and sold them for 7137 rupees How much did he gain?

22 One mountain is 15732 feet high, another is 3571 feet high How much is the one higher than the other?

23 A railway receives in a year 2684040 rupees Of this sum 1786064 rupees are for goods and the rest for passengers How much was received for passengers?

24 Of 17254120 Hindu population for Bengal Proper, 8624022 are males and the rest females, find their number

25 Queen Victoria was born in 1819 How old was she in 1895?

26 Three boys *A*, *B* and *C* at marbles won together 105, if the numbers that *B* and *C* won be added together they will make 82, and of this number *B* won 47 What did each boy win at play?

27 A gentleman gave 12462 rupees for a house and some land, the house alone was worth 9375 rupees, what was the value of the land?

28 The answer to a subtraction sum is 1026 and the top line 4387 What is the second line?

29 A man has 826 sovereigns in one box and 682 in another, he takes 176 from the former and puts them in the latter How many are in each box now?

30 When will the Prince of Wales, who was born in the year 1841, be as old as the Queen was in the year 1878, who was born in the year 1819? How old will the Queen then be?

54 A number preceded by the sign $+$ (*plus*), is called a **positive** number, and a number preceded by the sign $-$ (*minus*) is called a **negative** number. When *no* sign is affixed to a number, it is considered as *positive*.

55 An **expression** is one in which two or more numbers are connected by the sign $+$ or $-$, and the numbers thus connected are called its **terms**.

Thus, $4-3+2+1$ is an *expression*, 4, 3, 2 and 1 are *terms*, 4, 2 and 1 are *positive*, and 3 is *negative*.

56 If an addition and a subtraction, or *vice versa*, have to be performed in succession, we may invert their **order**, provided the resulting expression be possible.

Thus, since $9+5-3=11$ and $9-3+5=11$, $9+5-3=9-3+5$

57 Hence it is easily shewn that additions and subtractions may be performed in any *order*, and that the value of an expression made up of additions and subtractions may be obtained by taking the *difference* of the sums of *all* the positive and the negative numbers separately.

Ex. Find the value of $365-101+2+18-267$

Here, $365+2+18=385$, $101+267=368$, also $385-368=17$

Therefore the value required=17

58 The **complement** of a number is its defect from 10 units of the number's highest order.

Thus, the *complement* of 6 is 4 and of 659 is 341, for $10-6=4$, and $1000-659=341$.

Examples IX

Find the value of each of the following expressions —

1 $16-4+12-25+7-2$ 2 $751-9+1786-235-12-672$

3 $18+6-31+537-628-19+209$ 4 $467-84+49-36$

5 $1246-362-371+495+156-386+256$

6 $3210-67+59+401-342+491-382+459-87$

7 What number must be added to the sum of 750 and 3287 to make the result equal to the sum of 505, 650, 19 and 9003?

8 What is the difference between $23047+175-368+495-132$ and $10000-8406-704+7305$?

9 From the difference between 3285 and 456 subtract the difference between 19011 and 17455

10 A basket contained oranges, nuts and mangoes, in all 1769, there were 1696 oranges and nuts, and 1262 nuts and mangoes. How many more nuts were there than oranges?

11 Gopal goes up 16 steps of a ladder, which has 45 steps, then down 7 steps, then up 10, then down 2, then down 4, then up 11, then down 9, then up 7, then up 5, then down 8, what step from the top and bottom will he then be standing upon?

12 Write down the complements of 4, 7, 43, 86, 574, 998

III MULTIPLICATION

59 **Multiplication** is the method by which we find the sum of a given number repeated as many times as there are units in another given number.

60 The number to be repeated is called the **multiplicand**, the other the **multiplier**, and the sum found the **product**. The *multiplicand* and the *multiplier* are both called **factors** or *makers* of the *product*.

61 From the mode in which results are obtained in multiplication, it is manifest that Multiplication is merely a *compendious* method of performing the addition of two or more *equal* numbers.

Thus, to multiply 7 by 4 being the sum, arising from the number 7 repeated *four* times, we may determine the product as $7+7+7+7$ or 28. Here 7 is the *multiplicand*, 4 the *multiplier*, and 28 the *product*, also 7 and 4 are *factors* of 28.

62 *Multiplication* is either **simple** or **compound**.

(i) When the multiplicand is either an *abstract* number, or a *concrete* number of *one* denomination, it is called *Simple Multiplication*.

(ii) When the multiplicand is a *concrete* number of *more than one* denomination, but all of the *same* kind, it is called *Compound Multiplication*.

63 The operation of *Multiplication* is expressed by the sign \times , which is read *into* or *times* or *multiplied by*. Sometimes a dot is used instead of a \times .

Thus, 5×7 denotes the product of 5 and 7, and is read 5 *into* 7, or 5 *times* 7, or 5 *multiplied by* 7. Also $57 = 5 \times 7$. This must not be confounded with a dot placed near the top, as 57 (Art 332).

64 The operation intended by the word *Multiplication*, is defined in Art 59, and in the first place we will shew that the conclusions which it leads to, may be safely depended upon, as far as the *order* of the *factors* may influence the *product*.

Thus, to multiply 7 by 5, write down 1 in a horizontal line 7 times, and repeat this line 5 times. The sum 1 1 1 1 1 1 1 of each horizontal line is 7, and there are 5 1 1 1 1 1 1 such lines, therefore the sum of all the ones is 1 1 1 1 1 1 1 7×5 . Again, the sum of each vertical line is 1 1 1 1 1 1 1 5 and there are 7 such lines, therefore the sum 1 1 1 1 1 1 1 of all the ones is 5×7 , that is, 7×5 is the same as 5×7 .

By reasoning of this kind it is made to appear that the product has a *similar* or *symmetrical* relation to both its factors, because it remains the same if we interchange the *Multiplicand* and the *Multiplier*.

65 A number multiplied by 0 is 0, as also 0 multiplied by a number is 0; for a number taken *no* number of times is *nothing*, also *nothing* taken any number of times is *nothing*.

Thus, $5 \times 0 = 0$, as also $0 \times 5 = 0$.

66 The following Tables, which are termed the **Multiplication Tables**, present at one view the product arising from the multiplication of any two numbers not exceeding 20, and though the products of the *nine digits* form the *basis* of those of all numbers whatever, it is here extended for the sake of *practical* convenience, and should be carefully committed to memory.

Table 1

	1	2	3	4	5	6	7	8	9	10
Once	1	2	3	4	5	6	7	8	9	10
Twice	2	4	6	8	10	12	14	16	18	20
Thrice	3	6	9	12	15	18	21	24	27	30
4 times	4	8	12	16	20	24	28	32	36	40
5 times	5	10	15	20	25	30	35	40	45	50
6 times	6	12	18	24	30	36	42	48	54	60
7 times	7	14	21	28	35	42	49	56	63	70
8 times	8	16	24	32	40	48	56	64	72	80
9 times	9	18	27	36	45	54	63	72	81	90
10 times,	10	20	30	40	50	60	70	80	90	100

67 In Multiplication, one of the factors, namely, the *multiplier* must necessarily be an *abstract* number

Thus, if the factors are 7 rupees and 8 rupees, we could easily multiply together the abstract numbers 7 and 8, whose product is 56, but the *denomination* of this result as the product of 7 rupees and 8 rupees cannot be ascertained, and the *operation* is altogether *absurd*. Hence, the multiplication of *concrete* numbers *as such*, is altogether impossible. We can, however, multiply 7 rupees by the *abstract* number 8, and interpret the product 56 rupees as how many rupees there are in 8 times 7 rupees.

It is also absurd to speak of 7 multiplied by 8 rupees, but not of 7 times 8 rupees. Of the two factors that make 56 rupees, one must be *abstract*, the other *concrete*, but it does not matter which, for 7 times 8 rupees = 8 times 7 rupees. In no case do we multiply by rupees.

In certain cases, however, as will be seen hereafter, the meaning of multiplication may be so extended as to include some concrete multipliers. (Art 378)

Examples X (MENTAL MULTIPLICATION)

1 How much is

- (1) 7 times 6, 11 times 8, 9 times 7, 11 times 11, 8 times 9, 7 times 15?
- (2) 10 times 3, 9 times 12, 7 times 7, 12 times 14, 4 times 18, 6 times 8?
- (3) 8 times 11, 5 times 12, 11 times 12, 5 times 17, 6 times 19?

2 What is the product of—

- (1) 13 by 12, 8 by 9, 15 by 14, 18 by 17, 0 by 4, 12 by 4, 11 by 15?
- (2) 15 by 19, 17 by 12, 6 by 0, 0 by 11, 20 by 15, 16 by 18, 14 by 18?

3 How many are 16×19 , 13×15 , 19×19 , 12×12 , 17×19 , 20×13 , 13×14 , 14×18 , 17×15 , 15×20 ?

4 One book has 12 pages. How many pages will 8 such books have?

5 There are 11 boys in a class, each works 8 sums in an hour. How many sums do they all work together?

6 If one knife costs 14 pice, how many pice will 9 knives cost?

7 If there are 9 desks in a room, and 6 boys at each desk, how many boys will there be in the room?

8 What will 9 stools cost at 9 rupees each?

9 How many trees are in 18 rows, each row having 9 trees?

10 If I give 5 boys 8 marbles each, how many will be left out of 81, and out of 100?

11 A boy wrote 12 lines of dictation and there were 9 words in a line, how many words did he write altogether?

12 How many more are 9 tens than 4 twenties? 10 tens than 6 tens? 9 nines than 5 nines?

13 In one foot there are 12 inches, how many inches are there in 6, 8, 9, 11 feet?

14 There are 7 days in a week, how many days are there in 8, 11, 12 weeks?

15 A boy walks 3 miles in an hour. How many miles will he walk in 6 hours?

16 How many legs have 14 horses? How many feet have 9 ducks?

17 Ram is 8 years of age, his father is 4 times as old. How old is his father?

18 A man walked 4 miles in one hour. How many miles would he walk at the same rate in 16 hours?

19 Multiply 8 by 4 and take away 10, how much remains?

20 A window has 9 rows of panes, and 12 panes in each row. How many panes are there in the window?

SIMPLE MULTIPLICATION

68 When the Multiplier does not exceed 20, the multiplication is called **Short Multiplication**.

69 When the Multiplicand is a large number and the Multiplier a number of one figure, we have the following Rule —

RULE Write down the multiplier under the units' figure of the multiplicand, and draw a line underneath. Begin at the units' figure of the multiplicand, and multiply each figure in succession by the multiplier, setting down and *carrying* precisely as in Addition.

Ex Multiply 3468 by 7

$\begin{array}{r} 3468 \\ \times 7 \\ \hline 24276 \end{array}$	<p>Here 7 times 8 is 56. Set down 6 in the units' place and carry 5, 7 times 6 is 42, and $42+5=47$, set down 7 in the tens' place and carry 4, 7 times 4 is 28, and 4 carried is 32, put down 2 in the hundreds place and carry 3, lastly 7 times 3 is 21, and $21+3=24$, set down 24. The product is therefore <u>24276</u>.</p>
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70 Writing down the *local* values of the figures, the process will stand thus —

$3468 = 3 \text{ thousands} + 4 \text{ hundreds} + 6 \text{ tens} + 8 \text{ units},$
 $\therefore 3468 \times 7 = 7 \times 3 \text{ thousands} + 7 \times 4 \text{ hundreds} + 7 \times 6 \text{ tens} + 7 \times 8 \text{ units},$
 $= 21 \text{ thousands} + 28 \text{ hundreds} + 42 \text{ tens} + 56 \text{ units},$
 $= 21 \text{ thousands} + 28 \text{ hundreds} + 47 \text{ tens} + 6 \text{ units},$
 $= 21 \text{ thousands} + 32 \text{ hundreds} + 7 \text{ tens} + 6 \text{ units},$
 $= 24 \text{ thousands} + 2 \text{ hundreds} + 7 \text{ tens} + 6 \text{ units},$
 $= 24,276$

71 When the multiplier is greater than 9 but does not exceed 20, the multiplication can be effected easily in one line, by the help of the Rule in Art 69

Ex 1 Multiply 59867 by 15

Here, 15 times 7 is 105 put down 5 and carry 10, then 59867 15 times 6 is 90, and $90+10=100$, put down 0 and carry 10, then 15 times 8 is 120, and $120+10=130$ put down 0 and carry 13, then $15 \times 9=135$ and $135+13=148$ set down 8 and carry 14, lastly, $15 \times 5=75$ and $75+14=89$ set down 89 Thus the product is 398005

E1 2 Multiply 350974 by 18

Here, $18 \times 4=72$, put down 2 and carry 7, then $18 \times 7=126$ and $126+7=133$, put down 3 and carry 13, then 350974 $18 \times 9=162$, and $162+13=175$, set down 5 and carry 17, then 18 then $18 \times 0=0$ and $0+17=17$, place 7 and carry 1, then 6317532 $18 \times 5=90$ and $90+1=91$, put down 1 and carry 9, lastly $18 \times 3=54$, and $54+9=63$, put down 63 Thus the product is 6317532

72 When the multiplier is a simple number followed by one or more ciphers we have the following Rule —

RULE Multiply the multiplicand by the simple number, and to the right of the product place as many ciphers as there are ciphers to the right of the multiplier

E1 Multiply 5867 by 70, and by 18000

5867	(1) Here $5867 \times 70 = 5867 \times 7$ tens,
<u>70</u>	= 41069 tens,
410690	= 410690
5867	(2) Here $5867 \times 18000 = 5867 \times 18$ thousands,
<u>18000</u>	= 105606 thousands,
105606000	= 105606000

Examples XI

1 Multiply 284 by 2, 1475 by 3, 2867 by 4, 9048 by 2 6057 by 2, 80965 by 5, 49508 by 6 33069 by 7, 91537 by 12.

2 Multiply

- (1) 5849 separately by 2, 3, 4, 5, 6, 7, 8, 9 and 11
- (2) 38476 separately by 3, 5, 7, 9, 11, 13, 14, 15 and 19
- (3) 3870492 separately by 2, 5, 7, 9, 11, 12 and 15
- (4) 6508794 separately by 8, 7, 9, 11, 13, 15, 17 and 19
- (5) 987654321 separately by 2, 3, 4, 5, 6, 7, 8, 9, 11 and 12

3 Find the values of

- (1) 48508×8 , 69360×6 , 49216×11 , 69432×12
- (2) 38476×9 , 876549×12 , 378125×16 , 456932×18
- (3) 43275×17 , 46059×16 , 30748×19 , 600954×20
- (4) 4609758×14 , 4609758×19 , 56380477×18

4 Multiply

- (1) 980989 separately by 10, 100, 1000 and 10000
- (2) 72051 separately by 30, 40, 70, 90 and 100.

- (3) 91357 separately by 20, 200, 300, 5000 and 9000
 (4) 790785 separately by 120, 1500, 17000, 1300 and 190000
 (5) 900768 separately by 800, 1600, 14000 and 180000
- 5 By how much does 18 times 1118 exceed 17 times 1050 ?
 6 Find the sum of 19 times 2304 and 15 times 2045
 7 Multiply 123456789 separately by 1, 2, 3, 4, 5, 6, 7, 8 and 9, and add the several products together
 8 An estate contains 45068 bighas. Each bigha is worth 16 rupees. What is the value of the whole estate ?
 9 A railway train consists of 17 trucks. Each truck carries 12644 maunds weight. How many maunds does the whole train carry ?
 10 A man bought 305 cows at 12 rupees a head, and having spent on them for food 95 rupees, sold them at 16 rupees a head. What did he gain by his bargain ?
 11 Ram bought of Jadu 15 books at 13 annas each, and Jadu bought of Ram 19 books at 19 annas each. How many annas had Jadu to give to Ram ?
 12 Two persons start from the same place, and travel (i) in the same direction, (ii) in opposite directions. One travels at the rate of 93 miles a day and the other at 79 miles a day. How far will they be apart at the end of 7 days ?

73 When the multiplier is greater than 20, the multiplication is called **Long Multiplication**.

74 When the multiplicand and multiplier are both large numbers, we have then the following general Rule —

RULE. Place the multiplier under the multiplicand, so that units of the same order may be under one another and draw a line under the whole. Begin at the units' figure of the multiplier, and multiply by each of its figures in order, writing down each *partial* product so that its first figure shall be under the figure of the multiplier that produces it. Add together these *partial* products and the *sum* is the product required.

Ex. Multiply 7823 by 645

	7823	39115	Then multiply 7823 by 5 and set down the product
	645	31292,	so that 2 may come under the tens' place of the first
	59115		partial product, 9 in the hundreds' place and so on. Lastly,
	31292		multiply 7823 by 6 and set down the product 46938 so that
	46938		6 may be in the hundreds' place of the first partial product
	5045835		and so on. Add up the three lines of figures already
			obtained and their sum <u>5045835</u> is the required product.

75 The reasoning above employed can be rendered more clear by the following consideration

Since the above product is the sum of 7823 repeated 645 times—

and $645 = 600 + 40 + 5$, therefore by the use of Arts 69 and 72 we have the following process —

$$\begin{array}{r} 7823 \\ 645 \\ \hline 39115 \\ 312920 \\ 4693800 \\ \hline 5045835 \end{array}$$

$$645 = 600 + 40 + 5$$

$$\begin{array}{r} 7823 \times 5 = 39115 \\ 7823 \times 40 = 312920 \\ 7823 \times 600 = 4693800 \\ \hline 5045835 \end{array}$$

76 If one or more of the figures of the multiplier be 0, it is evident that the corresponding *partial product* will be 0 (Art 65) and the lines may be entirely omitted after placing down each 0 *once*, to give the proper value to the product arising from the next figure

Ex Multiply 4968 by 709

$$\begin{array}{r} 4968 \\ 709 \\ \hline 44712 \\ 347760 \\ 3522312 \end{array}$$

Here, in multiplying by 709, we first multiply by 9 and put down the result, then when we multiply by 7, we really multiply by 700, but not by 70, and so place the first figure of the second partial product under the hundreds' figure of the first, affixing one cipher in the ten's place

77 If the multiplicand, or multiplier, or both, end in ciphers, the ordinary process of Multiplication may be *shortened* or *facilitated* by the following Rule —

RULE Suppose the ciphers at the right of multiplicand, or multiplier or both omitted, find the product of the resulting numbers, and to the right of this product place as many ciphers as were supposed to be omitted in multiplicand, or multiplier or both together

Ex Multiply 47600 by 47, 257 by 64000, and 7900 by 83000

Here, omitting the ciphers on the right, or *supposing* them to be omitted, we have

$$\begin{array}{r} 47600 \\ 47 \\ \hline 3332 \\ 1904 \\ \hline 2257200 \end{array}$$

$$\begin{array}{r} 257 \\ 64000 \\ \hline 1028 \\ 1542 \\ \hline 16448000 \end{array}$$

$$\begin{array}{r} 7900 \\ 83000 \\ \hline 237 \\ 632 \\ \hline 655700000 \end{array}$$

where the ciphers are *annexed* at last to the right of the products obtained in the ordinary way, to give the other figures their proper local values

Thus, in the first case, when we multiply 6 by 7, we really multiply 600 by 7, and 600 multiplied by 7 gives 4200, therefore two ciphers are annexed after 2 in the product

In the second case, when we multiply 7 by 4, we really multiply 7 by 4000, and 7 multiplied by 4000 gives 28000, therefore three ciphers are annexed after 8 in the product

In the third case, when we multiply 9 by 3, we really multiply 900 by 3000, and 900 multiplied by 3000 gives 2700000, therefore five ciphers are annexed after 7 in the product

78 If the multiplicand and multiplier change places, the product must be the *same* as before, otherwise the *same* numbers would have *more* products than *one* (Art 64) Hence, it is convenient to make the larger number the multiplicand and the smaller number the multiplier

79 The following **Proofs** are generally adopted in Multiplication

(1) Interchange multiplicand and multiplier, the product ought to be the *same*

(2) **By Casting out the nines** We cast the nines out of a number thus add together all its figures, omitting every 9, and if the sum be greater than 9, replace it by the sum of its figures, and if the new sum be greater, replace it by the sum of its figures, and so proceed till we have a sum less than 9

Cast the nines out of multiplicand and multiplier Multiply the results, and *cast the nines out* of their product, noting the new result, now *cast the nines out* of the *product*, and if the result *coincide* with the one previously noted, we presume that the work is *correct*

80 We may mention here that the above test fails in the *three* following cases —

(1) If the order of figures in the product be misplaced, as 86 for 68

(2) If the errors made compensate each other, so far as the sum of the digits is concerned, as 65 for 83

(3) If 9 be written for 0, or 0 for 9, or either be inserted or omitted too often, as 59 for 50, or 597 for 57, or 708 for 78, and so on

E1 Multiply 5867 by 3478, annexing the proofs

$ \begin{array}{r} 5867 \quad 8 \\ 3478 \quad 4 \\ \hline 46936 \\ 41069 \\ 23468 \\ 17601 \\ \hline 20405426 \quad 5 \end{array} $	<div style="position: relative; height: 150px;"> <div style="position: absolute; top: 0; left: 50%; transform: translate(-50%, -50%);"> Multd and Multi 5 </div> <div style="position: absolute; bottom: 0; left: 50%; transform: translate(-50%, 50%);"> 5 Prod't </div> <div style="position: absolute; left: 20%; transform: rotate(-45deg);"> Multd 8 </div> <div style="position: absolute; right: 20%; transform: rotate(45deg);"> Multi 4 </div> </div>	$ \begin{array}{r} 3478 \\ 5867 \\ \hline 24346 \\ 20868 \\ 27824 \\ 17390 \\ \hline 20405426 \end{array} $
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Beginning at the left hand, we cast the *nines* out of the

(1) Multiplicand thus — 13, 19, 26, replace 26 by the sum of 2 and 6 or 8

(2) Multiplier thus — 7, 14, 22, replace 22 by the sum of 2 and 2 or 4
Now multiply 8 by 4, giving 32, which replace by the sum of 3 and 2 or 5, and note this result.

(3) Product thus — 6, 11, 15, 17, 23, replace 23 by the sum of 2 and 3 or 5

As this result coincides with the previous one, we presume the work is correct

Examples XII

1 Multiply 946 by 61, 869 by 89, 917 by 46, 909 by 88, 463 by 608, 417 by 739, 3259 by 497, and 692 by 73

2 Multiply

- (1) 47691 by 27, 28573 by 35, 716281 by 48, 39265 by 39
 (2) 129385 by 66, 138476 by 81, 480765 by 97, 829741 by 59
 (3) 5241763 by 123, 921846 by 158, 827911 by 376
 (4) 5086927 by 495, 254037 by 2980, 4785528 by 7802
 (5) 56380477 separately by 35, 48, 72 and 132
 (6) 67836479 separately by 356, 1378 and 78539
 (7) 70870096 separately by 404, 3009 and 000807
 (8) 279120 by 7350, 678000 by 87600, 80108 by 7770
 (9) 56348 by 50401, 876000 by 678000, 896385 by 6687400
 (10) 57483000 by 40, 900, 430, 24500 and 4670000

3 Find the product of —

(1)	(2)	(3)	(4)	(5)
45678	3124791	436712	1100785	4532815
9128	87223	4567	71053	751283
(6)	(7)	(8)	(9)	(10)
447002	8913243	11037500	110200570	275642
578444	231567	198075	200570	125255
(11)	(12)	(13)	(14)	(15)
447529123	4165348	79904451	81064270	123456789
8901234	700745	7640750	8745590	123456789

4 Find the values of —

- (1) 704745×615 , 464830×369 , 391525×861
 (2) 114575×2214 , 3523725×2563 , 925196×7896
 (3) 920685×7098 , 4105348×7000408 , 7650329×600509
 (4) $400795703206 \times 7005130502$, $8070906050493 \times 64032000905$
 (5) $670780260,504 \times 2005032057$, $1310275031496 \times 20156300170$

5 What is the difference between 23456 multiplied by 999 and the remainder in subtracting 4 times 23456 from 23456000?

6 A bigha of land costs 781 rupees. What will 203 bighas cost?

7 If there are 432 pages in a book, how many will there be in 89704 such books?

8 If I give 125 boys 79 marbles each, how many shall I have left out of 10000?

9 70432 copies of a newspaper are printed daily. How many are printed in a year of 314 days?

10 The cost of constructing a Railway is 61303 rupees per mile. What will 701 miles cost?

11 An army consists of 295 battalions of 34618 men each. What is the whole number of men in the army?

12 In a town there are 734 houses, 345 of them contain, on an average, 11 persons each and the rest 15 each. How many persons reside in the town?

13 If a master employs 73 workmen, each of whom receives 34 rupees per month, how many rupees does he pay away per month?

14 If of 20000 shells used in war, 3648 are 36 pounders, 11275 are 24 pounders, and the rest 18 pounders, what is the total weight (in pounds) of the whole?

15 A clock strikes 114 times in a day. How often will it strike in 365 days?

16 A town has 436 streets. Each street contains on an average 6422 inhabitants. What is the population of the town?

17 A directory contains 798 pages. There are 72 names in each page. How many names are in the directory?

18 343 paving-stones are required for every yard in a street. There are 18742 yards in the street. How many paving-stones will the whole street require?

19 The distance of the Earth from the Sun is found to be 11608 times the Earth's equatorial diameter, and that diameter is 7926 miles. Required the distance between the Earth and the Sun.

20 India contains about 1466576 square miles and the population is reckoned to be about 189 persons to every square mile, what is the whole population of the country?

81 To find the product of *more* than two numbers, multiply the product of two of the numbers by the third, the result by the fourth, and so on. The final result is called the **continued product** of so many factors.

Thus, the *continued product* of 3, 5, 8 and 47 = $3 \times 5 \times 8 \times 47 = 15 \times 8 \times 47 = 120 \times 47 = 5640$, and 3, 5, 8 and 47 are *factors* of 5640.

82 The continued product of any numbers will remain the *same*, however we may change the *order* of its factors.

Thus, since $4 \times 2 \times 5 \times 7 \times 3 = 8 \times 5 \times 7 \times 3 = 40 \times 7 \times 3 = 280 \times 3 = 840$, and $5 \times 4 \times 2 \times 3 \times 7 = 20 \times 2 \times 3 \times 7 = 40 \times 3 \times 7 = 120 \times 7 = 840$,

$$\therefore 4 \times 2 \times 5 \times 7 \times 3 = 5 \times 4 \times 2 \times 3 \times 7$$

Ex Find the continued product of 3471, 7 and 52

$$\begin{array}{r} 3471 \\ \times 7 \\ \hline 24297 \\ \times 52 \\ \hline 48594 \\ 121485 \\ \hline 1263444 \end{array}$$

Here, we first multiply 3471 by 7, and the product is 24297, again multiply 24297 by 52, and the product is 1263444, thus the continued product of the several factors is 1263444.

83 If *one* or *more* of the factors in any continued product be 0, the whole product is 0 (See Art 65)

Examples XIII

1 Find the continued products of —

- (1) 4, 7, 25, (2) 13, 15, 17 (3) 18, 19, 20 (4) 407, 18, 5
 (5) 729, 8, 61 (6) 7184, 6, 12 (7) 35, 32, 14, 29 (8) 35, 29, 43, 87
 (9) 33, 13, 15, 4, 56 (10) 27, 57, 35, 1277 (11) 156, 13, 365, 78
 (12) 18, 19, 35, 24, 12, 17 (13) 340, 255, 783 (14) 675, 225, 180, 125

2 A library contains 3275 volumes, and each volume on the average 493 pages, and each page 39 lines How many lines are there?

3 If the earth moves round the Sun at the rate of 68000 miles an hour, how far will it move in 365 days of 24 hours each?

4 If every page of a book contains 36 lines and each line on an average 11 words, how many words would there be in 157 pages?

5 If each of 36 trucks in a luggage train contains 18 barrels of cement, and each barrel 36 maunds, how many maunds is the train carrying?

6 How many yards of silk are there in 9 packages, each containing 8 parcels, each parcel 26 pieces, and each piece 53 yards?

7 In a school there are 10 classes each class has 4 desks, each desk holds 18 boys, how many boys are there in the school?

8 If 37 labourers earn 39 rupees each per day, how many rupees do they all earn in 36 working days?

9 If every man lived to marry and have 8 male children, how many great-great-grand children of the male sex could every one expect to have?

10 A Railway passenger train consists of 32 carriages, each carriage is divided into 12 compartments, in each compartment there are 5 benches and on each bench there is space for 8 persons, how many persons can the train carry?

84 When a number is multiplied by itself *once, twice, thrice, four, &c.* times, the product is called the **second, third, fourth, fifth, &c. power** of that number respectively The *second* and *third powers* of a number are commonly termed its **square** and **cube** respectively The number itself is called its **first power**

85 These *powers* are often indicated by small numerals 2, 3, 4, 5 &c, placed above the number to its right, which express how often the number is repeated in the product The small numerals so used are therefore called the **indices** or **exponents** of the several *powers*

Thus, $5^2 = 5 \times 5 = 25$, $\therefore 25$ is the *second power* or *square* of 5

$5^3 = 5 \times 5 \times 5 = 125$, $\therefore 125$ is the *third power* or *cube* of 5

$5^4 = 5 \times 5 \times 5 \times 5 = 625$, $\therefore 625$ is the *fourth power* of 5, and so on

86 If the three signs +, -, \times , occur in an expression, the

operation of Multiplication is to be performed first and then that of Addition or Subtraction

$$\text{Thus, } 4 \times 4 \times 3 + 3 \times 3 \times 2 - 4 \times 2 \times 1 + 2 \times 1 \times 0 = 48 + 18 - 8 + 0 \\ = 66 - 8 = 58$$

Examples XIV

1 Find the squares of —

- (1) 1, 2, 3, 4, 5, 25, 39, 46, 54, 86, 99 (2) 172, 237, 906, 987
(3) 729, 873, 1043, 5496 (4) 7342, 9384, 8796, 1234

2 Find the cubes of —

- (1) 1, 2, 3, 4, 5, 8, 27, 37, 48, 68, 77 (2) 88, 97, 123, 456
(3) 308, 876, 765, 999 (4) 987, 5386, 9876, 1234

3 Find the fourth powers of —

- (1) 678, 305, 987, 988 (2) 908, 3271, 8004, 9999

4 Find the values of —

- (1) $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2$ (2) $23^2 + 15^2 - 3^2$
(3) $5^3 - 4^2 - 8^2$ (4) $1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3 + 9^3$
(5) $2^4 + 3^4 - 1^4$ (6) $25^2 + 28^2 - 20^2 - 18^2 + 15^2$

5 Simplify the following expressions —

- (1) $8 \times 4 - 3 \times 6 + 4 \times 3 - 2 \times 1 + 5 \times 2 + 3 \times 7$
(2) $5 \times 6 \times 3 + 4 \times 3 \times 0 - 2 \times 1 \times 4 + 3 \times 6 \times 4 - 2 \times 2$
(3) $8 \times 6 \times 3 \times 1 - 3 \times 6 \times 2 \times 4 + 4 \times 6 \times 7 \times 4 - 7 \times 8 \times 2 \times 0$
(4) $-9 \times 6 \times 2 \times 3 + 7 \times 4 + 4 \times 6 \times 3 \times 5 - 3 \times 6 \times 7 \times 0 \times 5 + 2 \times 3 \times 4$
(5) $7^2 + 2 \times 3^2 + 3 \times 5^2 + 4 \times 9^2$ (6) $3^2 \times 2 + 2 \times 3 - 2^2 \times 3 + 6 \times 1^2$
(7) $23^2 - 11^2 + 115 \times 11^2 - 110^2 + 112^2$
(8) $3^2 + 3 \times 4 \times 5 + 5^3 - 4^3 - 2 \times 4 - 2^3 + 6^2 - 3$

IV DIVISION

87 Division is the method of finding *how many times* one given number is contained in another given number. The former of these numbers is called the **divisor**, the latter the **dividend**, and the number telling *how many times* the **quotient**. The number left after the operation is finished, is termed the **remainder**.

88 In dividing one number by another, we obviously take the latter number from the former, as often as we are able, according to the principle of Subtraction before explained. Hence *Division* bears the same relation to *Subtraction*, as *Multiplication* bears to *Addition*.

Thus, to divide 26 by 8, means that we are to find how many times 26 contains 8, and the operation at the side shews that 26 contains 8, 3 times with a remainder 2. Here 26 is called the *dividend*, 8 the *divisor*, 3 the *quotient* and 2 the *remainder*.

$$\begin{array}{r} 26(1, 1, 1 \\ 8 \\ \hline 18 \\ 8 \\ \hline 10 \\ 8 \\ \hline 2 \end{array}$$

89 Hence, by division we break up a given number into as many equal parts as there are units in another given number, and thus find one of these parts.

90 *Division* is of two kinds, **simple** and **compound**

(i) When the dividend and divisor are both *abstract* numbers or both *concrete* numbers of one and the same denomination, or when the divisor is an *abstract* number, and the dividend a *concrete* number of one denomination, it is called *Simple Division*

(ii) When the dividend is a *concrete* number of the same kind but of different denominations of that kind and the divisor an *abstract* number, or when both the dividend and divisor are *concrete* numbers of the same kind but of different denominations of that kind it is called *Compound Division*

91 When there is no remainder the division is said to be **exact** and since the Quotient tells how many times the Dividend contains the Divisor it follows that $\text{Dividend} = \text{Divisor} \times \text{Quotient}$. But when there is a Remainder, the division is called **inexact**, and the $\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$

92 The operation of *Division* is expressed by means of the sign $-$ and sometimes $/$, which is read *divided by* or simply *by*. It is also denoted by writing the dividend above the divisor with a line between them

Thus, $42-7$ denotes that 42 is to be divided by 7, and is read 42 *divided by* 7 or simply 42 *by* 7. Also $42/7$ and $42 \over 7$ mean 42 \div 7

93 In division, the quotient is an *abstract* number, if the dividend and divisor are both abstract or both concrete numbers, but the quotient is a *concrete* number, if the dividend is a concrete number and the divisor an abstract number. The divisor, if concrete, must be of the same kind as the dividend

Thus, 45 divided by 5, or 45 rupees divided by 5 rupees, gives the *abstract* number 9 as quotient, for 5 or 5 rupees taken 9 times gives 45 or 45 rupees, and 45 rupees divided by 5 gives the *concrete* number 9 rupees as quotient, for if 45 rupees be divided into 5 equal parts, each of these parts will contain 9 rupees. Also 15 rupees divided by 5 yards has no meaning, according to the definition of Division in Art 93

94 As Division is the *reverse* of Multiplication, it follows that, by a reversed process, the Multiplication Tables must furnish the means of obtaining the quotient, when the divisor does not exceed 20 and the dividend 400

Ex 1 Divide 96 by 8

Since $8 \times 12 = 96$, therefore $96 \div 8$ gives 12 as quotient

Ex 2 Divide 259 by 17

Since $17 \times 15 = 255$, and $259 - 255 = 4$ therefore $259 \div 17$ gives 15 as quotient and 4 as remainder.

Examples XV (MENTAL DIVISION)

1 How many times does 8 contain 2? 36 contain 3? 20 contain 4? 35 contain 5? 24 contain 6? 56 contain 7? 81 contain 9?

2 Divide 14 by 2, 12 by 3, 48 by 4, 20 by 5, 42 by 6
49 by 7, 32 by 8, 108 by 9, 90 by 10, 77 by 11, 96 by 12

3 Divide

- (1) 56 separately by 2, 3, 4, 5, 6, 7, 8, 9, 12 and 14
- (2) 98 separately by 2, 5, 7, 9, 13, 15, 17, 18 and 19
- (3) 168 separately by 2, 7, 8, 9, 6, 12, 11, 15 and 18
- (4) 288 separately by 4, 7, 9, 10, 6, 8, 12, 15 and 17
- (5) 342 separately by 3, 6, 8, 9, 4, 11, 13, 15, 16 and 18
- (6) 172 by 9, 141 by 11, 128 by 14, 257 by 16, 195 by 19

4 In 54, how many times is 8, and how many over? How many times is 15 contained in 195? In 240, how many times is 18, and how many over?

5 If 16 be taken 14 times from 228, what is left?

6 What is the 9th part of 36, 54, 108 and 144?

7 To how many boys can I give 9 marbles if I have 153?

8 At a cricket match 11 players make 132 runs. If each made the same number of runs, how many did each make?

9 A Patsala consists of 128 boys and they are made to stand in 8 rows, how many are there in each row?

10 If 320 rupees are shared equally among 16 men, how many does each man receive?

11 Divide 132 oranges equally among 7 girls and 5 boys

12 Divide 96 pencils equally among 8 boys

13 Bhuban spent 180 pice in oranges, buying them at the rate of 6 for 3 pice. how many oranges did he buy?

14 A boy, having a basket containing 214 oranges, distributed them equally between his 8 school-fellows and himself, the number which remained he gave to his school master, how many did the school master receive?

15 A man bought 11 cows at 18 rupees each, and sold them so as to gain 99 rupees, what did he sell each cow for?

16 How many seers of sugar at 5 annas each can be bought for 330 annas?

17 A woman bought 180 eggs at 3 for 2 pice and 275 more at 5 for 3 pice, and sold the whole lot at 13 for 19 pice, what does she gain or lose?

18 If 5 men can do a piece of work in 18 days, how long will it take 9 men to do the same work?

19 How many penknives, worth 8 annas each, ought to be exchanged for 144 pen-holders at one anna each?

20 A man walked 306 miles in 18 days, how many miles did he walk per day?

SIMPLE DIVISION

95 When the dividend is a large number, but the divisor less than 20, the division is called **Short Division** and can be done by the following Rule

RULE Place the divisor and dividend thus

divisor)
dividend

From the left of the dividend cut off a number not less than the divisor but less than 10 times the divisor, giving the first partial dividend. Find by the aid of the Multiplication Tables how often the divisor is contained in this dividend, put down the quotient under the units' figure of this dividend, and take notice of the remainder (whether it be any number or 0). On the right of this remainder, conceive in your mind to be placed the least number of the figures next following in the dividend which, affixed to the remainder, will make a number not less than the divisor. Proceed, as above, with this new partial dividend to find the next figure of the quotient, taking care to place after the first figure in the quotient a cipher for every figure brought down from the dividend which, affixed to the remainder, makes a number less than the divisor.

Continue this process till all the figures of the dividend have been thus brought down, and if there be any remainder at the end of the operation, write it as a remainder distinct from the quotient.

Ex 1 Divide 612459 by 7

$$\begin{array}{r} 7 \overline{)612459} \\ \underline{87494} \text{ rem } 1 \end{array}$$

From the left of the dividend cut off a number not less than 7 but less than 70 that is, cut off 61, our first partial dividend. Now 7 is contained in 61, 8 times and 5 over, put the 8 under the 1 in 61, and to the right of the remainder 5 affix the next figure of the dividend 2, making 52, the second partial dividend. But 7 is contained in 52, 7 times and 3 over, put 7 in the quotient, and to the right of the remainder 3 affix the next figure 4 making 34, the third new dividend, and so proceed.

The above operation is usually performed in saying —
 7 in 61, 8 and 5 over, in 52, 7 and 3 over, in 34, 4 and 6 over, in 65, 9 and 2 over, in 29, 4 and 1 over (as remainder)

Thus the quotient is 87494, and the remainder 1

Ex 2 Divide 61245 by 15

$$\begin{array}{r} 15 \overline{)61245} \\ \underline{4083} \end{array}$$

Here 15 in 6 goes no times, but 15 in 61 goes 4 times and 1 over, write 4 under the 1. Then 15 in 12 goes no times, but 15 in 124 goes 8 times and 4 over, write 0 under the 2 and 8 under the 4, lastly, 15 in 45 goes 3 times, write 3 under the 5.

Thus the quotient is 4083

96 The truth of the above method may be shewn thus —
 Since $61245 = 61 \text{ thousands} + 2 \text{ hundreds} + 4 \text{ tens} + 5 \text{ units},$
 $= 60 \text{ thousands} + 12 \text{ hundreds} + 4 \text{ tens} + 5 \text{ units},$
 $= 60 \text{ thousands} + 124 \text{ tens} + 5 \text{ units},$
 $= 60 \text{ thousands} + 120 \text{ tens} + 45 \text{ units}$

∴ 61245 divided by 15 gives as quotient 4 thousands + 8 tens + 3 units or 4083

Examples XVI

1 Divide

- (1) 462 separately by 3, 6, 8, 9, 10, 11 and 12
- (2) 682 separately by 3, 4, 6, 8, 9, 11, 14 and 15
- (3) 8425 separately by 5, 7, 8, 10, 13, 16, 17 and 19
- (4) 6876 separately by 2, 3, 7, 9, 11, 12 and 14
- (5) 35298 separately by 3, 5, 9, 7, 10, 12 and 18
- (6) 348 by 2, 4596 by 3, 276284 by 4, 84375 by 5
- (7) 53844 by 5, 536074 by 7, 95832417 by 8, 3158367 by 10,
- (8) 7163253651 by 9, 1234567890 by 11, 9876543 by 12
- (9) 27643532 by 14, 35762445 by 15, 47623554 by 18
- (10) 34672352 by 16, 987654321 by 17, by 18, by 19, by 20

2 If 1674 men are drawn up in 18 columns, how many men are there in each column?

3 I distributed 2160 marbles among a number of boys, and gave each boy 12 marbles, how many boys were there?

4 What is the 15th part of 135090? the 11th part of 101112?

5 A farmer has 1786 sheep divided into 19 equal flocks. How many sheep are there in each flock?

6 A farmer spent 1872 rupees in the purchase of oxen. Each ox cost 12 rupees. How many oxen did he buy?

7 If the sum of 18 and 30 be divided by their difference, and the quotient be multiplied by the product of 16 and 27, what is the result?

8 A man gives 14 cows and 35 sheep for 55 bags of potatoes worth 7 rupees per bag, if each sheep was worth 3 rupees, what did he get for each cow?

97 When the dividend and divisor are both large numbers, the division is called **Long Division** and can be performed by the following general Rule

RULE On either side of the dividend draw curved lines, place the divisor on the left and the figures of the quotient as they arise on the right, thus

divisor)dividend(quotient

Then try to find how often the first one or two figures on the left hand of the divisor are contained in the first one or more of those of the dividend, and place the result on the right as the first figure of the quotient, and the product arising from the multiplication of the divisor by this figure being subtracted from the dividend, *bring down* or *annex* to the right of the remainder the next figure of the dividend. Proceed as before, and continue the process till all the figures of the dividend have been brought down, then the quotient, and the remainder if any, will be obtained.

If at any stage of the process, the divisor is greater than the *partial* dividend, affix a *cipher* to the quotient and bring down the next figure of the dividend. Continue this process till the partial dividend is greater than the divisor and then proceed as before.

Ex 1 Divide 75035 by 349

249)75035(215

698
523
349
1745
1745

Here, the first figure 2 in the quotient is obtained by inquiring how often 3 is contained in 7, or 34 in 75, then, after multiplying 349 by 2, which from the places of the figures, represents 2 *hundreds*, and subtracting the product which is 698, from 750, we have a remainder 52, to this the next figure 3 of the dividend is *annexed* to form the partial dividend 523. Now seek how often 3 is contained in 5, or 34 in 52, and the quotient being 1, 1 *ten* is annexed to the 2 *hundreds* already obtained, multiplying 349 by 1, which means 1 *ten*, and subtracting the product 349 from 523, we get the remainder 174. Bring down the last figure 5 of the dividend to form the partial dividend 1745, and we find the corresponding quotient to be 5 units exactly, for 349 multiplied by 5 produces 1745, and the operation is then completed, leaving no remainder. Therefore the whole quotient is 215.

98 Supplying the auxiliary digits omitted in the above operation the process would stand thus —

349)75035(200+10+5

69800
5235
3490
1745
1745

Ex 2 Divide 39875365 by 8654

8654)39875365(4607

34616
52593
51924
66965
60578
6387

Here 3987 is less than 8654, but 39875 is greater, therefore take 39875 for the first partial dividend. It contains the divisor 4 times, put 4 in the quotient, multiply 8654 by 4, placing the product 34616 under 39875, and subtract, leaving 5259. To the remainder 5259 annex the next figure of the dividend 3, giving 52593, the second partial dividend. It contains the divisor 6 times, put 6 in the quotient, multiply 8654 by 6, placing the product 51924 under 52593, and subtract, leaving 669. Again, to 669 bring down the next figure 6, giving 6696 the third partial dividend. It contains the divisor 0 times, put 0 in the quotient, and the remainder is now 6696. Lastly to 6696 bring down the last figure 5, giving 66965, the fourth partial dividend. It contains the divisor 7 times, put 7 in the quotient, multiply 8654 by 7, placing the product 60578 under 66965, and subtract, leaving a remainder 6387. Thus the quotient is 4607 and the remainder is 6387.

99 When the divisor is terminated by one or more ciphers, we use the following Rule

RULE Cut off all the ciphers on the right of the divisor and as many figures from the right of the dividend — for the quotient, divide the remaining figures of the dividend by the remaining figures of the divisor (Aits 95, 97), and for the final remainder bring down to the particular remainder the figures cut off from the dividend

Ex Divide 20573296 by 80 and by 345000

$$(1) \begin{array}{r} 8,0 \overline{) 2057329, 6} \\ \underline{257166} - 16 \end{array}$$

$$(2) \begin{array}{r} 345,000 \overline{) 20573,296(59} \\ \underline{1725} \\ 3323 \\ \underline{3105} \\ 218296 \end{array}$$

In the first example, in dividing by 8 the remainder is 1, to which we bring down the figure cut off 6, giving 16 for the final remainder, and 257166 for quotient

In the second example, the remainder in dividing by 345 is 218, to which we bring down the figures cut off 296, giving 218296 for the final remainder and 59 for quotient

100 The Proofs usually adopted in division are the following —

(1) To the product of the divisor and quotient add the remainder (if any), if the result coincides with the dividend, we presume that the work is correctly performed

(2) By casting out the nines

(a) From the sums of the digits in the *divisor* and the *quotient* subtract 9 as many times as possible, and set down the remainders to the left and right of a cross sign

(b) Multiply the two remainders and from the product subtract 9 as often as possible Put down the remainder below the cross sign

(c) Lastly subtract the *remainder* from the *dividend* and from the sum of the digits of this difference subtract 9 as many times as possible and set down the remainder above the cross sign If the upper and lower figures agree, it is presumed that the operation is correct

Ex Find the quotient and remainder when 275487 is divided by 736

Division

$$\begin{array}{r} 736 \overline{) 275487(374} \\ \underline{2208} \\ 5468 \\ \underline{5152} \\ 3167 \\ \underline{2944} \\ 223 \end{array}$$

Thus the quotient is 374 and the remainder 223

Proofs

(1)

$$\begin{array}{r} 374 \\ \underline{736} \\ 2244 \\ \underline{1122} \\ 2618 \\ \underline{275264} \\ 223 \\ \underline{275487} \end{array}$$

(2) $7+3+6=16$, rem 7
and $3+7+4=14$, rem 5

$$\begin{array}{ccc} & 8 & \\ 7 & \times & 5 \\ & 8 & \end{array}$$

$7 \times 5 = 35$, rem 8
Also $275487 - 223 = 275264$,
and $2+7+5+2+6+4=26$,
rem 8

101 If all the four signs +, -, ×, ÷ are used together in an expression, the operations of *Division* and *Multiplication* are to be performed first and next those of *Addition* and *Subtraction*

Ex Find the value of $14 + 12 - 6 \times 4 - 3 \times 2 + 6 \times 72 - 12$
 The expression = $14 + 2 \times 4 - 3 \times 2 + 6 \times 6 = 14 + 8 - 6 + 36$
 $= 58 - 6 = 52$

Examples XVII

1 Divide —

- (1) 92483 by 23 (2) 79958 by 39 (3) 79796 by 79 (4) 588168 by 84
 (5) 79512587 by 43 (6) 69637856 by 32 (7) 67001228 by 49
 (8) 144157246 by 83 (9) 47073256 by 37 (10) 7417784 by 88
 (11) 579826952 by 76 (12) 9009196416 by 96 (13) 58762347 by 99
 (14) 17587694293 by 54 (15) 14528340631 by 84
 (16) 3708501975 by 81 (17) 96790123458 by 98

2 Find the values of —

- (1) 419352633—123 (2) 1721034655—144 (3) 47123419361—132
 (4) 3577926—506 (5) 27291888—478 (6) 87624792—843
 (7) 48310567—549 (8) 6430776444—876 (9) 137090807—996
 (10) 630762540981—652 (11) 632798014—7243
 (12) 519387042—2731 (13) 140167329—7038
 (14) 395494875—6007 (15) 2106144185—2375
 (16) 25413286—7960 (17) 8327976—5730
 (18) 64157660—1480 (19) 935384767—4836
 (20) 900370575—54321 (21) 183920748—37246
 (22) 2828882701578—38706 (23) 2919333978682—76913
 (24) 61190852817674—873156 (25) 163034794788—321567
 (26) 487264325876—5678909 (27) 876824985621—90956845
 (28) 56400003227—76589451 (29) 32899438654—100104325
 (30) 191776658604—68589649 (31) 4676705026675—154321235
 (32) 121932631112635269—123456789
 (33) 1630188053103649203285—2837154309
 (34) 560211975014967053000—700002030506
 (35) 1630188053103649203285—574585614865

3 Divide —

- (1) 23787693 by 5605, by 9089, by 40857, and by 57085
 (2) 81229 separately by 10, 20, 30, 40, 50, 80, 90
 (3) 342604 separately by 100, 400, 600, 800, 900
 (4) 78534826 separately by 800, 12000, 3200, 475000
 (5) 3854269734 separately by 310, 5900, 587000, 90900
 (6) 25413286 by 7900, 19054832 by 83000, 26799534687 by 7890000

4 Find the values of —

- (1) $192 + 16 + 720 - 18 + 795 - 15 - 1786 - 19$
 (2) $3871 - 49 + 6935 - 95 - 5432 - 56 - 1375 - 25 + 4590 - 45$
 (3) $56 + 81 - 3 + 8 \times 7 \times 9 - 12 \times 136 - 17 - 72 - 18 + 6 \times 3$
 (4) $12 \times 16 - 8 + 17 \times 6 - 18 \times 32 - 8 - 27 - 9 \times 7 + 8 \times 30 - 15 + 56 \div 14$
 (5) $15 \times 37153 + 73474 - 67152 - 4 + 40734 \times 2 - 5485 \times 75$

5 If a bag contains 103 potatoes, how many will be required to hold 7432274 potatoes ?

6 If each carriage contains 57 passengers, how many carriages are there in a train carrying 969 passengers ?

7 Each of 156 boys uses 12 pen nibs, and a box contains 144 nibs. How many boxes are required ?

8 A confectioner sells 23475 maunds of sweetmeats in a year of 313 days, how many maunds does he sell in a day ?

9 Supposing a Railway train to travel from Calcutta to Delhi, a distance of 924 miles, in 44 hours, what is the average speed per hour ?

10 The population of a country is 3083220 and its area is 7341 square miles. How many people are there on an average to each square mile ?

11 Find the number of pages in a book which has on an average 207 words on a page, and contains 201411 words altogether ?

12 How many minutes will a wheel be in turning round 895702 times, if it turn 158 times in a minute ?

13 What number multiplied by 79 will give the same product as 257 multiplied by 553 ?

14 A shopkeeper sold 267 shawls for 4005 rupees, gaining there by 4 rupees on each shawl, what had each shawl cost him ?

15 The population of a certain village is 21510, and one out of 45 dies annually. How many die in a year ?

16 Find how many times the numbers 11, 15, 19, and 23 must be equally repeated to make 13668.

17 Find the 532nd part of 1004416. What is the 365th part of 36865365 ?

18 How many pages contain 30888 words, every page having 52 lines of 9 words each ?

19 If 168465 maunds of rice be distributed equally among 11231 famine-stricken men, how many maunds will each receive ? And if the family of each consist of 5 persons, what will be the share of each person ?

20 The rays of light comes from the Sun to the Earth in 498 seconds, at what rate does light move per second, the distance of the Sun from the Earth being 93000000 miles ?

V THE USE OF BRACKET

102 Brackets, which are of several kinds, as (), { }, [], are used to denote that all numbers included within any pair of them are to be considered as forming but one number and are therefore to be equally affected by any number not included within the same pair of brackets.

Thus $(2+3+7)$ denotes that 2, 3 and 7 are to be taken as making one number, & what soever, outside the brackets affects 2 in any way, must also affect 3 and 7 in the same way.

A *vinculum* is a sign sometimes used instead of brackets. It consists of a *line* drawn over the numbers to be considered as forming one number.

Thus, $\overline{2+3}$ express the same thing as $(2+3)$

103 When two or more numbers, connected by the signs of operation are enclosed in a pair of brackets, the operations of arithmetic indicated inside the brackets are to be performed before the brackets are removed. Thus,

$$\text{Ex 1 } 7 - \overline{5-3} = 7 - 2 = 5$$

$$\begin{aligned} \text{Ex 2 } 22 - (4 \times 3 + 5 - 6 - 2) &= 22 - (12 + 5 - 3) \\ &= 22 - (17 - 3) = 22 - 14 = 8 \end{aligned}$$

104 When a number immediately precedes an expression included in a pair of brackets this number is to be multiplied by the number obtained after removing the brackets.

$$\text{Thus, } 7 + 4(5-2) - 6 \times 3 = 7 + 4 \times 3 - 18 = 7 + 12 - 18 = 19 - 18 = 1$$

105 When an expression is included in more than one pair of brackets, it is convenient to remove the innermost bracket first, then the innermost of those that remain, and so on, till all the brackets are removed.

$$\begin{aligned} \text{Thus, } 25 - \{ \{ 15 \times 10 - 2 \times 12 - 8(2 \times 12 - 10) \} - 2 \} \times (15 - 10 + 2) \\ &= 25 - \{ \{ 150 - 24 - 8(24 - 10) \} - 2 \} \times (15 - 12) \\ &= 25 - \{ \{ 150 - 24 - 8 \times 14 \} - 2 \} \times 3 \\ &= 25 - \{ \{ 126 - 112 \} - 2 \} \times 3 \\ &= 25 - [14 - 2] \times 3 = 25 - 7 \times 3 = 25 - 21 = 4 \end{aligned}$$

106 If the sign *plus* precedes a bracket, the bracket may be removed without affecting the result.

$$\begin{aligned} \text{Thus, since } 7 + (5 - 3) &= 7 + 2 = 9 \text{ and } 7 + 5 - 3 = 12 - 3 = 9, \\ \text{therefore, we have } 7 + (5 - 3) &= 7 + 5 - 3 \end{aligned}$$

107 If the sign *minus* precedes a bracket, the bracket may be removed, provided the signs of all the numbers, inside the bracket be changed from *plus* to *minus*, and from *minus* to *plus*.

$$\begin{aligned} \text{Thus, since } 29 - (7 - 5 + 3) &= 29 - (2 + 3) = 29 - 5 = 24, \\ \text{and } 29 - 7 + 5 - 3 &= 34 - 10 = 24, \\ \text{therefore, we have } 29 - (7 - 5 + 3) &= 29 - 7 + 5 - 3 \end{aligned}$$

108 The sign signifies *therefore*, and is often used in stating a method by which an answer has been obtained. The sign stands for *because* or *since*, and is used in stating a reason.

Examples XVIII

1 Find the values of —

$$(1) 10 + (5 - 3) - (17 - 8) + (16 - 11) + 25 - (6 - 3 + 1)$$

$$(2) 20 - \overline{10 - 3 - 6} + (15 - 3) - (16 - 9) - (5 + 6) + (4 + 9)$$

$$(3) 8 + 4(12 - 7) - 3(9 - 5) + 7(16 - 19 + 5) - (18 - 6 + 7)$$

$$(4) 3\{8 + 25 - 3(0 - 12)\} \qquad (5) 3\{8 + (25 - 3)\overline{20 - 12}\}$$

- (6) $287 - \{15 \times 10 - 2(12 - 8)(2 \times 12 - 10)\} - 2 \times 15 - \overline{10 - 2}$
 (7) $1520 - \{610 + \overline{703 - 608}\}$ (8) $605 - \{(95 - 11 - 19) + 237\}$
 (9) $86 - \{(59 - 48) + 16 - (59 - 49)\}$ (10) $168 - \{(70 - 39) + (90 - 83)\}$
 (11) $1246 - (362 - 156) - \{371 - (495 - 386)\}$

2 Find the values of —

- (1) $(1536 - 487) - 1392 - 29 + 7 \times 5$ (2) $5880 - (167 - 132) \times 6$
 (3) $(194 + 65) \times 7 + (352 - 220) - 11 - 952 - (91 - 35)$
 (4) $(67893 - 8637) - 823 + 7546 \times (2356 - 945) - (9870 \times 170)$
 (5) $\{(312570 \times 598 + 76125 \times 47 + 318 - 3) - 155146\} - (6139 \times 15)$

3 If the sum of 274 and 108 be multiplied by their difference and the product be divided by 166, what will be the quotient ?

4 If the sum of 103, 29, and 267 be divided by 19, and the quotient be multiplied by 57, and the product be diminished by 197, what will be the remainder ?

5 Multiply $(325 - 293)$ by $(306 - 17)$ and to the product add $(1000 + 99)$

6 From 34856 subtract (763×41) and to the remainder add $\{1998 - (663 - 441)\}$

7 Find the difference between 876 and $459 - 368 + 149$

8 What number subtracted from $(2471 + 56)$ will leave $(3863 - 1498)$ as remainder ?

9 Find the difference between

$$3210 + 401 - (67 - 59) \text{ and } 342 - (491 - 382)$$

10 From the sum of the greatest numbers of 9 and 10 digits subtract the difference of the least numbers of 10 and 11 digits

11 From the sum of the greatest numbers of 4, 5 and 6 digits subtract the sum of the least numbers of 3, 4 and 5 digits

12 Find the values of —

- (1) $6 + 8[3 \times 6 + \{3 + 7 - (8 + 3 - 6) - (2 \times 6 - 3 + 3 - 2)\}]$
 (2) $66 \times 37 - 8[(9 - 7) \times 6 - (27 + 12) - 13 + (17 + 15 + 39 - 50) \times 5 - 9 \times 7]$
 (3) $\{(7 + 75) \times 43 + (4698 + 171) - 9\} - \{(73 + 14 - 2) - (16 - 2 + 4 \times 7)\}$
 (4) $8[4 \times \{(360 \times 120) - (47 + 13) - 3\}] - \{(360 \times 120) + (65 - 25) - 5\} - 5401$
 (5) $108 - 9 \times [76 - 9\{63 - 7(9 \times 3 - 4 \times 8 + 5^2 - 10 \times 2) - (2^3 \times 9 - 2^4)\}]$
 $- 11 \times 12$
 (6) $23 \times 11 \times 3 + 7[206 \times (8 + 6 - 13) - \{(14 - 8) \times 7 - (15 + 5 - 11) \times 2$
 $+ (6^3 - 13 \times 2^3) + (6 \times 8 \times 15 - 5)\}] + (2 \times 5 + 3^2 - 3 \times 4 \times 7)$
 (7) $84 - 7[-11 - 4\{-17 + 3(8 - 9 - 5)\}]$
 (8) $5 \times \{4 - 2[4 - 2(4 + 3)]\} - 4 \times \{4 - 2[4 - 2(4 + 3)]\}$
 (9) $19 + 12 \times 15 - 120 - 4 + \{29 - 13 \times 2 + (14 - 9) \times 3\}$
 (10) $9 \times [125 - 5(7 - 2) \times 8(9 - 7) + 4\{7 + 2(3 + 8)\}]$

VI MISCELLANEOUS PROPOSITIONS.

(IN THE FUNDAMENTAL OPERATIONS)

109 Sum, difference, &c

(1) Given the difference between two numbers and the greater, to find the smaller number

RULE *Subtract the given difference from the greater number, and the result is the required smaller number*

E1 If 34060 be the difference between two numbers, and the greater number is 48752, what is the less number?

$$\text{The less number} = 48752 - 34060 = \underline{14692}$$

(2) Given the difference between two numbers and the smaller, to find the larger number

RULE *Add together the given difference and the smaller number, and the sum is the required larger number*

E1 The difference between two numbers is 14610 and the less is 4007, what is the larger number?

$$\text{The larger number} = 14610 + 4007 = \underline{18617}$$

(3) Being given the sum and difference of two numbers, to find the numbers

RULE *To find the larger number, add together the given sum and difference, and divide the result by 2. To find the smaller number, subtract the given difference from the given sum and divide the result by 2*

E1 1 The sum of two numbers is 25264, and their difference is 736, what are the numbers?

$$\text{The larger number} = (25264 + 736) \div 2 = 26000 \div 2 = \underline{13000}$$

$$\text{The smaller number} = (25264 - 736) \div 2 = \underline{12264},$$

$$\text{or, the smaller number} = (25264 - 736) - 2 = 24528 - 2 = \underline{12264}$$

E1 2 The price of a carriage with horse is 1590 rupees, and the price of the carriage is 324 rupees more than that of the horse. Find the price of each

Here, the sum of the two prices is 1590 rupees and the difference 324 rupees

$$\text{the price of the carriage} = (1590 + 324) \div 2 = \underline{957} \text{ rupees}$$

$$\text{And the price of the horse} = (1590 - 957) \text{ or } \underline{633} \text{ rupees}$$

(4) Being given the sums of every two of three given numbers, to find the numbers

RULE *Add together the three given sums, divide the result by 2, and from the quotient subtract separately the three given sums. The several differences are the required numbers*

Ex 1 The sum of the first and second of three numbers is 59, that of the first and third is 53, and that of the second and third is 42. Find the numbers

$$\left. \begin{aligned} (59+53+42)-2 &= 77, \\ \text{the first number} &= 77-42=35, \\ \text{the second number} &= 77-53=24, \\ \text{and the third number} &= 77-59=18 \end{aligned} \right\} \text{Ans}$$

Ex 2 At a game of cricket *A* and *B* together score 75 runs, *B* and *C* together score 90 runs, and *A* and *C* together score 51 runs, find the number of runs scored by each of them

Here, *A*, *B* and *C* together score $(75+90+51)-2$ or 108 runs

$$\left. \begin{aligned} A \text{ scored } (108-90) \text{ runs} &= 18 \text{ runs,} \\ B \text{ scored } (108-51) \text{ runs} &= 57 \text{ runs,} \\ \text{and } C \text{ scored } (108-75) \text{ runs} &= 33 \text{ runs} \end{aligned} \right\} \text{Ans}$$

(5) Having given the sum of three numbers, the excess of the first over the second, and the excess of the second over the third, it is required to find the numbers

RULE Subtract the sum of the excess of the second over the third and of the first over the third (which may be obtained by adding the two given excesses from the given sum, and divide the result by 3. The quotient is the least of the three required numbers

Ex Divide 53 rupees among *A*, *B* and *C*, so that *A* may receive 7 rupees more than *B*, and *B* 8 rupees more than *C*

Here, the sum of the three shares is 53 rupees, and the excess of *A*'s share over *C*'s is $8+7$ or 15 rupees,

$$\begin{aligned} \text{and } 53-(8+15) &= 53-23=30 \\ C's \text{ share} &= (30-3) \text{ rupees} = 10 \text{ rupees,} \\ B's \text{ share} &= (10+8) \text{ rupees} = 18 \text{ rupees,} \\ \text{and } A's \text{ share} &= (18+7) \text{ rupees} = 25 \text{ rupees} \end{aligned} \left\} \text{Ans}$$

110 Product, Quotient, Remainder, &c

(1) Given the product of two numbers and one of them, to find the other

RULE Divide the product by the given number, and the quotient thus obtained is the other required number

Ex The product of two numbers is 890368, and the smaller number is 256, what is the larger number?

$$\text{The larger number} = 890368 \div 256 = \underline{3478}$$

(2) Given the divisor, the quotient and the remainder, to find the dividend

RULE Multiply together the divisor and the quotient, and to the product add the remainder. The result is the dividend

Ex If the divisor be 3857, the quotient 489, and the remainder 1305, what is the dividend?

$$\text{The dividend} = 3857 \times 489 + 1305 = \underline{1887378}$$

(3) Given the dividend and the quotient, to find the divisor

RULE Divide the dividend by the quotient, and the result is the divisor

Ex The dividend is 342604 and the quotient 883, find the divisor

$$\text{The divisor} = 342604 \div 883 = \underline{388}$$

(4) Given the dividend, the quotient, and the remainder, to find the divisor

RULE From the dividend subtract the remainder, and divide the difference by the quotient. The result is the divisor

Ex 1 The dividend is 119376, the quotient 25 and the remainder 2076, what is the divisor?

$$\text{The divisor} = (119376 - 2076) \div 25 = 117300 \div 25 = \underline{4692}$$

Ex 2 A farmer having 2316 sheep, on putting an equal number of them into each of 25 fields, had 16 remaining. How many did he put into each of the fields?

$$\text{The required number} = (2316 - 16) \div 25 = 2300 \div 25 = \underline{92}$$

(5) To find the *least number* which must be added to a given number to make it exactly divisible by a second given number

RULE Divide the first given number by the second, and subtract the remainder from the second given number. The difference is the required number

Ex What least number must be added to 4856752 to make it exactly divisible by 2163?

$$4856752 \div 2163 \text{ gives } 2245 \text{ as quotient and } 817 \text{ as remainder}$$

$$\text{the number to be added} = 2163 - 817 = \underline{1346}$$

(6) To find the *least number* which must be subtracted from a given number to make it exactly divisible by a second given number

RULE Divide the first given number by the second, and the remainder is the required number

Ex 1 What least number must be subtracted from 90625 that it may be divisible by 727?

$$90625 \div 727 \text{ gives } 124 \text{ as quotient and } 477 \text{ as remainder}$$

$$\text{the number to be subtracted} = \underline{477}$$

(7) To find the *greatest number* of a given number of digits which is divisible by a given number

Proceed as in the following example —

Ex 1 Find the greatest number of five digits which is divisible by 529

The greatest number of 5 digits is evidently 99999
 99999 divided by 529 gives 189 as the quotient and 18 as the remainder

the reqd greatest number = $99999 - 18 = 99981$

(8 To find the *least* number of a given number of digits which is divisible by a given number

Proceed as in the following example —

E1 Find the least number of six digits which is divisible by 4325

The least number of 6 digits is evidently 100000
 100000 divided by 4325 gives 23 as the quotient and 525 as the remainder, and $4325 - 525 = 3800$,

the reqd least number = $100000 + 3800 = 103800$

111 Equidifferent Series

The numbers 1, 2, 3, 4, 5, etc., are called *natural* numbers, of which 1, 3, 5, etc., are *odd*, and 2, 4, 6, etc., are *even* numbers

(1) To find the sum of any number of the *natural* numbers beginning with 1

RULE *Multiply the last number by the next higher number, and divide the result by 2. The quotient is the required sum*

E1 Add together $1+2+3+4+5+\dots+40$

Here, the last number is 40, and the next higher number is 41
 the required sum = $40 \times 41 \div 2 = \underline{820}$

(2) To find the sum of any number of *odd* numbers beginning with 1

RULE *The square of the number of times the numbers are repeated, is the required sum*

E1 Add together $1+3+5+7+9+\dots+25$

Here, the number of times the numbers are repeated is 13
 the sum required = $13^2 = \underline{169}$

(3) To find the sum of any number of *even* numbers beginning with 2

RULE *Multiply the number of times the numbers are repeated by the same increased by 1. The product is the required sum*

E1 Add together $2+4+6+8+\dots+30$

Here, the number of times the numbers are repeated is 15
 the sum required = $15 \times 16 = \underline{240}$

(4) To find the sum of any given numbers increasing or decreasing by a fixed number

RULE *Multiply the sum of the two extreme numbers by the number of terms (or times repeated), and divide the result by 2. The quotient is the required sum*

Ex Add together $2+5+8+11+\dots+47$
 Here, the number of terms will be found to be 16
 \therefore the sum $\approx 16 \times (2+47) \div 2 = 16 \times 49 \div 2 = 392$

Examples XIX

- 1 What number subtracted from 850967 will leave 3876 ?
- 2 The difference between two numbers is 84489 and the larger is 123456, what is the smaller ?
- 3 The smaller of two numbers is $3087+56299$ and their difference is 22371, what is the larger number ?
- 4 The greater of two numbers is the sum of 505, 650, 19 and 4003 and the difference between them is $3287-750$. What is the less number ?
- 5 The sum of two numbers is 12640 and their difference 1608, what are the numbers ?
- 6 The sum of the ages of two men is 173 years and the difference between them is 15 years, what are their ages ?
- 7 The sum and difference of two numbers are 1426 and 364 respectively, find the numbers ?
- 8 A man bought a pair of horses and a carriage for 857 rupees, the carriage was worth 165 rupees more than the horses, what was the price of each ?
- 9 Two men having met on a journey, found that they had travelled 1200 miles and that one had travelled 360 miles more than the other, what distance had each travelled ?
- 10 Divide 168 marbles between two boys, giving to one 42 more than the other
- 11 Ram Gopal and Hari began to play at marbles. Ram and Gopal have 77 marbles between them, Gopal and Hari 63, and Ram and Hari 70. How many marbles has each ?
- 12 A basket containing oranges, apples and plums, has 15 more oranges than apples, and 8 more apples than plums. The whole number of fruits in the basket is 112. Find the number of each kind in the basket
- 13 Three persons A , B and C , are possessed of certain sums of money, such that A and B together have 120 rupees, A and C together have 140 rupees, and B and C together have 150 rupees. What is the sum possessed by each ?
- 14 Divide 4680 rupees, after giving away 180 rupees to the poor, between A , B and C , giving B 216 rupees more than A , and C 336 rupees more than B
- 15 The product of two numbers is 17037006 and one of them is 4858, what is the other ?

16 If the divisor be 3857, the quotient 489 and the remainder 1305, what is the dividend ?

17 A dividend is 16322853, the quotient is 1754 and the remainder is 129, what is the divisor ?

18 The quotient arising from the division of 183926157 by a certain number is 4938 and the remainder is 5409 Find the divisor

19 What least number must be added to 34568135 that the sum may be exactly divisible by 357 ?

20 What least number must be subtracted from 56854327 that the difference may be exactly divisible by 7323 ?

21 By what number must 109109109 be divided so that the quotient may be 51784, and 221 over ?

22 What number multiplied by 1617 will give 50696184 ?

23 What least number must we subtract from 57385 so that it can be exactly divided by 387 ? And what least number must we add ?

24 The sum of the product of two numbers and 355 is 87403, one of the numbers is 216, find the other number

25 What least number must be added to 30984051, that the sum may be exactly divisible by 288 ?

26 Add together —

$$(1) 1+2+3+4+ \quad +60$$

$$(3) 2+5+8+11+ \quad +29$$

$$(5) 2+4+6+8+ \quad +30$$

$$(7) 5+8+11+14+ \quad +53$$

$$(2) 1+2+3+4+ \quad +100$$

$$(4) 1+3+5+7+ \quad +31$$

$$(6) 2+6+10+14+ \quad +78$$

$$(8) 100+97+94+ \quad +43$$

27 A debt can be discharged in 52 weeks by paying one rupee the first week, 3 rupees the second week, 5 rupees the third week and so on Required the amount of the debt

28 A person goes 3 miles on the first day, 5 miles on the second, 7 miles on the third, and so on How far has he travelled in a month of 30 days ?

29 How many times will a clock strike in a day of 24 hours ?

30 Write down 576987, and under it write the eighth succeeding number, and under this latter the next eighth succeeding number and so proceed till nine numbers have been written down, find their sum

31 Find the greatest and least numbers of 5 digits which are divisible by 327

32 Find the least number of 6 digits which is divisible by 273

33 Find the product of the two greatest numbers of 5 digits

34 Divide the greatest number of 7 digits by the least number of 4 digits

35 Find the sum of the greatest and the least number that can be formed by the digits 3, 2, 0, 1, 5, 8 and 9 taken all together

112 Addition, Subtraction, &c

(1) To subtract a number from another consisting of 1 followed by ciphers only

RULE Put down as many nines as there are ciphers in excess of the number of figures in the subtrahend, then (beginning from the left) write down in order the differences of each of the figures from 9 except the units' figure, which subtract from 10

Ex Subtract 5736428 from 10000000000

Here are 10 ciphers in the minuend, and 7 figures in the subtrahend, hence put down 999. Again 5 from 9 is 4, 7 from 9 is 2, 3 from 9 is 6, 6 from 9 is 3, 4 from 9 is 5, 2 from 9 is 7, and 8 from 10 is 2. Therefore the required difference is 9994263572

(2) To subtract *mentally* the sum of several numbers from a given number

Proceed as in the following example —

Ex Subtract the sum of 1286, 495, 4758, 984 from 15812

15812

1286

495

4758

984

8289 *Ans*

Mentally thus 4, 12, 17, 23 and 9 = 32

carry 3, 11, 16, 25, 33 and 8 = 41

carry 4, 15, 20, 24, 26 and 2 = 28

carry 2, 6, 7 and 8 = 15

(3) To subtract *mentally* from a number the product of two other numbers one of which is less than 20

Proceed as in the following example —

Ex Subtract 8×549 from 6567

6567

549

8

2175 *Ans*

Mentally thus $8 \times 9 = 72$, and $5 = 77$

carry 7, add 8×4 , 39, and 7 = 46

carry 4, add 8×5 , 44, and 1 = 45

carry 4, 4, and 2 = 6

113 Multiplication by factors

To multiply one number by another which can be resolved into factors each less than 20

RULE Multiply the given number by each of the factors in succession, and the final product is the required one

Ex 1 Multiply 31729 by 648

$648 = 9 \times 9 \times 8$,

31729

9

285561

285561

9

2570049

2570049

8

20560392 *Ans*

Ex 2 Multiply 43896 by 357, and by 735, making in each case only two partial multiplications

$$\begin{array}{r}
 (1) \quad 43896 \\
 \quad \quad 357 \\
 \hline
 \quad \quad 307272 \\
 35 = 7 \times 5 \quad 1536360 \\
 \hline
 \quad \quad 15670872 \quad \text{Ans}
 \end{array}$$

$$\begin{array}{r}
 (2) \quad 43896 \\
 \quad \quad 735 \\
 \hline
 \quad \quad 307272 \\
 35 = 7 \times 5 \quad 1536360 \\
 \hline
 \quad \quad 32263560 \quad \text{Ans}
 \end{array}$$

Ex 3 Multiply 567224 by 48872, and 48872 by 567224, making in each case only three partial multiplications

$$\begin{array}{r}
 (1) \quad 567224 \\
 \quad \quad 48872 \\
 \hline
 \quad \quad 4537792 \\
 48 = 6 \times 8 \quad 27226752 \\
 72 = 8 \times 9 \quad 40840128 \\
 \hline
 \quad \quad 27721371328 \quad \text{Ans}
 \end{array}$$

$$\begin{array}{r}
 (2) \quad 48872 \\
 \quad \quad 567224 \\
 \hline
 \quad \quad 342104 \\
 56 = 7 \times 8 \quad 2736832 \\
 224 = 56 \times 4 \quad 10947328 \\
 \hline
 \quad \quad 27721371328 \quad \text{Ans}
 \end{array}$$

Examples XX

1 Subtract 57364 from 1000000, 542056 from 1000000000, 7859064 from 10000000000, and 79854 from 100000000

2 Subtract

- (1) $3671 + 45 + 467 + 2073$ from 10608
- (2) $469 + 10876 + 2468 + 13972$ from 38709
- (3) $1234567 + 1234 + 123 + 12345$ from 4567208
- (4) $3843 + 396 + 428 + 1543 + 2897$ from 12964

3 Subtract *mentally* —

- (1) 4×2016 from 8124, 6×1632 from 9798, 8×4506 from 46325
- (2) 9×18764 from 198765, 7×53197 from 3690756
- (3) 15×14567 from 3567824, 18×51987 from 37373784

4 Add 4×123 to 878, 9×2345 to 4675, 8×1071 to 8795

5 Multiply by factors —

- (1) 98989 by 44, 98909 by 72, 89088 by 96, 79797 by 63
- (2) 9785643 by 128, 6301246 by 256, 8725364 by 432
- (3) 9457283 by 792, 8465729 by 512, 5374896 by 588
- (4) 13245 by 1188, 246785 by 1872, 989045 by 15015

6 Multiply in *two* lines —

- (1) 4016 by 637, 3543 by 648, 47862 by 1629, 31127 by 14412
- (2) 324567 by 486, by 936, and by 13212, 617635 by 1089

7 Multiply in *three* lines —

- (1) 765389 by 64164, by 189279, and by 83256
- (2) 92135 by 10813212, 459896 by 864729, 1234567 by 4321089
- (3) 7893261 by 5678109, 5710987 by 105613212

8 Multiply 876043 by 1449117 and by 28917136 in *three* lines

114 Abbreviated methods of Multiplication

- (1) To multiply a number by 5

RULE *Annex one cipher to the right of the multiplicand, and divide the result by 2 The quotient is the required product*

Ex 1 Multiply 879324 by 5

$$\begin{array}{r} 2)8793240 \\ \hline 4396620 \end{array} = \text{the required product}$$

Ex 2 Multiply 6508 by 15

$$\begin{array}{r} 2)65080 = \text{product by 10} \quad (1) \\ \hline 32540 = \text{product by 5} \quad (2) \\ \hline 97620 = \text{the required product, adding (1) and (2)} \end{array}$$

(2) To multiply a number by 25

RULE *Annex two ciphers to the right of the multiplicand and divide the result by 4 The quotient is the required product*

Ex 1 Multiply 57943 by 25

$$\begin{array}{r} 4)5794300 \\ \hline 1448575 \end{array} = \text{the required product}$$

Ex 2 Multiply 7575 by 35

$$\begin{array}{r} 4)757500 \\ \hline 189375 = \text{product by 25} \quad (1) \\ \hline 75750 = \text{product by 10} \quad (2) \\ \hline 265125 = \text{the required product, adding (1) and (2)} \end{array}$$

Ex 3 Multiply 6213 by 75

$$\begin{array}{r} 4)621300 = \text{product by 100} \quad (1) \\ \hline 155325 = \text{product by 25} \quad (2) \\ \hline 465975 = \text{the reqd prod, subtracting (2) from (1)} \end{array}$$

(3) To multiply a number by 125

RULE *Annex three ciphers to the right of the multiplicand, and divide the result by 8 The quotient is the required product*

Ex Multiply 860978 by 125

$$\begin{array}{r} 8)860978000 \\ \hline 107622250 \end{array} = \text{the required product}$$

(4) To multiply a number by a number all the figures of which are nines

RULE *Annex as many ciphers to the right of the multiplicand as there are nines in the multiplier, and from the result subtract the number itself The difference is the required product*

Ex Multiply 6875 by 999

$$\begin{array}{r} 6875000 = \text{product by 1000} \quad (1) \\ \hline 6875 = \text{product by 1} \quad (2) \\ \hline 6868125 = \text{the reqd prod subtracting (2) from (1)} \end{array}$$

(5) To multiply a number by a number which differs by a small number from 100, 1000, 10000, &c, or from 50, 500, 5000, &c

Proceed as in the following examples —

Ex 1 Multiply 423571 by 98 and by 9997

$$(1) 98 = 100 - 2$$

$$423571 \times 100 = 42357100$$

$$423571 \times 2 = 847142$$

$$\text{the product} = \underline{41509958}$$

$$(2) 9997 = 10000 - 3$$

$$423571 \times 10000 = 4235710000$$

$$423571 \times 3 = 1270713$$

$$\text{the product} = \underline{4234439287}$$

Ex 2 Multiply 6854 by 496

$$\text{Here, } 496 = 500 - 4$$

$$6854 \times 500 = 6854000 - 2 = 3427000$$

$$6854 \times 4 = 27416$$

$$\text{the required product} = \underline{3399584}$$

(6) To multiply a number by 11

RULE *Add each figure to the figure on its left, beginning with 0 on the right, carrying 1 when necessary. The number thus formed is the required product*

Ex Multiply 75384 by 11

$$\begin{array}{r} 75384 \\ 11 \\ \hline 829224 \end{array}$$

Here, $0+4=4$, $4+8=12$, carry 1, $1+8+3=12$, carry 1, $1+3+5=9$, $5+7=12$, carry 1, $1+7=8$, but all the necessary workings are 4, 12, 12, 9, 12, 8

(7) To multiply a number by 625

RULE *Annex four cyphers to the right of the multiplicand and divide the result by 16. The quotient is the required product*

Ex Multiply 4837 by 625

$$\begin{array}{r} 16 \overline{) 48370000} \\ 3023125 \end{array}$$

$$\underline{3023125} = \text{the required product}$$

115 Squares, Cubes, &c

(1) To find the square of a number of two figures

RULE *Increase and diminish the number by the complement of its units figure and to the product of the two results thus obtained add the square of the complement. The number thus formed is the required square*

Ex 1 Find the square of 84 and 95

Here, the complement of 4 is 6 and of 5 is 5

$$\begin{array}{l} (1) 84+6=90 \text{ and } 84-6=78 \\ \text{the reqd square} = 90 \times 78 + 62 \\ \quad = 7020 + 36 \\ \quad = 7056 \end{array}$$

$$\begin{array}{l} 95+5=100 \text{ and } 95-5=90 \\ \text{the reqd square} = 100 \times 90 + 5- \\ \quad = 9000 + 25 \\ \quad = 9025 \end{array}$$

Ex 2 Find the square of 467

$$\begin{aligned} 467+67 &= 534, \quad 467-67=400 \\ \therefore 467^2 &= 534 \times 400 + 67^2 \\ &= 213600 + 67^2 \end{aligned}$$

$$\begin{aligned} \text{Again, } 67+7 &= 74, \quad 67-7=60 \\ \therefore 67^2 &= 74 \times 60 + 7^2 \\ &= 4440 + 49 = 4489 \end{aligned}$$

$$\text{Hence } 467^2 = 213600 + 4489 = 218089$$

(2) To find the difference of the squares of two numbers

RULE Multiply the sum of the numbers by their difference, and the product is the required difference

Ex Find the value of $(339)^2 - (319)^2$

$$\begin{aligned} \text{Here, } 339+319 &= 658 \text{ and } 339-319=20 \\ \text{the required difference} &= 658 \times 20 = \underline{13160} \end{aligned}$$

(3) To express the product of two numbers as the difference of two squares

RULE Find the sum and difference of the numbers and divide each result by 2. The difference of the squares of the two quotients is the required difference of two squares

Ex Express 81×53 as the difference of squares

$$\begin{aligned} \text{Here } (81+53)-2 &= 134-2=67 \text{ and } (81-53)-2=28-2=14, \\ \therefore \text{the required difference} &= (67)^2 - (14)^2 \end{aligned}$$

Examples XXI

1 Multiply —

- (1) 879326 separately by 5, 25, 75, 125 and 625
- (2) 63945 separately by 15, 35, 75 and 125
- (3) 87911365 separately by 5, 25, 75, 125 and 625
- (4) 4439854 separately by 99, 999, 9999 and 99999
- (5) 5792 separately by 96, 996, 9994 and 9998
- (6) 8734652 separately by 11, 121, 1331 and 99994

2 Find the squares of —

- (1) 37, 45, 48, 55, 65, 75, 64, 71, 83, 96 and 125
- (2) 108, 149, 156, 183, 215, 391, 478, 456 and 524

3 Express the following products as the difference of two squares — 65×53 , 96×74 , 126×84 , 245×197 , 478×316

4 Find the values of —

- (1) $(575)^2 - (425)^2$, $(101)^2 - (99)^2$, $(1639)^2 - (739)^2$, $(1811)^2 - (689)^2$
- (2) $(753)^2 - (625)^2$, $(1723)^2 - (277)^2$, $(2731)^2 - (269)^2$, $(678)^2 - (638)^2$

5 Divide —

- (1) $(8133)^2 - (8131)^2$ by 16264, $(5874)^2 - (3795)^2$ by 2079
- (2) $(2259)^2 - (1759)^2$ by 4018, $(3156)^2 - (968)^2$ by 2188

- 6 Find the greatest number of 8 digits which is divisible by 5293.
 7 Find the least number of 7 digits which is divisible by 7293
 8 Find the least number of 9 digits and the greatest number of 8 digits which are divisible by 37213 -

116 Division by factors

When the divisor is the product of two or more factors, we use the following Rule —

RULE *The quotient is obtained by dividing in succession by each of the factors of the divisor, and the final remainder at each step is obtained by multiplying its particular remainder by all the divisors preceding its own, and adding the preceding final remainder*

Ex 1 Divide 25872 by 56

$$56 = 7 \times 8$$

$$\begin{array}{r} 7 \overline{) 25872} \\ 8 \overline{) 3696} \\ \hline 462 \end{array}$$

Dividing in succession by 7 and 8, the quotient is 462

Ex 2 Divide 96500093 by 105

$$105 = 3 \times 5 \times 7$$

$$3 \overline{) 96500093}$$

$$5 \overline{) 32166697} \quad 2$$

$$7 \overline{) 6433339} \quad 2 \quad 8$$

$$\begin{array}{r} 919048 \quad 3 \quad 53 \end{array}$$

Dividing in succession by 3, 5 and 7, the particular remainders are 2, 2 and 3. The final remainder at the first step is 2, the final remainder at the second step is found thus $2 \times 3 + 2 = 6 + 2 = 8$. The final remainder at the third step is found thus $3 \times 5 \times 3 + 8 = 45 + 8 = 53$

Thus the quotient is 919048 and the remainder 53

117 Abbreviated methods of Division

(1) To divide a number by 5, 15, 35, 45, 55 or 65

RULE *Multiply the number by 2 and divide the product respectively by 10, 30, 70, 90, 110, or 130, as in Art 99. The result in each case gives the quotient and for the true remainder divide the remainder so obtained by 2*

Ex 1 Divide 86246 by 5 and 15623 by 45

(1) $\frac{86246 \times 2}{1,0 \overline{) 17249,2}}$

$$\begin{array}{r} 1,0 \overline{) 17249,2} \\ \hline 17249 \quad 2 \end{array}$$

Thus the quotient is 17249
and the remainder $2 - 2 = 0$

(2) $\frac{15623 \times 2}{9,0 \overline{) 3124,6}}$

$$\begin{array}{r} 9,0 \overline{) 3124,6} \\ \hline 3124 \quad 6 \end{array}$$

Thus the quotient is 347,
and the remainder $16 - 2 = 14$

(2) To divide a number by 25, 75, 175, 225, 275 or 325

RULE *Multiply the number by 4, and divide the result by 100, 300, 700, 900, 1100, or 1300 as in Art 99. The result in each*

We give the quotient and for the true remainder we divide the remainder so obtained by 4

Ex Divide 37057 by 25, and 905785 by 175

$$\begin{array}{r} (1) \quad 37057 \times 1 \\ 100 \overline{) 1482,28} \\ \underline{1482} \quad 28 \end{array}$$

$$\begin{array}{r} (2) \quad 905785 \times 4 \\ 7,00 \overline{) 36231,40} \\ \underline{5175} \quad 640 \end{array}$$

I thus the quotient is 1482, and the true remainder $28 - 4 = 7$; I thus the quotient is 5175, and the remainder $640 - 4 = 160$

'3' To divide a number by 125, 375 or 875

Rule Multiply the number by 8, and divide the product respectively by 1000, 5000, or 7000 as in Art 90 The result in each we give the quotient and for the true remainder we divide the remainder so obtained by 8

Ex Divide 905785 by 125, and 1607708 by 375

$$\begin{array}{r} (1) \quad 905785 \times 8 \\ 1,000 \overline{) 7246,280} \\ \underline{7246} \quad 280 \end{array}$$

$$\begin{array}{r} (2) \quad 1607708 \times 8 \\ 3,000 \overline{) 12861,664} \\ \underline{4287} \quad 664 \end{array}$$

Thus the quotient is 7246 and the remainder $280 - 8 = 35$; Thus the quotient is 4287, and the remainder $664 - 8 = 83$

(4) To divide a number by 625

Rule Multiply the number by 16, and divide the result by 10000, as in Art 90 The quotient is the required quotient and for the true remainder divide the remainder so obtained by 16

Ex Divide 3023173 by 625

$$\begin{array}{r} 3023173 \times 16 \\ 1,0000 \overline{) 4837,0768} \\ \underline{4637} \quad 768 \end{array}$$

I thus the quotient is 4837, and the remainder $768 - 16 = 48$

(5) The method of Long Division may be much shortened by the use of Art 112 (3)

Ex Divide 15218125 by 3854

$$\begin{array}{r} 3854 \overline{) 15218125} (3948 \text{ Ans} \\ \underline{36561} \\ 18752 \\ \underline{33365} \\ 2533 \text{ rem} \end{array}$$

Mentally thus $3 \times 4 = 12$ and $6 = 18$, carry 1, add 3×5 , 16 and $5 = 21$, carry 2, add 3×8 , 26 and $6 = 32$, carry 3, add 3×3 , 12 and $3 = 15$

Then bringing down the next digit in the dividend, repeat the process for the next digit in the quotient

Examples XXII

1 Divide by factors —

- (1) 1461408 by 32, 347808 by 56, 1556334 by 162
 (2) 7825687 by 64, 6598769 by 84, 8791605 by 88
 (3) 7654325 by 96, 12345678 by 68, 36925814 by 82
 (4) 76538959 separately by 28, 64, 72, 96, 39541234 by 256
 (5) 87625432 by 726, 17927618 by 476, 5213742 by 1142
 (6) 3790603808 separately by 132, 196, 378, 3246541 by 792

2 Divide —

- (1) 37964 separately by 5, 50, 500, 5000 and 25
 (2) 8754316 separately by 5, 15, 25, 35, 45, 55, 65 and 75
 (3) 90273189 separately by 125, 175, 225 and 275
 (4) 154725876 separately by 125, 375, 625 and 875
 (5) 68015637 by 8634, 57300652 by 5129 and 36942536 by 4204

118 Average, Shares, Barter, &c

(1) To find the average of two or more numbers

RULE Divide the sum of the numbers by their number and the quotient is the required average

Ex 1 The attendance at a school was 254 on Monday, 326 on Tuesday, 204 on Wednesday and 192 on Thursday. Find the average daily attendance of the 4 days

On Monday the attendance was	254	$976 - 4 = 244$
Tuesday	326	
Wednesday	204	the average was <u>244</u>
Thursday	192	
• in 4 days	<u>976</u>	

Ex 2 The yearly expenses of a person during the first 4 years was Rs 675, during the next 5 years Rs 825, and during the following 7 years Rs 977. What was his average expenses?

In 4 years the exp amt to	(Rs 675 × 4) or Rs 2700,	16) Rs 13664
5	(Rs 825 × 5) or Rs 4125,	Rs 854
7	(Rs 977 × 7) or Rs 6839,	the average
• in 16 years	<u>Rs 13664</u>	was <u>Rs 854</u>

(2) To divide a given number into parts, having certain given relations among them

Proceed as in the three following Examples

Ex 1 Divide 184 oranges between Ram and Gopal, giving Ram 7 times as many as Gopal.

If Gopal gets 1 orange, Ram gets 7 oranges, and $1+7=8$

∴ Gopal's share = $(184-8)$ or 23 oranges, }

and Ram's share = (23×7) or 161 oranges }

Ex 2 Divide 384 rupees among A, B, C and D, so that for every 5 rupees given to A, B gets 7 rupees, C 8 rupees, and D 12 rupees

$5+7+8+12=32$	$Rs\ 384-32=Rs\ 12$
A gets $Rs\ 12 \times 5 = Rs\ 60$,	C gets $Rs\ 12 \times 8 = Rs\ 96$,
B $Rs\ 12 \times 7 = Rs\ 84$,	and D $Rs\ 12 \times 12 = Rs\ 144$ }

Ex 3 Divide 1351 nuts among 13 men, 17 women, and 30 children, giving each woman 5 times the share of each child, and each man the share of a woman and a child

If each child gets 1, a woman gets 5 and a man $5+1$ or 6
Therefore 30 children get 30, 17 women get 5×17 or 85, and 13 men 6×13 or 78

Now, $30+85+78=193$, and $1351-193=7$

∴ the children will have 7×30 or <u>210</u> nuts	}
the women 7×85 or <u>595</u> nuts,	
and the men 7×78 or <u>546</u> nuts	

Ex 4 How many horses worth 132 rupees each, must be given for 1476 sheep worth 11 rupees each?

The cost of 1476 sheep at Rs 11 each = $Rs\ 1476 \times 11 = Rs\ 16236$

And $16236-132=123$ the required no of horses = 123

Ex 5 If a man can travel 2440 miles in 4 weeks, how many miles can he travel in 9 weeks?

In 4 weeks the man travels 2440 miles

∴ in 1 week $2440-4$ or 610 miles

∴ in 9 weeks 610×9 or 5490 miles

119 Backward process

In a backward process, beginning from the last number, we change Addition into Subtraction, Subtraction into Addition, Multiplication into Division and Division into Multiplication

Ex What number is that which if I divide by 6, to the quotient add 25, from the sum take 36 and multiply the remainder by 4, the product is 40?

The required number = $(40-4+36-25) \times 6 = (46-25) \times 6$
 $= 21 \times 6 = 126$

Examples XXXIII

1 What is the average age of 4 men whose ages are 47, 55, 39 and 77 respectively ?

2 In a school register of daily attendance the numbers for a certain week were —Monday 83, Tuesday 85, Wednesday 75, Thursday 80, Friday 78, Saturday 72 What was the average daily attendance ?

3 At a competitive examination there were 4 candidates at the age of 19, 3 at 20, 2 at 22, and 3 at 24 Find the average age

4 A man's income for 3 years is Rs 250 a year, for the next 5 years it is Rs 294, and for the next 4 years Rs 309 What is his average income for the 12 years ?

5 In the month of April, a man slept 7 hours on each of 16 nights, 6 hours on each of 8 nights, 8 hours on each of 5 nights and 10 hours on the last night How long did he sleep each night on an average during the month ?

6 Divide 1008 rupees among A , B and C , so that for every 2 rupees A gets, B shall get 3 rupees and C 4

7 Divide 2624 apples among A , B and C , so that for every 5 apples given to A , B may get 11, and C 16

8 The price of a carriage with horse is 1590 rupees, and the price of the carriage is 5 times that of the horse Find the price of the horse

9 If 23 men earn 1380 rupees in a month, how many men will earn 1980 rupees in the same time ?

10 A gentleman left 225,000 rupees to be divided amongst his 4 sons and 3 daughters in such a way that each son would receive three times as much as each daughter How much did each son and each daughter receive ?

11 Divide 33775 rupees among 13 men, 17 women and 30 children, giving each woman 5 times the share of each child, and each man the share of a woman and a child

12 24 cows are worth 864 rupees, and 45 horses are worth 2635 rupees, how many of such horses ought to be exchanged for 2520 of such cows ?

13 Divide 2954 rupees among A , B , C and D , so that for every 2 rupees given to A , B shall get 3 rupees, C 4 and D 5

14 A farmer had a horse worth 375 rupees and exchanged it for a yoke of oxen and three cows, the oxen he sold for 125 rupees, two of the cows at 85 rupees each and the other for 76 rupees How much did he lose by the bargain ?

15 Find a number such that if I divide it by 3, and then add 4, then divide the result by 2 and add 3, then multiply the result by 4 and subtract 5, the result of the whole will be 19

Examples worked out.

Ex 1 I have to divide 750 rupees among a number of boys and girls, giving 3 rupees to each boy and 2 rupees to each girl, there are as many boys as girls how many boys are there?

Here, 1 boy + 1 girl receive (3 + 2) or 5 rupees

150 × (1 boy + 1 girl) receive 5 × 150 or 750 rupees,
for 750 ÷ 5 = 150 the number of boys = 150 Ans

Ex 2 A man living at the rate of 750 rupees a year for 6 years finds that he is exceeding his income, and reduces his expenditure to 540 rupees a year, at the end of 4 years he finds that he is just out of debt, what is his income?

In 6 years his expenses amount to Rs 750 × 6 = Rs 4500

In 4 years Rs 540 × 4 = Rs 2160

in 10 years his income amounts to Rs 6660,

his debts of the first 6 years are paid off by the savings of the last 4 years,

his yearly income = Rs 6660 ÷ 10 = Rs 666 Ans

Ex 3 Two persons started at the same time from A and B. One left A for B travelling 5 miles an hour, and the other from B for A travelling 7 miles an hour. The distance between A and B is 108 miles. When and where did they meet?

While the first walks 5 miles, the second walks 7 miles, and the distance to be travelled by both before they meet is 108 miles

Now, 5 + 7 = 12, and 108 ÷ 12 = 9 they meet after 9 hours

Also, the distance from A where they meet = 5 × 9 or 45 miles

Ex 4 A man bought 75 cows at 50 rupees each, 94 cows at 43 rupees each and 106 cows at 48 rupees each, at what price per head must he sell the cows, so as to gain 595 rupees by his bargain?

The cost of 75 cows at Rs 50 each = Rs 75 × 50 = Rs 3750

94 cows at Rs 43 = Rs 94 × 43 = Rs 4042

106 cows at Rs 48 = Rs 106 × 48 = Rs 5088

the cost of 275 cows = Rs 12880

gain = Rs 595

the selling price of 275 cows = Rs 13475

the selling price of a cow = Rs 13475 ÷ 275 = Rs 49

17 Subtract the value of the second and fourth digits from that of the third and fifth digits in the number 123456

18 The quotient arising from the division of 256329 by a certain number is 354, and the remainder is 387 Find the divisor

19 A person, who was born in 1779, died at the age of 46 years his son died 27 years afterwards, and his daughter died 13 years after his son, in what year did the daughter die?

20 Of what number is 7036 both divisor and quotient?

21 What number is that, which being divided by 24, the quotient increased by 26 the sum diminished by the difference between 40 and 27, the remainder multiplied by 4, and the product divided by 11 will give 12 for a quotient?

22 The quotient being = 5 times divisor = 7 times remainder = 105, find the dividend

23 The quotient being 958 and the divisor 607, find the dividend What would the dividend be, had there been a remainder 44?

24 What least number must be subtracted from 2346, that the remainder may be divisible by 135? By what least number must the same be multiplied that the product may be divisible by 36?

25 A house and its furniture cost Rs 570600, the house is 5 times the furniture What is the cost of the house?

26 Find the number, which if I multiply by 7, then subtract 31, then divide the result by 3, then add 5, and then multiply by 4, the result is the square of 10

27 A merchant has three sorts of sugar, the first and second together weigh 12356 maunds, the third 7152 maunds less than the sum of the first and second, also the second weighs 1647 maunds less than the third Find the quantity of each sort

28 The product of two numbers is 1270374, and half of one of them is 3129, what is the other number?

29 There were 2244 pears on a tree The owner gathered 46 daily for 14 days, he divided the remainder between his son and daughter, giving the former 5 for every 3 that he gave the latter, how many pears did the son receive more than the daughter?

30 The Duke of Wellington died in the year 1852, aged 83, Napoleon was born in the same year as the Duke, and died in 1821, what was the Napoleon's age at the time of his death?

31 A speculator gained Rs 3560, and afterwards lost Rs 3479, he then gained Rs 6283, and then lost first Rs 1089, and then Rs 2361 by how much did his gains exceed his losses?

32 What least number must be subtracted from $72347 - 11 \times 7$ that the remainder may be divisible by $17 \times 9 - 3 \times 6$?

33 A merchant bought 122 maunds of oats at Rs 2 per maund,

and 256 maunds of an inferior sort at Rs 1 per maund and mixing the two sorts sold the whole for Rs 525 How much did he gain or lose ?

34 A man dies worth Rs 2427498 to be divided among his three sons He directed in his will that the eldest and second together shall get Rs 1937734, and the second and third together Rs 1196570 How many does each receive ?

35 How many words are there in a book of 347 pages, if there are 13 words in each line, and 40 lines in each page ?

36 A water tub has two pipes attached to it The first discharges 14 seers and the second 15 seers of water per minute When the tub is full, both the pipes are opened at once, and the tub becomes empty in 15 minutes Find the content of the tub

37 A is 27 years older than B , and 15 years younger than C who is 54 years of age, D is as old as the sum of A 's and B 's ages Is C older or younger than D ? How much ?

38 A has 74 marbles, B has 34 more than A , and C has 16 more than B , A gives B and C each 19, B gives A and C each 34 and C gives A and B each 10 How many marbles have A , B and C respectively after these exchanges ?

39 A person bought 68 bales of cloth containing 67048 yards, each bale contained 34 pieces, and each piece contained the same number of yards, find the number of yards in each piece

40 The nuts in a bag were divided among 59 boys and 27 girls each boy had 3 times as many as each girl, there were just nuts enough and one over to give the girls 7 nuts apiece How many nuts did the bag contain ?

41 A man's annual income is Rs 7836 His expenditure in January is Rs 632, in February and March Rs 1146, in April, May and June Rs 1698, and in each of the remaining 6 months Rs 595 on an average How much does he save in the year ?

42 A man divided his property worth Rs 12547 among his 4 sons in such a manner that the eldest received Rs 126 more than the second, the second Rs 131 more than the third, and the third Rs 121 more than the fourth How much did each receive ?

43 Three pipes are attached to a water-tub By two of these 36 and 24 maunds of water respectively enter into it every hour, while by the third 33 maunds go out in the same time If the tub can hold 2673 maunds of water, when will it be full, if all the pipes are opened together ?

44 Express 19141×1225 as the difference of two square numbers

45 If 256512 be divided by 105, using its factors 3, 5 and 7, find the true quotient and the true remainder

46 A gentleman left Rs 123600 to be divided among his two

sons, four daughters and one sister, in such a way that each daughter would receive twice as much as the sister, and each son one-half of what the three daughters would receive. What did the sister receive?

47 A man worth 30 lacs of rupees, having no heirs, divides his whole property among his four faithful servants A , B , C and D . He gives to B twice as much as he gives to A and Rs 1234 more, to C twice as much as A less Rs 2284, and to D Rs 32000. Find his bequest to A .

48 A and B walk at the rates of 10 and 13 miles per hour respectively. If they are walking towards each other, and if the distance between them be 207 miles, find when they will meet.

49 A says to B and C , I have Rs 1650, B replies if I had Rs 753 more than I have, I should have as much as you have. C adds, if I had Rs 105 more than I have, I should have as much as both of you. How many more rupees has C than B ?

50 To what number must 28 be added that the sum being multiplied by 25, the product will be 125625?

51 From what number must 302 be subtracted that the remainder being multiplied by 125, the product will be 321000625?

52 Divide Rs 40 between A and B in such a way that if A gets Rs 5, B shall get Rs 3.

53 Divide Rs 30 among A , B and C in such a way that if A gets Rs 1, B shall get Rs 2 and C Rs 3.

54 If the sum of 250 and 173 be multiplied by their difference, and the product be divided by 33, find the result.

55 Add together the six numbers you can form with the three figures 3, 4 and 5, taken all together, and multiply the sum by 597.

56 Add together all the numbers that you can form with the four digits 1, 2, 3 and 0 taken all together.

57 Arrange the nine digits 1, 2, 3, 4, 5, 6, 7, 8, 9, in three lines with three digits in each line, so that the sum of these digits may, taken in every possible direction, be 15.

58 Find the sum of all the numbers that you can form with the digits 1, 5, 7, only two digits being taken at a time.

59 By what number must 123456 be divided that if 15328 be added to the quotient and the sum divided again by 8, the quotient will be 7060?

60 A , B and C have between them 1467 marbles. B has three times as many as A , and C 131 marbles more than the sum of A and B . How many has each?

61 Divide Rs 5000, among A , B , C and D in such a manner, that if A gets Rs 2, B shall get Rs 3, C Rs 4 and D Rs 11.

62 Simplify—

(1) $920 - 23 \times 720 - (42 - 7) \times (78 - 13) - (5 \times 4)$

(2) $1250 \times (72 - 4) - (20 \times 5) \times (64 - 16) - (111 - 37)$

63 What least number must be added to $3243 - 3 \times 9$ that the sum may be divisible by $15 - 5 \times 8 \times 7$?

64 What number less than 365 added to 730320 will make the number exactly divisible by 365?

65 A man spends Rs 1485 annually for 6 years and runs into debt. He then reduces his expenses to Rs 1109 a year, and in 10 years just manages to clear off his debts. What is his yearly income?

66 Multiply 765389 by 64164, and by 189279, and by 83256, making in each case only three partial multiplications.

67 A volume of a work contains 6 parts of 128 pages each, and there are 46 lines in each page and 58 letters in each line. How many letters are there in 9 volumes?

68 A man spends Rs 600 a year for 5 years and saves some money, he then raises his expenditure during the next 7 years to Rs 720 a year, and finds all his savings spent. What does he earn each year?

69 The sum of the product of two numbers and 1420 is 349612, one of the numbers is 864. Find the other number.

70 Find the number which being divided by 24 gives a quotient which if increased by 36 and the sum multiplied by 24 gives a product that will be greater than 876 by 300.

71 If in dividing a number by 336, the operation be performed by short division by employing the factors 6, 7 and 8 in succession and the several remainders be 1, 2 and 3, find the complete remainder.

72 If two men start from the same place and travel in opposite directions, the one at the rate of 42 miles and the other 45 miles a day, how far apart will they be at the end of 12 days?

73 If two men start from the same place and travel in the same direction, the one at the rate of 512 miles and the other 540 miles a week, how far apart will they be at the end of 8 weeks?

74 A dividend is 4637064283, the quotient is 80496 and the remainder is 11707, what is the divisor?

75 If 20 men can do a piece of work in 11 days, how many days will it take 22 men to do it?

76 A, B, C and D have among them Rs 69, A, B and C have among them Rs 48, B and C Rs 31, B having Rs 15 more than C, how many more rupees have A and B than C and D?

77 The product of three numbers is 535500, one of the numbers is 75, another is 68. What is the third?

78 The product of three numbers is 8937992, the third number is double the second, and the sum of the second and third is 906. Find the first number.

79 Divide Rs 3975 among A , B , C and D so that B may have Rs 23 more than A , C Rs 45 more than A and B together, and D Rs 29 less than B and C together

80 A grazier bought a certain number of bullocks for Rs 4900 and sold a part of them for Rs 3840 at Rs 32 a head and gained on those he sold Rs 180. How much did he gain a head, and how many did he buy at first?

CHAPTER III

Compound Quantities

120 If one quantity contains another of the same kind an exact number of times, the first is said to be a **multiple** of the second, and the second a **submultiple** or **aliquot part** of the first

121 We have already seen that in considering quantities of the same kind, we take an arbitrary but well defined quantity of that kind as our **unit**, and finding *how many* times it is contained in each of them, we express them as whole numbers. But in this way very large quantities will be expressed by very high numbers, which give by inspection little idea of their relative values, to obviate this inconvenience we take such multiples of the *unit* as will enable us to avoid very high numbers. Thus, of length, we take a **yard** as our unit, but to measure long lengths we use the **mile**, a high multiple of the yard. Hence has arisen the custom of using large units for large quantities and small units for small quantities. Thus we say that the price of a chair is 8 rupees, that of a book is 14 annas, and that of a pencil is 2 pice.

122 Since it is the custom to use more than one unit for things of the same kind, it would be convenient to select one quantity as the principal or **standard** unit and thence derive the various minor or **auxiliary** units, either by dividing this unit into a number of equal parts or by multiplying it a number of times. The *standard* unit of any quantity and its *auxiliary* units are called its **denominations**.

123 In the preceding Chapter we have considered only such *abstract* numbers, or such *concrete* numbers of one denomination as are formed by figures whose local values are always regulated by the same fixed number *ten*, but the rules given can easily be extended to *concrete* numbers of different denominations, wherein the local values of the figures are connected by more numbers than one, as, for instance, to **rupees**, **annas** and **pies**, where twelve pies are equivalent to one anna, which is the next higher denomination, sixteen annas to one rupee, which is the next denomination in order, the *different* numbers 12 and 16 connecting the denominations, in the same manner, as the *fixed* number 10 was supposed to connect the denominations of Integers

Here, the standard unit **rupee** is divided into 16 equal parts to obtain the auxiliary unit *anna*, and into 16×12 or 192 equal parts to obtain the auxiliary unit *pie*. Thus, the rupee, anna and pie are the various denominations of money.

124 The processes employed in cases of this nature are Reduction, and the fundamental operations are then called Compound Addition, Compound Subtraction, Compound Multiplication and Compound Division, each of which will be exemplified in order, and the various Tables, which furnish us with a list of the relative magnitudes of the different *auxiliary* units, and by means of which the above operations are conducted, are given below in order.

TABLE I MONEY

British Indian Money

125	3 Pies (<i>p</i>) or 2 half-pice make 1 Pice (<i>ps</i>)	
	2 Pice	" 1 Half anna
	4 Pice or 12 pies	" 1 Anna (<i>1a</i>)
	16 Annas	" 1 Rupee (<i>Re</i> 1 or 1/)
	1, Rupees	" 1 Sovereign

126 Accounts in Bengal are kept by the following Table

4 Cowries make 1 Ganda	4 Pans make 1 Chouk
5 Gandas " 1 Buri (Paisa)	4 Chouks " 1 Kahan or
4 Buris " 1 Pan (Anna)	Rupee

Also 1 Cowry or *Bat* = 3 *Krantis* = 4 *Kags* = 5 *Tals* = 7 *Dwips* = 9 *Dantis* = 27 *Jabs* = 80 *Tils* = 320 *Ramus* = 1280 *Bahars* or *Ghuns* = 25600 *Bindus*

Therefore 1 Cowry = 4 *Kags*, 1 *Kag* = 20 *Tils*, 1 *Til* = 16 *Ghuns*
1 *Ghun* = 20 *Bindus*

127 The following Tables are in use in different parts of India

IN BEHAR, N-W P AND PUNJAB	IN BOMBAY
5 Cowries make 1 Adhi	100 Raes make 1 Quarter
2 Adhis " 1 Damri	4 Quarters " 1 Rupee
2 Damris " 1 Chhadam	IN MADRAS
2 Chhadams " 1 Adhela	1 Pagoda = Rs 3 8a
2 Adhelas " 1 Paisa	IN CEYLON
2 Paisas " 1 Taka	100 Cents make 1 Rupee
2 Takas " 1 Anna	

In British India the common medium of exchange is *silver*. The principal coin made of it is called a **Rupee**. The Rupee weighs 1 tola or 180 grains, and consists of 11 parts of silver and 1 of alloy. The weight of a gold *Mohur* is the same as that of a Rupee and is 180 grs. It consists of 11 parts of gold and 1 of alloy. The values of gold coins are variable, and therefore they are not used in mercantile transactions except the Sovereign, whose value is 15 rupees. The *Cowry*

is a shell brought from the Laccadive and Maldive Islands, and is used for very small payments. They vary in value according to supply in market but they are generally reckoned at 80 to a pice.

N B—The cowries as shells are now going out of use, but cowries (called *kaias*) are in use in keeping accounts.

Of the copper coins, a half-anna weighs 200 grains, a pice weighs 100 grains, and a half pice 50 grains.

15 *Sicca* Rupees = 16 Rupees. The *Doctor's* Gold Mohur = 16 Rupees, the *Lawyer's* Gold Mohur = 17 Rupees.

Gold coins (*obsolete*), Five-rupee piece, Ten rupee piece, Gold Mohur, Double Gold Mohur.

Silver coins (*current*) Two anna piece, Four anna piece or Quarter-rupee, Eight-anna piece or Half rupee, Rupee.

Copper coins (*current*) Pie, Half-pice, Pice or Paisa, Double paisa or Half-anna.

Note *Re* 1 = 2 half-rupees = 4 quarter-rupees or four-anna pieces = 8 two-anna pieces, and 1 anna = 2 double paisas. Also *Re* 1 = 64 pice = 192 pies.

English Money

128	2 Farthings (<i>q</i>) make	1 Half-penny ($\frac{1}{2}d$)
	2 Half-pence	1 Penny (<i>d</i>)
12	Pence	1 Shilling (1 <i>s</i> or 1/)
20	Shillings	1 Pound (£1)

[1, 2, 3 farthings are usually denoted by $\frac{1}{4}d$, $\frac{1}{2}d$, $\frac{3}{4}d$ respectively]

Money as expressed by means of these denominations is called **Sterling money**, in order to distinguish it from stocks, shares, &c. The **Standard** gold coin of England is made of a metal consisting of 22 parts of *pure gold*, and 2 parts of *copper*. Each of these 24 parts is called a *Carat*. Pure gold is said to be 24 carats *fine* and standard gold 22 carats *fine*. The *Pound sterling* is represented by a gold coin called a **sovereign**, and from 40 pounds Troy of standard gold are coined 1869 sovereigns, and the value of gold of the *Mint-Fineness*, called 22 carat gold, is £3 17*s* 10½*d* per ounce.

The **Standard** silver coin consists of 37 parts of *pure silver*, and 3 parts of *copper*. A pound Troy of this metal furnishes 66 *shillings*, and the *Mint-Price* of standard silver is 5*s* 6*d* per ounce. The silver coinage is not a *legal tender* for more than 40*s*, the gold coinage being the *general* standard of value.

In the **copper coinage**, 24 pennies are made from an Avoirdupois pound of copper. This coin is not a *legal tender* for more than 12*d*. The coins now *current* in England are the following —

Copper coins Farthing, Half-penny, Penny.

Silver coins Three penny piece, Four penny piece, Six-penny piece, Shilling, Florin (2*s*), Half-crown (2*s* 6*d*), Crown (5*s*).

Gold coins Half-sovereign, Sovereign.

The following coins were formerly in use, but now they are *obsolete*.

Silver coins Groat (4*d*), Tester (6*d*).

Gold coins Noble (6s 8d), Angel (10s) Half Guinea (10s 6d)
 Mark or Merk (13s 4d), Guinea (21s), Carolus (23s), Jacobus
 (25s), Moidore (27s)

Note 1 shilling = 2 six pences = 3 four penny pieces = 4 three penny pieces
 Also 1 half crown = 5 six pences, 1 half guinea = 21 six-pences, 1 guinea = 42 six-pences

Also £1 = 4 crowns - 8 half crowns = 10 florins = 40 six pences = 80 three-pences = 240d = 960q

I. REDUCTION.

129 When a quantity is expressed in one denomination only it is called a **simple quantity**, as 7 rupees 5 yards

When a quantity is expressed in several denominations, it is called a **compound quantity**, as Rs 8 2a 3p, 5 yards 2 feet 3 inches

130 **Reduction** is the process by which we convert or change (1) a simple or a compound quantity into terms of its lower denominations, or (2) a simple quantity into terms of its higher denominations, so that the *real* or *absolute* values remain unaltered

131 *To express a quantity in terms of its lower denominations (Descending Reduction)*

RULE Multiply the number in the highest denomination by the number of units of the next inferior denomination contained in one unit of the highest, and to the product add the number (if any) of the inferior denomination in the quantity proposed, and repeat this for each succeeding denomination till the required one is obtained

Ex 1 Reduce Rs 315 to pice

$$\begin{array}{r} \text{Rs } 315 \\ 16 \\ \hline 5040a \\ 4 \\ \hline 20160ps \end{array}$$

$$\text{Rs } 1 = 16a$$

$$\text{Rs } 315 = (315 \times 16a) = 5040a$$

$$\text{Again, } 1a = 4ps$$

$$5040a = (5040 \times 4)ps = 20160ps$$

$$\text{Rs } 315 = 20160ps$$

Ex 2 Reduce Rs 5 14a 6p to pice

$$\begin{array}{r} \text{Rs } 5 \quad 14a \quad 6p \\ 16 \\ \hline 94a \quad (5 \times 16 + 14) \\ 12 \\ \hline 1134p \quad (94 \times 12 + 6) \end{array}$$

$$\text{Rs } 5 = 5 \times 16a = 80a$$

$$\text{Rs } 5 \quad 14a = 94a$$

$$\text{Again, } 94a = 94 \times 12p = 1128p$$

$$\text{Rs } 5 \quad 14a \quad 6p = 94a \quad 6p = 1128p + 6p = 1134p$$

Ex 3 Reduce £25 13s 6½d to farthings

$$\begin{array}{r} \text{£ } 25 \quad 13s \quad 6\frac{1}{2}d \quad \text{£1} = 20s \\ 20 \\ \hline 513s \\ 12 \\ \hline 6162d \\ 4 \\ \hline 24651q \end{array}$$

$$\text{£ } 25 = 25 \times 20s = 500s$$

$$\text{£ } 25 \quad 13s = 500s + 13s = 513s$$

$$\text{Again, } 1s = 12d, \quad 513s = 513 \times 12d = 6156a$$

$$513s \quad 6d = 6156d + 6d = 6162d$$

$$\text{Again, } 1d = 4q, \quad 6162d = 6162 \times 4q = 24648q$$

$$\text{£ } 25 \quad 13s \quad 6\frac{1}{2}d = 24648q + 3q = 24651q$$

Here, the *denominations* are separated by a point as (), and this is necessary to distinguish them from *ordinary* numbers, which do not require it, because their local values are all fixed and certain

Examples XXIV

1 Reduce to *annas* —

- (1) Rs 17, Rs 19, Rs 42, Rs 45, Rs 69, Rs 54 Rs 95
- (2) Rs 87, Rs 120, Rs 245, Rs 160, Rs 9 12a, Rs 20 14a
- (3) Rs 36 6a, Rs 53 13a, Rs 87 11a, Rs 79 15a, Rs 234 11a

2 Reduce to *pies* —

- (1) Rs 34, Rs 56, Rs 97, Rs 146, Rs 342, Rs 496
- (2) Rs 84 5a, Rs 76 12a, Rs 265 9a, Rs 804 13a, Rs 945 6a
- (3) Rs 15 8a 3p, Rs 7 13a 11p, Rs 8 0a 5p, Rs 9 10a 9p
- (4) Rs 425 7a 9p, Rs 550 3a 11p Rs 1250 5a 7p, Rs 5050 14a 1p, Rs 456 14a 11p, Rs 31 10a 1p, Rs 343 8a 7p

3 Reduce (i) to *pie* and (ii) to *pies* —

- (1) Rs 52, Rs 19, Rs 112 6a, Rs 36 11a 2p, Rs 20 8a 3p
- (2) Rs 87 10a 1p, Rs 172 5a 2p, Rs 225 9a 2p, Rs 476 12a 1p, Rs 782 0a 3p, Rs 13 10a 3p, Rs 215 7a 3p

4 Reduce (i) to *gandas* and (ii) to *cowries* (*kaias*) —

- (1) Rs 19 Rs 34, Rs 56, Rs 78, Rs 105, Rs 84 7a
- (2) Rs 102 15a 1p, Rs 24 14a 3p, Rs 405 13a, Rs 75 7a 5a
- (3) Rs 48 9a 10gan, Rs 53 13a 17gan, Rs 9570 14a 16gan

5 Reduce to *cowries* (*kaias*) —

Rs 53 13a 17gan 2cow, Rs 68 9a 11gan 1cow, Rs 18 6a 12gan 2cow, Rs 5942 0a 17gan 3cow

6 Reduce (i) to *pie* and (ii) to *pies* —

- (1) 175 half rupees, 370 quarter rupees, 845 two anna pieces
- (2) 425 double paisas, 3116 two anna pieces, 2415 half rupees
- (3) 34212 quarter-rupees, 20157 double paisas, 67950 four anna pieces, 827 eight-anna pieces

7 Reduce (i) to *half rupees*, (ii) to *quarter rupees* and (iii) to *two anna pieces*

- (1) Rs 729, Rs 925, Rs 1228, Rs 1427, Rs 4243, Rs 97403
- (2) Rs 858 8a, Rs 9726 8a, Rs 73246, Rs 57509

8 Reduce (i) to *half annas* and (ii) to *half pie* —

Rs 75 6a, Rs 132 0a, Rs 150 0a 2p, Rs 3005 10a 2p

9 Reduce —

- (1) A lac of rupees to *paisas*, Rs 7125 4a to *four anna pieces*, Rs 6075 8a to *two anna pieces*, Rs 1250 7a 2p to *double-paisas*, Rs 9864 8a to *eight anna pieces*
- (2) Rs 1325 9a 1p to *half-paisas*, Rs 3116 14a 6p to *double-paisas*, Rs 2415 10a 9p to *half pie*

10 Reduce to *shillings* —

- (1) £345, £498, £795, £1402, £9086, £8092
 (2) £71 1s, £490 18s, £790 13s, £3456 17s, £6403 7s

11 Reduce to *pence* —

- (1) £65, £98, £156, £405, £1849, £5043, £9236
 (2) £134 15s, £198 13s, £416 11s, £526 5s, £926 7s
 (3) £2 6s 8d, £40 10s 6d, £11 7s 9d, £374 11s 8d
 (4) £655 13s 6d, £71 13s 5d, £343 13s 5d, £1274 19s 9d

12 Reduce to *farthings* —

- (1) £4 8s 4½d, £7 13s 11½d, £13 19s 0¼d, £29 10s 11d
 (2) £101 9s 2¼d, £153 3s 4½d, £600 6s 3d, £83920 16s 2¼d

13 Reduce (i) to *half-pence* and (ii) to *farthings* —

- (1) 15s 6d, 18s 9d, 13s 11d, 19s 6d, 8s 10d, 17s 5d
 (2) £4080, £8608, £8734, £726 18s, £517 13s, £2125 6s
 (3) £79 14s 8d, £47 19s 9½d, £389 12s 8½d, £879 18s 0½d
 (4) £1560 10s 4½d, £2145 18s 7½d, £9136 15s 9½d
 (5) 3899 half sovereigns, 4807 crowns, 8608 half crowns, 6530 florins,
 5869 six pences, 6958 groats, 8009 three penny pieces,
 9076 guineas, 3089 half-guineas, 7632 four-penny pieces
 1445 moidores, 2047 nobles, 3286 florins, 1983 six-pences

14 Reduce (i) to *three penny pieces*, (ii) to *four penny pieces*, and (iii) to *six-pences* —

- (1) £95, £128, £8076, £1857, £9083, £9072
 (2) £11 14s, £144 17s, £2145 11s, £4265 15s, £3264 17s

15 Reduce —

- (1) 95 guineas 17s 9½d to *farthings*, £450 16s 6d to *six pences*
 (2) £570 12s to *florins*, £382 7s 6d to *half crowns*, £589 15s to *crowns*, £3500 17s 6d to *half crowns*
 (3) £99 9s 9d to *three-pences*, 5573 half-crowns to *pence*
 (4) 9571 half-crowns to *six-pences*, 9100 half-crowns to *three-pences*

16 Reduce to *farthings* —

- (1) 71 *gui* 16s 2½d, 937 *flor* 1s 2½d, 2902 *ci* 1s 3½d
 (2) 150 *half-sov* 7s 2½d, 79924 *gui* 12s 2½d, 7255 *flor* 1s 3½d

17 For how many children can a treat be provided with Rs 32 8a at 2 annas a head?

18 How many two pice stamps can I buy for Rs 5 6a 2ps?

19 If the cost of a telegram is 3d a word, how many words can be sent for £1. 3s 3d?

20 A poor woman had only Rs 2 1a 8p to live upon. She spent daily 4 pies for her food. How many days did she live upon?

132 To express a simple quantity in terms of its higher denominations (Ascending Reduction)

RULE Divide the number by the number of units which make one unit of the next higher denomination, setting down the remainder (if any) as of the same denomination as its dividend, and continue this process till we come to the required denomination

Ex 1 Reduce 1560 *pis* to *rupees*, &c.

$$\begin{array}{r} 4 \overline{) 1560 \text{ pis}} \\ 16 \overline{) 390} \\ \hline \text{Rs } 24 \text{ 61} \end{array} \quad \begin{array}{l} 4 \text{ pice} = 1 \text{ anna} \\ 16 \text{ annas} = 1 \text{ rupee} \\ \therefore \text{ the result is } \underline{\text{Rs } 24 \text{ 61}} \end{array}$$

Ex 2 Reduce 30857 *p* to *rupees*, *annas* and *pis*

$$\begin{array}{r} 12 \overline{) 30857 \text{ p}} \\ 16 \overline{) 2571 - 5 \text{ p}} \\ \hline \text{Rs } 160 - 11 \text{ a} \end{array} \quad \begin{array}{l} 12 \text{ pice} = 1 \text{ anna} \\ 16 \text{ annas} = 1 \text{ rupee} \\ \text{the result is } \underline{\text{Rs } 160 \text{ 11 a } 5 \text{ p}} \end{array}$$

Ex 3 Reduce 97403 *q* to *pounds*

$$\begin{array}{r} 4 \overline{) 97403 \text{ q}} \\ 12 \overline{) 24350 - 3 \text{ q}} \\ 20 \overline{) 2029 - 2 \text{ d}} \\ \hline \text{£ } 101 - 9 \text{ s} \end{array} \quad \begin{array}{l} 4 \text{ q} = 1 \text{ a} \\ 12 \text{ d} = 1 \text{ s} \\ 20 \text{ s} = \text{£ } 1 \\ \text{the result is } \underline{\text{£ } 101 \text{ 9 s } 2 \text{ d}} \end{array}$$

Ex 4 Reduce 36173 half pence to *guineas*

$$\begin{array}{r} 2 \overline{) 36173 \text{ half pence}} \\ 12 \overline{) 18086 - 1 \text{ half penny}} \\ 21 \left\{ \begin{array}{l} 3 \overline{) 1507 - 2 \text{ d}} \\ 7 \overline{) 502 - 1} \end{array} \right. \quad \begin{array}{l} 2 \text{ half penny} = 1 \text{ d} \\ 12 \text{ d} = 1 \text{ s} \\ 21 \text{ s} = 1 \text{ gu} \\ \text{the result is } \underline{71 \text{ gu } 16 \text{ s } 2 \frac{1}{2} \text{ a}} \end{array} \end{array}$$

Examples XXV

1 Reduce to *rupees*, *annas* and *pice* —

- (1) 25325 *p*, 57509 *p*, 51039 *p*, 679298 *p*, 37921 *p*, 456786 *p*
 (2) 643294 *p*, 732394 *p*, 1982345 *p*, 967573 *p*, 1043324 *p*

2 Reduce to *rupees*, *annas* and *pice* —

987945 *ps*, 1234567 *ps*, 547321 *ps*, 894956 *ps*, 5537792 *ps*

3 Reduce to *rupees*, *annas*, &c. —

- (1) 8320 *gandas*, 7680 *cowries* (*karas*), 379498 *gandas*, 40768 *buris*
 (2) 1045673 *double paisas* 2067544 *half paisas*, 1077760 *cowries* (*karas*)
 (3) 342876 *buris*, 596824 *paisas*, 23679 *double paisas*, 103678 *half paisas*, 1155440 *cowries* (*karas*)

4 Reduce to *rupees* —

- (1) 1648 *half rupees*, 1892 *quarter rupees*, 2530 *two anna pieces*
 (2) 2896 *annas*, 5952 *paisas*, 920320 *gandas*, 24320 *cowries* (*karas*)

5 Reduce to *pounds, shillings and pence* —

- (1) 6913*d*, 60748*d*, 90231*d*, 2733*d*, 89900*d*, 157362*d*
 (2) 147040*q*, 284051*q*, 123290*q*, 350000*q*, 80563979*q*
 (3) 345679*z*, 1300013*z*, 10000019*z*, 284079*z*, 415739*z*, 3650324*z*

6 Reduce to *£ s d* —

- (1) 890045 half sovereigns, 971112 crowns, 40073 florins, 886101 half crowns, 85730 half pence, 15029 three pences
 (2) 15137 four penny pieces, 82556 florins, 28892 half crowns, 2857 four-penny pieces, 987653 half pence, 47285 guineas
 (3) 23645 moidores, 49726 half guineas, 183491 six pences, 281062 three pences, 40340 firlings

7 How much money will be required to buy 37528 penny stamps?

8 A dealer bought 436 coconuts at 9 pice each how many rupees, &c. had he to pay for them?

9 I distributed among 1682 beggars a sum of money, giving them two pice a head, what sum did I spend?

10 If during a festival 2250 people on an average cross the Hugh Bridge daily, each paying 2 pice, what is the collection of the ferry farmer, if the festival lasts for 16 days?

133 There are some cases in Reduction where we cannot pass directly step by step from the given denominations to the one proposed. We must in such cases pass through an intermediate denomination common to both, and it will be advisable to keep such common denomination as high as possible. Then, find by division what quantity of the proposed denomination is equivalent to the given quantity.

Ex 1 Reduce £253 9*s* 10*d* to half crowns

$$\begin{array}{r} \text{£}253 \text{ } 9\text{s } 10\text{d} \\ 20 \\ \hline 5069 \\ 12 \\ \hline 3,016 \text{ } 8\text{s } 8\text{d} \\ 2027 - 28\text{d} \end{array}$$

1 half crown = 2*s* 6*d* = 12*d*
 the result is 2027 half crowns and 28*d*
 or 2*s* 4*d* over

Ex 2 Reduce Rs 31 10*s* 2*p* to £ s d, when 1*q* = 2 pice

$$\begin{array}{r} \text{Rs } 31 \text{ } 10\text{s } 2\text{p} \\ 16 \\ \hline 5060 \\ 12 \\ \hline 6074\text{p} \end{array} \quad \begin{array}{r} 2 \text{ } 6074\text{p} \\ 3037\text{q} \end{array} \quad \begin{array}{r} 4 \text{ } 3037\text{q} \\ 12 \text{ } 759 - 1\text{q} \\ \hline 2,0 \text{ } 6,3 - 3\text{d} \\ \hline \text{£}3 - 3\text{s} \end{array}$$

∴ Rs 31 10*s* 2*p* = £3 3*s* 3*d*

134 Proof Descending and ascending Reductions are in verse processes, if therefore we perform one process on a given

quantity, and on the result the other process, we ought to get the original quantity

Thus, if by the descending process we find that £25 13s 6½d = 24651q, we ought by the ascending process to find that 24651q = £25 13s 6½d

Examples XXVI

1 Reduce (i) to *guineas* and (ii) to *half guineas* —

£63, £105, £96 16s, £876 15s, £538, £10728

2 Reduce (i) to *crowns* and (ii) to *half-crowns* —

£265 10s, £589 15s, £437 10s, £620 5s, £5189 15s

3 Reduce to *crowns* —

10987 guineas, £89000, £36 17s 6d, 18756 four-penny pieces

4 Reduce to *half crowns* —

£48 17s 6d, £382 7s 6d, £583 2s 6d, 670 half guineas

5 Reduce to *guineas* —

28906 florins, 10728½ half crowns, 23810 crowns, 760 half crowns, £647 0s 11d, £375 16s 0½d

6 Reduce to *half guineas* —

325 crowns, 10867 half sovereigns, 3150 four penny pieces, £3240 10s 6d, 147 half crowns

7 Reduce to £ s d, (1d = 11p) —

Rs 35 9a 3p, Rs 707 11a 7p, Rs 2510 8a 4p

8 Reduce to Rs a p, (1q = 2 pies) —

£32 14s 7d, £96 17s 6d, £903 17s 6½d, 54 half-guineas, 107 florins, 17 half-crowns

9 If a guinea be equal to Rs 10 8a, find the number of two anna pieces contained in 1760 guineas

10 Reduce 7500 Sicca rupees to *current rupees* and 6432 rupees to *Sicca rupees*

II COMPOUND ADDITION

135 Keeping in mind what was said in Art 123, we need no additional inquiry to inform us that the fundamental operations on *Compound Quantities* must be performed as in *Integers*, with this difference, that instead of carrying and borrowing *tens*, we must do the same with the *different numbers* which connect their parts together, and we shall therefore merely enunciate the rule for each at the beginning of the portion of the work appropriated to it

136 **Compound Addition** is the method of finding a single quantity which is equal to two or more quantities of the same kind. This single quantity is called the **sum** of the given quantities

RULE Arrange the quantities under one another according to their denominations, so that units of the same denomination may be in the same vertical column, and draw a line below them. Add together the numbers of the lowest denomination, reduce the sum to the next higher denomination, set down the remainder, if any, under the column, and carry the quotient to the first figure of the next column. Repeat the process with all the columns.

Ex 1 Add together Rs 14 15a 10p, Rs 54 14a 9p, Rs 156 11a 2p, and Rs 34 14a 10p

$$\begin{array}{r}
 \text{Rs} \quad a \quad p \\
 14 \quad 15 \quad 10 \\
 54 \quad 14 \quad 9 \\
 156 \quad 11 \quad 2 \\
 34 \quad 14 \quad 10 \\
 \hline
 \text{Rs } 261 \quad 8 \quad 7 \quad \text{Ans}
 \end{array}$$

$10p + 9p + 2p + 10p = 31p = 2a \ 7p$
 Carry 2a, $2a + 15a + 14a + 11a + 14a = 56a = \text{Rs } 3 \ 8a$
 Carry Rs 3, $\text{Rs } 3 + \text{Rs } 14 + \text{Rs } 54 + \text{Rs } 156 + \text{Rs } 34 = \text{Rs } 261$

Ex 2 Add together £156 8s 9½d, £33 15s 11½d, £204 0s 1½d, £5275 17s 8d and £105 18s 6½d

$$\begin{array}{r}
 £ \quad s \quad d \\
 156 \quad 8 \quad 9\frac{1}{2} \\
 33 \quad 15 \quad 11\frac{1}{2} \\
 204 \quad 0 \quad 1\frac{1}{2} \\
 5275 \quad 17 \quad 8 \\
 105 \quad 18 \quad 6\frac{1}{2} \\
 \hline
 £5776 \quad 1 \quad 1\frac{1}{2} \quad \text{Ans}
 \end{array}$$

$39 + 39 + 29 + 29 = 106 = 2\frac{1}{2}d$ Carry 2d,
 $2d + 9d + 11d + 8d + 6d = 36d = 3s \ 1d$
 Carry 3s, $3s + 8s + 15s + 17s + 18s = 61s = £3 \ 1s$
 Carry £3, $£3 + £156 + £33 + £204 + £5275 + £105 = £5776$

Examples XXVII

1 Add together —

(1)	(2)	(3)	(4)	(5)	(6)
As p	As p	As p	As p	As p	As p
9 7	12 3	9 8	13 4	12 3	15 4
12 3	13 7	11 2	7 8	14 4	11 10
9 4	4 9	13 4	9 10	3 7	4 5
11 10	7 10	3 7	13 8	2 6	8 11

(7)	(8)	(9)	(10)
Rs a p	Rs a p	Rs a p	Rs a p
2 2 2	8 11 9	42 10 9	67 10 6
3 4 3	10 5 9	54 12 6	71 12 9
5 7 9	9 12 5	67 9 3	62 14 9
8 10 6	12 11 6	75 11 6	73 13 8
9 12 7	15 6 8	72 6 7	85 7 5

(11)	(12)	(13)	(14)
<i>Rs a p</i>	<i>Rs a p</i>	<i>Rs a p</i>	<i>Rs a p</i>
5 11 3	12 12 3	6 14 9	47 5 2
9 10 10	19 4 10	14 0 3	1 15 9
2 14 9	4 15 8	15 15 5	65 6 0
3 5 11	7 9 5	27 12 11	88 15 3
1 6 7	23 7 6	7 14 4	14 15 10
11 13 6	25 0 2	29 0 5	34 14 10
<u>15 7 7</u>	<u>8 14 3</u>	<u>104 13 1</u>	<u>54 14 9</u>

(15)	(16)	(17)	(18)
<i>Rs a ps</i>	<i>Rs a ps</i>	<i>Rs a p</i>	<i>Rs a p</i>
7 11 2	27 11 2	378 9 10	98 0 9
8 14 3	9 14 3	4 7 4	448 6 5
13 12 1	4 10 1	56 8 8	3839 4 0
315 10 2	156 8 2	464 0 3	97 3 2
23 7 2	215 13 2	368 6 8	136 3 7
625 15 3	18 7 1	535 7 1	4837 1 9
24 0 1	106 14 0	97 3 2	28 10 9
129 13 3	315 0 2	893 15 9	234 11 6
<u>56 8 1</u>	<u>57 14 3</u>	<u>14 10 7</u>	<u>536 12 11</u>

(19)	(20)	(21)	(22)
<i>Rs a p</i>	<i>Rs a p</i>	<i>Rs a p</i>	<i>Rs a p</i>
1135 4 3	1325 10 9	3004 7 6	74037 9 4
1243 6 9	7602 11 3	907 5 2	80668 12 0
1575 8 8	3006 7 7	1235 10 7	50087 13 4
2007 7 7	4040 8 6	2727 11 5	136 7 4
3445 9 10	3050 12 5	3647 12 9	3270 2 6
4002 10 11	2225 13 8	7532 9 8	5971 14 8
997 11 10	110 6 6	2121 13 10	58065 9 4
1005 9 9	965 14 11	3333 15 8	360 2 6
2220 13 7	1097 13 4	2025 7 6	943 5 4
<u>997 15 3</u>	<u>2110 6 9</u>	<u>1605 0 10</u>	<u>72459 4 0</u>

2 Add together —

(1)	(2)	(3)	(4)	(5)	(6)
<i>s d</i>	<i>s d</i>	<i>s d</i>	<i>£ s d</i>	<i>£ s d</i>	<i>£ s d</i>
3 7½	19 8½	19 10½	37 13 6	3 7 6½	18 15 7½
14 6½	1 9½	18 4½	29 12 4	69 11 10	76 14 2
2 11	15 9	9 5	6 3 9	13 0 4½	25 10 2½
15 8½	13 3½	15 6½	55 17 2	37 13 2½	13 13 3½
<u>13 4</u>	<u>10 9½</u>	<u>14 9½</u>	<u>7 10 10</u>	<u>26 15 7</u>	<u>66 4 7½</u>

(7)			(8)			(9)			(10)		
£	s	d	£	s	d	£	s	d	£	s	d
156	14	8½	8	19	10½	7	19	3½	2769	10	8½
9	16	4½	1379	17	6½	16	12	4	36	11	2½
83	18	10½	897	16	9½	276	4	7½	472	13	10
17	19	7	89	18	11	77	7	9½	4792	18	4½
186	7	9½	4357	8	11½	8760	10	6	3279	15	8½
8	15	6½	52765	15	8½	795	15	3½	24	8	11
3548	19	9½	99	19	11½	20	4	4	429	17	5½
95	8	8½	67	5	10	813	11	7½	4198	15	4½

3 Add together —

(1)			(2)			(3)		
Rs	a	p	Rs	a	p	Rs	a	p
3672	6	9	8274	5	7	527	9	8
4278	13	6	329	8	6	8436	10	2
236	4	1	115	2	9	4167	9	8
5982	14	6	—	5	10	129	8	3
3716	8	4	2756	7	4	927	7	7
410	7	10	9	15	7	8	1	2
6759	0	5	8136	14	1	72	7	9
4917	0	0	725	4	6	129	0	5
127	12	6	87	9	11	7283	8	6
218	8	5	351	15	4	5132	12	3
29	15	8	9027	5	9	710	10	6
6374	8	11	4578	9	3	656	8	2
7109	15	7	271	2	5	12	3	7
492	7	5	12	9	7	9215	8	6

(4)			(5)			(6)		
£	s	d	£	s	d	£	s	d
7214	18	7½	4614	13	3½	9241	12	5½
829	2	1½	12	1	5½	159	3	9½
3484	19	11	6078	11	3	63	17	10½
151	3	9½	85	7	3½	4375	19	1
10	14	3½	813	19	10	88	6	7½
2607	17	10½	7913	5	8½	797	15	9
263	6	6	24	6	8	972	13	3
90	18	8½	1012	14	6½	2356	11	6½
485	13	7	820	12	4	38	5	8½
7324	7	4½	537	9	11½	125	18	5
934	16	1½	125	16	8	6316	4	2½
78	15	10½	8416	15	1½	244	3	7

4 A cash box contained 89 sovereigns, 35 half-sovereigns, 19 half-crowns, 25 florins, 31 shillings and 15 six penny bits, find the value of the coins in £ s d

5 A tradesman bought goods to the value of £1368 12s 6d he paid for carriage, £25 16s 9d, and other charges, £2 15s 8½d; he gained by the sale of the goods £269 15s 3½d, how much did he sell the goods for?

6 A stationer bought some books for Rs 79 12a 6p, some paper for Rs 161 4a 3p, some pens for Rs 14 10a and some envelopes for Rs 12 8a 6p. How much must he charge for all these articles, so as to gain exactly Rs 100 by his bargain?

7 A collection was once made in a district for a charitable purpose. The following coins were obtained: 99 gold mohurs, 1875 rupees, 990 eight anna pieces, 5891 four anna pieces, 1276 two anna pieces, 90617 half anna pieces and 81516 pice. What did the collection amount to in Rs a pies?

8 Add together 53 guineas, 107 sovereigns, 161 half-guineas, 55 half sovereigns, 223 half crowns, 505 four penny pieces, and 603 farthings.

III COMPOUND SUBTRACTION

137 Compound Subtraction is the method of finding what quantity is left when a smaller quantity is taken from a greater of the same kind. The quantity thus left is called the difference of the given quantities.

RULE Write the less number below the greater, so that units of the same denomination may be under one another, and draw a line below. Begin at the right hand and subtract (if possible) each number in the lower line from the corresponding one in the upper and place the remainder underneath. But if, in any case, the number in the lower line be greater than the one above it, add to the upper one as many units of the same denomination as make one unit of the next higher denomination, and then subtract, taking care to add 1 to the next number in the lower line. Proceed thus through all the columns.

Ex 1 Subtract Rs 47 12a 9p from Rs 72 15a 3p

Rs	a	p	
72	15	3	3p is less than 9p, so add 12p, to 3p, and 1a to 15a, 15p - 9p = 6p
47	12	9	to 12a, 15p - 9p = 6p
<hr/>			15a - 13a = 2a, and Rs 72 - Rs 47 = Rs 25
Rs 25	2	6	

Ex 2 Subtract £207 13s 8½d from £304 2s 10½d

£	s	d	
304	2	10½	2d is less than 8d, so add 4d to 2d and 1d to 8d, 6d - 3d = 3d, or ½d
207	13	8½	10d - 9d = 1d
<hr/>			2s is less than 13s, so add 20s to 2s and £1 to £304, 22s - 13s = 9s, £304 - £208 = £96
£96	9	1½	

Examples XXVIII

1 Perform the operation of subtraction in the following —

(1)	(2)	(3)	(4)	(5)
<i>Rs a p</i> 55 15 10 47 8 11	<i>Rs a p</i> 106 12 9 77 15 10	<i>Rs a p</i> 57 6 3 46 9 10	<i>Rs a ps</i> 75 11 2 49 12 3	<i>Rs a ps</i> 126 3 1 82 8 3
(6)	(7)	(8)	(9)	
<i>Rs a p</i> 150 4 10 24 5 9	<i>Rs a p</i> 1000 8 4 488 15 6	<i>Rs a p</i> 269 5 11 189 13 10	<i>Rs a p.</i> 4172 8 5 2008 14 9	
(10)	(11)	(12)	(13)	
<i>Rs a p</i> 772 13 9 347 15 11	<i>Rs a p</i> 5400 14 7 3216 15 10	<i>Rs a p</i> 3406 4 7 2958 13 9	<i>Rs a p</i> 4658 7 6 4139 9 8	
(14)	(15)	(16)	(17).	
<i>Rs a p</i> 50 48 14 11	<i>Rs a p</i> 575 10 6 89 11 9	<i>Rs a p</i> 1000 101 10 8	<i>Rs a p</i> 7071 15 10 5707 10 11	

2 Perform the operation of subtraction in the following —

(1)	(2)	(3)	(4)	(5)
<i>s d</i> 17 9 11 8½	<i>s d</i> 17 5½ 5 9½	<i>s d</i> 19 0½ 14 11½	<i>s d</i> 18 3½ 11 7½	<i>s d</i> 10 3½ 4 7¾
(6)	(7)	(8)	(9)	
<i>£ s d</i> 58 15 3½ 19 4 7½	<i>£ s d</i> 95 14 2 37 6 3½	<i>£ s d</i> 586 17 1½ 298 13 1¼	<i>£ s d</i> 100 14 7 50 14 7½	
(10)	(11)	(12)	(13)	
<i>£ s d</i> 98 6 2½ 67 11 4½	<i>£ s d</i> 100 3 3 95 15 6½	<i>£ s d</i> 611 17 2½ 492 18 8½	<i>£ s d</i> 743 0 4½ 275 15 5½	
(14)	(15)	(16)	(17)	
<i>£ s d</i> 525 14 7½ 345 17 8½	<i>£ s d</i> 536 8 7½ 89 13 9½	<i>£ s d</i> 837 14 2½ 358 18 6¼	<i>£ s d</i> 86 15 9½ 9 18 11½	

3 Subtract —

- (1) Rs 979 15a 9p from Rs 5707 15a 7p
- (2) Rs 2102 13a 11p from Rs 4365 10a 9p
- (3) Rs 6779 14a 8p from Rs 7865 12a 6p
- (4) £554 12s 7½d from £1739 7s 6½d
- (5) £1975 13s 9½d from £3003 10s 4d
- (6) Rs 55734 12a 4p from Rs 88659 8a 3p
- (7) The sum of Rs 14 3a 5p and Rs 9 8a 7p from Rs 53 11a 6p
- (8) The sum of £5 6s 4½d, £31 15s 10½d, £43 18s 5½d, and £25 16s 4½d from £371 14s 6½d

4 What must be added to £157 16s 9½d to make £355 13s 4d ?

5 After spending Rs 237 14a 3p, how much has a man left out of Rs 532 10s ?

6 A man has 50 guineas in his purse, what would he have left after paying bills amounting to £49 8s 11½d ?

7 A tradesman, in making out a bill, copied 16s 3d for £16 3s and £10 8s for 10s 8d. By what amount was the bill wrong ?

8 By how much is Rs 803 11a 3p greater than Rs 213 8a 4p ?

9 A borrowed from B Rs 387 5a 8p and then Rs 39 9a 1p, repaid him Rs 28 7a and again borrowed Rs 625 13a 11p, find what will be the amount of his debt still due if he makes payment of Rs 967 3a 7p

10 Find the value of Rs 20 15a 11p + Rs 26 11a 3p - Rs 17 12a 5p + Rs 59 13a 6p - Rs 13 10a 4p + Rs 18 3a 7p - Rs 28 12a 9p - Rs 10 14a 3p

11 A man has Rs 5000 in the bank, he draws Rs 2500 on Monday, Rs 1175 4a on Wednesday, and Rs 959 6a on Saturday. What has he left in the bank ?

12 A boy took the sum of 19s 11½d three times out of a bag containing £5. What was left ?

13 A house and furniture are worth Rs 1001 11a 10p. The house costs Rs 750 14a 11p. What is the value of the furniture ?

14 A, B and C together owe £107 11s 8d, the sum of the debts of A and B is £70 5s 5d, and of B and C £80 16s 1d. How much does each owe ?

15 A, who has Rs 5 4a, gives B Rs 3 7a 6p and C Rs 2 9a, but he receives from D Rs 10 10a 8p, and from E Rs 3 11a 6p less than he received from D, how much has he after these payments ?

16 A tradesman's cash in hand on Monday morning was £5 13s 6d. His cash receipts on Monday amounted to £2 15s 6½d.

and on the following days of the week were, respectively, £4 18s 4d, £5 13s 6d, £5 10s 10d, £4 12s 11d, and £16 9s 8d. His cash outlay during the week amounted to £24 17s 5d. What cash had he remaining at the end of the week?

17 COMPOUND MULTIPLICATION

138 Compound Multiplication is the method by which we find the sum of a compound quantity repeated as many times as there are units in a given number. The sum found is called the **product**.

139 When the Multiplier is not greater than 20

RECIPE. Place the Multiplier under the lowest denomination of the multiplicand and draw a line below. Beginning with the lowest denomination multiply by the given multiplier, and find the number of the next higher denomination contained in the product, put down the remainder (if any) and carry the quotient to the next product, and repeat the process till all the denominations are multiplied.

Ex. 1. Multiply Rs 72 11a 9p by 7

Rs	a	p	
72	11	9	
			7
Rs 507	2	3	

$9 \times 7 = 63p = 5a \text{ } 3p$, carry 5
 $11a \times 7 = 77a$ with $5a = 82a =$
 $Rs \text{ } 5 \text{ } 2a$, carry $Rs \text{ } 5$
 $Rs \text{ } 72 \times 7 = Rs \text{ } 504$, with $Rs \text{ } 5 = Rs \text{ } 509$

Ex. 2. Multiply £9 10s 7½d by 17

£	s	d	
9	10	7½	
			17
£169	13	11½	

$3d \times 17 = 51d = 4s \text{ } 3d$, carry 4s
 $7s \times 17 = 119s$, $119s + 4s = 123s = 10s$
 $11d$ carry 10s
 $10s \times 17 = 170s$, $170s + 10s = 180s = £16$
 $9s \times 17 = £153$, $£153 + £16 = £169$

140 When the Multiplier is a number greater than 20, and can be resolved into two or more factors none of which is greater than 20, multiply by each of these factors in succession, and the last result will be the product required. (Art 51)

Ex. Multiply Rs 60 13s 9p by 28 and £9 19s 7½d by 42

$$28 = 4 \times 7$$

Rs	s	p
60	13	9
Rs 397	7	0
Rs 2796	1	0

$$42 = 6 \times 7$$

£	s	d
9	19	7½
£59	17	10½
£419	5	1½

141 When the multiplier exceeds or falls short of a product by a small number, multiply by such product and then by this number and add or subtract for the required product.

Ex. Multiply Rs 240 7a 10p by 29, and £17 8s 5½d by 139

$$29 = 28 + 1 = 4 \times 7 + 1$$

$$139 = 144 - 5 = 12 \times 12 - 5$$

R_s	α	p
240	7	10

£ s d
17 8 54

$$\begin{array}{r} \text{Rs } 961 \quad 15 \quad \frac{4}{4} \\ \hline 7 \end{array}$$

$$\begin{array}{r} 12 \\ \hline £209 \quad 1 \quad 3 \\ 12 \end{array}$$

$$\begin{array}{r} Rs\ 6733\ 11 \\ 240\ 7\ 10 \end{array} \quad \begin{array}{r} 4\ \text{product by } 28 \\ 1 \end{array}$$

$\begin{array}{r} \text{£}2508 \ 15 \ 0 \\ 87 \ 2 \ 2\frac{1}{4} \end{array}$ product by 144
5

Rs 6974 3 2 29

£2421 12 9 $\frac{3}{4}$ 139

142 When the Multiplier is a very large number

RULE Multiply by 10 as many times in succession as there are figures in the multiplier less 1, then multiply the given quantity by the units' figure of the multiplier, the first product by the tens' figure, the second product by the hundreds' figure and so on. The sum of these partial products will give the required product.

Ex Multiply £16 12s 9½d by 7249

\mathcal{L}	s	d
16	12	$9\frac{1}{10} \times 9 =$
		10

£ s d
149 14 11½ product by 9

$$\begin{array}{r} \text{£}166 \quad 7 \quad 8\frac{1}{2} \times 4 = \\ \hline 10 \end{array}$$

665 10 10 40

$$\begin{array}{r} \text{£}1663 \quad 17 \quad 1 \times 2 = \\ 10 \end{array}$$

3327 14 2 200

$$\overline{\text{£16638}} \quad \overline{10 \quad 10} \times 7 =$$

116469	15	10	7000
<u>£120612</u>	15	9½	7249

143 When the multiplier is a large number, as in the above example, and we are told to proceed by Compound Multiplication, the following is the simplest method

£	s	d
16	12	9½
<hr/>		
		7249
£120612	15	9½
<hr/>		

$$\begin{array}{r}
 4) \begin{array}{r} 72499 \\ 1812 \\ \hline 65241 \\ \hline 67053 \\ 5587 \\ \hline 86988 \end{array} \quad \begin{array}{l} 19 \\ \\ 9d \\ 9d \end{array} \\
 12) \begin{array}{r} 67053 \\ 5587 \\ \hline 86988 \end{array} \quad \begin{array}{l} \\ 9d \\ \\ 12s \end{array} \\
 2,0) \begin{array}{r} 9257,5 \\ 4628 \\ \hline 115984 \\ \hline 1120612 \end{array} \quad \begin{array}{l} 15r \\ \\ 15r \\ 16 \end{array}
 \end{array}$$

144 In compound multiplication we may reduce the multiplicand to the lowest denomination contained in it, then multiply this result by the multiplier, and then reduce the product back again. This method is generally tedious.

Ex. Multiply £5045 6s 2½d by 4342

$$\begin{aligned} & \text{£5045 6s } 2\frac{1}{2}\text{d} = 48434979 \\ & \text{and } 48434979 \times 4342 = 210301039149 \\ & \text{and } 210304639749 = \underline{\underline{\text{£21006733 6s } 1\frac{1}{2}\text{d}}}} \text{ Ans} \end{aligned}$$

Examples XXIX

1 Multiply —

- (1) Rs 18 8s 1p by 2, Rs 42 10s 6p by 3, Rs 67 11s 6p by 8
- (2) Rs 51 11s 7p by 4, Rs 67 13s 9p by 7, Rs 58 2s 7p by 6
- (3) Rs 05 12s 8p by 5, Rs 84 11s 5p by 11
- (4) Rs 45 14s 10p by 9, Rs 69 3s 4p by 18
- (5) £10 18s 7½d by 5, £3 0s 7½d by 12, £87 8s 11½d by 10
- (6) £37 19s 9½d by 9, £374 12s 10½d by 7
- (7) £549 13s 7½d by 11, £49 13s 0½d by 19
- (8) £497 10s 7½d separately by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12
- (9) Rs 46 10s 9p 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12
- (10) £6 12s 5p 13, 14, 16, 18 and 20
- (11) Rs 104 12s 5p 13, 14, 15, 17, 19 and 20
- (12) £66 10s 7½d 15, 16, 17, 18 and 19

2 Multiply (by factors) —

- (1) Rs 191 8s 7p by 24, Rs 320 14s 10p by 21
- (2) Rs 586 13s 10p by 64, Rs 142 0s 9p by 132
- (3) Rs 1025 12s 3p by 72, Rs 133 6s 6p by 75
- (4) Rs 205 11s 3p by 108, Rs 140 2s 6p by 144
- (5) Rs 249 15s 5p by 198, Rs 8967 8s 1p by 321
- (6) £96 18s 3d by 96, £68 7s 4½d by 35, £13 7s 4½d by 275
- (7) £67 0s 7½d by 77, £4 8s 9½d by 121, £13 15s 6½d by 132
- (8) Rs 277 5s 2p by 216, Rs 1250 10s 1p by 221

3 Multiply (by factors and parts) —

- (1) Rs 77 2s 4p by 23, Rs 13 15s 4p by 62, £9 19s 7½d by 31
- (2) Rs 3 15s 10p separately by 67, 71 and 79
- (3) Rs 398 15s 2p 67, 59 and 41
- (4) £150 18s 9½d 89, 93 and 113
- (5) £808 12s 7½d 79, 83 and 131
- (6) Rs 19 11s 6p 379 and 845
- (7) Rs 8 14s 2p 1234 and 5678
- (8) Rs 37 15s 6p 9803 and 5840
- (9) £5015 6s 2½d 923, 956 and 2765

4 Multiply —

- (1) £324 12s 6½d by 394, £2 16s 9½d by 702

- (2) Rs 19 4a 6p by 3210, Rs 23 6a 2p by 3684
 (3) £9 15s 10½d by 4508, £3 18s 11½d by 57089
 (4) Rs 42 4a 4p separately by 3005 and 7052
 (5) £567 13s 8½d 8736 and 98736

5 Find the values of —

- | | |
|-----------------------------|--------------------------------|
| (1) 19 things at 3a 2p each | (2) 156 things at 11a 6p each. |
| (3) 96 9a 5p | (4) 315 15a 8p |
| (5) 428 16s 10½d | (6) 728 7s 7½d |
| (7) 943 Rs 4 2a | (8) 625 £1 13s 6d |
| (9) 729 Rs 7 5a 3p | (10) 829 Rs 8 11a 5p |
| (11) 1502 19s 5½d | (12) 2014 17s 6d |

6 Make out the following bills —

- (1) 17 yards of calico at 6a 6p per yard, 143 yards of long cloth at 12a 10p per yard, 14 yards of muslin at Rs 2 3a 6p per yard, 204 yards of flannel at 14a 9p per yard, 456 yards of linen at Re 1 12a per yard, and 755 yards of silk at Rs 3 5a 4p per yard
- (2) 40 seers of Assam Tea at Rs 3 3a 4p per seer, 65 lbs of China Tea at Rs 2 5a 4p per lb, 35 seers of coffee at Re 1 12a 4p per seer, 145 seers of sugar at 7a 4p per seer, and 122 seers of best sugar at 10a 4p per seer
- (3) 23 yards of silk at 5s 4½d per yard, 5 yards of velvet at 13s 6d per yard, 8 yards of velveteen at 3s 11½d per yard, 13 yards of linen at 3s 2d per yard, 19 yards of flannel at 1s 9d per yard, and 26 yards of calico at 11½d per yard

7 A man distributed a certain sum of money to 79 poor persons and gave £17 12s 9½d to each, find the sum of money distributed

8 A bankrupt's estate can pay 14a 10½p in the rupee, what will a creditor receive who has lent 3125 rupees, and how much will he lose?

9 How much money must be added to £1000 that each of 33 people may receive £35 3s 4d?

10 A gowala exchanges 59 calves each worth Rs 15 10a for 37 cows each worth Rs 26 4a, ought he to receive, or to pay any money? How much?

11 If I spend £2 7s 1½d a day, how much is that in a year of 365 days?

12 There are 53 chests of drawers, in each chest there are 4 drawers, in each drawer there are 10 compartments and in each compartment there are deposited £32 5s 6d. How much money is deposited in the chests?

V COMPOUND DIVISION

145 Compound Division is the method by which (1) we break up a compound quantity into as many *equal parts* as there are

units in a given number, and thus find the value of one of these parts, (2) we find how many times one compound quantity is contained in another of the same kind. The first method is called **Partition** and the second **Quotition**.

146 In the first case the divisor is an abstract number, and the quotient telling *the value of each part* is a compound quantity of the same kind as the dividend. In the second case the divisor is a compound quantity of the same kind as the dividend, and the quotient telling *how many times* is an abstract number.

147 *When the divisor is an abstract number*

RULE Place the dividend and divisor as in Simple Division. Find how often the divisor is contained in the highest denomination of the dividend, put down the quotient, and reduce the remainder (if any), to the next inferior denomination. Add to it the number of that denomination in the dividend, and repeat the division. Continue the process step by step through all the denominations.

(1) When the divisor does not exceed 20 the division can be performed *mentally* thus —

Ex Divide Rs 436 5s 4p by 11

$$\begin{array}{r}
 \text{Rs} \quad \text{s} \quad \text{p} \\
 11 \overline{) 436 \quad 5 \quad 4} \\
 \text{Rs} \quad \underline{39 \quad 10 \quad 8}
 \end{array}
 \quad
 \begin{array}{l}
 \text{Rs } 436 - 11 \text{ is Rs } 39 \text{ with Rs } 7 \text{ over} \\
 \text{Rs } 7 = 112\text{s}, \text{ with } 5\text{s} = 117\text{s}, \\
 117\text{s} - 11 \text{ is } 10\text{s} \text{ and } 7\text{s} \text{ over} \\
 7\text{s} = 84\text{p}, \text{ with } 4\text{p} = 88\text{p}, \text{ which} - 11 \text{ is } 8\text{p}
 \end{array}$$

(2) When the divisor is a number larger than 20

Proceed as in the following Examples

Ex Divide £52 10s 7½d by 41, and Rs 3441 5s 9p by 129

$$\begin{array}{r}
 \text{(1)} \quad \begin{array}{r}
 \text{£} \quad \text{s} \quad \text{d} \\
 41 \overline{) 52 \quad 10 \quad 7\frac{1}{2}} \\
 \underline{41} \\
 11 \\
 \underline{20} \\
 230 \left(5\text{s} \right. \\
 \underline{205} \\
 25 \\
 \underline{12} \\
 207 \left(7\text{d} \right. \\
 \underline{287} \\
 20 \\
 \underline{4} \\
 82 \left(2\text{q} \right. \\
 \underline{82}
 \end{array}
 \end{array}
 \quad
 \begin{array}{r}
 \text{(2)} \quad \begin{array}{r}
 \text{Rs} \quad \text{s} \quad \text{p} \\
 129 \overline{) 3441 \quad 5 \quad 9} \\
 \underline{258} \\
 861 \\
 \underline{774} \\
 87 \\
 \underline{16} \\
 1397 \left(10\text{s} \right. \\
 \underline{129} \\
 107 \\
 \underline{12} \\
 1293 \left(10\text{p} \right. \\
 \underline{129} \\
 3\text{p}
 \end{array}
 \end{array}$$

the required quotient is Rs 26 10s 10p and 3p over

∴ the required quotient is £1 5s 7½d

Ex Divide Rs 1179 2a 8p by 100, and £9797 5s 6d by 900

$$\begin{array}{r} \text{(1)} \quad \text{Rs} \quad \text{a} \quad \text{p} \\ 100 \overline{) 11,79 \quad 2 \quad 8} \\ \underline{16} \\ \text{a } 12,66 \\ \underline{12} \\ \text{p } 8,00 \end{array}$$

$$\begin{array}{r} \text{(2)} \quad \text{£} \quad \text{s} \quad \text{d} \\ 9 \overline{) 9797 \quad 5 \quad 6} \\ \underline{9000} \\ 100 \text{ £ } 10,88 \quad 11 \quad 8\frac{1}{2} \quad 6q \\ \underline{20} \\ \text{£ } 17,71 \\ \underline{12} \\ \text{d } 8,60 \\ \underline{4} \\ \text{q } 2,42 \end{array}$$

The final remainder
is $42 \times 9 + 6$ or $384q$
or 8s

∴ Quotient = Rs 11 12a 8p

∴ Quotient = £10 17s 8½d and 8s over

Examples XXX

1 Divide —

- (1) Rs 11 13a 8p by 2, Rs 393 14a 4p by 7, Rs 328 15a 4p by 5
- (2) Rs 5161 9a 4p by 3, Rs 440 5a 6p by 9, Rs 436 5a 4p by 11
- (3) Rs 5392 1a 4p by 8, Rs 576 8a by 12, Rs 1721 7a 10p by 14
- (4) £26 15s 3½d by 2, £87 16s 8½d by 9, £614 2s 6½d by 7
- (5) £79 13s 9d by 12, £147 11s 6½d by 15, £95 2s 3½d by 11
- (6) £241 8s 8½d by 63, £1990 10s 9d by 42, £75 1s 10½d by 45
- (7) Rs 8370 15a separately by 17, 51 and 126
- (8) Rs 12342 12a 2p 19, 59 and 325
- (9) Rs 3253 15a 23, 87 and 712
- (10) £1302 18s by 144, £890 12s 6d by 125
- (11) £75 6s 4½d by 103, £4718 14s 8d by 132
- (12) £7549 17s 6d by 859, £77573 18s 9½d by 4578

2 Divide by the *short* method —

- (1) £239 14s 4½d separately by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12
- (2) Rs 1088 12a separately by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12
- (3) Rs 1877 7a 4p by 14, Rs 2757 9a by 18, Rs 7023 2a by 20
- (4) £623 5s 4½d by 13, £318 10s 7d by 14, £139 13s 8d by 16

3 Divide by using factors —

- (1) Rs 517 11a 4p by 35, Rs 34 11a by 45, Rs 3868 3a 6p by 99
- (2) Rs 3639 1a 6p by 81, Rs 3191 6a by 132, Rs 5761 8a by 144
- (3) £579 18s by 45, £1328 13s 6d by 56, £453 11s 6½d by 77
- (4) £374 10s 3d by 108, £576 3s by 144, £386 16s 5½d by 99

4 Divide .—

- (1) Rs 2625 1a 8p by 10
- (2) £176 16s 8d by 10
- (3) Rs 3395 13a 4p by 100
- (4) £72 12s 11d by 100

- (5) £876 2s 11d by 100 (6) Rs 1151 9a 2p by 1000
 (7) £9658 17s 3½d by 1000 (8) Rs 4579 2a 8p by 400
 (9) Rs 6925 by 800, Rs 3625 by 6000, Rs 11375 by 2400
 (10) £1556 5s by 3600, £513 8s 9d by 3100, £2559 7s 6d by 18900

5 Divide —

- (1) Rs 73298 3a 8p separately by 842, 912 and 8317
 (2) Rs 84566 2a 8p 392, 573 and 7856
 (3) Rs 56789 15a 8p by 9357 (4) Rs 98767 5a 2p by 10048
 (5) £6011656 5s 8½d by 2331 (6) £467325 10s 1½d by 2803
 (7) £530866 17s 6d by 2772 (8) £4420895 or 3½d by 3001

6 Find, to the nearest *pie* or *farthing*, the result of dividing —

- (1) Rs 33 9a 4p by 9 (6) Rs 2684 2a 9p by 241
 (2) Rs 511 8a 5p by 97 (7) Rs 523 6a 8p by 100
 (3) Rs 29 10a 3p by 31 (8) £1867 16s 8½d by 407
 (4) £150 4s 9d by 12 (9) £15104 19s 2d by 100
 (5) £74 6s 10½d by 23 (10) £2160 18s 11d by 1000

7 If Rs 2757 9a be equally divided among 18 people, how much will each receive ?

8 A man spends Rs 5611 14a in a year of 365 days, how much does he spend in a week of 7 days ?

9 After buying 15 books I have £2 15s 7½d left out of £7. What was the price of each book ?

10 The cost of 720 goats is Rs 712 8a, what is the cost of each goat ?

11 205 sovereigns, all equally light, are worth £201 15s 11½d, find the worth of each

12 A cattle-dealer bought 11 cows at Rs 8 4a each, after spending Rs 26 4a in feeding them, he sells 3 of them for Rs 11 4a each, at what price must he sell each of the others to gain Rs 23 by the bargain ?

151 When the divisor is a compound quantity of the same kind as the dividend

RULE Reduce the dividend and the divisor to the same denomination, and then proceed as in Simple Division

Ex 1 Divide, Rs 113 14a 6p by Rs 12 10a 6p

Rs 113 14a 6p = 21870p, Rs 12 10a 6p = 2430p
 the quotient required = $21870 \div 2430 = 9$ Ans

Ex 2 How many cricket balls each worth 5s 7½d can I buy with £134 14s 4½d ?

£134 14s 4½d = 129330q, 5s 7½d = 270q
 the number of balls = $129330 \div 270 = 479$ Ans

Examples XXXI

1 Divide —

- (1) Rs 175 9a 4p by Rs 1 12a 8p, Rs 854 2a 8p by Rs 20 13a 4p
 (2) Rs 438 7a by Rs 6 5a 8p, Rs 4012 2a by Rs 25 11a 6p
 (3) £28 2s 6d by 12s 6d, £150 7s 5d by 6s 3½d
 (4) £286 3s 2d by £1 11s 1¼d, £144 13s 11½d by 9s 11¼d
 (5) Rs 22831 1a 6p by Rs 66 2a 10p, £4808 14s by £7 8s 5d
 (6) Rs 200157 8a 10p by Rs 576 13a 2p, £131 4s 4½d by 10s 7½d

2 How often is

- (1) Rs 760 6a 8p contained in Rs 6843 12a ?
 (2) Rs 3 12a 10p Rs 2771 11a 6p ?
 (3) Rs 2 15a 4p Rs 2366 10a 8p ?
 (4) £35 16s 7¼d £9961 7s 6½d ?
 (5) £2579 0s 0¾d £399745 9s 8¼d ?

3 Find the quotient and the remainder in the division of —

- (1) Rs 9607 15a 10p by Rs 26 5a 2p
 (2) Rs 1225 1a 9p by Rs 55 10a 8p
 (3) £568 13s 8d by £1 8s 6d
 (4) £339 14s 7½d by £4 11s 9¼d

4 How many dollars worth 4s 1½d each must be given in exchange for £235 10s 9d ?

5 To how many persons may Rs 607 12a be distributed giving Rs 46 12a to each ?

6 How many hats each costing £1 2s 3½d can be bought for £134 17s 3½d ?

7 How many cows at Rs 108 12a each can I buy with the proceeds of selling 87 horses at Rs 1151 4a each ?

8 How many days must a labourer work at 2s 1½d a day to earn £51 ?

9 I buy a number of books at 2s 9½d each and sell them at 3s 3d each. If I thereby make a profit of £2 4s, how many books must I buy ?

10 I buy 60 gallons of wine at £1 3s 6d a gallon and £1 10s is gained by selling it at £1 2s 6d a gallon. How much water is added ?

II MEASURES OF WEIGHT

152 Indian Bazar Weight

4 Sikis make 1 Tola
 5 Sikis " 1 Kancha

4 Kanchas or 5 Tolas make	1 Chhatak (<i>ch</i>)
4 Chhataks	" 1 Poa
4 Poas or 16 Chhataks	" 1 Seer (<i>sr</i>)
5 Seers	" 1 Pasari
8 Pasaris or 40 Seers	" 1 Maund (<i>md</i>)

The weight of a rupee is called 1 *tola* A seer = 80 tolas

49 Bazar maunds = 54 Factory maunds

BOMBAY LOCAL WEIGHT	MADRAS LOCAL WEIGHT
4 Dhans make 1 Ratika	180 Grains make 1 Tola
8 Ratikas " 1 Masha	3 Tolas " 1 Palam
4 Mashas " 1 Tank	8 Palams " 1 Seer
72 Tanks " 1 Seer	5 Seers " 1 Vis
40 Seers " 1 Maund	8 Vis " 1 Maund
20 Maunds " 1 Kandi	20 Maunds " 1 Kandi

49 Bazar maunds = 144 Bombay maunds 175 Bazar maunds =

576 Madras maunds 25 Bombay maunds = 28 Madras maunds

153 English Standard Weight (*Avoirdupois*)

16 Diams (dr)	make 1 Ounce (oz)
16 Ounces	" 1 Pound (lb)
28 Pounds	" 1 Quarter (qr)
4 Quarters or 112 lbs	" 1 Hundred weight (cwt)
20 Hundred-weights	" 1 Ton
1 Stone = 14 lbs	1 Cental = 100 lbs

A stone of butcher's meat = 8 lbs	A sack of flour = 280 lbs
A sack of Coal = 2 cwt	A barrel of " = 196 lbs
A barrel of Gunpowder = 100 lbs	A peck of " = 44 lbs
A pack of wool = 240 lbs	A quartern loaf = 4 lbs
A Firkin of Butter = 56 lbs	A pocket of Hops = 168 lbs
A Great Pound of Silk = 24 oz	Two Fodders of Lead = 39 cwt

A pound (*Avoirdupois*) = 7000 grains (*Troy*), 7 Bazar maunds = 576 lbs (*Avoir*), 1 Bombay maund = 28 lbs (*Avoir*), 1 Madras maund = 25 lbs (*Avoir*), 3 Factory maunds = 2 cwt, 35 seers = 72 lbs (*Avoir*)

The Jeweller's Tables

INDIAN JEWELLER'S WEIGHT	ENGLISH TROY WEIGHT
4 Dhans make 1 Rati (<i>ra</i>)	24 Grains (gr) make 1 Penny-weight (dwt.)
6 Ratis " 1 Anna (<i>a</i>)	
8 Ratis " 1 Masha (<i>ma</i>)	20 Penny weights " 1 Ounce (oz Tr)
12 Mashas } " 1 Tola or Bhari	12 Ounces or } " 1 Pound (lb Tr)
or 16 annas }	5760 grains }

1 Tola = 180 grs Troy, 1 Bazar maund = 100 lbs Troy, 1 pound = 32 tolas, a *Carat* = $3\frac{1}{8}$ grs (for weighing diamonds)

[Gold, silver, jewels and precious stones are weighed by the Troy weight]

Measures of Weight for Medicines

BENGAL PHYSICIANS' WEIGHT	ENGLISH APOTHECARIES WEIGHT
4 Dhans make 1 Rati	20 Grains make 1 Scruple (ð)
10 Ratias " 1 Masha	3 Scruples " 1 Dram (ð)
8 Mashas " 1 Tola	8 Drams " 1 Ounce (ð)
	12 Ounces " 1 Pound (lb)

[The Apothecaries' weight is now out of use, except in selling drugs by retail.]

144lbs (Avoii) = 175lbs (Troy or Apoth), 1 lb (Troy or Apoth) = 5760 grains, 1 lb (Avoii) = 7000 grs Troy, 1 lb (Avoii) + the weight of a double pice (200 grs) = half a seer

Note The term 'carat' applied to gold has a relative meaning only any quantity of pure gold, or of gold alloyed with some other metal, being supposed to be divided into 24 equal parts, called *carats*; if the gold be pure, it is said to be 24 carats fine, if 22 parts be pure gold and 2 parts alloy, it is said to be 22 carats fine

Standard gold is 22 carats fine, *Jeweller's* gold is 18 carats fine

Ex 1 Reduce 14cwt 3qrs 24lbs to *ounces*, and 32250 kanchas to *maunds*

$$\begin{array}{r}
 \text{(1) cwt qrs lbs} \\
 14 \quad 3 \quad 24 \\
 \underline{4} \\
 50 \text{ qrs} \\
 28 \\
 \hline
 1676 \text{ lbs} \\
 16 \\
 \hline
 26816 \text{ or Ans}
 \end{array}$$

$$\begin{array}{r}
 \text{(2)} \\
 4) 32250 \text{ kan} \\
 16) 8062 \text{ ch} \quad 2 \text{ kan} \\
 40) 503 \text{ sr} \quad 14 \text{ ch} \\
 \quad 12 \text{ md} \quad 23 \text{ sr}
 \end{array}$$

the result = 12 mds 23 sr 14 ch 2 kan

Ex 2 Reduce 425095 grs of gold to lbs &c, and 11ka 13mds 3 vis 5 palams 2 tolas to *tolas* (Mad)

$$\begin{array}{r}
 \text{(1)} \\
 24 \left\{ \begin{array}{l} 4) 425095 \text{ grs} \\ 6) 106273 \quad 3 \\ 2,0) 17712 \text{ dwts} \quad 1 \end{array} \right\} 7 \text{ grs} \\
 12) 885 \text{ or} \quad 12 \text{ dwts} \\
 \quad 73 \text{ lbs} \quad 9 \text{ oz}
 \end{array}$$

the result = 73 lbs 9 oz 12 dwts 7 grs

$$\begin{array}{r}
 \text{(2)} \quad \text{ka md vis pa tolas} \\
 11 \quad 13 \quad 3 \quad 5 \quad 2 \\
 20 \\
 \hline
 233 \text{ mds} \\
 8 \\
 \hline
 1867 \text{ vis} \\
 40 \\
 \hline
 74685 \text{ palams} \\
 3 \\
 \hline
 224057 \text{ tolas Ans}
 \end{array}$$

Examples XXXII ✓

(Indian Bazar and Avondupois Weights)

1 Reduce (i) to *lanchas* and (ii) to *tolas* —

(1) 20 mds 13 sr 7 ch, 12 mds 15 sr 10 ch, 75 mds 32sr 15 ch

(2) 46 mds 25 sr 12 ch, 25 mds, 45 mds 12 sr 8 ch

2 Reduce to *lanchas* —

(1) 30 mds 27 sr 12 ch 2 kan, 45 mds 30 sr 8 ch 1 kan

(2) 210 mds 15 sr 2 ch 3 kan, 220 mds 17 sr 3 kan

3 Reduce to *maunds* &c —4123000 kan, 30205676 kan, 15025276 tolas, 4876235 poas,
4320578 ch, 4362508 tolas, 782504 poas4 Reduce to *tolas* —

2mds 5vis 4sr, 5kan 15mds 4vis, 8kan 14mds 7vis 5palams

5 Reduce to *dhans* —2 mds 14 sr 57 ta, 8 kan 16 mds 2, sr 55 ta 3 m, 10 kan
10 mds, 39 sr 16 ta 3 m 21at6 Reduce 156728306 tolas to *kandis*, 460879025 dhans to
kandis, 786250 tanks to *landis*, 4586 seers to *landis*

7 Reduce —

(1) 11 cwt 2 qrs 17 lbs 15 oz 10 ounces, 3 cwt 13 lbs to ounces

(2) 6 tons 5 st to ounces, 4 tons 15 cwt 2 qrs 12lbs to pounds

8 Reduce to *drams* —(1) 2 tons 10 cwt 1 qr 2 lbs 3 oz 3 drs, 3 tons 14 cwt 3 qrs
25 lbs 11 oz 9 dis, 3 tons 3 qrs 3 oz 27 lbs

(2) 8 st 11lbs 9 drs, 16lbs 12 oz 13 dis, 18 cwt 73 lbs 9 drs

9 Reduce to *tons, cwt, &c*

(1) 87654 lbs, 378539 oz, 1693539 drs 65437 drs

(2) 2345820 drs 1008001 oz, 237023 oz, 59653007 sr

(3) 21633lbs, 17739853 oz, 5390054 drs, 713969416 drs

10 Add together —

(1) mds	sr	ch	(2) mds	sr	ch	lan	(3) mds	sr	ch	lan
25	10	5	65	10	10	2	115	30	7	1
110	2	3	72	15	8	2	202	27	10	2
115	20	12	102	14	10	3	323	15	12	3
97	27	15	125	30	13	2	222	9	5	2
102	15	7	207	32	15	1	313	32	14	3

(4) tons	cwt	qrs	lbs	oz	(5) cwt	qrs	lbs	oz	(6) tons	cwt	qrs	lbs
16	0	3	5	15	32	2	15	12	32	12	2	25
8	16	0	0	14	47	0	25	7	18	15	0	20
28	8	1	27	6	5	3	17	10	23	10	1	16
210	6	3	14	11	23	1	19	15	14	18	1	27
17	17	0	15	12	1	2	10	8	25	4	0	3
412	15	3	18	13	9	3	0	14	35	12	2	19

11 Perform the operation of subtraction in the following —

- | | | | | | | | | | | | | | |
|-----|-----------|-----------|-----------|-----|------------|-----------|-----------|-----------|-----|------------|-----------|-----------|-----------|
| (1) | mds | sr | ch | (2) | mds | sr | ch | kan | (3) | mds | sr | ch | kan |
| | 530 | 10 | 12 | | 672 | 12 | 10 | 0 | | 427 | 10 | 10 | 1 |
| | <u>93</u> | <u>27</u> | <u>15</u> | | <u>127</u> | <u>24</u> | <u>14</u> | <u>3</u> | | <u>212</u> | <u>25</u> | <u>14</u> | <u>3</u> |
| (4) | cwt | qrs | lbs | (5) | tons | cwt | qrs | lbs | (6) | cwt | qrs | lbs | oz |
| | 47 | 0 | 12 | | 75 | 7 | 1 | 16 | | 112 | 2 | 23 | 8 |
| | <u>32</u> | <u>3</u> | <u>22</u> | | <u>41</u> | <u>14</u> | <u>2</u> | <u>10</u> | | <u>59</u> | <u>0</u> | <u>27</u> | <u>10</u> |

12 Multiply —

- | | | | | | |
|-----|---------|--------|--------|---------------|-------------------|
| (1) | 110 mds | 20 sr | 12 ch | separately by | 24, 36 and 72 |
| (2) | 225 mds | 22 sr | 13 ch | 2 kan | 144, 126 and 360 |
| (3) | 20 tons | 3 cwt | 12 lbs | | 132 and 143 |
| (4) | 25 tons | 18 cwt | 2 qrs | 15 lbs | 144 and 1728 |
| (5) | 8 tons | 87 lbs | 13 drs | | 18, 29 47 and 133 |

13 Divide —

- | | | | | | | | | | | | | | | | | | | |
|-----|----------|--------|-----------|---------------|--------------|---------------|--------|----------|------|-------|-----------|-------|------|-------|-----------|-------|-------|-------|
| (1) | 252 mds | 10 sr | 12 ch | separately by | 63 and 84 | | | | | | | | | | | | | |
| (2) | 1230 mds | 22 sr | 15 ch | | 112 and 336 | | | | | | | | | | | | | |
| (3) | 3125 mds | 10 sr | 10 ch | 2 kan | 167 and 4008 | | | | | | | | | | | | | |
| (4) | 48 tons | 17 cwt | 3 qrs | 27 lbs | 1 oz | 9, 17 and 500 | | | | | | | | | | | | |
| (5) | 30 tons | 15 cwt | 2 qrs | 15 lbs | | 144 and 864 | | | | | | | | | | | | |
| (6) | 1061 cwt | 2 qrs | by 37 cwt | 3 qrs | 18 lbs | , 89 cwt | 22 lbs | by 3 cwt | 1 qr | 6 lbs | , 404 mds | 35 sr | 2 ch | 2 kan | by 23 mds | 32 sr | 10 ch | 2 kan |

14 If 41 cwt cost £52 10s 7½d, what is the price of a cwt ?

15 A chest of tea weighing 1 cwt 1 qr 15 lbs cost £22 8s 10½d, what is the cost of 1 lb ?

16 At a school feast the children on the average ate 9 oz of cake a piece, and 84 lbs 6 oz of cake were eaten, how many children were there in the school ?

(Indian Jewellers and Troy Weights)

1 Reduce to *dhans* —

25 tolas 10 m 41 3 dh, 150 tolas 14 a 5 r, 162 tolas 13a 31 2 dh

2 Reduce to *tolas* —

56430 dhans, 53426 ratas, 37484 dhans, 32458 ratas

3 Reduce to *grains* —

- | | | | | | | | | | |
|-----|--------|-------|---------|---------|----------|----------|---------|---------|---------|
| (1) | 12 lbs | 10 oz | 15 dwts | 14 grs | , 15 lbs | 11 oz | 17 grs | , 9 lbs | 18 dwts |
| (2) | 16 lbs | 1 roy | , 9 oz | 17 dwts | 22 grs | , 165 oz | 280 grs | | |

4 Reduce to *lbs, etc* (1 roy) —

13600 grs, 146320 grs, 400903 dwts, 6739 oz, 873521 grs

5 Add together —

(1) tolas m r dh	(2) tolas a r dh	(3) lbs oz dwts grs
45 10 7 3	47 10 3 2	64 11 16 14
63 8 6 2	52 9 2 1	21 10 12 13
58 9 5 1	65 8 5 3	2 0 1 16
62 11 7 3	77 13 4 0	12 10 0 18
39 8 4 2	82 14 5 3	24 11 12 0
<u>112 6 3 1</u>	<u>75 12 4 2</u>	<u>14 1 0 1</u>

6 Perform the operation of subtraction in the following —

(1) tolas m r dh	(2) tolas a r dh	(3) lbs oz dwts grs
530 8 4 2	579 17 3 2	81 10 9 18
<u>327 12 6 3</u>	<u>380 14 5 3</u>	<u>14 11 12 19</u>

(4) lbs oz dwts grs	(5) lbs oz dwts grs	(6) tolas a r dh
225 8 14 15	13 0 7 18	467 10 3 0
<u>167 11 18 19</u>	<u>11 11 15 23</u>	<u>279 13 5 2</u>

7 Multiply —

- (1) 115 tolas 7 m 5 r 1 dh separately by 72 and 80
 (2) 210 tolas 10 a 2 r 2 dh 132 and 143
 (3) 22 lbs 7 oz 12 dwts 20 grs 64 and 96
 (4) 83 lbs 17 dwts 5 grs 26, 131 and 257

8 Divide —

- (1) 1125 tolas 8 m 6 r 3 dh separately by 132 and 144
 (2) 1020 tolas 12 a 4 r 2 dh 172 and 516
 (3) 606 lbs 4 oz 15 dwts 20 grs 131 and 500
 (4) 110 lbs 10 oz 14 dwts 16 grs 136 and 272
 (5) 2025 lbs 2 oz 18 dwts 8 grs by 5 lbs 6 oz 280 grs

9 If 28 lbs 9 oz of gold be worth £1343 6s 10½d, what is the worth of 1 ounce?

10 A certain number of forks, each weighing 3 oz 5 dwts and double that number of spoons, each weighing 3 oz 10 dwts are made out of 10 bars of silver, each weighing 3 lbs 5 oz, find the number of spoons

*(Native Physicians' and Apothecaries' Weights)*1 Reduce to *dhans* —

25 tolas 6 m 8 r 3 dh, 32 tolas 5 m 9 r 2 dh, 8 tolas 7 m 5 r 2 dh

2 Reduce to *tolas* —

1224 ratis, 13200 dhans, 426507 dhans, 2406 ratis

3 Reduce to *grains* —

3 lbs 5 3 19 grs, 2 lbs 4 drs 2 scr, 18 lbs 2 oz 4 drs 2 scr 12 grs

4 Reduce to *pounds &c* —

270083 grs, 26984 D, 92200 grs, 51960 grs, 17599 grs

5 Add together —

(1) tolas	m	r	dh	(2) oz	drs	scrs	grs	(3) lbs	3	3	3	grs
25	7	8	3	11	4	2	11	15	3	5	1	17
32	5	7	2	10	3	0	4	18	10	6	2	5
49	7	5	2	16	0	1	14	20	9	1	2	12
55	3	6	1	10	0	1	16	25	7	3	0	18
60	6	5	3	6	2	2	18	36	5	4	1	14
79	4	6	2	14	5	1	0	26	8	6	2	15

6 Perform the operation of subtraction in the following —

(1) tolas	m	r	dh	(2) lbs	oz	drs	scrs	grs	(3) lbs	3	3	3	grs
125	6	3	0	28	7	1	2	4	75	7	3	0	
58	7	6	2	12	8	2	1	12	49	10	6	2	

7 Multiply —

(1) 32 tolas 6 m 8 r 3 dh	separately	by	132 and 143
(2) 45 lbs 7 oz 3 drs 2 scrs 8 grs			16 and 64
(3) 7 lbs 6 3 14 grs			53, 71 and 2500

8 Divide —

(1) 65 tolas 7 m 6 r 2 dh	separately	by	72 and 81
(2) 120 lbs 9 oz 5 drs 2 scrs 12 grs			120 and 132
(3) 270 lbs 5 3 6 3 2 scrs			46, 53 and 1000
(4) 7 lbs by 2 3 2 3 and 1234 lbs 68 3 by 44 lbs 23 3			

9 How many pills, each containing 6 3 2 grs can be made out of 2 lbs 11 3 6 3 of rhubarb

154 To convert from one system of weights into another

(1) To convert Indian weight into Troy, multiply the weight in tolas by 3 and divide by 8, the result will be the weight in oz Troy. Or multiply the weight in tolas by 180, the result will be the weight in grains Troy.

Conversely, to convert Troy weight into Indian weight, multiply the weight in oz Troy by 8 and divide by 3, the result will be the weight in tolas. Or divide the weight in grs Troy by 180, the result will be the weight in tolas.

(2) To convert Indian weight into Avoirdupois, multiply the weight in chhataks by 9 and divide by 70, the result will be the weight in lbs Avoirdupois. Or multiply the weight in seers by 72 and divide by 35, the result will be the weight in lbs Avoirdupois. Or multiply the weight in maunds by 36 and divide by 49, the result will be the weight in cwt Avoirdupois.

Conversely, to convert Avoirdupois weight into Indian weight, multiply the weight in lbs Avoirdupois by 70 and divide by 9, the result will be the weight in chhataks. Or multiply the weight in lbs Avoirdupois by 35 and divide by 72, the result will be the weight in seers. Or multiply the weight in cwt by 49 and divide by 36, the result will be the weight in maunds.

(3) To convert lbs Avoir into Troy, multiply the weight in lbs Avoir by 7000, the result will be the weight in *grains* Troy. Conversely, to convert Troy weight into Avoir, multiply the weight in lbs Troy by 144 and divide by 175, the result will be the weight in lbs Avoir.

(4) As the weight in grains of both Apoth and Troy weights is the same, therefore the one may be taken for the other.

Ex 1 Convert 9 cwt 3 qrs 6 lbs into *Indian weight*

$$9 \text{ cwt } 3 \text{ qrs } 6 \text{ lbs} = 1098 \text{ lbs} = 1098 \times 70 = 76860 \text{ ch} \\ = 13 \text{ mds } 13 \text{ sr } 12 \text{ ch} \quad \text{Ans}$$

Ex 2 Convert 6 mds 26 sr 14 ch into *cwt, ch, (Avoir)*

$$6 \text{ mds } 26 \text{ sr } 14 \text{ ch} = 4270 \text{ ch} = 4270 \times 9 = 38430 \text{ lbs (Avoir)} \\ = 549 \text{ lbs} = 4 \text{ cwt } 3 \text{ qrs } 17 \text{ lbs} \quad \text{Ans}$$

Ex 3 Reduce 1 cwt 2 lbs (Avoir) to *Troy weight*

$$1 \text{ cwt } 2 \text{ lbs} = 114 \text{ lbs} = 114 \times 7000 \text{ grs} = 798000 \text{ grs} \\ = 138 \text{ lbs } 6 \text{ oz } 10 \text{ dwts} \quad \text{Ans}$$

Examples XXXIII

1 Reduce to *tolas* —

$$1440 \text{ grs}, 7 \text{ lbs } 7 \text{ oz } 17 \text{ dwts } 12 \text{ grs} = 16 \text{ lbs } 6 \text{ oz}, 2 \text{ oz } 5 \text{ dwts}$$

2 Reduce to *grains* (Troy) —

$$(1) 16 \text{ sr } 8 \text{ ch}, 25 \text{ sr } 14 \text{ ch } 3 \text{ tolas}, 1 \text{ md } 5 \text{ sr } 14 \text{ ch} = 4 \text{ vis } 15 \text{ palams } 2 \text{ tolas}$$

$$(2) 8 \text{ tons } 8 \text{ cwt } 98 \text{ lbs } 3045 \text{ grs}, 425 \text{ tons } 19 \text{ cwt } 100 \text{ lbs } 15 \text{ oz } 200 \text{ grs}, 1 \text{ cwt } 1 \text{ qr } 25 \text{ lbs}$$

3 Reduce 20 lbs Avoir to *Troy weight*, 16 dwts to *Apoth weight*, 5 drs Apoth to *Troy weight*, 525 lbs Troy to *mds, sr, &c*

4 Reduce 96 tolas to *oz Troy*, 37400157 grains Troy to (Madras) *mds, vis, &c*, 309432159 lbs Avoir to *mds, sr, &c*

5 Reduce to *maunds, sr, &c* —

$$14 \text{ cwt } 1 \text{ qr } 24 \text{ lbs}, 10 \text{ cwt } 3 \text{ qrs } 20 \text{ lbs}, 3 \text{ tons } 12 \text{ cwt } 1 \text{ qr } 8 \text{ lbs}, 3 \text{ tons } 19 \text{ cwt } 8 \text{ lbs}, 4 \text{ cwt } 3 \text{ qrs } 8 \text{ lb}$$

6 Reduce to *tons, cwt, &c* —

$$7 \text{ mds}, 15 \text{ mds } 38 \text{ sr } 12 \text{ ch}, 9 \text{ mds } 7 \text{ sr } 8 \text{ ch}, 10 \text{ mds } 20 \text{ sr}, 53 \text{ mds } 15 \text{ sr } 21 \text{ mds } 35 \text{ sr}$$

7 Reduce 1137 lbs 6 oz Troy to *lbs Avoir*, 2 cwt 3 qrs 17 lbs and 5 cwt 18 lbs 14 oz to *Troy weight*

8 Convert 6 tons 10 cwt 65 lbs into *Madras maunds*, 8 tons 2 cwt 9 lbs into *Bombay maunds*, 368 Bombay maunds and 140 Madras maunds into *tons, cwt, &c*

9 How many 2 lbs packets of tea can be made from a chest weighing 7 cwt 3 qrs 16 lbs?

10 Each ton of ore obtained from a gold mine yields on an average 2 or 1 dwt 15 grs of fine gold. How much fine gold will be obtained from 293 tons?

11 How many coins each weighing 1 or 8 dwt can be made of 770 lbs of metal?

12 A truck is loaded with 120 sacks, each sack weighs 7 sr 10 ch, and contains 84 seers of grain. What is the weight of the whole in maunds and seers?

13 How many pounds Avon are equal to 175 lbs Troy?

14 Multiply 88 ka 12 mds 16 sr (Bombay) separately by 99, 66 and 144, and 4 ka 5 mds 15 sr by 3268.

15 A train consists of 29 trucks of equal weight, 9 of them weigh 53 tons 1 cwt 1 qr 3 lbs. What do the rest of them weigh?

16 Convert 2 qrs 16 lbs into *seers*, 10 cwt 1 qr 13 lbs into *maunds*, and 15 lbs 2 oz 5 dwts 20 grs into lbs *Avon*.

17 Express 576 lbs *Avon* as lbs *Troy*, 58 lbs 4 oz *Troy* as lbs *Avon*, and 16 dwts 16 grs in *Apoth weight*.

18 Reduce 9720 grs *Troy* to *tolas* and find how many lbs are there in 12288 *tolas*?

19 How many times is a weight of 6 tons 7 cwt 27 lbs 5 oz contained in 159 tons 1 cwt 10 lbs 13 oz?

20 What is the whole weight of 217 waggon loads each containing 2 tons 13 cwt 3 qrs 13 lbs?

21 797 tons 19 cwt 2 qrs 14 lbs is divided among a certain number of people so that each receives 5 tons 3 cwt 2 qrs 15 lbs. How many of them were there?

22 84 poor men have distributed equally among them 252 mds 10 sr 12 ch of rice, what share will each receive?

23 If 5 ka 15 mds 30 sr of a certain article can be bought for a rupee, what quantity can be bought for 2384 rupees?

24 21 tons 3 cwt 1 qr 17 lbs 5 or 8 drs of rice are to be packed in bags of equal size. How many bags will be required if each hold 24 lbs 6 or 8 drs?

25 Reduce 2457600 dhans to *maunds*.

26 Multiply 109 ka 13 mds 6 sr (Madras) separately by 72, 35 and 750, and 5 ka 15 mds 30 sr by 4503.

27 Divide —

(1) 6 mds 6 sr 27 ta (Bombay) by 73

(2) 311 ka 10 mds 36 sr 4 palams (Madras) by 503

28 Divide 64 ka 7 mds 12 sr by 15 mds 13 sr (Madras)

29 Divide 160 ka 10 mds 39 sr by 15 mds 3 sr (Bombay)

30 If standard gold contained 12 parts of pure gold to 1 part of copper, and 247 oz Troy were coined into 960 sovereigns, what would be the weight of pure gold in a sovereign?

31 How many bars of gold each weighing 5 oz 13 dwts 21 grs can be made out of a bar weighing 88 lbs 8 oz 14 dwts 15 grs?

32 Find the weight of 73 iron bars, each weighing 17 cwt 2 qrs 19 lbs 5 oz

33 How many bars of iron each weighing 11 lbs 10 oz 11 drs must be taken to make up a weight of 4 tons 8 cwt 3 lbs 6 oz 15 drs?

34 Express in *Troy weight* the weight of a silver dish weighing 3 sr 2 poas, and of 6 scruples of soda

35 Which is the heavier, a pound of gold or a pound of feathers? and by how much?

III MEASURES OF LENGTH

155 Indian Lineal Measure

3 Yabs	make	1 Anguli
4 Angulis	,	1 Mushti
3 Mushtis	"	1 Bighat (<i>span</i>)
2 Bighats or 24 angulis	,	1 Hath or Cubit
4 Haths	,	1 Danda or Dhanu
2000 Dandas or 8000 haths	"	1 Kros or Kos
4 Kros	"	1 Yo jan

156 English Lineal Measure

3 Bailey corns (in length)	make	1 Inch (<i>in</i> , or <i>i'</i>)
12 Inches	"	1 Foot (<i>ft</i>)
3 Feet	"	1 Yard (<i>yd</i>)
5½ Yards	"	1 Rod, Pole (<i>po</i>) or Perch
40 Poles, or 220 yds	"	1 Furlong (<i>fur</i>)
8 Furlongs, or 1760 yds	"	1 Mile (<i>m</i>)
3 Miles	"	1 League (<i>lea</i>)

1 Gaid = 2 cubits, 1 Ilahi Gaj (N-W P) = 33 in, 1 Kros = 4000 yds,
1 Karam (Madras) = 3 cubits, 1 Kathi (Bombay) = 9 ft 1 half yard
= 1 ft 6 in

Cloth Measure

IN BENGAL		ENGLISH	
3 Angulis	make 1 Gnah	2½ Inches	make 1 Nail (<i>nl</i>)
8 Girahs	" 1 Hath	4 Nails	" 1 Quarter (<i>qr</i>)
2 Haths or 16 girahs	" 1 Gaj	4 Quarters	" 1 Yard
IN BOMBAY		3 Quarters	" 1 Flemish ell
2 Angulis	make 1 Tasu	5 Quarters	" 1 English ell
24 Tasus	" 1 Gaj	6 Quarters	" 1 French ell

Examples XXXIV

- 1 Reduce (i) to *haths* or *cubits* and (ii) to *angulis* —
15 kros 1008 dandas, 6 yojan 2 kros 1780 dandas, 20 bi
4 kat, 25 bi 15 kat 3 cubits, 10 kros 875 dandas 3 haths
- 2 Reduce to *gay*, &c —
34256 angulis, 94605 gnahs, 420367 angulis, 7035 girahs
- 3 Reduce to *inches* —

- (1) 3 fui 135 yds 4 in, 5 mi 200 yds 3 in, 512 yds 2 ft 9 in, 4 lea
- (2) 2 mi 7 fui 15 po 1 yd 1 ft 6 in, 13 lea 1 mi 4 fur 37 po 1 ft 8 in
- (3) 31 mi 4 fur 115 yds 1 ft 8 in, 25 mi 6 fui 17 po 4 yds 3 in
- (4) 25 mi 459 yds 31 in, 25 fur 39 po 3 yds 2 ft 8 in

4 Reduce 7 mi 5 fur 32 po 4 yds to *yards*, 2 lea 2 mi 7 fui to *yards*, 5 mi 3 fur 208 yds 1 ft to *feet*, 15 mi 5 fui 31 po to *poles*

- 5 Reduce to *miles*, &c —

- (1) 57383 yds, 1847638 ft, 268543 in, 304935 ft, 53628 ft
- (2) 1081080 in, 231031 vds, 517900 in, 36090 ft, 2000000 in

- 6 Reduce 163810 ft to *liagns*, 152017634 in to *mils*

- 7 Reduce —

- (1) 20 yds 3 qrs 1 nl to *nails*
- (2) 5 miles to *fathoms*
- (3) 35 ells 4 qrs to *nails*
- (4) 16 ells 1 qr 3 nls 1 in to *in*
- (5) 500 fathoms to *yards*
- (6) 5 furlongs to *fathoms*
- (7) 35 kros to *cubits*
- (8) 5 miles to *links*
- (9) 1 gay 1 hath 1 girah to *angulis*
- (10) 16 haths 9 in to *feet*

- 8 Reduce —

- (1) 2897 in of cloth to *yards*
- (2) 567912 cubits to *bigahs*, &c
- (3) 201494 jabs to *dandas*
- (4) 74310 tasu to *gay*, &c
- (5) 25 kros to *miles*, and *yards*
- (6) 75 miles to *lios* and *haths*
- (7) 1 kros 1999 dandas 1 gay 1 hath 7 girahs 2 angulis to *angulis*

- 9 Add together —

- | | | |
|------------------------|------------------------|-------------------------|
| (1) yds ft in | (2) po yds ft in | (3) mi fur po yds |
| 22 2 7 | 7 3 1 11 | 14 3 17 2 $\frac{1}{2}$ |
| 54 1 9 | 12 2 $\frac{1}{2}$ 2 4 | 23 5 33 4 |
| 67 2 10 | 9 4 0 7 | 37 1 24 5 |
| 85 0 11 | 2 3 $\frac{1}{2}$ 1 9 | 43 7 3 1 $\frac{1}{2}$ |
| 92 1 3 | 10 1 2 8 | 75 6 36 2 $\frac{1}{2}$ |
| (4) mi po vds in | (5) vds qrs nls | (6) ells qrs nls |
| 3 54 2 $\frac{1}{2}$ 7 | 25 3 2 | 35 2 3 |
| 12 113 0 9 | 37 0 3 | 42 4 5 |
| 6 0 4 $\frac{1}{2}$ 11 | 54 1 1 | 37 2 2 |
| 25 44 3 8 | 49 2 3 | 25 4 3 |

10 Perform the operation of subtraction in the following —

(1) m ₁ fur po	(2) fur po yds	(3) m ₁ po yds	(4) yds qrs nls
21 0 7	6 37 4	6 0 0	18 2 0
11 5 18	5 18 4 ¹	3 37 2 ¹	8 3 3

11 Multiply —

- (1) 5 fur 78 yds 2 ft 7 in by 56, 42 yds 2 qrs 2 nls by 40
 (2) 13 lea 2 m₁ 6 fur 25 po separately by 42 and 97
 (3) 25 m₁ 6 fur 23 po 3 yds 2 ft 8 in 56 and 83
 (4) 20 dandas 1 hath 7 girahs 4, 5 and 12

12 Divide —

- (1) 3179 lea 1 m₁ 5 fur 16 po by 46, 485 yds 3 qrs 3 nls 2 in by 11
 (2) 478 m₁ 6 fur 19 po 2 yds 1 ft 10 in separately by 96 and 4397
 (3) 679 m₁ 7 fur 125 yds 2 ft 6 in 11 and 120
 (4) 275 dandas 1 gay 4 girahs 3, 5, 10 & 12
 (5) 1027 m₁ 1 fur 6 po by 17 m₁ 5 fur 27 po

13 If 67 pieces of cloth measure 2335 yds 2 qrs 7 in, what is the length of 1 piece?

14 If a person complete a journey of 422 m₁ 3 fur 38 po in 37 days, what distance does he travel per day?

15 Find the aggregate of 4 kros 1 gay 1 ha 7 gir, 5 kros 1978 dan 4 gir, 2 kros 150 dan 1 gay 1 ha 2 gir, 11 kros 25 dan 1 ha 5 gir and 6 kros 1 gay 3 gir

16 How many lengths each equal to 9 po 3 yds 1 ft 3 in will make up 1 mile 6 fur 26 po 4 yds 2 ft 9 in?

IV MEASURES OF SURFACE

158

Land Measure in Bengal

20 Square cubits or Gandas make 1 Chhatak

16 Chhataks " 1 Katha

20 Kathas " 1 Bigha

IN N.W.P.

20 Kachvansi make 1 Bisvansi

20 Bisvansi " 1 Bisva

20 Bisvas " 1 Bigha

IN THE PUNJAB

9 Sarsi make 1 Marla

20 Marlas " 1 Kanai

4 Kanals " 1 Bigha

2 Bighas " 1 Ghuma

IN BOMBIAY

39¹/₄ Square cubits make 1 Kathi

20 Kathis " 1 Pand

20 Pands " 1 Bigha

6 Bighas " 1 Rukeh

20 Rukehs " 1 Chahur

IN MADRAS

144 Sq Inches make 1 Sq ft

2400 Sq feet " 1 Ground or

Manai

24 Grounds " 1 Cawn

484 Cawnies " 1 Sq mile

1 Bengal Bigha = 1600 sq yds , 1 N-W P Bigha = 3025 sq yds ,
 1 Punjab Bigha = 1620 sq yds , 1 Bombay Bigha = 3927 sq yds
 Also 1 Madras Cawn = 6400 sq yds = 4 Bengal Bighas

159 English Square Measure

144 Square Inches (<i>sq in</i>)	make	1 Square Foot (<i>sq ft</i>)
9 Square Feet	,	1 Square Yard (<i>sq yd</i>)
30½ Square Yards	"	1 Square Pole (<i>sq po</i>)
40 Square Poles	"	1 Rood (<i>ro</i>)
4 Roods or 4840 sq yds	"	1 Acre (<i>ac</i>)
640 Acres	"	1 Square Mile (<i>sq mi</i>)

10,000 Sq links make 1 Sq chain , 484 Sq yds make 1 Sq chain
 10 Sq chains , 1 Acre , 100,000 Sq links 1 Acre

A Rod of Brickwork = 272½ sq ft A Rod of Building = 36 sq yds
 A Square of Flooring, Roofing, &c = 100 sq ft A Yard of Land = 30 ac
 A Hide of Land = 100 ac One sq chain = 10,000 sq links

40 ac = 121 Bengal Bighas , 5 ac = 8 N W P Bighas , 81 ac = 242
 Punjab Bighas , 160 ac = 121 Madras Cawnies Also 1 sq mi = 1936
 Bengal Bighas = 1024 N-W P Bighas = 484 Madras Cawnies

160 To reduce square poles to square yards, we have to multiply by 30½, but since 30½ sq yds is 121 qr-sq yds, we multiply the sq poles by 121 and divide by 4. In the converse operation, to divide by 30½, we multiply the sq yds by 4 and divide by 121. The remainder in each case is qr-sq yds and note that 1 qr-sq yd is 2½ sq ft = 2 sq ft 36 sq in.

Also in reducing acres and roods to sq yards, multiply by 4840, and 1210 respectively, unless prevented by the form of the question. To reduce square yards to acres, divide by 4840.

Note 1 qr sq yd = 2 sq ft 36 sq in , 2 qr sq yds = 4 sq ft 72 sq in 3 qr sq yds = 6 sq ft 108 sq in Also 1 sq po = 30 sq yds 2 sq ft 36 sq in

Ex 1 Reduce 3 ac 2 ro 23 sq po 10 sq yds 8 sq ft 18 sq in to sq inches

$$\begin{array}{r}
 3 \text{ ac } 2 \text{ ro } 23 \text{ sq po } 10 \text{ sq yds } 8 \text{ sq ft } 18 \text{ sq in} \\
 \underline{4} \\
 14 \text{ ro} \\
 \underline{40} \\
 563 \text{ sq po} \\
 \underline{121} \\
 4)70543 \\
 \underline{17635} \text{ sq yds } + 3 \text{ qr sq yds} \\
 = 17635 \text{ sq yds } 6 \text{ sq ft } 108 \text{ sq in} \\
 \underline{10} \quad \underline{8} \quad \underline{18} \\
 17646 \text{ sq yds } 5 \text{ sq ft } 126 \text{ sq in}
 \end{array}$$

$$\begin{array}{r}
 17646 \text{ sq yds } 5 \text{ sq ft } 126 \text{ sq in} \\
 \underline{9} \\
 156819 \text{ sq ft} \\
 \underline{144 = 12 \times 12} \\
 22870062 \text{ sq in } \text{Ans}
 \end{array}$$

E1 2 Reduce 9532482 sq inches to *acres*

$$\begin{array}{r}
 144 \left\{ \begin{array}{l} 12) 9532482 \text{ sq in} \\ 12) \underline{794373} \quad 6 \\ 9) \underline{66197} \text{ sq ft } 9 \end{array} \right\} 114 \text{ sq in} \quad \therefore \text{the result} \\
 \begin{array}{r} 7355 \text{ sq yds } 2 \text{ sq ft} \\ \quad \quad \quad 4 \end{array} = 1 \text{ ac } 210 \text{ } 3 \text{ sq po } 17 \text{ qr-sq yds } \\
 \quad \quad \quad 2 \text{ sq ft } 114 \text{ sq in} \\
 121 \left\{ \begin{array}{l} 11) \underline{29430} \text{ qr-sq yds} \\ 11) \underline{2674} \quad 6 \\ 40) \underline{243} \text{ sq po } 1 \\ 4) \underline{60} \text{ } 3 \text{ sq po} \end{array} \right\} 17 \text{ qr-sq yds } + 2 \text{ sq ft } 36 \text{ sq in} \\
 \quad \quad \quad 114 \text{ sq in} \\
 \quad \quad \quad 1 \text{ ac } 210 \text{ } 3 \text{ sq po } 4 \text{ sq yds } 5 \text{ sq ft } 6 \text{ sq in}
 \end{array}$$

Examples XXXV

1 Reduce to *gandas* or *square cubits* —

5 bi 3 kat 6 ch , 45 bi 9 kat 7 ch , 25 bi 15 kat 4 ch 15 ga ,
135 bi 11 kat , 425 bi 17 kat 13 ch 17 ga , 29 bi 17 kat

2 Reduce to *bighas* —

357628 ch. , 10486 ga , 8326675 sq cubits , 4675900 ga , 125720 ch

3 Reduce to *kachvansi* —

24 bi 15 bisv , 136 bi 14 bisv 17 bisvansi , 86 bi 7 bisv , 423 bi
10 bisv 12 bisvansi 15 kachv

4 Reduce to *square inches* —

8 sq mi 340 caw , 15 sq mi 285 caw 12 grounds , 25 sq mi 375 caw
20 grounds 1452 sq ft , 3 caw 13 manies 5 sq ft

5 Reduce to *sq karam* or *sarsai* —

26 ghm 1 bi , 42 ghm 1 bi 3 ka 15 marlas , 42 bi 2 ka 4 sai

6 Reduce to *kathis* —

163 bi 7 pands 3 ka , 4 cha 108 bi 15 pands , 42 bi 112 ka

7 Reduce —

- | | |
|---------------------------------------|--|
| (1) 246053 kachvansi to <i>bighas</i> | (2) 34512876 kathis to <i>bighas</i> |
| (3) 43276850 sq in to <i>catomes</i> | (4) 403207654 kathis to <i>chahurs</i> |
| (5) 1130692 manies to <i>sq miles</i> | (6) 8740361 sq salsai to <i>ghumas</i> |

8 Reduce to *sq inches* —

- (1) 17 sq yds 8 sq ft , 3 sq yds 6 sq ft 75 sq in ; 29 sq yds ,
54 sq yds 8 sq ft 104 sq in , 3 ro 17 po 21 sq yds 8 sq ft
(2) 17 ac 14 po , 1 ac 2 ro 3 po 4 sq yds , 3 ro 22 po 21 sq yds
8 sq ft 116 sq in , 56 ac 2 ro 25 po 37 sq yds 5 sq ft 73 sq in
(3) 38 ac 2 ro 35 po , 324 sq po , 3 sq mi , 4 ac 26 po , 42 ac

9 Reduce to *acres* —

- (1) 16553 sq po , 13678 sq yds , 170184 sq ft , 82973 sq po ,
895487 sq yds , 2709437 sq ft

- (2) 123456789 sq in 94501362 sq in , 455462764 sq in .
72013512032 sq in , 355433005 sq in

10 Reduce —

- (1) 14 ac to sq *hills* (2) 1803 ac to sq *miles*
(3) 5200000 sq yds to sq *miles* (4) 428 sq chains to sq *miles*
(5) 5621 sq po to sq *chains* (6) 535 sq miles to *bighas*

11 Reduce (*Bengal bighas*) —

5445 *bighas* to *acres* , 2560 ac to *bighas* , 9680 bi to *acres* ,
14400 ac to *bighas* , 7260 bi to *acres* , 92360 ac to *bighas*.

12 Reduce 629200 *Bengal bighas* to *N W P bighas* , 9720 *Bengal bighas* to *Punjab bighas* , 320780 *Bengal bighas* to *Madras cawmes* , 768000 *N W P bighas* to *Bengal bighas* and 28800000 *Punjab bighas* to *Bengal bighas*

13 Add together —

(1) bi	ka	ch	(2) sq yds	sq ft	sq in	(3) ac	ro	po
30	15	10	32	2	98	29	3	28
19	17	12	12	8	120	35	3	35
25	18	13	19	7	47	45	0	25
31	12	15	23	6	135	17	1	20
28	8	9	45	7	85	19	2	16

(4) ro	sq	po	sq yds	(5) ac	ro	po	sq yds	(6) ac	ro	sq yds	sq ft	sq in
74	19	15	35	1	23	12 ¹	25	11	0	8	23	
6	34	11 ¹	9	2	15	27 ¹	36	39	11	0	136	
17	0	27 ¹	11	1	24	11	7	0	27	6	0	
23	39	16 ¹	42	0	35	2 ¹	18	20	23	7	9+	

14 Perform the following subtractions —

(1) bi	kat	ch	(2) ac	ro	po	(3) ac	ro	po	sq yds
125	8	9	96	1	19	45	1	29	25 ¹
76	12	13	29	3	30	39	3	18	27 ¹

15 Multiply —

- (1) 120 bi 14 kat 10 ch by 99 , 1539 bi 15 kat 7 ch by 102
(2) 17 ac 1 ro 31 po by 72 , 210 27po 15sq yds 8sq ft by 6 and bi 10
(3) 37 ac 3 ro 19 po 28 sq yds 4 sq ft 103 sq in by 8 and by 75

16 Divide —

- (1) 112 bi 18 kat 14 ch by 99 , 1539 bi 15 kat 7 ch by 102
(2) 82 bi 16 kat 12 ch by 72 , 130 ac 1 ro 28 po by 120
(3) 854 ac 3 ro 27 po 8 sq yds 8 sq ft 45 sq in by 9 and by 246
(4) 166 ac 2 ro 6 po 30 sq yds 5 sq ft by 7 ac 38 po 17 sq yds
1 sq ft , 935 bi 12 kat 12 ch by 55 bi 12 ch

- 17 How many allotments each equal to 2 ro 5 po 13 sq yds
6 sq ft 108 sq in can be formed out of 158 ac 2 ro 20 po ?

18 A certain district contains 514164 ac and another 95805 ac
How many sq miles does the one contain more than the other ?

V MEASURES OF SOLIDITY

161 Bengal Measure of Solidity

13824 Cubic Angulis make 1 Cubic Cubit or 1 hath
8 Cubic Cubits , 1 Cubic yard
8 Cubic yards or 64 cub cubits ,, 1 Chouka

162 English Measure of Solidity

1728 Cubic Inches (*cub in*) make 1 Cubic Foot (*cub ft*)
27 Cubic feet ,, 1 Cubic yard (*cub yd*)

1 Cub hath = 5832 cub in A Load of rough Timber = 40 cub ft
A Load of squared Timber = 50 cub ft A Ton of Shipping = 42 cub ft
A Stack of wood = 108 cub ft A Cord of wood = 128 cub ft

Examples XXXVI

1 Reduce to *cub cubits* —

42 choukas 54 cub cubits , 87 choukas 62 cub cubits ,
146 choukas 32 cub cubits , 144 choukas

2 Reduce to *cub in* —

24 cub yds 7 cub ft 144 cub in , 18 cub yds 1274 cub in ,
12 cub yds 23 cub ft , 23 cub yds 1000 cub in

3 Reduce to *cub yds* —

200000 cub in , 138297 cub in , 141721 cub in , 863005 cub in

4 Reduce to *choukas* —

36248742 cub cubits , 4308756 cub cubits , 862097 cub cubits

5 Reduce 1053 choukas 28 cub cubits to *cubic angulis*

6 Add together —

(1) Chouka cub yds cub hath	(2) c yds c ft c in	(3) c yds c ft c in
- 18 6 4	53 7 1249	328 15 323
27 5 7	27 23 472	237 19 484
134 4 5	29 16 1384	785 10 1259
49 3 2	45 18 1186	546 0 342
<u>234 3 6</u>	<u>33 9 1324</u>	<u>729 11 1075</u>

7 Perform the following subtractions —

(1) c yds c ft c in	(2) c yds c ft c in	(3) c yds c ft c in
49 15 542	150 0 0	527 0 1
<u>39 23 736</u>	<u>59 25 1001</u>	<u>279 1 259</u>

8 Multiply —

- (1) 2 cub yds 5 cub ft 704 cub in by 11 and by 23
 (2) 275 cub yds 17 cub ft 125 cub in by 56

9 Divide —

- (1) 372 cub yds 1236 cub in by 64
 (2) 6739 cub yds 2 cub ft 466 cub in by 19 and by 509
 (3) 18809 cub yds 1 cub ft 1156 cub in by 723 cub yds 11c ft 84c in

10 A certain number of bins, each containing 8 cub yds 152 cub in, contain 1512 cub ft 1064 cub in, find the number

VI MEASURES OF CAPACITY

163 1st Tables of Corn or Dry Measure

Indian

BENGAL MEASURE		BOMBAY MEASURE	
5 Chhataks make	1 Kunka	36 Tanks make	1 Tipari
2 Kunkas	" 1 Khunchi	2 Tiparis	" 1 Seer
2 Khunchis	" 1 Rek	4 Seers	" 1 Payli
2 Reks	" 1 Pali	16 Paylis	" 1 Phara
2 Palis	" 1 Doan	8 Pharas	" 1 Kandī
2 Doans	" 1 Kati	25 Pharas	" 1 Muda
8 Katis	" 1 Arhi	MADRAS MEASURE	
20 Arhis	" 1 Bish	8 Ollaks make	1 Paddi
16 Bishes	" 1 Kahan	8 Paddis	" 1 Markal
16 pa or 8 do	" 1 Maund (<i>md</i>)	5 Maikals	" 1 Phara
20 Doans	" 1 Sali	80 Pharas	" 1 Garce

In Bengal, lime is measured thus 1 Phara = $27' \times 20' \times 9'$,
 6 Pharas = 5 cub hath, 80 Pharas = 100 mds, 1 maikal (Madras)
 = 750 cub in

English

		COAL MEASURE	
2 Quarts (<i>qt</i>) make	1 Pottle (<i>pot</i>)	4 Pecks make	1 Bushel
2 Pottles or 4 qts	" 1 Gallon (<i>gal</i>)	3 Bushels	" 1 Sack
2 Gallons	" 1 Peck (<i>pk</i>)	12 Sacks or 36 bus }	" 1 Chaldron
4 Pecks	" 1 Bushel (<i>bus</i>)		
2 Bushels	" 1 Strike (<i>str</i>)		
4 Bushels	" 1 Coomb (<i>co</i>)		
2 Coombs or 8 bus	" 1 Quarter (<i>q</i>)		
5 Quarters	" 1 Load (<i>ld</i>)		
2 Loads or 10 qts	" 1 Last		

A gallon (*Imperial*) contains 277 274 cub in, hence a bushel (*Imperial*) consisting of 8 gallons, contains $8 \times 277\ 274$ or 2218 192 cub in

164

2nd Tables of Liquid Measure

Indian

4 Chhataks	make	1 Poa
4 Poas	"	1 Seer
40 Seers	"	1 Mund

The weight of a seer for this measure varies in different localities from 40 tolas to 112 tolas

English

WINE MEASURE		ALE AND BEER MEASURE	
4 Gills (<i>gi</i>) make 1 Pint (<i>pt</i>)		2 Pints make 1 Quart	
2 Pints " 1 Quart (<i>qt</i>)		4 Quarts " 1 Gallon	
4 Quarts " 1 Gallon (<i>gal</i>)		36 Gallons " 1 Barrel (<i>bar</i>)	
63 Gallons " 1 Hogsheaf (<i>hhd</i>)	1½	Barrels or } " 1 Hogsheaf	
2 Hogsheafs } " 1 Pipe (<i>fir</i>)	2	54 gallons } " 1 Butt	
or 126 gallons }	2	Hogsheafs " 1 Butt	
2 Pipes " 1 Tun	2	Butts " 1 Tun	
10 Gallons = 1 Anker		9 Gallons = 1 Firkin	
18 Gallons = 1 Runlet		18 Gallons = 1 Kilderkin	
42 Gallons = 1 Tierce			
84 Gallons or 2 Tierces = 1 Puncheon			

A pint of pure water weighs a pound and a quarter, therefore a gallon of distilled water weighs 10lbs (Avoir), when the barometer is at 30 in and the air at a temperature of 62° Fah thermometer. Hence the weight of a cubic foot of water is very nearly 1000 oz (Avoir)

165

English Apothecaries Measure

60 Minims (<i>m</i>) or drops make 1 Fluid Dram (<i>fl dr</i>)	
8 Fluid Drams " 1 Fluid Ounce (<i>fl oz</i>)	
20 Fluid Ounces " 1 Fluid Pint (<i>O Octarius</i>)	
8 Pints " 1 Gallon (<i>C, Congius</i>)	

A tea-spoonful = 1 fluid dram A desert-spoonful = 2½ fluid drams
A table-spoonful = 4 fluid drams 1 Fluid ounce = 1 ounce (Avoir)

Examples XXXVII

1 Reduce to *chhataks* 2 mds 3 do 2 pa 3 ch, 1 md 3 do 1 khun, 8 kah 14 bus 16 arh, 125 mds 6 do 1 pa 1 rek, 14 kah 10 do, 17 salis 58 pa 2 reks

2 Reduce 3842 ch to *maunds*, 201372 kunikas to *maunds*, 48762035 ch to *maunds*, 467032000 ch to *lahans*, 246780 reks to *maunds*, 346760 khun to *doans*

3 Reduce 125 pharas to *tanks*, 416 mudas to *tanks*, 1 ka 3 ph 5 paylis 1 tipari 26 tanks to *tank*, 6932843 tiparis to *mudas*, 54038764 tanks to *landis*

4 Reduce 205 pharas to *ollaks*, 1 grace 45 pharas 2 markals 3 paddis to *ollaks*, 28 pharas 4 markals 54 ollaks to *ollaks*, 256284 ollaks to *garus*, 123456 ollaks to *pharas*, 2368 paddis to *pharas*, 98760 ollaks to *markals*

5 Reduce to *gallons* 2 qrs 7 bus 2 pks, 3 lds 3 qrs 3 pks, 54 qrs 7 bus 6 gal, 64 lasts 1 ld 3 qrs 7 bus 1 pk.

168 The number of days in the *Calendar Months* are recollect ed by means of the following lines —

Thirty days hath September,
April, June and November,
February has twenty eight alone,
And all the rest have thirty one,
But leap-year coming once in four,
February then has one day more

<i>Bengali Months</i>		<i>English Months</i>	
1	Baisakh (বৈশাখ)	1	January = 31 days
2	Jaistha (জ্যৈষ্ঠ)	2	February = 28 "
3	Asharh (আষাঢ়)	3	March = 31 "
4	Srawan (শ্রাবণ)	4	April = 30 "
5	Bhadra (ভাদ্র)	5	May = 31 "
6	Aswin (অশ্বিন)	6	June = 30 "
7	Kirtick (কর্তিক)	7	July = 31 "
8	Agrahayan (অগ্রহায়ণ)	8	August = 31 "
9	Pous (পৌষ)	9	September = 30 "
10	Magh (মাঘ)	10	October = 31 "
11	Falgun (ফাল্গুন)	11	November = 30 "
12	Chaitra (চৈত্র)	12	December = 31 "

Mahomedan Names Muharram (মহরর), Safar (শফর), Rabiulaval (রবিয়ল অউয়ল), Rabiassani (রবিয়সানি), Jamadiul awal (জমাদিয়ল অউয়ল), Jamadiyassani (জমাদিয়সানি), Rajab (রজব), Saban (শাবন), Remyan (রমজান), Saoyal (শওয়াল), Jelkad (জেলকাদ), and Jelhija (জেলহিজ)

A Bengali month is generally supposed to consist of 30 days, but this is not strictly correct. Some months are 29 days some 30, some 31 and some 32

THE HINDU CALENDAR

169 The Hindu **Chandra Batsar** (Lunar year) consists of 354 days 8 hrs 48 min 57 sec. It is therefore shorter than the **Saur Batsar** (Solar year) by 10 days 21 hrs, 23 min 12 sec. After a period of 32 $\frac{1}{2}$ months the difference amounts to a month, consequently to make the Lunar year system correspond with the Solar year system, a month is intercalated on the occurrence of two conjunctions of the Sun and Moon in the same sign of the Zodiac. The intercalated month and the month preceding it go by the same name. The intercalated month is called **Mala** or **Intercalary Mas**. This is done in those parts of India where the lunar year and lunar month are reckoned. In Bengal, where solar year and solar month are reckoned, a month is rejected in a period of every 32 $\frac{1}{2}$ lunar months as unfit for any religious festival, in order to make the religious festivals of particular months recur in those months. The rejected month is called *Usha Ma*.

THE ENGLISH CALENDAR

170 The interval of time between two passages of the Sun across the meridian of any place when taken at its *mean magnitude*, is termed a *day* or a *mean solar day*, which is supposed to be divided into 24 equal portions called *mean solar hours*. It appears from the observations and calculations of Astronomers that the time between the Sun's leaving a certain point (First point of Aries) in his path called the *Ecliptic* and returning to it again, consists of 365 242218 such days or of 365 days, 5 hours, 48 minutes, $47\frac{1}{2}$ seconds, very nearly, which is therefore termed a *Solar Year*.

For the purposes of civil life it would be exceedingly inconvenient that one year should commence at one time of the day and another at a different time, and this circumstance gave rise to the invention of the *civil year*, which will be explained in the next Articles.

171 When the Science of Astronomy was much less perfect than it is at present, the length of the solar year was much less accurately known, and accordingly we find that in the time of *Julius Caesar* it was supposed to consist of 365 days 6 hours, or of $365\frac{1}{4}$ days, *exactly*. On this supposition, it is evident that if out of *four* years in succession, any *three* consisted of 365 days each and the remaining one of 366 days, the Sun would have returned at the end of those *four* years to the place in the *Ecliptic* which it occupied at their commencement.

The scheme was called the *Julian Calendar*, and if the hypothesis had been correct, it would have been attended with much convenience, the additional *day* was called *Intercalary*, and the *year* in which it was added or inserted was termed *Bissextile*.

The regulation, applied to the years of the *Christian Era*, was so managed that whenever the number of years was divisible by 4, the corresponding year consisted of 366 days and was called *Leap year*, the month of *February* having 29 days in that year, and each of the remaining three years 28 days, without interfering at all with their order.

Hence also, the remainder after the division of any other number of years by 4, was the number of years since a leap year occurred up to that year. Thus, in the year 1893 this remainder is 1, and accordingly it is 1 year since the last leap-year happened and it is 3 years before the next will occur, according to this scheme.

172 Since the true solar year is 365 242218 days and not 365 25 days, it is evident that the reckoning of time according to the Julian Calendar would place the end of the year *after* the time when the Sun had returned to the point of the *Ecliptic* occupied by it at the beginning of the year and consequently *in advance* of the course of the *Seasons*, but the error in one year is $365\frac{1}{4} - 365\frac{242218}{365} = 0.07782$ of a day. Therefore in 400 years the error would amount to 0.07782×400 or 31.128 days.

Now, according to the Julian Calendar 400 years would comprise 100 Leap-years, and since we find that this reckoning falls nearly 3 days *after* the true time, if there were only 97 Leap years in 400 years, the Julian year would very nearly agree with the true solar year, and it is accordingly ordained that whenever the *numbers* expressing the *Centuries* as 16, 17, 18, 19, &c, denoting 1600, 1700, 1800, 1900, &c, are *not* divisible by 4, the corresponding year shall *not* be a Leap year although according to the Julian Computation it would, as, 1600 would be a Leap year, but 1700, 1800, 1900 would not

The Calendar thus corrected though not absolutely accurate, is well adapted to every *practical* purpose, as the error in 5000 years will not amount to much more than *twenty eight* hours. The correction was first promulgated in Europe by *Pope Gregory* in the 1582 and the calendar has since been called the *Gregorian Calendar*, but it was not introduced into *Protestant Countries* till a much later period. In *England* it was adopted on the *second* day of September 1752 when the error amounted to 11 days, and it is called the *New Style* to distinguish it from the Julian Calendar which is now termed the *Old Style*.

The New Style has not yet been adopted in Russia, so that since 1752 they have had one more leap-year (1800) than we have, and they are now 12 days behind us. Thus Old Michaelmas and Old Christmas taking place 12 days after New Michaelmas and New Christmas.

173 The Civil year thus fixed and determined is then subdivided into twelve Calendar Months, as described in the Table. The word *Month* however is used in different senses, sometimes to denote a *twelfth* part of a year, sometimes as equivalent to 4 weeks or 28 days, and accordingly a year is equivalent to 13 months and 1 day, or to 52 weeks and 1 day, with the addition of another day when it happens to be Leap year.

174 To reduce *prahars* to *dandas*, multiply by 15, and divide the product by 2, the remainder (if any) is a half-danda or 30 pals. Conversely to reduce *dandas* to *prahars*, multiply by 2 and divide the product by 15, the remainder (if any) is equal to so many half-dandas.

Ex Reduce 8sap 5da 3pr 4dan 45 pals to *bipals*, 266330 sec to *days*, and 2 yrs 15 da 6 hrs to *minutes*.

(1) 8 sap 5¹da 3 pr 4 dan 45 pals

$$\begin{array}{r} 7 \\ 61 \text{ da} \\ 8 \\ \hline 491 \text{ pr} \\ 15 \\ \hline 27365 \end{array}$$

3682 dan 4 1 half dan

$$= 3682 \text{ dan } 30 \text{ pals}$$

$$\begin{array}{r} 4 \quad 45 \\ \hline 3687 \text{ dan } 15 \text{ pals} \end{array}$$

$$\begin{array}{r} 60 \\ \hline 221235 \text{ pals} \end{array}$$

$$\begin{array}{r} 60 \\ \hline 13274100 \text{ bipals} \end{array}$$

Ans

(2) 6,0)26633,0 sec

$$\begin{array}{r}
 6,0 \overline{) 443,8} \quad 50 \text{ sec} \\
 24 \left\{ \begin{array}{l} 3 \text{ } 73 \\ 8 \overline{) 24} \end{array} \right. \quad \begin{array}{l} 58 \text{ min} \\ 1 \end{array} \left. \vphantom{\begin{array}{l} 3 \text{ } 73 \\ 8 \overline{) 24} \end{array}} \right\} 1 \text{ hr} \\
 \quad \quad \quad 3 \text{ da}
 \end{array}$$

the result

$$= 3 \text{ da } 1 \text{ hr } 58 \text{ m } 50 \text{ sec}$$

(3) 2 yrs 15 da 6 hrs

$$\begin{array}{r}
 365 \\
 745 \text{ da} \\
 24
 \end{array}$$

$$17886 \text{ hrs}$$

$$60$$

$$1073160 \text{ min } \text{Ans}$$

Examples XXXVIII

1 Reduce to *anupals* —

- (1) 5 dan 30 pa (2) 12 pr 6 dan 40 bip (3) 8 sap 5 da 5 pr
 (4) 3 ba 6 m 5 da 5 dan (5) 4 sap 6 da 6 pr 50 pa 40 bip
 (6) 6 pr (7) 13 sap (8) 12 days (9) 10 da 5 pr 45 anu
 (10) 46 ba 267 da 57 dan 43 pa 51 bip

2 Reduce to *seconds* —

- (1) 27 wks 5 da 15 hrs , 6 hrs 25 min 32 sec , 5 wks 3 da
 (2) 3 yrs 147 da 15 hrs , 76 da 19 hrs 43 min 57 sec
 (3) 2 da 4 hrs 51 min 50 sec , 4 mo 2 wks 23 hrs , 3 leap years

3 Reduce 15 yrs 26da 2hrs 27min to *minutes* , 19yrs 153da
 8 hrs to *hours* , 3 yrs 315 da to *minutes*

4 Reduce —

- (1) 563472 pals to *dins* , 59018732 anupals to *dins*
 (2) 8463045 bipals to *prahars* , 74632508 anupals to *dandas*
 (3) 673985643 anupals to *days* , 36438005 dan to *batsars*

5 Reduce —

- (1) 72015 hours to *weeks* , 2706359 sec to *weeks* , 38567 min to *days*
 (2) 123456 sec to *hours* , 3456794 sec to *days* , 579574 min to *years*

6 Reduce to *years* —

$$71871900 \text{ sec} , 1301416510 \text{ sec} , 713969410 \text{ sec} , 413419320 \text{ sec}$$

7 Add together —

(1) sap	da	pr	(2) dan	pal	bip	anu	(3) din	dan	pal	bip	anu
36	5	6	41	36	57	51	60	57	19	21	27
24	4	2	39	48	39	47	73	40	23	17	13
48	6	5	49	55	13	58	9	55	19	18	29
2	3	4	59	26	49	38	37	20	40	19	24
18	6	7	21	50	28	19	47	30	59	29	34

(4) hrs	min	sec	(5) da	hrs	min	sec	(6) wks	da	hrs	min
15	42	45	35	14	32	30	10	5	14	31
57	36	40	47	16	25	27	18	4	12	38
32	12	14	54	18	52	57	25	0	10	14
16	37	45	43	21	37	29	75	6	23	59
5	51	41	62	22	58	57	53	4	19	23
24	19	40	40	15	20	32	40	0	17	25

8 Perform the following subtractions —

(1) sap da pi	(2) dins dan pal bip	(3) da hrs min sec
527 5 5	80 50 40 20	17 1 0 17
418 6 7	50 55 50 36	7 17 13 22
(4) da hrs min sec	(5) wks da hrs	(6) yis da hrs min sec
24 14 46 31	7 3 18	7 129 13 26 17
4 21 18 52	4 6 20	3 273 18 34 29

9 Multiply —

- (1) 7 dins 5 dan 30 pal 15 bip by 74, and by 140
 (2) 9 ba 8 ma 27 da 45 dan 56 pal 38 bip 52 anu by 43, 67
 (3) 43 days 18 hrs 45 min by 77, and by 147
 (4) 17 wks 4 da 13 hrs 27 min 36 sec by 9, and by 79
 (5) 17 years 110 da 17 hrs 57 sec by 144

10 Divide —

- (1) 694 dins 7 pr 3 dan 30 pal by 32
 (2) 2056 ba 5 ma 27 da 44 dan 15 pal by 87
 (3) 17 wks 5 da 18 hrs 25 min by 49
 (4) 878 wks 4 da 15 hrs 37 min 36 sec by 9, and by 56

11 How many days are there (the last day mentioned in each case being excluded) from

- (1) April 5, 1863 to Nov 3, 1863 ? (2) Dec 31, 1863 to Dec 31, 1864 ?
 (3) Sep 21, 1863 to March 1, 1864 ? (4) Nov 16, 1882 to Sep 5, 1884 ?

12 How many bipals are there in a year of 365 days 6 hours ?

- 13 A solar year = 365 days 5 hrs 48 min 47½ sec (1) how many more seconds are there in a solar year than in a common year ?
 (2) How many seconds less than in a leap year ?

14 How many portions of time each equal to 1 day 7 hrs 45 min 56 sec are contained in 346 days 18 hrs 34 min 32 sec ?

15 If the 1st of April is a Monday, on what day of the week will Christmas fall that year ?

VIII MEASURES OF ANGLES

175

English Angular Measure

60 Seconds (60")	make 1 Minute (')
60 Minutes	" 1 Degree (°)
90 Degrees	" 1 Right Angle (1 rt g ^{le})

IX MEASURES OF NUMBERS

176

BENGALI TABLE

ENGLISH TABLE

4 Units	make 1 Ganda	12 Units	make 1 Dozen
5 Gandas	" 1 Buri	12 Dozen	" 1 Gross
4 Buris	" 1 Pan	12 Gross	" 1 Great Gross
16 Pans	" 1 Kahan	20 Units	" 1 Score (Kuti)
		120 Units	" 1 Long Hundred.

FOR PAPER

24 Sheets = 1 Quire, 20 Quires = 1 Ream, 10 Reams = 1 Bale

Examples XXXIX

 1 Reduce to *seconds* —

 (1) $172^{\circ} 8' 25''$ (2) $275^{\circ} 30' 26''$ (3) $144^{\circ} 12' 38''$ (4) $57^{\circ} 7' 45''$

 2 Reduce to *right angles, degrees &c* —

(1) 206265" (2) 865408' (3) 718276" (4) 42861' (5) 78205

 3 Add together, $175^{\circ} 32' 45''$, $75^{\circ} 59' 27''$, $114^{\circ} 28' 47''$, $105^{\circ} 45''$, $144^{\circ} 12' 38''$, $160^{\circ} 52' 58''$, and $175^{\circ} 20' 46''$

 4 Subtract $149^{\circ} 53' 56''$ from $277^{\circ} 30' 47''$

 5 Multiply $24^{\circ} 12' 16''$ by 42, $19^{\circ} 14' 25''$ by 36

 6 Divide $25^{\circ} 25' 32''$ by 16, $144^{\circ} 44' 7''$ by 22

7 In 56 reams of paper, how many sheets?

 8 Reduce 67835 kahans 11 pans 18 ga 3 units to *units*

 9 Reduce 7297865 units to *kahans*, 9 scores to *dozens*

10 Multiply 9 kahans 2 pans 17 ga 2 units by 82, and by 346

Examples XL

(Recapitulatory Exercises)

1 In 340 pistoles at 17s 6d each, how many pounds sterling?

2 How many moidores of 27s each, are equivalent to 198 guineas and to £500638 1s?

3 In £453 16s 8d, how many pieces of coin valued at 3d each? How many at 11s 8d each?

4 What number of weights of 14 oz 13 dis each, are equivalent to 25 cwt 2 qrs 13 lbs 14 oz 12 drs?

5 If I spend £2 7s 1½d a day, how much is that in 28 weeks, and also in a year of 365 days?

6 If each of 114 persons receive £1 18s 6½d, what is received by them all?

7 If the clothing of 754 soldiers come to £3178 11s 7½d, how much is that for each man?

8 If a person complete a journey of 422 mi 3 fur 38 po in 37 days, what distance does he travel each day?

9 A year being equivalent to 365 days 6 hours, find the number of years, &c in 295402374 seconds

10 Multiply 4 dins 3 pr 2 dan 25 bip 15 anu by 401

 11 Reduce 9367875 angulis to *lros*, 14978631 gandas to *bighas*

12 Find how often a rod 2 ft 10 in in length, must be applied to measure 10 miles 140 yds

13 Find the number of yards in 40 pieces of cloth, each containing 42 yds 2 qrs 2 nls

14 If a soldier's pay for a year of 365 days be £9 2s 6d how much is that for a day?

15 If a person's yearly income be £65 12s 6d, and he lay by £20 a year, how much does he spend each day?

16 How many pounds of silver are there in a half-dozen of dishes, each weighing 51 oz 10 dwts and 1 dozen of plates, each weighing 15 oz 15 dwts 22 grs?

17 Express 452 dan 48 pal 45 bip in *English measure*.

18 If 145 sheep cost £69 3s 4d, what is the price of a score at the same rate?

19 If 8 packages of cloth, each consisting of 4 parcels, each parcel of 10 pieces, and each piece of 26 yards, cost Rs 66560, what is the price of a yard?

20 The sum of £263 8s 11½d is distributed equally among a number of persons so that the share of each is £37 12s 8½d, find the number of persons.

21 A boy's school, to and from which he walks daily, is distant from his home 1 kros 250 dan 1 gay 1 hath 7 girahs. How many girahs does he walk every day?

22 Reduce 35 tons 19 cwt 99 lbs 12 oz 135 grs to *grains*.

23 Reduce 294322493 sq in to *acres, &c*.

24 Find the weight of copper coin required to pay a debt of £1000, when 3 pennies weigh 1 oz.

25 Which is the heavier, 1 lb of gold or 1 lb of sugar?

26 If 28 lbs 9 oz of gold be worth £1343 6s 10½d, what is the worth of 1 ounce?

27 Among how many boys can I distribute £14 9s 9d, giving to each boy a half crown, a florin, 1 four penny piece, and also a three penny piece?

28 If a man's net income be £1785 12s 6d, how much may he spend on an average per day to the nearest farthing, so as not to run into debt?

29 Reduce 5792685 inches to *miles, &c*.

30 Jadu was born at 6 o'clock A.M., 24th June, 1872, how old will he be at 3 o'clock P.M., 10th Jan., 1898?

31 Find the sum of 32 cwt 2 qrs 15 lbs 12 oz, 47 cwt 25 lbs 9 oz, 5 cwt 3 qrs 17 lbs 10 oz, 23 cwt 1 qr 19 lbs 15 oz, and 9 cwt 3 qrs 14 oz, divide the sum by 4 cwt 2 qrs 18 lbs 8 oz.

32 If 1s 5½d be the unit of money, what will be the measure of £7 17s 6d and of £20 1s 0½d?

33 If 2 ft 6 in be the unit of length, what number will represent (i) 10 miles, (ii) 25 miles 760 yds?

34 If 6 hrs 32 min 10 sec be the unit of time, what will be the measure of 74 days 1 hr 49 min 20 sec?

35 If 2 lbs 5 oz be the unit of weight, what number will measure 5 cwt 6 lbs 9 oz ?

36 If 5 sec be the unit of time, what will be the measure of 3 hrs 5 sec. and of 15 hrs 20 min ?

37 22nd September 1897 was Wednesday What day of the week was 22nd September 1797 and what day of the week will 22nd September 1997 be ?

38 19th September 1897 was Sunday, What day of the week was 23rd January 1807 and what day of the week will 23rd January 1907 be ?

X MISCELLANEOUS PROPOSITIONS

(IN COMPOUND QUANTITIES)

177 The Unitary Method (*Simple Cases*)

If the value, weight, length, &c of any number of units be given, we can by Compound Division find that of one unit of the same kind, and the value, weight, length, &c of one unit being found, we can by Compound Multiplication find that of any number of units of the same kind. The solution which combines these two processes is called **The Method of Reduction to the Unit or The Unitary Method**

(1) The value, weight, length, &c of one unit being given, we can by *Compound Multiplication* find the value, weight, length, &c of any number of units of the same kind

Ex The price of a maund of sugar is Rs 10 15a 6p, find the price of 35 maunds

$$\begin{array}{r} \text{Rs } 10 \quad 15a \quad 6p \\ \quad \quad \quad 35 \\ \hline \text{Rs } 383 \quad 14a \quad 6p \end{array}$$

$$\begin{array}{l} \text{The price of 1 maund} = \text{Rs } 10 \quad 15a \quad 6p \\ \text{the price of 35 mds} = \text{Rs } 10 \quad 15a \quad 6p \times 35 \\ \quad \quad \quad = \text{Rs } 383 \quad 14a \quad 6p \end{array}$$

(2) The value, weight, length, &c of any number of units being given, we can *Compound Division* find the value, weight, length, &c of one unit of the same kind

Ex If 30 mds of rice cost Rs 134 1a, what is the price per maund ?

$$\begin{array}{r} 30 \overline{) \text{Rs } 134 \quad 1a} \quad (4Rs \\ \underline{120} \\ 14 \\ \underline{10} \\ 225(7a \\ \underline{210} \\ 15 \\ \underline{12} \\ 180(6p \\ \underline{180} \end{array}$$

$$\begin{array}{l} \text{The price of 30 mds} = \text{Rs } 134 \quad 1a \\ \text{the price of a md} = \text{Rs } 134 \quad 1a \div 30 \\ \quad \quad \quad = \text{Rs } 4 \quad 7a \quad 6p \end{array}$$

- (3) The value, weight, &c of a certain number of units being given, to find the value, weight, &c of a certain other number of units of the same kind

Proceed as in the following Examples —

Ex 1 If 7 yards of cloth cost Rs 26 4a, what will be the cost of 15 yds of the same ?

$$\begin{array}{r} 7)Rs\ 26\ 4a \\ \underline{Rs\ 3\ 12a} \\ 15 \\ \underline{Rs\ 56\ 4a} \end{array}$$

$$\begin{array}{l} 7\text{ yards cost Rs } 26\ 4a \\ 1\text{ yard costs Rs } 26\ 4a \div 7 = Rs\ 3\ 12a \\ 15\text{ yards cost Rs } 3\ 12a \times 15 = \underline{Rs\ 56\ 4a} \end{array}$$

Ex 2 If 7lbs of tea cost 15s 9d, what will be the cost of 12lbs ?

$$\begin{array}{r} 7)15s\ 9d \\ \underline{2s\ 3d} \\ 12 \end{array}$$

$$\begin{array}{l} 7\text{lbs cost } 15s\ 9d \\ 1\text{lb costs } 15s\ 9d \div 7 = 2s\ 3d \\ 12\text{lbs cost } 2s\ 3d \times 12 = \underline{\pounds 1\ 7s} \end{array}$$

£1 7s

- (4) The value, weight, &c of a certain number of units being given, to find the number of units of the same kind corresponding to some other value, weight, &c

Proceed as in the following Examples —

Ex 1 If 12 maunds of rice cost Rs 35, find how many maunds of the same can be bought for Rs 20 6a 8p

$$\begin{array}{l} 12)Rs\ 35 \\ \underline{Rs\ 2\ 14a\ 8p} \\ =\text{the price of a maund} \end{array} \quad \begin{array}{l} Rs\ 20\ 6a\ 8p = 3920p, \quad Rs\ 2\ 14a\ 8p = 560p \\ \text{the no of mds required} = 3920 - 560 \\ = \underline{7}\text{ Ans} \end{array}$$

Ex 2 If 25 men finish a piece of work in 16 days, in how many days will 20 men finish it ?

$$\begin{array}{r} 25 \\ 16 \\ 20)400\text{ days} \\ \underline{20\text{ days}} \end{array} \quad \begin{array}{l} 25\text{ men finish the work in } 16\text{ days,} \\ 1\text{ man will finish in } (25 \times 16)\text{ or } 400\text{ days} \\ 20\text{ men will finish in } 400 - 20\text{ or } \underline{20}\text{ days Ans} \end{array}$$

Ex 3 How many men can perform in 24 days a piece of work which 15 men can perform in 40 days ?

$$\begin{array}{r} 15 \\ 40 \\ 24)600\text{ men} \\ \underline{25\text{ men}} \end{array} \quad \begin{array}{l} \text{In } 40\text{ days the work is done by } 15\text{ men} \\ \text{in } 1\text{ day the work is done by } (15 \times 40)\text{ or } 600\text{ men} \\ \text{in } 24\text{ days, the work is done by } 600 - 24\text{ or } \underline{25}\text{ men} \end{array} \quad \text{Ans}$$

Note In questions such as the two above, it should be noticed that to a *diminution* in the number of men corresponds an *increase* in the number of days and *vice versa*

Examples XLI

- 1 What is the value of 72 reams of paper, at 13s 8d a ream ?
- 2 Find the cost of 120 ounces of silver, at 5s 3½d an ounce
- 3 What will be the price of 1 lb, when 1 cwt costs £137 18s ?
- 4 If 41 cwt cost £52 10s 7½d, what is the price of a cwt ?
- 5 If 6 chairs cost Rs 32 12a, what will 3 dozen cost ?
- 6 If a workman's wages for 12 days be Rs 14 4a 6p, what would it amount to in 18 days ?
- 7 If 4 yards of flannel cost Rs 3 13a 4p, what is the cost of 57 yards of the same ?
- 8 If 42 bighas of land be rented for Rs 640 8a, what would be the rent of 61 bighas ?
- 9 If a man earn Rs 15 12a in 6 days, in how many days will he earn Rs 189 ?
- 10 If I travel by Railway 85 miles for Rs 7 15a 6p, how far may I travel for Rs 9 6a ?
- 11 If 13 sheep cost Rs 175 8a, how many may be purchased for Rs 2160 ?
- 12 If 7 seers of tea cost Rs 7 9a 4p, what will be the cost of 1 md 24 sr 8 ch ?
- 13 A clerk's salary is Rs 1916 4a per annum, what ought he to receive for 60 days' service ?
- 14 How much land may be rented for Rs 705 4a, if 5 acres are rented for Rs 46 10a 8p ?
- 15 How many men can perform in 12 days a piece of work, which 15 men can perform in 20 days ?
- 16 If 3 mds 12 sr 8 ch of sugar cost Rs 16 9a, what will 2 mds 14 sr 10 ch cost ?
- 17 Find the quantity of rice which can be purchased for Rs 86 3a 9½p, when 70 mds 10 sr cost Rs 270 12a 1p
- 18 If 3 cwt, 69 lbs cost £14 3s 6d, how much may be bought for £23 12s 6d ?
- 19 If 2 cwt 3 qrs 7 lbs cost £5 17s 8½d, what is the cost of 9 cwt ?
- 20 In how many days would 171 men perform a piece of work, which 108 men can perform in 266 days ?

178 Revolution of Wheels

A wheel in making one revolution passes over a length of ground exactly equal to its circumference. Hence, if we multiply

the circumference by the number of revolutions made, we shall find the distance passed over, and conversely, if we divide the distance passed over by the circumference, we shall find the number of revolutions, or by the number of revolutions we shall find the circumference

E1 1 A carriage wheel is 4 yds 2 ft 7 in in circumference, and makes 1456 revolutions on a journey. What is the length of the journey?

$$1456 = 8 \times 13 \times 14$$

$$(220 \text{ yds} = 1 \text{ furlong})$$

∴ the distance passed over is
4 mi 0 fur 37 yds 2 ft 4 in

mi	fur	yds	ft	in
		4	2	7
				8
		38	2	8
				13
		2	65	8
				14
4	0	37	2	4

E1 2 A wheel makes 131 revolutions in passing over 669 yds 1 ft 8 in, what is its circumference?

$$\text{The circumference} = 669 \text{ yds } 1 \text{ ft } 8 \text{ in} - 131 = \underline{5 \text{ yds } 4 \text{ in}}$$

E1 3 How many revolutions will a carriage wheel 3 yds 2 ft 6 in in circumference, make in a journey of 7 miles 3 fur 34 po 4 yds 1 ft?

$\begin{array}{r} 3 \text{ yds } 2 \text{ ft } 6 \text{ in} \\ \underline{3} \\ 11 \text{ ft} \\ \underline{12} \\ 138 \text{ in} \end{array}$	$\begin{array}{r} 7 \text{ mi } 3 \text{ fur } 34 \text{ po } 4 \text{ yds } 1 \text{ ft} \\ \underline{8} \\ 59 \text{ fur} \\ \underline{40} \\ 2394 \text{ po} \\ \underline{11} \\ 2)26334 \\ \underline{13167 \text{ yds} + 4 \text{ yds}} \\ =13171 \text{ yds} \\ \underline{3} \\ 39514 \text{ ft} \\ \underline{12} \\ 474168 \text{ in} \end{array}$	$\begin{array}{r} 138)474168(3436 \\ \underline{414} \\ 601 \\ \underline{552} \\ 496 \\ \underline{414} \\ 828 \\ \underline{828} \end{array}$
--	--	---

the number of revolutions required = 3436 Ans

Examples XLII

1 If a wheel 5 yds 2 ft 4 in in circumference makes 1080 revolutions on a journey, how far will the carriage go?

2 If a wheel 5 yds 1 ft 6 in in circumference makes 64640 revolutions, what space will it pass over?

3 How many revolutions will the wheel of a carriage, 4 ft 7 in in circumference, make in 2 mi 4 fur?

4 A wheel makes 514 revolutions in passing over 1 mi 467 yds 1 ft, what is its circumference?

5 A boy's hoop is 3 yds 10 in round, how many miles of ground will it pass over in 2501 turns?

6 The fore wheel of a carriage is 4 ft 6 in round, and the hind wheel 1 foot longer, how many more turns will the former make than the latter in a distance of 30 miles?

7 A wheel makes 1510 revolutions in passing over 2 mi 456 yds 1 ft, what is its circumference?

8 How many revolutions will a wheel 4 yds 2 ft in circumference make on a journey of 12 mi 696 yds 2 ft?

9 The circumference of the fore wheel of a carriage being 8 ft 3 in, and that of the hind wheel 11 ft 11 in, how many more revolutions would be made by the fore wheel than by the hind wheel in going a distance of 52 miles?

10 The driving wheel of a locomotive is 5 yds 2 ft 9 in in circumference, and makes on an average 3 revolutions a second, find the rate of the train per hour.

11 The fore-wheel of a carriage which is 2 yds 2 ft 6 in in circumference makes 4350 more revolutions than the hind wheel in going over a distance of 19 miles 2 fur 120 yds, what is the circumference of the hind-wheel?

12 Find the circumference of the wheel of a locomotive which makes on an average 4 revolutions in a second and which performs a journey of 76 miles in 1 hour 36 min.

13 A wheel revolves 1028 times in going 2 mi 934 yds 2 ft. What is its circumference?

14 In going over a distance of 205 miles the fore wheel turns 98100 times and the hind-wheel 78720 times. How much longer is the circumference of the hind-wheel than that of the fore wheel?

15 The circumference of the fore wheel of a carriage is 8 ft and that of the hind-wheel is 10 ft, in what distance will the fore-wheel make 100 revolutions more than the hind wheel?

179 Averages

The **Average** or **Mean** of any number of given quantities of the same kind, is that quantity which when substituted for each of the given quantities makes their sum the same. Hence, to find the *Average* of any number of quantities we divide the sum of them by their number.

Fr The receipts at a Railway Station are as follow Jan, Rs 2458 14s 6p, Feb, Rs 2019 6s, March, Rs 2857 4s 8p, April,

Rs 3051 1*a* 4*p* , May, *Rs* 3463 13*a* 4*p* , and June, *Rs* 4007 10*a* , find the average receipts per month

<i>Rs</i>	<i>a</i>	<i>p</i>
2458	14	8
2019	6	0
2857	4	8
3051	1	4
3463	13	4
4007	10	0
6)17858	2	0
<i>Rs</i> 2976	5	8

The sum of the receipts for the 6 months is found to be *Rs* 17858 2*a* , hence the average month's receipt is found by dividing this sum by 6, and is

Rs 2976 5*a* 8*p*

180 Nearest money

When there is a remainder after division, we observe, that if the quotient be multiplied by the divisor the product will be *less* than the dividend, also that if the quotient be increased by 1 and be then multiplied by the divisor the product will be *greater* than the dividend. Hence, in all cases, a nearest sum can be found, which will be exactly divisible by the divisor. Also a quotient correct to the nearest lowest denomination (Art 149)

Ex 1 Find the nearest sum of money to £197 11*s* 6*d* that can be divided by 23 without remainder

	£	<i>s</i>	<i>d</i>		£	<i>s</i>	<i>d</i>
23)	197	11	6	(8	11	9½
	184						
	13						
	20						
	271	(11 <i>s</i>					
	23						
	41						
	23						
	18						
	12						
	222	(9 <i>d</i>					
	207						
	15						
	4						
	60	(2 <i>q</i>					
	46						
	149						

From the work it appears that if the given sum be diminished by 14*q*, or 3½*d*, there will be no remainder, or if it be increased by 9*q* or 2½*d*, so as to make the last partial dividend 69, there will be no remainder, hence the nearest sum required is £197 11*s* 6*d* + 2½*d* or

£197 11*s* 8½*d* Ans

Ex 2 If £197 11*s* 6*d* be given for 23 pieces of cloth, find to the nearest penny the price given for each piece

From the last *Ex*, it appears that £8 11*s* 9*d* a piece would give 15*d* too little, and £8 11*s* 10*d* would give 8*d* too much, hence, to the nearest penny the price would be £8 11*s* 10*d* Ans

Examples XLIII

1 On Sunday I spent no money, on Monday Rs 43 14s, on Tuesday Rs 51 12s 8d, on Wednesday, Rs 46 14s 6d on Thursday Rs 52 8s, on Friday Rs 32 15s 6d on Saturday Rs 26 4s, find my average daily expenditure during the week

2 The daily receipts of a grocer for the week are as follow — Monday Rs 47 10s 2d, Tuesday Rs 56 8s 4d, Wednesday Rs 78 7s, Thursday (being a holiday) nothing, Friday Rs 39 7s 4d, and Saturday Rs 159 13s 2d, find his average daily receipts (1) excluding Thursday and (2) including Thursday

3 Find the least sum of money that must be subtracted from £653 14s 8d to make the remainder divisible by 37

4 Deduct Rs 26 13s 6d from Rs 562 8s, and divide the resulting sum equally among 29 persons to the nearest pie, how much will each person receive and how much will remain over?

5 The average price of a quarter of wheat for 19 years was 56s 8d a quarter, for the first five years the average price was 61s 3½d a quarter for the next 4 years 58s 0½d, for the next 7 years 53s 5½d, find the average of the last 3 years

6 Find the nearest sum of money to Rs 3339 10s 10d that can be divided by 29 without remainder

7 The mean height of 6 mountains is 10357 feet, find what the height of the seventh mountain must be, in order that the mean height of the seven mountains may be 10543 ft

8 120 tons of coal are purchased for £87 16s 9d, find to the nearest farthing the price at which they must be retailed per ton, so that no loss may be incurred

9 Find the least sum of money that must be added to Rs 3658 12s 4d to make the sum divisible by 127

10 A tradesman's average annual income from 1830 to 1850 was Rs 3744 13s 4d. In 1830 his income was Rs 3699 6s 8d and in 1851 his income was Rs 3600 8s 8d what was his average annual income from 1831 to 1851 (inclusive)?

181 Gain and Loss

The price at which an article is bought is called its **cost price**, that at which it is sold its **selling price**. If the selling price be greater than the cost price, it is **gain**, if less, it is **loss**. Hence the difference between the two prices is the *gain* or *loss*.

- (1) Given the quantity sold, and also the cost and selling prices, to find the gain or loss

Ex 1 A person bought 524 yards of cloth at Rs 7 14s 6p per yard and retailed it at Rs 8 2s 4p per yard, what was his profit?

$$\begin{array}{r} \text{Selling price per yard} = \text{Rs } 8 \quad 2s \quad 4p \\ \text{Cost} \qquad \qquad \qquad = \text{Rs } 7 \quad 14s \quad 6p \\ \hline \end{array}$$

$$\text{gain per yard} = 3s \quad 10p$$

$$\text{gain on 524 yards} = 3s \quad 10p \times 524 = \text{Rs } 125 \quad 8s \quad 8p \quad \text{Ans}$$

Ex 2 A trader bought 1763 yards of cloth at 6s 11d per yard and retailed it at 5s 3½d per yard, what was his loss?

$$\begin{array}{r} \text{Cost price per yard} = 6s \quad 11d \\ \text{Selling price} \qquad \quad = 5s \quad 3\frac{1}{2}d \\ \hline \end{array}$$

$$\text{loss per yard} = 1s \quad 7\frac{1}{2}d$$

$$\text{loss on 1763 yards} = 1s \quad 7\frac{1}{2}d \times 1763 = \text{£}143 \quad 4s \quad 10\frac{1}{2}d \quad \text{Ans}$$

(2) Given the gain or loss, and the cost and selling prices, to find the quantity sold

Ex 3 A mercer bought some gloves at 2s 2½d a pair, and by selling them at 3s 6d per pair, gained £9 6s, how many pairs did he buy?

$$\begin{array}{r} \text{Selling price per pair} = 3s \quad 6d \\ \text{Cost} \qquad \qquad \qquad = 2s \quad 2\frac{1}{2}d \\ \hline \end{array}$$

$$\therefore \text{gain per pair} = 1s \quad 3\frac{1}{2}d = 62d$$

$$\text{Now, the whole gain} = \text{£}9 \quad 6s = 8928d$$

$$\text{the number of pairs bought} = 8928 \div 62 = \underline{144} \quad \text{Ans}$$

Examples XLIV

1 A person bought 500 yds of cloth at Rs 7 14s per yard and retailed it at Rs 8 2s per yard, what was his profit?

2 A person gave Rs 200 for 48 cwt of goods, what does he gain by selling them at Rs 5 a cwt?

3 A man buys 35 sheep for Rs 360 and 30 more for Rs 450, what will he gain or lose by selling them at Rs 15 4s each?

4 A merchant bought 35 pieces of cloth measuring on an average 29 yards each at 3s 10½d a yard, and sold them at 5s 7d a yard, what profit did he make?

5 I bought 360 yds of cloth at Rs 2 10s 8p per yard, of which I sold 210 yds at Rs 3 9s 4p per yard, but the article advancing in price, sold the remainder at Rs 4 8s per yard, what did I gain on the whole?

6 I buy 84 books at Rs 1 15s 8p each, and sell them at a profit of Rs 70, what is the selling price of each?

7 A shopkeeper purchases 35 reams of scribbling paper at Rs 7 4s per ream, the carriage of the paper costs Rs 4 12s. He

sells it at $8\text{ s } 8\text{ p}$ a quire with the exception of the outside quires of each ream, which he sells at 5 s a quire Find his gain

8 A grocer gave $\text{Rs } 500$ for 16 cwt 2 qrs 18 lbs of sugar, and he lost $\text{Rs } 72\ 6\text{ s}$ by retailing it, at what rate did he sell it per lb?

9 I buy a number of books at $\text{Rs } 1\ 6\text{ s } 4\text{ p}$ each and sell them at $\text{Rs } 1\ 10\text{ s}$ each If I thereby make a profit of $\text{Rs } 22$, how many books do I buy?

10 A person gives $\text{Rs } 556\ 8\text{ s}$ for a certain number of gallons of wine He sells it at $\text{Rs } 2\ 10\text{ s}$ a gallon, and thereby, makes a profit of $\text{Rs } 56\ 12\text{ s}$ How many gallons does he buy?

11 Find the cost of 20 dozen bottles of wine at $\text{Rs } 2\ 7\text{ s } 8\text{ p}$ per bottle, and if 3 bottles be spoiled, what will the merchant gain by selling the remainder at $\text{Rs } 2\ 10\text{ s } 8\text{ p}$ per bottle?

12 A cabinet dealer bought chairs at $\text{Rs } 11\ 15\text{ s}$ a piece, and lost $\text{Rs } 9\ 12\text{ s}$ by selling each at $\text{Rs } 11\ 2\text{ s}$ How many chairs did he buy?

13 A person lays out $\text{£ } 45\ 9\text{ s } 4\text{ d}$ in spirits at $5\text{ s } 4\text{ d}$ a gallon, 19 gallons leaked out in the carriage, he however sold the remainder at $7\text{ s } 6\text{ d}$ a gallon, what profit did he make?

14 A merchant bought 7 pieces of cloth, each 27 yards, for $\text{£ } 55\ 12\text{ s}$, and sold 56 yards at $5\text{ s } 3\frac{1}{2}\text{ d}$ per yard and the rest at $6\text{ s } 8\text{ d}$ per yard Find his whole gain

15 A merchant laid out $\text{Rs } 633$ in spirits which he bought at $\text{Rs } 6\ 6\text{ s } 8\text{ p}$ a gallon, he retailed it at $\text{Rs } 8\ 4\text{ s}$ a gallon, making a profit of $\text{Rs } 115\ 8\text{ s}$ How many gallons must he have lost by leakage?

182 Barter and Exchange

When we barter we give or take one sort of goods in exchange for another of a different sort which is regarded as an equivalent Hence, to find how much of the first sort be given in exchange for a fixed quantity of the second, we must *first* find the money value of the second sort and *then* find what quantity of the first sort is of equal value

Ex 1 How many pounds of tea at $3\text{ s } 2\frac{1}{2}\text{ d}$ a lb must a grocer give in exchange for 35 yards of cloth at $12\text{ s } 4\frac{1}{2}\text{ d}$ a yard?

$12\text{ s } 4\frac{1}{2}\text{ d}$	5949	$3\text{ s } 2\frac{1}{2}\text{ d}$	$154)20790(135$
$\frac{12}{148\text{ d}}$	$\frac{35}{207909}$	$\frac{12}{38\text{ d}}$	$\frac{154}{539}$
$\frac{4}{5949}$		$\frac{4}{1549}$	$\frac{462}{770}$
			$\frac{770}{770}$

\therefore the number of lbs of tea = 135 Ans

Ex 2 What weight of sugar at $3a$ a lb must be given in exchange for a chest of tea weighing 84 lbs at $Re\ 1\ 9a$ a lb?

$$\begin{array}{r} Re\ 1\ 9a \\ \underline{16} \\ 25a \end{array} \qquad \begin{array}{r} 25a \\ \underline{84} \\ 2100a \end{array}$$

$$\begin{array}{r} 3)2100 \\ \underline{700} \end{array}$$

the number of lbs of sugar = 700 Ans

Examples XLV

1 How many dollars of $4s\ 1\frac{1}{2}d$ each must be given in exchange for 4950 thalers of $2s\ 11\frac{1}{2}d$ each?

2 How many francs of $9\frac{1}{2}d$ each will be given in exchange for 475 thalers at $2s\ 11\frac{1}{2}d$ each?

3 How many lbs of tea at $Re\ 1\ 9a\ 8p$ a lb must be given in exchange for 46 yards of silk at $Rs\ 4\ 0a\ 2p$ a yard?

4 A man exchanges 45 sheep at $Rs\ 22\ 14a$ each and 37 pigs at $Rs\ 36\ 12a$ each for 13 oxen at $Rs\ 173\ 4a$ each, the difference being paid or received in money, how much does he pay or receive?

5 The Calcutta rupee is worth $1s\ 11\frac{1}{2}d$ each, how many must be given for £9895 16s 8d?

6 How much coffee at $1s\ 10\frac{1}{2}d$ a lb should be given in exchange for 72 lbs of tea at $3s\ 4d$ per lb?

7 How many yards of cloth worth $3s\ 7\frac{1}{2}d$ a yard must be given in exchange for 144 yards worth $18s\ 1\frac{1}{2}d$ a yard?

8 How many Rubles at $3s\ 4\frac{1}{2}d$ each are equal in value to 378 Napoleons, at $15s\ 9\frac{1}{2}d$ each?

9 What quantity of tea at $Rs\ 2\ 6a\ 6p$ per lb, must be given in exchange for 5 cwt 2 qrs of sugar at $Rs\ 3\ 15a$ per stone?

10 A person exchanged 18 dozen of wine for a gold snuff-box weighing 8 oz 13 dwts 10 grs valued at £4 10s an oz. What did he value his wine at per dozen?

11 A gives B 98 gallons of brandy worth $Rs\ 12\ 12a$ a gallon, and gets in return $Rs\ 409\ 8a$ and 576 yards of cloth, what is the value of the cloth per yard?

12 A man sold 53 horses at $Rs\ 168\ 11a\ 4p$ each, and with the money he received for them and $Rs\ 990$ more he bought 355 cows and a certain number of calves, he gave for 198 of the cows $Rs\ 22\ 2a$ a head, and for the rest of the cows $Rs\ 18\ 6a\ 8p$ a head, and for the calves $Rs\ 14\ 6a$ a head. How many calves did he buy?

183 Allotment

By **allotment** we divide a given quantity in a certain way into a proposed number of parts and thus ascertain the actual amount of each part

Ex 1 How many sovereigns, half-sovereigns, crowns, florins, shillings, six-pences and three-pences, and of each an equal number are there in £67 16s 3d?

s	d	£	s	d
20	0	67	16	3
10	0	20		
5	0	1356s		
2	0	12		
1	0	16275d		
	6	465) 16275 (35		
	3	1395		
38s	9d	2325		
12		2325		
465d				

Since every collection of one of each of these coins amounts to 38s 9d, therefore there will be as many coins of each kind as £67 16s 3d, contains 38s 9d. Hence the number of coins of each kind = 35 Ans

Ex 2 An equal number of men, women and boys earned Rs 556 8a in 6 weeks, each man earned Re 1 2a 8p a day, each woman 10a and each boy 6a 8p, how many were there of each?

Rs	a	p	= a man's daily earning	Rs	a	Rs	a
1	2	8		92	12	556	8
10	0		= a woman's	16		16	
6	8		= a boy's	1484a		8904a	
Rs 2	3	4	= total daily earnings			1484) 8904 (6	
		7				8904	
Rs 15	7	4	= weekly earnings				
		6					
Rs 92	12	0	= earnings of 6 weeks				

no of each sort = 6 Ans

Examples XLVI

1 Divide £39 into four equal numbers of guineas, half-guineas, crowns and half crowns respectively

2 An equal number of gold-mohurs, rupees, eight-anna pieces, four-anna pieces, two anna pieces and pice amount to Rs 447 4a 1p, how many of each sort are there?

3 An equal number of guineas, pounds, half guineas, crowns, half crowns and six pences amount to £714, how many of each are there?

4 An equal number of rupees, half-rupees, quarter-rupees, two anna pieces, double-paisas and paisas amount to Rs 803 5a 2p, find the number of each

5 At the end of a week £54 3s is paid in wages to an equal number of men, women and boys a man is paid 4s 6d, a woman 3s 3d and a boy 1s 9d a day, how many of each class are there?

6 Tithes of the value of £448 10s are commuted for an equal number of bushels of wheat, barley and oats, how many bushels of

each kind will be received when wheat is sold at 7s 2d a bushel, barley at 4s 9d, and oats at 3s 5d ?

7 Rs 750 is paid in wages at the end of the week to a certain number of men, twice as many women, and three times as many children, each man earns Rs 2 1a 4p a day each woman Re 1 6a and each child Re 1 2a 8p, how many children are there ?

8 A bag contains a certain number of rupees, twice as many half-rupees, five times as many quarter rupees, and eight times as many two anna pieces, and the value of the whole sum in the bag is Rs 272 Find the number of each

9 One farm produced 111 times as much rice as another, both farms produced 1776 mds 10 sr, how much did the smaller farm produce ?

10 How many packets of tea of 1 lb 8 oz and 1 lb 12 oz respectively, an equal number of each, can be made out of a chest of tea, in which the tea weighs 1 cwt 1 q1 3 lbs 1

184 Mixtures

When several articles of the same kind but of different qualities or value are mixed together to form a **compound**, it is called a **mixture**. The parts forming the compound are called **ingredients** or **components** of the compound

(1) Given the quantity and price of each of the component parts, to find the price of the mixture

Ex 1 A mixture is made of 9 gallons of spirit at Rs 6 4a per gal, 16 gallons at Rs 9 6a and 90 gallons at Rs 11 2a, what is the value of a gallon of it ?

$$\begin{aligned}\text{The cost of 9 gals} &= \text{Rs } 6 \ 4a \times 9 = \text{Rs } 56 \ 4a \\ 16 \text{ gals} &= \text{Rs } 9 \ 6a \times 16 = \text{Rs } 150 \ 0a \\ 90 \text{ gals} &= \text{Rs } 11 \ 2a \times 90 = \text{Rs } 1001 \ 4a\end{aligned}$$

$$\text{the cost of 115 gals} = \text{Rs } 1207 \ 8a$$

$$\therefore \text{the cost of 1 gal} = \text{Rs } 1207 \ 8a \div 115 = \text{Rs } 10 \ 8a \quad \text{Ans}$$

Ex 2 A man buys 16 lbs of tea at Rs 2 2a per lb, 12 lbs at Rs 2 5a 4p per lb, and 24 lbs at Rs 2 6a 10p per lb. At what price per lb must he sell the mixture, so as to gain Rs 35 12a on the whole ?

$$\begin{aligned}\text{The cost of 16 lbs} &= \text{Rs } 2 \ 2a \times 16 = \text{Rs } 34 \ 0a \\ 12 \text{ lbs} &= \text{Rs } 2 \ 5a \ 4p \times 12 = \text{Rs } 28 \ 0a \\ 24 \text{ lbs} &= \text{Rs } 2 \ 6a \ 10p \times 24 = \text{Rs } 58 \ 4a\end{aligned}$$

$$\therefore \text{the cost of } 52 \text{ lbs} = \text{Rs } 120 \ 4a$$

$$\text{Gain} = \text{Rs } 35 \ 12a$$

$$\text{selling price of 52 lbs} = \text{Rs } 120 \ 4a + \text{Rs } 35 \ 12a = \text{Rs } 156 \ 0a$$

$$\text{selling price per lb} = \text{Rs } 156 \div 52 = \text{Rs } 3 \quad \text{Ans}$$

- (2) To find the quantity to be added to a mixture under certain conditions

Ex 3 A pipe of wine containing 126 gallons is bought for £112, how much water must be added to it to allow of its being sold at 17s 6d a gallon?

$$£112 = 112 \times 20 \times 12d = 26880d, \quad 17s \ 6d = 210d$$

Now the quantity sold for £112 at 17s 6d a gal = $(26880 - 210)$ or 128 gallons

the quantity of water mixed = $(128 - 126)$ or 2 gallons *Ans*

Ex 4 If a person gives Rs 556 8a for 184 gallons of wine, how much water must be added to it, if he wishes to sell it at Rs 2 10a a gallon and make a profit of Rs 36 12a?

$$\begin{aligned} \text{The selling price of the mixture} &= Rs \ 556 \ 8a + Rs \ 36 \ 12a \\ &= Rs \ 593 \ 4a = 9492a \end{aligned}$$

Also the selling price per gal = Rs 2 10a = 42a

the quantity sold = $(9492 - 42)$ or 226 gallons

the quantity of water added = $(226 - 184)$ or 42 gallons *Ans*

Examples XLVII

1 A grocer mixes 40 lbs of tea at Re 1 3a a lb, 48 lbs at Re 1 5a 6p a lb and 64 lbs at Re 1 9a 10p a lb, find the value of 1 lb of the mixture

2 A grocer mixes 3 cwt 24 lbs of sugar at 6½d per lb with 2 cwt 64 lbs at 4½d, at what price per lb must he sell the mixture so as not to lose by the sale?

3 A tea merchant mixes 25 lbs of tea at 14a a lb, 40 lbs at Re 1 3a 4p, and 27 lbs at Re 1 9a 4p, at what rate per lb must he sell the mixture, so as to gain Rs 23 2a on the transaction?

4 How many lbs of tea-dust (which cost him nothing) must be put in the above mixture, to enable him to sell the tea at Re 1 3a 4p per lb and gain at the same time Rs 4 4a on the transaction?

5 A trader buys 756 cwt of sugar at Rs 19 7a 8p per cwt with which he mixes 1921 cwt of sugar which cost him Rs 21 per cwt, at how much per lb must he sell the mixture in order to make a profit of Rs 7396 1a 4p?

6 A grocer mixes 19 lbs of tea at 1s 10½d per lb, 26 lbs at 2s 3½d per lb, and 27 lbs at 2s 6½d per lb, at how much per lb must he sell the mixture so as to gain £2 3s 4d on his outlay?

7 A spirit merchant mixes 26 gallons of wine at 12s 3d a gallon with 39 gallons at 13s 4d a gallon, how many gallons of water must he add to the mixture so as to sell it at 10s 9d a gallon?

8 A man bought 150 eggs at 2 a penny, 150 more at 3 a penny, and mixed them and sold the whole at 5 for 2d, how much does he lose?

9 A grocer buys 4 cwt of sugar at $6d$ per lb, and 8 cwt at $4\frac{1}{2}d$ per lb. He sells 6 cwt at $5\frac{1}{2}d$ per lb, at what rate per lb must he sell the remainder so as neither to gain nor lose?

10 A merchant bought 84 gallons of whisky at Rs 8 6a a gallon, and sold it at Rs 8 4a a gallon, making a profit of Rs 105. How many gallons of water did he add to the whisky?

185 Income and Expenditure

Income including taxes and other rates is called **gross income**, but excluding these, it is **net income**. What a man lays by out of his income after meeting all necessary expenses, is called his **savings**.

Ex 1 On the reduction of the income tax from $9d$ in the pound to $4d$, a person saves £29 15s 10d, find his gross income.

$$£29\ 15s\ 10d = 7150d = \text{savings}$$

He saves $(9-4)$ or $5d$ in every £ of his income

$$\text{gross income required} = £(7150 \div 5) \text{ or } £1430 \quad \text{Ans}$$

Ex 2 A man has a yearly income of Rs 4867 8a and sets aside Rs 630 for charity, insurance and other purposes. What is the greatest sum he can spend per week, without getting into debt?

$$\begin{array}{r}
 \text{Rs} \quad a \quad p \\
 4867 \quad 8 \quad 0 = \text{yearly income} \\
 630 \quad 0 \quad 0 = \text{charity, \&c} \\
 \hline
 4237 \quad 8 \quad 0 = \text{yearly expenditure} \\
 13 \overline{) 4237 \quad 8 \quad 0} \\
 \underline{13 \quad 1059 \quad 6 \quad 0} \\
 \text{Rs } 81 \quad 7 \quad 10 \quad 8p
 \end{array}$$

Hence we see that he may spend Rs 81 7a 10p every week, and have $8p$ over at the end of the year. If he spends Rs 81 8a per week he will run into debt.

Examples XLVIII

1 A man's annual income is Rs 10,000 and his daily expenses are Rs 18 10a 4p, how much does he save in 9 years?

2 A man's income in the year 1895 was Rs 5250, out of which he saved Rs 1691 4a, what was his average daily expenditure?

3 A man spends Rs 105 14a in a week, how much does he spend in a year of 365 days?

4 If a person spends in 4 months, as much as he earns in three, how much can he lay by annually, supposing that he earns Rs 2505 every 6 months?

5 What annual income would enable a person to spend 8s 9d a day and save £7 16s 10½d, every calendar month?

6 If a person has an income of £535 17s 6d a year, and he spends daily £1 3s 10½d, how much will he save at the end of the year?

7 Find the salary of a person who pays £7 9s 4d income tax, when the tax is 7d in the pound

8 A person after paying an income tax of 4p in the rupee, has Rs 8567 11a 4p remaining, find his gross income

9 If a person's yearly income be £65 12s 6d and he lay by £20 a year, how much does he spend per day?

10 A person has an income of Rs 6706 12a 6p, and for the first 7 months he spends on an average Rs 588 6a 6p a month, how much must he spend during each of the remaining 6 months, so as not to run into debt?

186 Division of money

When a given sum of money is divided among a number of persons in a proposed way, the amounts they severally receive are called their respective shares

Ex 1 Divide £16 5s 6d among A, B and C, so that A may have £1 2s 6d more than B, and B 16s 9d more than C

Here B has	£	s	d	more than C,
and A	0	16	9	
A	1	2	6	more than B
	1	19	3	more than C

Now, if we take away these sums, to be subsequently given to B and A respectively, their shares will be equal to that of C

Hence we have

£	s	d	£	s	d	£	s	d	£	s	d
16	9		16	5	6	4	9	10	4	9	10
1	19	3	2	16	0	16	9		1	19	3
£2	16	0	3)13	9	6	£5	6	7	£6	9	1
			£4	9	10						

∴ A's share = £6 9s 1d, B's share = £5 6s 7d,
and C's share = £4 9s 10d

Ex 2 Divide Rs 117 11a among A, B and C, so that A may receive twice as much as B, and B twice as much as C

If C's share is 1, B's share is 2 and A's share is 4

Now, 1+2+4=7,

$$\begin{array}{r} 7)Rs\ 117\ 11a \\ Rs\ 16\ 13a \end{array}$$

C's share	= Rs 16	13a	} Ans
B's share = Rs 16	13a × 2 = Rs 33	10a	
and A's share = Rs 16	13a × 4 = Rs 67	4a	

Ex 3 Divide Rs 2415 among *A*, *B* and *C* in such a way that for every Rs 20 that *A* gets, *B* gets Rs 15, and *C* gets Rs 11, how much does each receive?

$$20 + 15 + 11 = 46,$$

$$\begin{array}{r} 45 \text{ Rs } 2415 \\ \hline \text{Rs } 52 \quad 8a \end{array}$$

$$\left. \begin{array}{l} A's \text{ share} = \text{Rs } 52 \quad 8a \times 20 = \text{Rs } 1050 \\ B's \text{ share} = \text{Rs } 52 \quad 8a \times 15 = \text{Rs } 787 \quad 8a \\ \text{and } C's \text{ share} = \text{Rs } 52 \quad 8a \times 11 = \text{Rs } 577 \quad 8a \end{array} \right\} \text{Ans}$$

Examples XLIX

1 Divide Rs 24 9a 4p among *A*, *B* and *C*, so that *B* may have Rs 3 5a 4p more than *A*, and *C*'s share may be double of *B*'s

2 Divide Rs 73 4a 6p between two men so that one may receive as much again as the other

3 Divide Rs 1845 9a 6p equally among 39 persons, and supposing 15 of them to have received their portions, and of the rest only 21 to appear, how much might be given to each of these?

4 Divide £20 2s 6d into two sums of money, one of which contains as many half-crowns as the other contains shillings

5 Divide Rs 24515 among *A*, *B* and *C*, so that *A* may have Rs 1786 12a more than *B*, and *C* Rs 3257 5a less than *B*

6 Divide Rs 2509 14a among *A*, *B* and *C*, so that *B* may receive 3 times, and *C* 5 times, as much as *A*

7 Divide £189 5s 7½d among 3 men, so that one of them may have 15 guineas more than either of the other two

8 A purse and the money it contains are worth Rs 19 4a, and the money is 10 times the value of the purse, how much does the purse contain?

9 Divide Rs 690 between *A*, *B* and *C*, so that where *A* receives Rs 10, *B* may receive Rs 30, and where *B* receives Rs 20, *C* may receive Rs 50

10 The sum of Rs 473 6a 4p has to be divided among 5 persons, so that the first has 20 shares, the second 17, the third 12, the fourth 8, and the fifth 5, how much will each receive?

11 Divide £119 16s 3d among 36 persons, in such a way that 17 of them may each receive 18s 9d more than each of the rest

12 Divide Rs 68427 3a 4p among 3 persons, so that the first shall have Rs 5687 2a 8p more than the second, and the second Rs 7289 1a 4p more than the third

187 Men, Women and Boys

Ex 1 Divide Rs 156 4a among 7 men, 9 women and 11 boys,

so that each man may receive three times as much as a boy, and each woman twice as much as a boy

The 7 men will receive as much as 7×3 or 21 boys and the 9 women as much as 9×2 or 18 boys, therefore 7 men, 9 women and 11 boys will receive as much as $21 + 18 + 11$ or 50 boys. Thus,

7 men	= 21 boys	50 {	$\begin{array}{r} Rs \quad a \\ 5 \overline{)156 \quad 4} \\ 10 \overline{)31 \quad 4} \\ \hline Rs \quad 3 \quad 2 \end{array}$
9 women	= 18		
11 boys	= 11		
	<u>50</u>		

Hence a boy's share = $Rs \quad 3 \quad 2a$,

a woman's = $Rs \quad 6 \quad 4a$, and a man's = $Rs \quad 9 \quad 6a$

Ex. 2 A man and a woman together have $Rs \quad 40 \quad 6a \quad 8p$, a woman and a boy together have $Rs \quad 30 \quad 8a$, a man and a boy together have $Rs \quad 35 \quad 7a \quad 6p$, find how much a man, a woman and a boy together have

Here, adding the three given items, we have

twice a man's money + twice a woman's money + twice a boy's money
 $= Rs \quad 40 \quad 6a \quad 8p + Rs \quad 30 \quad 8a + Rs \quad 35 \quad 7a \quad 6p = Rs \quad 106 \quad 6a \quad 2p$

a man + a woman + a boy together have $Rs \quad 106 \quad 6a \quad 2p \div 2$
 $= \underline{Rs \quad 53 \quad 3a \quad 1p} \quad Ans$

Examples I

1 Divide £2 10s 10¹d between 3 men and 2 women, giving to each of the men 3 times as much as to each of the women

2 A gentleman divided $Rs \quad 103 \quad 2a$ among 12 men, 16 women and 30 children, he gave to each man twice as much as to each woman, and to each woman three times as much as to each child. What did each woman receive?

3 Divide $Rs \quad 3993 \quad 8a$ among one man, one woman and 15 boys in such a way that the man's share is 10 times and the woman's share 3 times as much as that of each boy, what is the value of the share of each?

4 Divide $Rs \quad 5501 \quad 9a$ among 4 men, 6 women and 8 boys, giving to each man double that of a woman and to each woman triple that of a boy

5 Divide £15 6s among 12 men, 17 women and 26 children, in such a way that a man shall receive 3 times as much as a child and a woman twice as much as a child, what does a woman receive?

6 Divide $Rs \quad 1151 \quad 4a$ among 20 women and 25 men, so that each woman may receive $Rs \quad 7 \quad 8a$ more than each man, how much will each woman receive?

7 If 20 men, 40 women and 50 children receive $Rs \quad 3500$ among them for 7 weeks' work, and 2 men receive as much as 3 women or 5 children, what sum does a woman receive per week?

8 The weekly wages at a mill amount to Rs 1862 In the mill a certain number of women are employed at Re 1 6a 8p, a day, five times as many men at Rs 2 12a a day, and 6 times as many boys at Re 1 2a 8p a day, how many men are employed?

9 A and B together have Rs 48 14a 9p, B and C together have Rs 45 10a 6p, A and C together have Rs 54 8a 11p, how much has C?

10 A goat and a lamb are together worth Rs 6 10a a goat and a calf are together worth Rs 10 4a 8p, and a calf and a lamb are together worth Rs 8 5a 6p, find the price of a goat, of a lamb and of a calf

Examples worked out

Ex 1 A man has a certain number of pice, twice as many two anna pieces, three times as many four anna pieces and four times as many rupees If the total amount be Rs 501 9a, find the number of coins of each kind

Here, 1 pice + 2 two anna pieces + 3 four anna pieces + 4 rupees = $(1 + 16 + 48 + 256)$ pice = 321 pice, and Rs 501 9a = 32100 pice

the number of pice = $(32100 \div 321)$ or 100

Hence, no of two anna coins = $2 \times 100 = 200$, the no of four anna coins = $3 \times 100 = 300$, and the no of rupees = $4 \times 100 = 400$ Ans

Ex 2 A man died on June 2 Monday, 1890, having lived 23025 days exclusive of the day of his death Find the day and date of his birth

A year = 365 days, therefore $23025 \text{ days} \div 365 = 63 \text{ years } 30 \text{ days}$ Now in these 63 years, 16 are leap years (which = 366 days), therefore $23025 \text{ days} = 63 \text{ years} + (30 - 16) \text{ days}$ or 63 years 14 days

Again, $1890 - 63 = 1827$, and reckoning 14 days backwards from June 1, we come to May 19

Hence, the man was born on May 19, 1827

Now 23025 divided by 7 gives a remainder 2, therefore he was born on Saturday, reckoning 2 days backwards from Sunday

Ex 3 The total expenses of a family when rice is at Rs 4 per maund are Rs 55, when rice is at Rs 3 12a per maund, they are Rs 52 8a (other expenses remaining the same), find his total expenses when rice is at Rs 4 4a per maund

Here, a decrease of $(Rs 4 - Rs 3 12a)$ or 4a per md in the price of rice makes a decrease of $(Rs 55 - Rs 52 8a)$ or Rs 2 8a = 40a in the family expenses

Hence, quantity of rice consumed by the family = $\frac{40}{4}$ or 10 mds Therefore the expenditure on rice = Rs (4×10) or Rs 40 and the other expenses = $Rs (55 - 40) = Rs 15$

Now, the price of 10 mds at Rs 4 4a per md = Rs 4 4a \times 10 = Rs 42 8a

Hence the required expenses = Rs 42 8a + Rs 15 = Rs 57 8a

Ex 4 A corn merchant mixed 10 mds of rice worth Rs 4 per md with a certain quantity worth Rs 3 8a per md, and selling the mixture at Rs 3 12a per md gained Rs 10 on the whole. How many mds of the second kind did he mix?

By selling the first sort of rice at Rs 3 12a per md he incurs a loss of (Rs 4 - Rs 3 12a) or 4a per mround, therefore the loss in 10 mds = $10 \times 4a = 40a =$ Rs 2 8a

Now, gain per md on the second sort = (Rs 3 12a - Rs 3 8a) = 4a and as he shall have to make altogether Rs 10 + Rs 2 8a or Rs 12 8a = 200a

\therefore the quantity required = $\frac{200a}{4a}$ or 50 mds *Ans*

Ex 5 A grocer mixed milk worth Rs 7 per md with twice as much worth Rs 5 8a per md and having sold the mixture at Rs 6 4a per md, cleared Rs 10 8a on the whole. How much did he mix of each sort?

The cost of 1 md of first + 2 mds of second = Rs 7 \times 1 + Rs 5 8a \times 2 = Rs 18

\therefore the cost of 1 md of the mixture = Rs 18 \div 3 = Rs 6

The gain per md = Rs 6 4a - Rs 6 = 4a and the total gain is Rs 10 8a = 168a

\therefore the whole mixture contains $\frac{168a}{4a}$ or 42 mds

Now, $1 + 2 = 3$, quantity of first sort = $42 \div 3 = 14$ mds } *Ans*
and second = $14 \times 2 = 28$ mds

Ex 6 A supply of water suffices for 60 days if 10 mrounds leak off every day, but only for 55 days if 15 mrounds leak off daily. Find the total quantity of water in the supply

In the first case 60×10 or 600 mds leak off altogether, while in the second 55×15 or 825 mds leak off

for (60 - 55) or 5 days' use (825 - 600) or 225 mds of water are required

\therefore for daily use (225 \div 5) or 45 mds of water are required

Now, taking the first case, we find that the supply lasts for 60 days, and in that time (60 \times 45) or 2700 mds are required for use, and 60 \times 10 or 600 mds leak off

Hence the total quantity reqd. = (2700 + 600) mds = 3300 mds *Ans*

Ex 7 On changing 3 four-anna pieces, I received 36 coins in single and double pice. How many did I get of each?

Here 3 four-anna pieces = 12a = 48 pice

Now had all been single pice, I would have received 48, but as I received $(48 - 36)$ or 12 single pice less, and the difference between a double and a single pice is one pice,

$$\left. \begin{array}{l} \text{the number of double pice} = 12 \\ \text{and} \quad \text{single pice} = 36 - 12 = 24 \end{array} \right\} \text{Ans}$$

Ex 8 A man has three estates, and his incomes from the second and third are respectively twice and thrice as much as from the first. He has to pay an income-tax of 8 pies in the rupee for the first, 12 in the rupee for the second, and 12 4p in the rupee for the third. If the total income tax be Rs 80, how much income does each estate yield?

Supposing his income from first to be $Rs\ 1$, his income from second = $Rs\ 2$, and from third $Rs\ 3$

Also, for the first he should have to pay (1×8) or $8p$ in the *Re*
 second (2×12) or $24p$ in the *Re*
 third (3×16) or $48p$ in the *Re*

the total tax amounts to $(8+24+48)$ or 80 pias in the rupee

Also Rs 80 = $(80 \times 16 \times 12)$ pies, the total tax

Hence, income from first = Rs $(80 \times 16 \times 12 - 80) = \text{Rs } 192$
 second = $192 \times 2 = \text{Rs } 384$
 third = $192 \times 3 = \text{Rs } 576$ } Ans

Ex. 9 *A* gave *B* as many sovereigns as is expressed by the sum of all the numbers that can be formed by different arrangements of the digits 2, 4 and 7 taken all together, and *B* gave *A* as many six-pences as is expressed by the sum of all the numbers that can be formed by different arrangements of the figures 4, 5, 8 and 9 taken all together. Who is the gainer and by how much?

The sum of all the numbers that can be formed by different arrangements of the digits 2, 4 and 7 taken all together = $2 \times (2+4+7) \times (10^2 + 10 + 1) = 2 \times 13 \times 111 = 2886$ [See Ex 9, Page 70]

Similarly, the sum of the numbers formed by the different arrangements of the digits 4, 5, 8 and 9 taken all together = $6 \times (4 + 5 + 8 + 9) \times (10^3 + 10^2 + 10 + 1) = 6 \times 26 \times 1111 = 173316$

Hence A gave B 2886 sov or £2886, and B gave A 1733¹⁶ six-pences or £4332 18s

Therefore A is the gainer by $(£4332\ 18s - £2886)$ or $£1446$

Miscellaneous Examples II

1 From 261 times Rs 352 1a 4p take Rs 90892 8a and divide the remainder by 89

2 How many Napoleons of 15, 9½ ℓ each can be obtained for 5685 thalers of 2s 11½ ℓ each?

3 How many Nobles are equivalent to £195 13s 4d?

4 13 rupees, 9 half crowns and 17 three penny pieces amount to £2 16s, find the value of a Rupee Find the value of a lac of rupees in English money (1 lac=1,00,000)

5 A dealer bought 9 horses at Rs 118 13a 4p each, one died and the others he sold at a profit on each of Rs 21 1a 8p Find his gain

6 The value of a mark being 13s 4d, and that of a moidore 27s, shew that there are twice as many farthings in 57 marks and 57 moidores, as there are drams in 1 cwt 3 qrs 19 lbs 8 oz 8 drs of sugar

7 To a certain stock-in trade *A* and *B* together contributed Rs 22 10a, *B* and *C* together Rs 25 8a and *A* and *C* together Rs 27 6a, how much did each contribute?

8 A boy receiving 4a per week has 2a stopped every third week, if there are 39 weeks in a school year, how much does he realize in 4 years?

9 *A* has Rs 1002 7a 8p and *B* 128786 pies, if *A* receive from *B* 22222 pies and *B* from *A* Rs 115 15a 6p, how much will *A* have more than *B*?

10 Of 21 people 13 lose Rs 1163 13a 6p each and 8 lose Rs 930 1a 9p each What is the average loss per man?

11 *A* and *B* having an equal share in a heap of potatoes containing 86 maunds, *A* takes 24 mds and *B* the rest, paying *A* Rs 27 11a 4p What is the worth of a maund of potatoes?

12 A grocer's bill amounts to Rs 1897 8a It happens to be made up of equal sums for tea at Re 1 14a 8p per seer, sugar at 4a per seer, rice at 3a per-seer, and coffee at 11a per seer How many seers are there of each sort?

13 A person mixes together 10 lbs of tea at Re 1 4a per lb, 12 lbs at Re 1 6a and 14 lbs at Re 1 8a per lb He reserves 6lbs of the mixture for himself and sells the remainder at Re 1 13a per lb How much does he gain?

14 A manufacturer employs 50 men and 35 boys who work respectively 12 and 8 hours a day during 5 days of the week, and half the time the other day, each man receives 4a and each boy 1a 4p an hour What is the whole amount of wages for a year?

15 What quantity of water must I add to a pipe of wine which costs Rs 900, to reduce its price to Rs 5 a gallon?

16 The yearly expense of a school is Rs 18993 11a, there is an endowment yielding Rs 4850 15a and subscriptions Rs 743 The rest is to be made up by the fees of the pupils of whom there are 217, what must each of them pay on an average?

17 In what time will a tradesman, who gains $10\text{a } 8\text{p}$ a day and spends 5a of it, be able to pay off a debt of $\text{Rs } 208 \text{ } 9\text{a } 8\text{p}$?

18 A man's weekly income is $\text{Rs } 18 \text{ } 7\text{a}$ and his quarterly expenditure is $\text{Rs } 182$. How much will he save at the year's end? (A year = 52 weeks)

19 I buy 80 lbs of black tea at $\text{Rs } 2 \text{ } 2\text{a}$ per lb and 20 lbs of green at $\text{Rs } 2 \text{ } 12\text{a}$ per lb and mix them, at what rate must I sell the mixture so as to gain $1\text{a } 4\text{p}$ in the rupee?

20 Divide two fields, one of 6 ac 3 po 13 sq yds, the other of 4 ac 37 po 27 sq yds, between A, B and C, so that A's no of ro = B's no of sq po = C's no of sq yds

21 September 17, 1893, was Sunday. What day of the week was September 17, 1891?

22 A wine merchant bought 2 pipes of wine at $\text{£}2 \text{ } 13\text{s } 4\text{d}$ per gallon. How much water must he mix with it that by selling a gallon of the mixture for $\text{£}2 \text{ } 6\text{s } 8\text{d}$, he may gain on the whole $\text{£}14$?

23 A factor bought 25 pieces of cloth for $\text{Rs } 185000$ at $\text{Rs } 4 \text{ } 10\text{a}$ per yard. How many yards are there in each piece?

24 A house and its furniture are together worth $\text{£}3367 \text{ } 2\text{s } 6\text{d}$, if the house is worth 8 times the furniture. What is the house worth?

25 A man's total expenses are $\text{Rs } 44$, when rice sells at $\text{Rs } 2 \text{ } 8\text{a}$ per maund, and $\text{Rs } 46 \text{ } 4\text{a}$ when rice sells at $\text{Rs } 2 \text{ } 11\text{a}$ per md. What are his expenses when rice sells at $\text{Rs } 3 \text{ } 3\text{a}$ per maund?

26 Two persons buy mangoes at 16 per rupee, one sells at 12 per rupee and the other 16 for $\text{Rs } 1 \text{ } 4\text{a}$. How much profit does one make more than the other?

27 A man spending daily $\text{Rs } 2 \text{ } 10\text{a } 6\text{p}$ lays by $\text{Rs } 150 \text{ } 2\text{a}$ in the year 1897, find his daily income.

28 I received 320 pieces in half-rupees and quarter-rupees in exchange for 100 rupees. How many of each did I get?

29 A and B gave equal sums in buying 15 horses and 22 cows. A took 5 horses and 17 cows and B the rest. If a horse cost $\text{Rs } 56 \text{ } 8\text{a}$ and a cow $\text{Rs } 35 \text{ } 10\text{a}$, how should they settle the account?

30 A man was born on the 15th of May 1762, and died on the 17th of June 1825. How many days did he live, exclusive of the day of his death?

31 A goldsmith manufactured 2 lbs 3 dwts 8 grs of gold into rings, each containing 9 dwts 16 grs, he sold the rings at $\text{Rs } 25$ each, how much did he receive for them?

32 A piano, table and carpet cost $\text{Rs } 632 \text{ } 12\text{a}$, the piano and table cost $\text{Rs } 547 \text{ } 6\text{a}$, and the table and carpet cost $\text{Rs } 260 \text{ } 2\text{a } 8\text{p}$. Find the price of each.

33 A grocer buys 40 lbs of tea at $Rs\ 1\ 12a$ per lb and also some cheaper tea, he mixes the two kinds of tea and by selling all the tea for $Rs\ 236\ 4r$ at $Rs\ 1\ 11a$ per lb gains $Rs\ 32\ 14a\ 8p$ on his outlay, how many lbs of the cheaper tea does he buy, and at what price per lb?

34 1 twice A 's money = 3 times B 's money, and the difference of their moneys is $Rs\ 12\ 10a$. How much has each?

35 A bag contains a certain number of rupees, twice as many half rupees, 4 times as many quarter rupees and 8 times as many two-anna pieces, and total amount in the bag is $Rs\ 100$. How many of each are there?

36 A , B and C contributed equal sums in purchasing 22 horses, 28 cows and 56 sheep. A took 7 horses, 9 cows and 19 sheep, B took 8 horses, 8 cows and 17 sheep, and C the rest. If the price of a horse be $Rs\ 68\ 8a$, of a cow $Rs\ 44\ 10a$ and of a sheep $Rs\ 7\ 6r$, which of them shall have to pay and which to receive, and how much?

37 A landowner has three estates. The first estate yields an income of $Rs\ 3000$, the second $Rs\ 4200$ and the third $Rs\ 6250$. If the rate of tax be $1a$ in the rupee for the first, $1a\ 4p$ in the rupee for the second and $1a\ 3p$ in the rupee for the third, how much tax has he to pay altogether?

38 Divide $Rs\ 7890$ among A , B and C in such a way that A may receive $Rs\ 125$ more than twice as much as B , and C $Rs\ 250$ more than thrice as much as B .

39 A certain weight of gold worth $Rs\ 20\ 14a\ 6p$ per tola is mixed with an equal weight worth $Rs\ 18\ 6a\ 6p$ per tola. Determine the weight of gold, so that by selling the mixed gold at $Rs\ 19\ 14a\ 6p$ per tola, a goldsmith may clear $Rs\ 12\ 8a$ on the whole.

40 In making 50 benches, the cost of each foot wood is $Rs\ 1\ 2a$, for labour $13a$, for polish $2a$ and for screws $1a$. How much is gained on each bench by selling the whole lot for $Rs\ 112\ 8a$?

41 The 15th of May 1890 was Thursday. What day of the week was the 27th April 1790?

42 The cost of maintaining a family is $Rs\ 122\ 8a$ when milk sells at $2a$ per seer, and $Rs\ 125\ 12a$ when milk sells at $2a\ 3p$ per seer. Find the monthly consumption of milk in the family and the amount of other expenses, supposing the latter to be unchanged.

43 A besieged garrison has a supply of water for 50 days. Owing to a leak, however, in the bottom of the reservoir, 5 gallons waste every day, and then the supply suffices for 10 days less. Find for how many days the supply would suffice if 20 gallons leak off every day.

44 A *gowala* mixes 12 mds 16 sr of milk at Rs 6 9a per md with 22 mds 24 sr at Rs 7 8a per md. He then adds 1 md 20 sr of water and sells the mixture at 6 seers per rupee. How much does he gain or lose?

45 Divide Rs 10256 12a among three men, so that the first shall get Rs 1251 4a more than the second, and Rs 152 less than the third.

46 8 men, 16 women and 24 boys earned Rs 136 in 8 days. A woman earns daily 2a more than a boy, and a man daily earns as much as a woman and a boy together. Find how much a man, a woman and a boy daily earn.

47 If 50 pieces of coin consisting of single and double pice make up 1 rupee, find the number of each coin.

48 A man died on the 7th August, Thursday, 1890, having lived 21000 days (exclusive of the day of his death). Find the day and date of his birth.

49 A certain English landowner has three estates, for which he has to pay a total tax of £180. His income from the second and third estates are respectively twice and four times his income from the first. The rates of tax for the three are respectively 1s 2d, 1s 3d and 1s 4d in the £. Determine his income from each estate.

50 A pays B as many rupees as is expressed by the sum of the numbers formed by all the different arrangements of the figures 2, 3 and 4 taken all together, and B pays A as many double pice as is expressed by the sum of the numbers formed by the figures 1, 2, 3 and 4 taken all together and arranged in all possible ways. Who shall be the gainer and by how much?

51 Divide Rs 51 10a among 8 boys, 4 women and 3 men in such a manner that a woman shall receive 2a more than twice as much as a boy, and a man 4a more than as much as a boy and a woman together.

52 A man died on the 1st of August, Friday morning, 1890. He had lived 10000 days. Find the date and day of his birth.

53 Divide Rs 1780 13a into three such parts that the first part shall be Rs 125 3a more than the sum of the second and third, and the second part Rs 17 12a more than the third.

54 If the monthly expenditure of a family be Rs 57 8a, when rice is at Rs 4 6a per maund and Rs 58, when rice is at Rs 4 8a per maund, what should the expenditure be when rice would be at Rs 4 12a per maund?

55 What sum of money is that which being multiplied by 16, Rs 24 added to the product, the sum divided by 13, and Rs 3 13a added to the quotient, the sum is Rs 7 13a?

56 An equal number of men, women and boys together earned

Rs 62 8a in 5 days A boy earns *2a* a woman *3a* and a man *5a* daily Find the number of boys

57 A goldsmith mixes a certain number of tolas of gold worth *Rs 20 8a* per tola with twice that quantity worth *Rs 19 6a* per tola On selling the mixed gold at *Rs 20* per tola, he gained *Rs 15* How much of each kind did he mix?

58 Sound travels at the rate of 1142 ft per second, what is the distance of a thunder-cloud when the sound of thunder follows the flash of lightning after an interval of 9 seconds?

59 A gives B 112 gallons of brandy at *32s 6d* a gallon, and receives in return *£40 12s 6d* and 780 yds of cloth What is the price of the cloth per yard?

60 There are 6 presses at work striking off sovereigns, half-sovereigns, florins, shillings, six-pences and four-penny-pieces respectively, and each at the rate of 2500 per hour, find the value of the money struck off in 13 days of 9 hours each

61 What is the difference in seconds between the Mahomedan year of 354 days 8 hrs 48 min and the Hindu year of 365 days 6 hrs 12 min 30 sec?

62 If 6 hats cost as much as 25 pairs of gloves, worth *Re 1 10a* a pair, how many hats can be bought for *Rs 616 2a 4p*?

63 If telegraph posts are placed 66 yards apart and a railway train passes one in every three seconds, how many miles an hour is the train running?

64 A person observed the flash of a canon 7 seconds before he heard the report, how far was the cannon distant, supposing that sound moves at the rate of 1142 ft. per second?

65 In how many days of 8 hours each will a person be able to count 10 lacs of rupees at the rate of 80 per minute? How many will remain to be counted on the morning of the 26th day?

66 How much water must be mixed with 30 seers of milk worth *2a* per seer, in order to reduce its price to *1a 6p* per seer?

67 By the payment of *2s 1d* in London a banker will give credit at Calcutta for a rupee, how many rupees may be received in Calcutta for the payment of *£5025 6s 3d* in London?

68 If 5 oz of silk can be spun into a thread 2 fur 20 po long, what weight of silk would supply a thread sufficient to reach to the Moon if the distance be 240000 miles?

69 A ship's crew of 50 men have a supply of water for 30 days at 2 seers a head, if they lose 125 seers, and find that they will be 50 days at sea, what must be each man's daily allowance?

70 A landowner has four estates, for which he has to pay a tax of *Rs 760* The second, third and fourth yield respectively twice,

thrice and four times as much income as the first. If the tax be levied at 10, 9, 8 and 6 pies in the rupee respectively, find the amount of his income from each estate.

71. A tradesman in India exchanges with a merchant in China as many mounds of sugar as is expressed by the sum of all the numbers that can be formed by the different arrangements of the digits 7, 8 and 9 taken all together, for as many pounds of tea as is expressed by the sum of all the numbers similarly formed by the digits 3, 0, 5 and 7 taken all together. How much tea does the Indian merchant get in return for 37 mds of sugar?

72. A man's monthly expenditure consists of 5 mds of rice, 1 md 20 sr of flour, 15 sr of ghee and 2 mds 15 sr of milk. When rice costs Rs 3 10a 6p per md, flour Rs 4 12a per md, ghee Rs 37 8a per md and milk Rs 5 8a per md the total expenses amount to Rs 130 10a. If the prices of other articles remain the same, what would his family expenses amount to, when rice would sell at Rs 4 12a, flour at Rs 5 4a, ghee at Rs 41 8a, and milk at Rs 6 4a per mound?

73. A total weight of 12 mds 10 sr consists of a certain number of 10 seer-weights, three times as many of 5 seer-weights, 4 times as many of 2½ seer weights, 6 times as many of 1 seer weights, 8 times as many of half seer weights and 16 times as many of pound weights. Find the number of each kind of weights.

74. A certain number of sovereigns, twice as many crowns, 5 times as many half crowns, 8 times as many shillings and 12 times as many six-pences together amount to £28 5s, find the numbers of each coin.

75. A man mixed 3 mds of milk at Rs 4 8a per md with a certain quantity worth Rs 4 4a per md and three times that quantity worth Rs 3 12a per md. He sold the mixture at Rs 4 2a per md and thus cleared Rs 15 on the whole. How much of the second and third sort did he mix?

CHAPTER IV

Numbers, Measures and Multiples

I NUMBERS

188. Numbers which follow a regular order increasing by 1 are called **consecutive** numbers. The consecutive numbers commencing at 1 are called **natural** numbers.

Thus, 4, 5, 6, 7, 8, &c are *consecutive*, and 1, 2, 3, 4, 5, 6, &c are *natural* numbers.

189 Numbers are either **even** or **odd**

Numbers are called **even** when they can be divided by 2 without a remainder, and **odd** when they cannot be so divided

Thus, 4, 8, 10, 16, &c are *even* and 3, 5, 7, 13, &c are *odd* numbers

190 A measure or factor of a number is *any* number which divides it without a remainder. It is said to *measure* the number by the **units** contained in the *quotient*

Thus, 4 is a *measure* or *factor* of 24, because it is contained exactly 6 times in 24. All numbers have 1 for a measure

191 An aliquot part of a number is *any* measure of it

Thus, 4 is an *aliquot part* of 20, for 4 is a measure of 20

192 A multiple of a number is *any* number which contains it an exact *number of times*

Thus, 108 is a *multiple* of 12, because 12 is contained exactly 9 times in 108

193 A measure is sometimes called a **submultiple**

Thus, 4 is a *submultiple* of 16

194 Numbers are either **prime** or **composite**

A **prime number**, or a **prime**, is a number which can be divided exactly only by itself and by unity. A **composite number** is a number which can be separated into *factors* each greater than unity, or which, in other words, arises from the multiplication of *two or more* other numbers, termed *factors*

Thus, 2, 5, 7, 11, &c are *primes*, and 4, 8, 10, 12, &c are *composite* numbers

195 Two numbers are **prime** to each other, when their only common measure is 1

196 One number is **divisible** by another when it can be divided by that other number exactly

Thus, 20 is *divisible* by 5, for 20 contains 5 exactly 4 times

197 The following RULES are important, and should be carefully committed to memory

(1) If a number divide a product of two factors and be prime to one of them, it must divide the other

Thus, if 4 divide 9×24 , and 4 is prime to 9, then 4 must divide 24, for 4 is a measure of 24

(2) If a number is divisible separately by two others which are prime to each other, it is divisible by their product

Thus, if 240 be divisible by 3, and by 4, where 3 and 4 are prime to each other, it will be divisible by 3×4 , for $240 = (3 \times 4) \times 20$

- (3) If one number is divisible by another, any multiple of the first is also divisible by the second

Thus, 10 is divisible by 2 and 5, hence any number ending with 0, being a multiple of 10, is divisible by 2 and 5

100 is divisible by 4 and 25, therefore all numbers ending with two ciphers are divisible by 4 and 25

1000 is divisible by 8 and 125, hence all numbers ending with three ciphers are divisible by 8 and 125

Again, $1001 = 7 \times 11 \times 13$, and therefore 1001 is divisible by 7, 11, and 13. Hence all numbers like 7007 or (7×1001) , 18018, or (18×1001) , 325325 or (325×1001) are all divisible by 7, 11 and 13

- (4) If each of two numbers is divisible by a third, their sum or difference is also divisible by the third

Thus, $8654 = 8650 + 4$ and is divisible by 2, if 4 is,
 $4235 = 4240 - 5$ 5, if 5 is,
 $7336 = 7400 - 64$ 4, if 64 is,
 $78664 = 78000 + 664$ 8, if 664 is,

$86184 = 86086 + 98$ and is divisible by 7, if 98 or $(184 - 86)$ is,
 $429275 = 429429 - 154$ 11, if 154 or $(429 - 275)$ is,
 $186459 = 186186 + 273$ 13, if 273 or $(459 - 186)$ is

- (5) If each of two numbers is divisible by a third, then the sum or difference of any multiple of the first and of any multiple of the second is also divisible by the third

Thus, $627 = 600 + 20 + 7 = 6(99 + 1) + 2(9 + 1) + 7$
 $= 6 \times 99 + 2 \times 9 + 6 + 2 + 7,$

627 is divisible by 3, if $6 + 2 + 7$ is
 $7362 = 7000 + 300 + 60 + 2 = 7(999 + 1) + 3(99 + 1) + 6(9 + 1) + 2$
 $= 7 \times 999 + 3 \times 99 + 6 \times 9 + 7 + 3 + 6 + 2,$

7362 is divisible by 9, if $7 + 3 + 6 + 2$ is
 $82654 = 80000 + 2000 + 600 + 50 + 4$
 $= 8(9999 + 1) + 2(1001 - 1) + 6(99 + 1) + 5(11 - 1) + 4$
 $= 8 \times 9999 + 2 \times 1001 + 6 \times 99 + 5 \times 11 + 8 - 2 + 6 - 5 + 4,$

82654 is divisible by 11, if $(8 + 6 + 4) - (2 + 5)$ is
 (for 9999, 1001, 99 and 11 are all divisible by 11)

198 Criteria of Divisibility

A number is divisible by

- 2, if its *last* digit is divisible by 2, as 450, 326
- 3, if the *sum* of the digits is divisible by 3, as 267, 531
- 4, if its *last two* digits are divisible by 4, as 600, 520, 924
- 5, if its *last* digit is 0 or 5, as 370, 865
- 6, if it is divisible by both 2 and 3, as 318, 588
- 8, if its *last three* digits are divisible by 8, as 3000, 5240, 2816.

- 9, if the *sum* of its digits is divisible by 9, as 648, 702
 10, if its *last* digit is 0, as 4570, 2300
 11, if the difference between the sum of its digits in the *odd* and in the *even* places is 0 or is divisible by 11, as 1067, 2695, 19613
 12, if it is divisible by both 3 *and* 4, as 708, 1164

For 7 and 13, see Art. 197 (4)

199 There is no direct method for determining primes, and so we give below a list of the prime numbers from 1 to 227

1	11	29	47	71	97	113	149	173	197
2	13	31	53	73	101	127	151	179	199
3	17	37	59	79	103	131	157	181	211
5	19	41	61	83	107	137	163	191	223
7	23	43	67	89	109	139	167	193	227

200 To ascertain what numbers are prime

(i) Every number whose last digit is 0, 2, 4, 6, or 8 is divisible by 2 (Art. 198), and therefore every such number except 2 itself is not a prime. Every number whose last digit is 0 or 5 is divisible by 5, and therefore every such number except 5 itself is not a prime. Hence the last digit of every prime number except 2 and 5, must be 1, 3, 7 or 9.

(ii) If then the last digit of the given number be 1, 3, 7 or 9 try as divisors one after another the primes 3, 7, 11, 13, &c., if there is a remainder in each case the given number is a prime. It is not necessary to try a divisor whose square is greater than the given number.

Ex. Are 689 and 947 primes?

(1) 689 is not divisible by 3 (for $6+8+9=23$), nor by 7 (by trial), nor by 11 (for $6+9-8=7$), but is divisible by 13, therefore 689 is *not* a prime.

(2) 947 is not divisible by 3, 7, 11, 13, 17, 19, 23 or 29, and we need not try the next divisor 31, for the square of 31 is greater than 947. Hence 947 is a prime.

201 To resolve or decompose a composite number into its prime factors is to find those prime numbers which when multiplied together produce the given number.

Thus, $210=2 \times 3 \times 5 \times 7$, $504=2 \times 2 \times 2 \times 3 \times 3 \times 7=2^3 \times 3^2 \times 7$

202 When the factors obtained are all primes, the number is said to be resolved or decomposed into its prime or elementary factors.

203 No number can be resolved into prime factors in more than one way.

204 To resolve a number into its prime factors

RULE Divide in succession by each of the primes 2, 3, 5, 7, 11, &c., which can be used as divisors, and in each case as often as

possible, until we obtain a quotient which is a prime, these divisors and the last quotient expressed in the form of a product make up the given number

Ex 1 Resolve 44856 into prime factors

$2 = 8 \overline{) 44856}$ The last two digits form 56, which is divisible
 $3^2 = 9 \overline{) 5607}$ by 8, the sum of the digits $= 4+4+8+5+6=27$
 $7 \overline{) 623}$ Hence the number is divisible by 8 and 9 or 2^3
 89 and 3^2
 Also $623 = 7 \times 89$, and that 89 is a prime

$$44856 = 2^3 \times 3^2 \times 7 \times 89$$

Ex 2 Decompose 8862777 into its prime factors

$3^2 = 9 \overline{) 8862777}$ The sum of the digits $= 45$, which is divisible by
 $3 = 9 \overline{) 984753}$ 9 or 3^2 , the sum of the digits of the quotient $= 36$,
 11) 109417 also $(8+6+7+7) - (8+2+7) = 11$ Hence the num-
 7)9947 ber is divisible by 9, 9 and 11
 7)1421
 7)203
 29
 Again, in 9947, we have $947 - 9 = 938$, which is
 divisible by 7 in like manner, again by 7, and 203
 $= 7 \times 29$ and 29 is a prime

$$8862777 = 9 \times 9 \times 11 \times 7 \times 7 \times 7 \times 29 = 3^4 \times 11 \times 7^3 \times 29$$

Examples LI

1 Resolve *mentally* the following into elementary factors —

(1) 6, 10, 14, 21, 35, 28, 45, 64, 81, 96, 72

(2) 56, 30, 280, 144, 224, 285, 198, 176, 342

2 Decompose the following numbers into their prime factors —

(1) 320, 460, 462, 315, 612, 715, 846, 945, 735

(2) 1188, 1309, 1827, 1331, 1456, 1485, 3675, 4620

(3) 5250, 55020, 16632, 47089, 53599, 88725, 11025

(4) 514250, 190463, 259811, 508079, 4149173, 4057690

(5) 7507500, 73896433, 11176704, 119189070, 125023500

3 Ascertain which of the following numbers are prime and the prime factors of those which are composite —

(1) 31, 53, 86, 96, 167, 131, 275, 480, 856, 873

(2) 397, 289, 461, 727, 667, 851, 953, 971, 997

(3) 1009, 1517, 1729, 4576, 2501, 4717, 3389

4 Determine which of the following numbers are divisible by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 and 13 respectively —

(1) 165, 216, 324, 425, 639, 936, 868, 512, 795

(2) 3164, 4228, 11172, 2859, 11509, 14916, 53729

(3) 1235, 6467, 38187, 123456 777777, 601830

(4) 2702344, 50707338, 6013580247 726441196

5 How many prime numbers are there between ?—

(1) 16 and 96

(2) 53 and 100

(3) 110 and 230

(4) 330 and 350

(5) 556 and 600

(6) 790 and 1008

6 By what numbers may 179, 313 and 799 be divided that the remainders may be 3, 5 and , respectively ?

II GREATEST COMMON MEASURE

205 A common measure or common factor of two or more numbers is any number, which will divide each of them without leaving a remainder.

Thus, each of the numbers 2, 3 and 6 is a *common measure* or *common factor* of 18 and 30, for each of the numbers 2, 3 and 6 divides 18 and 30 exactly.

206 The *greatest* number that divides each of two or more numbers exactly is called their **Greatest Common Measure (G C M)** or **Highest Common Factor (H C F)**.

Thus, 6 is the *greatest common measure* of 18 and 30, for it is the greatest number capable of dividing each of them exactly.

207 If one number measure each of two others, it will measure their sum and difference, also, any multiples of each their sums and differences.

Thus, 4 is a common measure of 20 and 12, and their sum $= 20 + 12 = 32 = 1 \times 32$ their difference $= 20 - 12 = 8 = 4 \times 2$ a multiple of 20 $= 20 \times 5 = 100 = 1 \times 25$, of 12 $= 12 \times 7 = 84 = 4 \times 21$

also, $100 + 48 = 148 = 4 \times 37$, $100 - 48 = 52 = 4 \times 13$,

each of which evidently comprises the number 4 as a measure or factor and similarly of more numbers.

Examples LII

Find, by inspection, the G C M of —

1 4 and 6	2 6 and 9	3 8 and 12	4 9 and 24
5 20, 32	6 48, 27	7 42, 28	8 48, 54
9 61, 64	10 30, 15	11 45, 57	12 42, 56
13 21, 28, 35	14 30, 25, 45	15 32, 40, 48	16 30, 35, 40

208 The G C M of two or more numbers may often be found by resolving each number into its prime factors and then taking the product of all the prime factors common to them

Ex Find the G C M of 63 and 168

$$63 = 7 \times 9 = 7 \times 3 \times 3, \quad 168 = 7 \times 24 = 7 \times 3 \times 8 = 7 \times 3 \times 2 \times 2 \times 2$$

Therefore the factors common to 63 and 168 are 7 and 3, hence the G C M $= 7 \times 3 = \underline{21}$ *Ans*

209 In finding the G C M of two or more numbers, it is sufficient to find the prime factors of one of the numbers, and then find by trial which of these factors divide each of the remaining numbers exactly, the product of all these common factors is the required G C M

Ex Find the G C M of 492, 744 and 1044

The prime factors of 492 are 2, 2, 3 and 41, of these factors 2, 2 and 3 divide 744 and 1044 exactly, but 41 does not divide them

Hence, the required G C M is $2 \times 2 \times 3$ or 12 *Ans*

Examples LIII

Find, by *method of factors*, the G C M of —

- | | | |
|-------------------------------|------------------------|------------------|
| 1 45 and 72 | 2 64 and 96 | 3 48 and 72 |
| 4 56 and 140 | 5 81 and 171 | 6 74 and 259 |
| 7 205 and 287 | 8 325 and 425 | 9 230 and 414 |
| 10 490 and 546 | 11 308 and 506 | 12 247 and 323 |
| 13 1216 and 424 | 14 620 and 2108 | 15 45, 72 and 81 |
| 16 162, 729 and 4374 | 17 1326, 3094 and 4420 | |
| 18 372, 994 and 3132 | 19 504, 5292 and 3040 | |
| 20 102, 612, 476, 816 and 428 | | |

210 When numbers can easily be resolved into their prime factors we have shown in Art 208, that their G C M is formed by the product of the **least powers** of those factors which are common to all the given numbers, but when the numbers are large and their prime factors cannot be readily determined, we use a different method

211 To find the G C M of two numbers, whose prime factors cannot be readily ascertained, we use the following Rule

RULE Divide the greater of the numbers by the less, then the first divisor by the remainder, then the second divisor by the second remainder, and repeat this operation till there is no remainder, the last divisor will be the G C M required

Ex Find the G C M of 9756 and 8496

8496)9756(1

8496

1260)8496(6

7560

936)1260(1

936

324)936(2

648

288)324(1

288

36)288(8

288

The first divisor is 8496 and the first remainder 1260

The second divisor is 1260 and the second remainder 936

The third divisor is 936 and the third remainder 324, and so on

The final divisor is 36

the required G C M is 36

212 To find the C C M of three or more numbers

RULE Find the G C M of the first two numbers, then the G C M of this G C M and the third number, then the C C M of this last C C M and the fourth number, and continue this process to the last number, the last G C M is the required G C M of the given numbers

Ex Find the G C M of 741, 1131, 1183 and 1989

741)1131(1

741

390)741(1

390

351)390(1

351

39)351(9

351

39)1183(30

117

13)39(3

39

13)1989(153

13

68

65

39

39

the required G C M is 13 Ans

Examples LIV

Find the G C M of —

1 126 and 444

2 646 and 950

3 54 and 258

4 366, 128

5 3556, 3444

6 5187, 5850

7 4833, 6237

8 9367, 14501

9 3252, 4248

10 2145, 3471

11 4081, 5141

12 1441, 1572

13 6441, 10283

14 13667, 14186

15 43365, 44688

16 12925, 63305

17 11050, 35581

18 109056, 179712

19 125075, 225025

20 105945, 945105

21 428571, 999999

22 143278, 1278142

23 385629, 7855323

24	1257214, 11215246	25	703037, 5134083
26	3876519, 3101729671	27	41615795893, 877267019106
28	6186, 10310, 15465	29	12018, 20030, 30045
30	1617, 2871, 4213	31	13338, 14136, 15903
32	16442, 24663, 41105	33	2697, 3441, 1271
34	204, 1190, 1445, 2006	35	12558, 20769, 47403, 12581
36	5040, 23940, 28350, 31773	37	11573, 19397, 28036
38	70843288, 852706430 and 686138242		
39	1070784, 1180608, 1455168 and 1520376		
40	22680, 49140, 154980, 429660 and 925932		

213 Numbers which have no common measure greater than unity, are said to be **prime to each other**

Thus, 15 and 29 are prime to each other

Ex Are 1726 and 1623 prime to each other ?

$$1623 \overline{) 1726} (1$$

$$\underline{1623}$$

$$103 \overline{) 1623} (15$$

$$\underline{103}$$

$$593$$

$$\underline{515}$$

$$78$$

$$78 \overline{) 103} (1$$

$$\underline{78}$$

$$25 \overline{) 78} (3$$

$$\underline{75}$$

$$3 \overline{) 25} (8$$

$$\underline{24}$$

$$1$$

∴ 1726 and 1623 are prime to each other

214 Every common measure of two numbers is a measure of their G C M

Thus, 2 and 3 being common measures of 18 and 30, is a measure of 6, the G C M of 18 and 30

215 The numbers of which the G C M is required must refer to the same unit, and the G C M refers to that unit

Thus the G C M of Rs 429 and Rs 715 is Rs 143, the G C M of 224 feet and 336 feet is 112 feet

Examples LV

1 Are the following numbers prime to each other ?—

(1) 5789 and 7337

(2) 3375 and 5836

(3) 49561 and 97073

(4) 58573 and 84329

(5) 9367 and 14501

(6) 19001 and 46253

(7) 2698705 and 54987262

(8) 18432, 21952 and 42895

2 Find the G C M of —

- (1) 8029 and 23791 (2) 441441 and 844272 (3) 181896 and 147576
(4) 39835 and 162424 (5) 218707, 526769 and 695822

Examples worked out

Ex 1 Find the greatest number that will divide 2293, 4245 and 5348 leaving the remainders 18, 20 and 23 respectively

$$2293 - 18 = 2275, 4245 - 20 = 4225, 5348 - 23 = 5325$$

The reqd no is the G C M of 2275, 4225 and 5325 = 25 Ans

Ex 2 Two bills, one amounting to Rs 78 12a and the other to Rs 420 are to be paid in coins of one kind, what is the largest coin that can be used

$$Rs\ 78\ 12a = 1260a, Rs\ 420 = 6720a$$

the largest coin required is the C C M of 1260a and 6720a
= 420a = Rs 26 4a Ans

Ex 3 The sum of two numbers is 1144, and their G C M is 143, how many pairs of such numbers can be formed? Form them

$$1144 - 143 = 8$$

Now $8 = 1 + 7 = 2 + 6 = 3 + 5 = 4 + 4$, and no more

Of these parts the only pairs of numbers that are prime to each other are 1, 7 and 3, 5. Hence *only two* pairs of numbers can be formed

Thus, the first pair = 1×143 and 7×143 , or 143 and 1001, }
and the 2nd pair = 3×143 and 5×143 , or 429 and 715 } Ans

As regards the other pairs that can be formed, 143 will be a common measure, but not the G C M

Ex 4 The product of two numbers is 3240, and their G C M is 18, how many pairs of such numbers can be formed? Form them

$$3240 - 18^2 = 10, \text{ and } 10 = 1 \times 10 \text{ or } 2 \times 5$$

Hence *only two* pairs of numbers can be formed

Thus, the 1st pair = 18×1 and 18×10 , or 18 and 180, }
and the 2nd pair = 18×2 and 18×5 or 36 and 90 } Ans

Ex 5 What number is that which, when divided by 6, the quotient again by 6, and that quotient again by 6, will give the C C M of 35 and 135?

The C C M of 35 and 135 is 5

Now the question is, what number is that which, when divided by 6, the quotient again by 6, and that quotient again by 6, will give 5?

Since, 6, 6 and 6 are the three divisors and 5 the last quotient,

∴ the first dividend or the required number is $5 \times (6 \times 6 \times 6)$
= 1080 Ans

Examples LVI

- 1 What is the greatest sum of money contained exactly in Rs 34 7a 6p and Rs 70 12a 6p ?
- 2 Find the greatest number that will divide 35 and 61, leaving remainders 3 and 5 respectively
- 3 What number is that which, when divided by 12, the quotient again by 12, and that quotient again by 12, will give the C C M of 148 and 772 ?
- 4 Find the greatest weight in grains, that will measure both pounds Avoir and pounds Troy
- 5 The sum of two numbers is 928, and their G C M is 58, form as many pairs of numbers as convenient
- 6 What is the greatest unit of time with which 15 hrs 12 min and 1 day 3 hrs 33 min can be both represented by integers ?
- 7 Find the greatest number that will divide 1624, 2878 and 4220 leaving 7 as remainder after each division
- 8 The product of two numbers is 5700, and their G C M is 5, find as many pairs of numbers as convenient
- 9 In working out a question in the C C M of two numbers, I found the different remainders were 2388, 180, 48, 36 and 12, and the first two quotients 1 and 9, find the numbers, and the last three quotients
- 10 In solving a question in the G C M of two numbers, the quotients are 5, 1, 18, 1, 3, 1 and 2 The last divisor is 15 Find the numbers
- 11 The sum of two numbers is 1394, and their G C M is 34, how many pairs of numbers can be formed ?
- 12 The product of two numbers is 4608, and their G C M is 16, how many pairs of numbers can be formed ?
- 13 What highest number will divide 287, 480 and 599 leaving the remainders 2, 5 and 10 respectively ?
- 14 What is the greatest number by which, when 399, 695, 548, 1003 are divided, the respective remainders are 3, 2, 8 and 4 ?
- 15 Two bills, one of £4 13s 8d and the other of £6 9s 4d are to be paid in the same coin Find the largest coin that can be used
- 16 A has Rs 679, B Rs 5901 and C Rs 6734, they agree to lay it out for sheep, at the highest price per head that will allow each exactly to invest his money, how much can they pay a head and how many can each purchase ?
- 17 Find the two numbers nearest to 10000 that have 169 for their G C M
- 18 A national school-master divided his scholars, consisting of 221 boys and 143 girls, into the largest possible equal classes so

that each class of boys should contain the same number as each class of girls. Find the number of classes.

19 A person wishes to distribute 805 mangoes, 1311 guavas, and 1978 plantains, equally among a number of beggars. Find the greatest number receiving the charity in this way.

20 A labourer was engaged for a certain number of days for Rs 10 15s 8p, but being absent on some of these days he was paid only Rs 3 3s 8p, shew that his daily wages cannot exceed 10s 4p.

21 Find the greatest number of 4 digits and the least number of 5 digits that have 124 for their G.C.M.

22 Find the greatest and the least number of 6 digits that have 251 for their common measure. What is their G.C.M.?

III LEAST COMMON MULTIPLE

216 A common multiple of two or more numbers is *any* number which is divisible by each of them separately.

Thus, 96 is a *common multiple* of 2, 3, 4, 6, 8 and 12, because it is divisible by each of them.

217 The Least or Lowest Common Multiple (L.C.M.) of two or more numbers is the *least* number that can be divided by each of them without a remainder.

Thus, 24 is the *Least Common Multiple* of 2, 3, 4, 6, 8 and 12, for it is the *least* number that the above numbers can divide without leaving a remainder.

218 The L.C.M. of two or more numbers may be obtained by resolving them into their prime factors, and taking the product of the highest powers of all the factors that are found in the given numbers.

Ex. Find the L.C.M. of 8, 12, 16, 20, 25 and 30.

$$\begin{aligned} 8 &= 2 \times 2 \times 2 = 2^3, & 12 &= 2 \times 2 \times 3 = 2^2 \times 3, \\ 16 &= 2 \times 2 \times 2 \times 2 = 2^4, & 20 &= 2 \times 2 \times 5 = 2^2 \times 5, \\ 25 &= 5 \times 5 = 5^2, & 30 &= 2 \times 3 \times 5 = 2 \times 3 \times 5 \end{aligned}$$

Here the factors that occur in the given numbers are 2, 3 and 5, of which the highest power of 2 is 2^4 , and that of 5 is 5^2 , therefore the L.C.M. is $2^4 \times 3 \times 5^2 = 16 \times 3 \times 25 = 1200$ *Ans*

Examples LVIII

1 Find *mentally* the L.C.M. of —

- | | | | |
|-----------------|----------------|----------------|-----------------|
| (1) 6, 8 | (2) 8, 16 | (3) 10, 15 | (4) 18, 30 |
| (5) 12, 27 | (6) 10, 18 | (7) 16, 24 | (8) 12, 15 |
| (9) 3, 4, 5 | (10) 2, 5, 7 | (11) 3, 4, 16 | (12) 5, 8, 20 |
| (13) 16, 12, 24 | (14) 7, 10, 24 | (15) 5, 12, 15 | (16) 20, 40, 60 |

2 Find, by resolving into factors, the L C M of —

- | | | |
|---------------------------------|-----------------------------|--------------------|
| (1) 12, 16, 18 | (2) 16, 24, 30 | (3) 24, 56, 84 |
| (4) 15, 35, 16, 56 | (5) 25, 60, 84, 15 | (6) 81, 27, 45, 18 |
| (7) 756, 6435 | (8) 729, 1681 | (9) 1008, 2064 |
| (10) 756, 350, 9075 | (11) 735, 1575, 2205 | (12) 225, 336, 360 |
| (13) 196, 350, 728, 924 | (14) 11573, 19397, 28036 | |
| (15) 72, 96, 144, 180, 450, 540 | (16) 44, 126, 280, 198, 330 | |

219 To find the L C M of two large numbers which cannot easily be resolved into prime factors, we use the following Rule

RULE Find the G C M of the two numbers, and then multiply *either* of the numbers by the quotient arising from dividing the *other* by the G C M. The product will be the I C M of the numbers

Ex Find the I C M of 209 and 304

Here, the G C M is 19 Also $209 \div 19 = 11$

the L C M $= 11 \times 304 = 3344$ *Ans*

220 To find the I C M of three or more numbers which cannot be readily resolved into factors, use the following Rule

RULE First find the L C M of two of the numbers as in Art 219, then the L C M of this and another and so on, until all are taken. The last L C M is the L C M required

Ex Find the I C M of 64, 250 and 432

The G C M of 64 and 250 is 2, and their L C M is 8000

The G C M of 8000 and 432 is 16, and the I C M is 216000

Hence, the L C M required $= 216000$ *Ans*

Examples LVIII

Find the L C M of —

- | | | |
|--------------------------|---------------------------|-----------------|
| 1 289, 323 | 2 849, 1132 | 3 508, 889 |
| 4 420, 798 | 5 1287, 6281 | 6 7247, 9365 |
| 7 12432, 36075 | 8 15863, 21489 | 9 24, 39, 376 |
| 10 84, 672, 472 | 11 629, 851, 253 | 12 64, 720, 960 |
| 13 1003, 2301, 4017 | 14 14491, 16641, 3707 | |
| 15 2523, 5887, 203, 8631 | 16 1175, 4747, 5875, 9447 | |

221 When the L C M of several small numbers is required, the easiest method is that given by the following Rule

RULE Arrange the given numbers in a horizontal line from left to right, with a comma placed between every two. Divide by any one of the prime numbers 2, 3, 5, 7, 11 which will divide any two

at least of the given numbers exactly, set down the quotient so obtained and the undivided numbers in a line below, separated as before. Proceed in the same way with the numbers in the second, and each succeeding line, till we come to a line where no two numbers have a common divisor. The product of the numbers in the last line and of the several divisors is the L. C. M. of the given numbers.

Note The work may often be shortened by *striking out* in the same line every number which exactly measures any other number in that line.

Ex. Find the L. C. M. of 2, 3, 8, 9, 15, 21 and 35;

2) 2, 3, 8, 9, 15, 21, 35
 3) 1, 3, 4, 9, 15, 21, 35
 5) 1, 1, 4, 3, 5, 7, 35
 7) 1, 1, 4, 3, 5, 7, 7
 1, 1, 4, 3, 1, 1, 1

3) 2, 3, 8, 9, 15, 21, 35
 8, 3, 8, 7, 35
 the L. C. M. = $3 \times 8 \times 3 \times 35$
 = 2520 *Ans*

∴ the L. C. M. = $2 \times 3 \times 5 \times 7 \times 4 \times 3$
 = 2520 *Ans*

In the first line 2 is contained in 8, and 3 in 9, and struck off
 In the second line 5 and 7 are both contained in 35, and struck off

Examples LIX

Find the L. C. M. of —

- | | | | | | |
|----|--|----|--------------------------------|----|--------------------|
| 1 | 12, 15, 16 | 2 | 8, 16, 20 | 3 | 15, 25, 105 |
| 4 | 9, 15, 18, 20 | 5 | 8, 12, 15, 20 | 6 | 34, 68, 17, 2 |
| 7 | 16, 9, 12, 18 | 8 | 36, 56, 75, 72 | 9 | 81, 27, 45, 18 |
| 10 | 15, 35, 16, 56 | 11 | 15, 20, 24, 21, 35 | 12 | 24, 28, 36, 22, 16 |
| 13 | 3, 9, 7, 15, 28, 42 | 14 | 8, 18, 28, 36, 54, 72, 90 | | |
| 15 | 9, 12, 15, 18, 21, 24, 27, 30 | 16 | 32, 63, 25, 36, 42, 49, 84 | | |
| 17 | 12, 18, 28, 35, 60, 84, 100 | 18 | 15, 16, 18, 20, 24, 25, 27, 30 | | |
| 19 | 48, 64, 27, 81, 33, 110, 165 | 20 | 48, 64, 27, 33, 110, 165, 240 | | |
| 21 | 35, 52, 63, 77, 132, 117, 143 | 22 | 27, 91, 42, 39, 63, 156, 234 | | |
| 23 | 27, 36, 54, 72, 84, 96, 215, 248, 324 | | | | |
| 24 | 18, 24, 35, 48, 56, 60, 72, 90, 120 | | | | |
| 25 | 7, 11, 21, 63, 91, 99, 117, 143 | | | | |
| 26 | 24, 35, 52, 60, 91, 108, 126, 156, 315 | | | | |
| 27 | 26, 30, 34, 39, 51, 65, 78, 85, 102, 195, 255 | | | | |
| 28 | 27, 87, 189, 126, 145, 210, 203, 261, 385 | | | | |
| 29 | 8, 9, 10, 11, 12, 14, 15, 18, 21, 24, 28, 35, 36, 40, 42, 44, 45, 50 | | | | |
| 30 | The first 12 numbers, the even numbers from 10 to 28 inclusive | | | | |

222 Every common multiple of two numbers is a multiple of their L. C. M.

Thus, 48 a common multiple of 8 and 12 is a multiple of 24, the L C M of 8 and 12

223 If two numbers are prime to each other, then L C M is their product

Thus, the L C M of 13 and 15 is $13 \times 15 = 195$

224 Since the L C M of two numbers is their product divided by their G C M (Art 219), therefore the L C M \times the G C M of two numbers is equal to their product. Hence, if the G C M, the L C M, and one of the two numbers be given, we can find the other number by multiplying the G C M and the L C M and dividing the product by the given number

Ex The G C M and the L C M of two numbers are 11 and 11803 respectively, and one of them is 319, what is the other?

Here, the G C M \times the L C M $= 11 \times 11803 = 129833$

the required number $= 129833 \div 319 = 407$ Ans

225 (1) To find the *least* number that will contain each of two or more given numbers exactly

RULE The required *least* number is the L C M of the given numbers

Ex 1 Find the least number that is divisible by 40, 63, 112

The required number $=$ the L C M of 40, 63, 112 $= 5040$ Ans

Ex 2 Five bells toll at intervals of 5, 8, 9, 10 and 12 seconds respectively, what interval will elapse between two of their successive tollings together?

The L C M of 5, 8, 9, 10, 12 is 360

the required time $= 360$ sec or 6 min Ans

(2) To find the *least* number which, when divided by each of several given numbers, leaves the same remainder

RULE Find the L C M of the several given numbers and to it add the given remainder. The sum is the required *least* number

Ex Find the least number which, when divided by 4, 18, 21 and 20, leaves in each case a remainder 3

The L C M of 4, 18, 21 and 20 is 1260

the required number $= 1260 + 3 = 1263$ Ans

Examples LX

1 Find the least number which, when divided by 6, 8 and 9, gives in every case the remainder 5

2 What is the smallest sum that can be paid either in guineas, or in half crowns, or in florins or in half-sovereigns?

3 Five bells begin to toll simultaneously and they toll at intervals of 4, 6, 8, 9 and 10 seconds. After what time will they again toll simultaneously?

4 Find the least number which, when divided by 675, 1050 and 4368, will leave the same remainder 32

5 Find the least weight that can be weighed by either pounds Avoir or pounds Troy

6 Six men fire at a target at intervals of 2, 5, 7, 10, 12 and 14 minutes respectively. After what time will they all *first* fire simultaneously, and how many times will each man have fired?

7 Seven bells are tolling, and they toll at intervals of 3, 5, 7, 8, 9, 10 and 12 seconds respectively. What interval will elapse between their once tolling together and tolling together again?

8 A can go round a circular course in 6 minutes, B in 8, C in 12, D in 15, and E in 18, if they all start together from the same place at the same time (7h 13m A.M.) when will they be together again?

9 Find the least sum of money that can be paid in pence, shillings, florins, half crowns, crowns, sovereigns or half sovereigns

10 The G.C.M. and L.C.M. of two numbers are 124 and 10540 respectively, and one of the numbers is 620, find the other

11 A heap of pebbles can be made up exactly into groups of 25, but when made up into groups of 18, 27 and 32, there is always a remainder of 11, find the least number of pebbles such a heap can contain

12 A basket contains a number of oranges ascertained to be between 500 and 900. If 2 fruits are taken away, the remainder may be distributed equally among 3, 4, 5, 6 or 7 boys. Find the number of oranges in the basket

13 A book is divided into four parts, each part being divided into chapters. The number of pages in each part is the same. Each chapter in the first part contains 20 pages, each chapter in the second 40, each chapter in the third 60, and each chapter in the fourth 80. Find the number of pages and chapters in the book, the number of pages in the book is known to be between 900 and 1000

14 Three horses are running round a race course of 5280 yards, the first horse runs 440 yards a minute, the second 352 yards, and the third 264 yards, find the time between their once coming all together, and their coming all together again

15 What is the least number which when increased by 17, is divisible by 22, 25, 33, 44 and 45 separately?

16 The G.C.M. and the L.C.M. of two numbers are 19 and 49077 respectively, and one of them is 779, find the other

17 What is the least number which, when diminished by 145, is exactly divisible by 24, 27, 28, 32, 36 and 56?

18 What is the least number which, when divided by all the digits except the first, leaves the remainder 1?

19 The G.C.M. of two numbers of 4 digits is 221, and their L.C.M. is 46189, determine the numbers

20 Find all the numbers between 250 and 600 that have 1728 for their L.C.M.

21 Find the least sum of money that can be paid in coins worth either 8 pies, half rupees, rupees, 5 *sikhs*, 10 *sikhs*, 14 *sikhs*, Rs 5 4a, Rs 10 8a

22 There is an island 48 miles in circumference. Four persons A, B, C and D begin to walk continually round it starting from the same place at the same time. They walk 3, 4, 6 and 8 miles per hour respectively. How soon will they all be again together at the starting point?

23 Five men run round a circular park in 4, 5, 6, 7 and 8 hours respectively. If they all start at the same time from the same point, find the least number of hours in which they will again be at that point together.

24 Three round pillars are 10 ft 5 in, 14 ft 7 in and 6 yds 9 in respectively in circumference, find the length of the shortest rope that can be wrapped round each an exact number of times.

25 The circumferences of the wheels of a carriage are 7 ft 4 in and 11 ft, what is the least distance in which both the wheels will make an exact number of revolutions?

26 A cask is required to be exactly filled by any one of the following measures, 1 seer, 2 seers, 3 seers, 5 seers, 6 seers or 9 seers, find the smallest cask for this purpose.

27 I have travelled between 700 and 760 miles, had I travelled 20 miles less, I could have completed my journey in a train which goes at the rate of 40 miles an hour, or in a carriage which goes at the rate of 16 miles an hour, or on foot at the rate of 6 miles an hour in an exact number of hours. Find the distance I have travelled.

28 Find the least number of 8 digits that is divisible by 15, 18, 25, 35, 40 and 55. Also the greatest number of 5 digits that is divisible by 14, 20, 35, 45 and 75.

CHAPTER V

The Doctrine of fractions

(USUALLY TERMED VULGAR FRACTIONS)

226 When a magnitude contains its unit a number of times exactly, the resulting number is called an *integer* or *whole number* (Art 7). Hence all *whole numbers*, or *integers*, being supposed to be formed by the *repetition* of the unit, may therefore be regarded as the result of the *multiplication* of that element, but if the unit be considered capable of *division* into any number of *equal* portions, the quantities thence arising must be viewed in the light of *broken* magnitudes, and these are therefore termed *Fractions* or more generally, *Vulgar Fractions*, in order to distinguish them from fractions of a different *form*, whose nature will be discussed in the next chapter.

I NOTATION AND NUMERATION OF FRACTIONS

227 A Fraction denotes a part or parts of a unit, it is expressed in figures by two numbers placed one above the other with a bar or line between them

228 If we suppose the *unit* to be divided into 2, 3, 4, 5, &c., equal portions, *one* of the portions in each case is represented by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ &c., which may be regarded as the **primitive fractions** of their respective denominations and are called the **reciprocals** of the natural numbers 2, 3, 4, 5, &c., also the fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, &c., are read, *one-half*, *one-third*, *one-fourth*, *one-fifth*, &c.

229 If *two* or *more* of these equal portions be taken together, the **aggregates** thence arising are expressed by repeating the unit as *often* as such portions are repeated, in the *form* of their sum, the number below the line remaining the same

Thus, if the primitive fraction $\frac{1}{2}$ be taken *twice* there will arise a new fraction expressed by $\frac{2}{2}$, if $\frac{1}{2}$ be repeated *thrice* there results a new fraction expressed by $\frac{3}{2}$, again, if $\frac{1}{2}$ be taken *four times*, the new fraction will be $\frac{4}{2}$, and similarly of all the other primitive fractions; also, the fractions $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, &c., are read *two-thirds*, *three-fourths*, *four-fifths*, &c., and all quantities of this *form* are called **Simple Fractions**

230 Hence, the number *below* the line denotes the number of equal portions into which the unit is supposed to be divided, and is therefore called the **denominator** and the number *above* the line expressing the number of such equal portions intended to be taken, is therefore termed the **numerator**. The numerator and denominator are called the **terms** of a fraction

Thus, of the fraction $\frac{5}{7}$, whose *terms* are 5 and 7, the denominator 7 implies that the unit is supposed to be divided into *seven* equal portions, and the numerator 5 shews that *five* of such equal portions are here the object of our consideration

231 The sum of a whole number and a fraction is called a **Mixed number**, as, $4 + \frac{5}{7}$ or rather $4\frac{5}{7}$, for the addition sign is almost always omitted

232 From what has been said above it appears, that a fraction expressed in figures is read by first reading the numerator and then the denominator with the termination "ths", thus $\frac{5}{7}$ is read *five-sevenths*. The exceptions are that fractions with denominator 2 or 3 are read as so many *halves* or *thirds*, and with denominator 4 as so many *quarters* as well as *fourths*. A mixed number is read by connecting the integer and the fraction by "*and*", thus $4\frac{5}{7}$ is read *four and five-sevenths*

233 From Art 230, it follows, that if the numerator be less than the denominator, the value of the fraction is less than the unit, if the numerator be equal to the denominator, the value of the frac-

tion is the unit, and if the numerator be greater than the denominator, the value of the fraction is greater than the unit

234 Every whole number or integer may be expressed as a fraction whose denominator is 1

Thus, $7 = \frac{7}{1}$, for the unit is divided into 1 part, comprising the whole unit, and 7 of such parts, that is 7 units, are taken

235 A fraction also expresses the quotient of the numerator by the denominator

Thus, $\frac{5}{7} = 5 \div 7$, since 1 unit is 7-sevenths, therefore 5 units is 35 sevenths, and therefore 5 divided by 7 is 35 sevenths divided by 7, and is therefore 5-sevenths, that is $5 \div 7 = \frac{5}{7}$. Hence $\frac{5}{7}$ is not only read 5 *sevenths* but also 5 *by 7*

Similarly, $\frac{8}{4} = 8 \div 4 = 2$, $\frac{7}{7} = 7 \div 7 = 1$, and so on

236 From the last Art it follows, that if we multiply a fraction by its denominator we get its numerator

Thus, since $\frac{5}{7}$ is the seventh part of 5, $\frac{5}{7}$ repeated 7 times gives 5, or $\frac{5}{7} \times 7 = 5$, and 5 may therefore be expressed in a *Fractional Form* by $\frac{5}{1}$

237 If we take a fractional magnitude and considering it as a new unit, divide it into any number of equal parts and take one or more of these parts, we shall obtain a **fraction of a fraction**, as $\frac{1}{4}$ of $\frac{3}{8}$

238 When fractions are represented in the manner above explained, they are called **Vulgar fractions**, (i.e.) *common* or *ordinary* fractions

239 We make the following distinctions in fractions —

- (1) A **proper fraction** is one in which the numerator is less than the denominator, thus $\frac{3}{4}$, $\frac{5}{7}$, $\frac{2}{8}$ are *proper* fractions
- (2) An **improper fraction** is one in which the numerator is either equal to or greater than the denominator, thus $\frac{7}{7}$, $\frac{9}{8}$, $\frac{13}{4}$ are *improper* fractions
- (3) A **simple fraction** is one in which numerator or denominator are both whole numbers, thus $\frac{3}{4}$, $\frac{10}{7}$ are *simple* fractions
- (4) A **compound fraction** is a fraction of a fraction, thus $\frac{3}{4}$ of $\frac{5}{8}$, $\frac{2}{3}$ of $\frac{1}{5}$ of $\frac{10}{9}$ are *compound* fractions
- (5) A **complex fraction** is one in which numerator or denominator or both are not whole numbers, thus $\frac{\frac{2}{3}}{\frac{5}{7}}$, $\frac{2\frac{1}{2}}{8}$, $\frac{3}{4\frac{1}{2}}$,

$\frac{2\frac{1}{2}}{3\frac{1}{2}}$, $\frac{2\frac{1}{2} + 1\frac{1}{2}}{2^1 - 1\frac{1}{4}}$ are *complex* fractions

240 The **reciprocal** of a fraction is the fraction formed by interchanging its terms, thus the *reciprocal* of $\frac{5}{7}$ is $\frac{7}{5}$, of 5 or $\frac{5}{1}$ is $\frac{1}{5}$.

241 We are hence enabled to find the results of the multiplication and division of a fraction by an integer, and these may be integers or fractions

(1) To *multiply* a fraction by a whole number, only multiply the *numerator* by it

Thus, $\frac{4}{13} \times 3 = \frac{4 \times 3}{13} = \frac{12}{13}$, because in $\frac{12}{13}$, *three times* as many parts of the unit are implied, as there are in $\frac{4}{13}$

(2) To *divide* a fraction by a whole number, only multiply the *denominator* by it

Thus, $\frac{2}{7} \div 5 = \frac{2}{7 \times 5} = \frac{2}{35}$, because the same number of parts are indicated in $\frac{2}{7}$ and $\frac{2}{35}$, and each part in the former is *five times* as great as each part in the latter, by Art. 230

Examples LXI

1 What fraction do we form in dividing a unit into 13 equal parts, and taking 11 of them, into 1000 equal parts, and taking 101?

2 Express in figures —

One-seventh, one quarter, seven-halves, thirty-four thirds, forty five seventy ninths, seven eighths, seven, and a half, nine, and seven-ninths, sixteen, and four twenty oneths, two hundred and three elevenths, ninety four, and five seventeenth

3 Express in words —

$\frac{2}{3}$, $\frac{3}{4}$, $\frac{7}{11}$, $\frac{17}{100}$, $3\frac{1}{2}$, $8\frac{1}{10}$, $24\frac{1}{11}$ and $125\frac{1}{1000}$

4 Multiply —

(1) $\frac{4}{8}$ and $\frac{3}{4}$ each separately by 2, 3, 5, 7, 9, 11, 12, 13 and 18

(2) $\frac{1}{2}$, and $\frac{2}{100}$ 36, 68, 80, 95, 112 and 157

5 Divide —

(1) $\frac{7}{7}$ and $\frac{1}{13}$ each separately by 2, 3, 5, 7, 9, 11, 12, 13 and 18

(2) $\frac{3}{2}$ and $\frac{2}{100}$ 36, 68, 80, 95, 112 and 157

II TRANSFORMATION OF FRACTIONS

242 If the numerator and denominator of a fraction be both multiplied or both divided by the same number, the value of the fraction will not be altered

For, if the fraction $\frac{3}{7}$ be multiplied by 5, the product is $\frac{15}{7}$ and again if this be divided by 5, the quotient is $\frac{3}{7}$, by Art. 241, but since these two operations are the *reverse* of and therefore *neutralize* each other, it follows that—

$$\frac{3}{7} = \frac{15}{35} = \frac{3 \times 5}{7 \times 5}, \text{ and also, that } \frac{15}{35} = \frac{3}{7} = \frac{15 \div 5}{35 \div 5}$$

243 It is clear from the above, that a whole number may be expressed in the form of a fraction with *any* denominator we please

$$\text{Thus, } 5 = \frac{5}{1} = \frac{5 \times 2}{1 \times 2} = \frac{10}{2} = \frac{20}{4} = \frac{35}{7} = \&c$$

Also, a fraction may be transformed into another with a *given* denominator or numerator, provided it be a *multiple* or *sub-multiple* of the denominator or numerator of the proposed fraction

Ex 1 Convert $\frac{7}{8}$ into a fraction with 96 for its denominator and reduce $\frac{24}{40}$ to a fraction with denominator 5

$$(1) \frac{7}{8} = \frac{7 \times 12}{8 \times 12} = \frac{84}{96}, \quad (2) \frac{24}{40} = \frac{24-8}{40-8} = \frac{3}{5}$$

Ex 2 Convert $\frac{5}{6}$ into a fraction with numerator 55, and $\frac{56}{64}$ into a fraction with numerator 7

$$(1) \frac{5}{6} = \frac{5 \times 11}{6 \times 11} = \frac{55}{66}, \quad (2) \frac{56}{64} = \frac{56-8}{64-8} = \frac{7}{8}$$

$$244 \text{ Since } \frac{5}{8} \times 4 = \frac{20}{8} = \frac{5 \times 4}{2 \times 4} = \frac{5}{2},$$

therefore, to *multiply* a fraction by an integer, it appears to be immaterial whether the numerator be multiplied, or the denominator be divided, by it, and since

$$\frac{8}{9} \div 4 = \frac{8}{36} = \frac{2 \times 4}{9 \times 4} = \frac{2}{9},$$

therefore, to *divide* a fraction by a whole number, it amounts to the same thing whether we multiply the denominator, or divide the numerator by it

245 Now referring to Art 241, we see that we have a choice of two methods both in the multiplication and division of a fraction by an integer, and we prefer the latter in accordance with the direction "*Divide when you can, multiply when you are obliged*"

Examples LXII

1 Reduce each of the whole numbers 3, 5, 7, 8, 15, 18, 20, 25 to a fraction with the denominator 13

2 Convert 26, 117 and 125 into fractions with denominators 13, 25 and 35 respectively

3 Convert $\frac{1}{2}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{4}{5}$, $\frac{5}{6}$ and $\frac{7}{8}$ into fractions having 120 for their denominator

4 Express $\frac{1}{12}$, $\frac{1}{15}$, $\frac{1}{20}$ and $\frac{1}{24}$ as fractions having 756 for their common numerator

5 Express $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$ each as a fraction with denominator 9 Also express each as a fraction with numerator 5040

6 Convert $\frac{1}{5}$, $\frac{1}{40}$, $\frac{1}{15}$, $\frac{1}{14}$ and $\frac{1}{20}$ into equivalent fractions with denominators 5, 40, 15, 14 and 20 respectively

246 To express a mixed number as an improper fraction

RULE. Multiply the integer by the denominator of the fraction to the product add the numerator and the result will be the new numerator, which placed over the given denominator will form the *improper* fraction required

Ex. Represent $3\frac{1}{2}$ as an improper fraction

$$3\frac{4}{5} = \frac{3 \times 5 + 4}{5} = \frac{15 + 4}{5} = 1^9 \quad \text{Ans}$$

247 To represent an improper fraction as a whole or mixed number

RULE. Divide the numerator by the denominator and the quotient will be the integral part, and the fractional part will be formed by placing the remainder over the given denominator. If there be no remainder, the fraction is equivalent to the integer thus found.

Ex Reduce $\frac{5}{6}$ and $\frac{7}{8}$ to whole or mixed numbers

(1) $8 \overline{) 32}$
 $\underline{4}$
 $32 - 32 = 0$
 $\therefore \frac{32}{8} = 4$ Ans

Or $\frac{32}{8} = \frac{8 \times 4}{8 \times 1} = \frac{4}{1}$ (Art 242) = 4 (Art 234)

(2) $11 \overline{) 327}$
 $\underline{29 \ 6}$
 $11 = 29 \frac{8}{11}$ Ans

1 or $\frac{327}{11} = \frac{319+8}{11} = \frac{319}{11} + \frac{8}{11}$ (Art 229)
 $= 29 + \frac{8}{11} = 29 \frac{8}{11}$ (Art 231)

248 The complete quotient of one number divided by another is the mixed number obtained by the above Rule

Thus, the *complete quotient* of 79 divided by 15 is the mixed number $5\frac{4}{15}$, for $79 \div 15 = 5\frac{4}{15}$.

Examples LXIII

1 Express *orally* the following as improper fractions —

- (1) $1\frac{1}{2}, 2\frac{1}{2}, 3\frac{1}{2}, 8\frac{1}{2}, 9\frac{1}{2}, 6\frac{1}{2}, 5\frac{1}{2}, 7\frac{1}{2}, 4\frac{1}{2}, 9\frac{1}{2}$
 (2) $13\frac{1}{2}, 15\frac{1}{2}, 16\frac{1}{2}, 19\frac{1}{2}, 14\frac{1}{2}, 20\frac{1}{2}, 17\frac{1}{2}$

2 Convert into improper fractions —

- [illegible]

3 Express *orally* as mixed or whole numbers —

- [illegible]

4 Represent the following as mixed or whole numbers —

- $$(1) \quad \frac{440}{13}, \frac{2417}{19}, \frac{3797}{29}, \frac{9999}{31}, \frac{30471}{37}, \frac{523}{23}, \frac{747}{45}, \frac{775}{31}$$

$$(2) \quad \frac{3003}{217}, \frac{4531}{171}, \frac{6984}{481}, \frac{52504}{572}, \frac{51637}{152}, \frac{9999}{347}, \frac{19585}{144}$$

$$(3) \quad \frac{76845}{999}, \frac{830526}{9891}, \frac{907133}{7816}, \frac{4003187}{99999}, \frac{1516461}{30125}, \frac{1001010111}{100001}$$

5 Express the reciprocals of the following fractions, as mixed numbers -

$$\frac{7}{15}, \frac{15}{49}, \frac{17}{65}, \frac{100}{6874}, \frac{87}{3415}, \frac{9}{4567}, \frac{152}{51847}, \frac{1251}{59956}$$

6 Express $41\frac{7}{8}$, $25\frac{1}{2}$, $9\frac{5}{8}$, and $10\frac{9}{10}$ as fractions, with denominators 240 and 720

7 Find the respective values of -

$$(1) \quad 2\frac{1}{4} \times 8, \frac{1}{2} \times 17, 6\frac{3}{8} \times 7, 10\frac{7}{17} \times 17, 6\frac{1}{11} \times 11, 2\frac{1}{8} \times 13$$

$$(2) \quad \frac{1}{5} - 9, \frac{1}{10} - 7, \frac{1}{12} - 13, \frac{1}{20} - 11, 6\frac{9}{12} - 12, 9\frac{1}{7} - 15$$

249 To express a compound fraction as a simple one

A **Compound Fraction** is made up of two or more simple fractions connected by the word *of*, as $\frac{1}{3}$ of $\frac{1}{5}$ of $3\frac{9}{10}$

RULE Multiply all the numerators together for the numerator of the simple fraction, and all the denominators together for its denominator

Ex 1 Convert $\frac{1}{3}$ of $\frac{1}{5}$ of $3\frac{9}{10}$ into a simple fraction

$$\left. \begin{aligned} \frac{1}{3} \text{ of } \frac{1}{5} \text{ of } 3\frac{9}{10} &= \frac{1 \times 4 \times 6}{3 \times 5 \times 7} & \text{For } \frac{1}{3} \text{ of } 3\frac{9}{10} &= \frac{1}{3} \times 5 = \frac{5}{3} \\ &= \frac{24}{105} \text{ Ans} & \text{and } \frac{1}{5} \text{ of } \frac{5}{3} &= \frac{1}{3} \times 4 = \frac{4}{3} \\ \text{whence, } \frac{1}{3} \text{ of } \frac{1}{5} \text{ of } 3\frac{9}{10} &= \frac{1}{3} \text{ of } \frac{4}{3} = \frac{4}{9} - 3 = \frac{24}{105} \end{aligned} \right\} \text{ (Ait 241)}$$

$$\frac{1}{3} \text{ of } \frac{1}{5} \text{ of } 3\frac{9}{10} = \frac{1 \times 4 \times 6}{3 \times 5 \times 7} = \frac{24}{105}$$

Note 1 Before applying the above Rule mixed numbers must be expressed as improper fractions

Note 2 If there are factors common to both numerator and denominator, they may be *cancelled* or struck out, before obtaining the final result, for this is in fact simply dividing the numerator and denominator of a fraction by the same number (Ait 242)

Ex 2 Reduce $\frac{3}{8}$ of $2\frac{1}{2}$ of $5\frac{1}{5}$ to a simple fraction

$$\begin{aligned} \frac{3}{8} \text{ of } 2\frac{1}{2} \text{ of } 5\frac{1}{5} &= \frac{3}{8} \text{ of } \frac{5}{2} \text{ of } 7\frac{1}{5} = \frac{3 \times (5 \times 5) \times (4 \times 19)}{8 \times (3 \times 4) \times (3 \times 5)} \\ &= \frac{3 \times 5 \times 5 \times 4 \times 19}{5 \times 3 \times 4 \times 3 \times 5} = \frac{19}{3} = 6\frac{1}{3} \text{ Ans} \end{aligned}$$

(dividing numerator and denominator by the factors 3, 5, 5, 4 common to both)

Examples LXIV

Reduce the following compound fractions to simple ones —

- 1 $\frac{1}{4}$ of $\frac{1}{8}$, $\frac{1}{8}$ of $\frac{1}{16}$, $\frac{1}{16}$ of $\frac{1}{32}$, $\frac{1}{32}$ of $\frac{1}{64}$, $\frac{1}{64}$ of $\frac{1}{128}$
- 2 $\frac{1}{2}$ of $\frac{1}{3}$, $\frac{1}{3}$ of $\frac{1}{4}$, $\frac{1}{4}$ of $\frac{1}{5}$ of 9, $\frac{1}{5}$ of $\frac{1}{6}$ of $\frac{1}{7}$, $\frac{1}{7}$ of $\frac{1}{8}$ of $\frac{1}{9}$
- 3 $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{1}{5}$, $\frac{1}{5}$ of $\frac{1}{6}$ of $\frac{1}{7}$ of $\frac{1}{8}$ of $\frac{1}{9}$ of $\frac{1}{10}$
- 4 $\frac{1}{2}$ of $\frac{1}{3}$ of 4 of $\frac{1}{2}$ of $\frac{1}{3}$ of 6, $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{1}{5}$ of $\frac{1}{6}$ of $\frac{1}{7}$ of $\frac{1}{8}$ of $\frac{1}{9}$
- 5 $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{1}{5}$ of $\frac{1}{6}$ of $\frac{1}{7}$ of $\frac{1}{8}$ of $\frac{1}{9}$ of $\frac{1}{10}$ of $\frac{1}{11}$ of $\frac{1}{12}$ of $\frac{1}{13}$ of $\frac{1}{14}$ of $\frac{1}{15}$ of $\frac{1}{16}$ of $\frac{1}{17}$ of $\frac{1}{18}$ of $\frac{1}{19}$ of $\frac{1}{20}$
- 6 $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{1}{5}$ of $\frac{1}{6}$ of $\frac{1}{7}$ of $\frac{1}{8}$ of $\frac{1}{9}$ of $\frac{1}{10}$ of $\frac{1}{11}$ of $\frac{1}{12}$ of $\frac{1}{13}$ of $\frac{1}{14}$ of $\frac{1}{15}$ of $\frac{1}{16}$ of $\frac{1}{17}$ of $\frac{1}{18}$ of $\frac{1}{19}$ of $\frac{1}{20}$
- 7 $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{1}{5}$ of $\frac{1}{6}$ of $\frac{1}{7}$ of $\frac{1}{8}$ of $\frac{1}{9}$ of $\frac{1}{10}$ of $\frac{1}{11}$ of $\frac{1}{12}$ of $\frac{1}{13}$ of $\frac{1}{14}$ of $\frac{1}{15}$ of $\frac{1}{16}$ of $\frac{1}{17}$ of $\frac{1}{18}$ of $\frac{1}{19}$ of $\frac{1}{20}$
- 8 $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{1}{5}$ of $\frac{1}{6}$ of $\frac{1}{7}$ of $\frac{1}{8}$ of $\frac{1}{9}$ of $\frac{1}{10}$ of $\frac{1}{11}$ of $\frac{1}{12}$ of $\frac{1}{13}$ of $\frac{1}{14}$ of $\frac{1}{15}$ of $\frac{1}{16}$ of $\frac{1}{17}$ of $\frac{1}{18}$ of $\frac{1}{19}$ of $\frac{1}{20}$
- 9 $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{1}{5}$ of $\frac{1}{6}$ of $\frac{1}{7}$ of $\frac{1}{8}$ of $\frac{1}{9}$ of $\frac{1}{10}$ of $\frac{1}{11}$ of $\frac{1}{12}$ of $\frac{1}{13}$ of $\frac{1}{14}$ of $\frac{1}{15}$ of $\frac{1}{16}$ of $\frac{1}{17}$ of $\frac{1}{18}$ of $\frac{1}{19}$ of $\frac{1}{20}$
- 10 $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{1}{5}$ of $\frac{1}{6}$ of $\frac{1}{7}$ of $\frac{1}{8}$ of $\frac{1}{9}$ of $\frac{1}{10}$ of $\frac{1}{11}$ of $\frac{1}{12}$ of $\frac{1}{13}$ of $\frac{1}{14}$ of $\frac{1}{15}$ of $\frac{1}{16}$ of $\frac{1}{17}$ of $\frac{1}{18}$ of $\frac{1}{19}$ of $\frac{1}{20}$
- 11 $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{1}{5}$ of $\frac{1}{6}$ of $\frac{1}{7}$ of $\frac{1}{8}$ of $\frac{1}{9}$ of $\frac{1}{10}$ of $\frac{1}{11}$ of $\frac{1}{12}$ of $\frac{1}{13}$ of $\frac{1}{14}$ of $\frac{1}{15}$ of $\frac{1}{16}$ of $\frac{1}{17}$ of $\frac{1}{18}$ of $\frac{1}{19}$ of $\frac{1}{20}$
- 12 $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{1}{5}$ of $\frac{1}{6}$ of $\frac{1}{7}$ of $\frac{1}{8}$ of $\frac{1}{9}$ of $\frac{1}{10}$ of $\frac{1}{11}$ of $\frac{1}{12}$ of $\frac{1}{13}$ of $\frac{1}{14}$ of $\frac{1}{15}$ of $\frac{1}{16}$ of $\frac{1}{17}$ of $\frac{1}{18}$ of $\frac{1}{19}$ of $\frac{1}{20}$
- 13 $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{1}{5}$ of $\frac{1}{6}$ of $\frac{1}{7}$ of $\frac{1}{8}$ of $\frac{1}{9}$ of $\frac{1}{10}$ of $\frac{1}{11}$ of $\frac{1}{12}$ of $\frac{1}{13}$ of $\frac{1}{14}$ of $\frac{1}{15}$ of $\frac{1}{16}$ of $\frac{1}{17}$ of $\frac{1}{18}$ of $\frac{1}{19}$ of $\frac{1}{20}$
- 14 $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{1}{5}$ of $\frac{1}{6}$ of $\frac{1}{7}$ of $\frac{1}{8}$ of $\frac{1}{9}$ of $\frac{1}{10}$ of $\frac{1}{11}$ of $\frac{1}{12}$ of $\frac{1}{13}$ of $\frac{1}{14}$ of $\frac{1}{15}$ of $\frac{1}{16}$ of $\frac{1}{17}$ of $\frac{1}{18}$ of $\frac{1}{19}$ of $\frac{1}{20}$
- 15 $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{1}{5}$ of $\frac{1}{6}$ of $\frac{1}{7}$ of $\frac{1}{8}$ of $\frac{1}{9}$ of $\frac{1}{10}$ of $\frac{1}{11}$ of $\frac{1}{12}$ of $\frac{1}{13}$ of $\frac{1}{14}$ of $\frac{1}{15}$ of $\frac{1}{16}$ of $\frac{1}{17}$ of $\frac{1}{18}$ of $\frac{1}{19}$ of $\frac{1}{20}$
- 16 $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{1}{5}$ of $\frac{1}{6}$ of $\frac{1}{7}$ of $\frac{1}{8}$ of $\frac{1}{9}$ of $\frac{1}{10}$ of $\frac{1}{11}$ of $\frac{1}{12}$ of $\frac{1}{13}$ of $\frac{1}{14}$ of $\frac{1}{15}$ of $\frac{1}{16}$ of $\frac{1}{17}$ of $\frac{1}{18}$ of $\frac{1}{19}$ of $\frac{1}{20}$

250 A fraction is in its lowest terms, or in its simplest form when there are no factors common to both numerator and denominator. This will be the case when the numerator and denominator are prime to each other.

251 To reduce a fraction to its lowest terms

RULE Divide the numerator and denominator by their G C M

Ex Express the fraction $\frac{825}{960}$ in its lowest terms

The G C M of 825 and 960 is 15

$$\frac{825}{960} = \frac{825-15}{960-15} = \frac{55}{64} \quad \text{Ans}$$

252 In many instances, it is unnecessary to find the G C M at first, the fractions being reducible to lower terms by successive divisions of the numerators and denominators by common factors discovered by inspection, or by employing the tests of divisibility given in Art 198

Ex Reduce $\frac{4968}{5904}$ to its lowest terms

$$\frac{4968}{5904} = \frac{2484}{2952} = \frac{1242}{1476} = \frac{621}{738} = \frac{207}{246} = \frac{69}{82} \quad \text{Ans}$$

from three successive divisions of the numerator and denominator by 2, and then from two successive divisions by 3, and these are the terms which would have been obtained from dividing at once by 72 which is their G C M

253 In examples like the following, it is convenient to break

up both the numerator and denominator into factors, and then *cancel* those which are common to both

Note It should be remarked that when a factor is *cancelled*, it is to be replaced by 1 and not by 0

Ex Reduce $\frac{35 \times 63}{60 \times 77}$ to its lowest terms

$$\frac{35 \times 63}{60 \times 77} = \frac{(5 \times 7) \times (3 \times 3 \times 7)}{(3 \times 5 \times 4) \times (7 \times 11)} = \frac{5 \times 7 \times 3 \times 3 \times 7}{3 \times 5 \times 7 \times 4 \times 11} = \frac{3 \times 7}{4 \times 11} = \frac{21}{44} \quad \text{Ans}$$

Examples LXV

1 Reduce to their lowest terms (by *inspection*) —

- (1) $\frac{3}{4}, \frac{1}{2}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}$
 (2) $\frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}$
 (3) $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}$

2 Reduce the following fractions to their lowest terms —

- (1) $\frac{135}{957}, \frac{203}{315}, \frac{455}{1645}, \frac{256}{1024}, \frac{444}{703}, \frac{925}{1025}, \frac{768}{2592}, \frac{1476}{1764}$
 (2) $\frac{3094}{3042}, \frac{3444}{3556}, \frac{5239}{6076}, \frac{5565}{8533}, \frac{7568}{9504}, \frac{1775}{2350}, \frac{3565}{4930}, \frac{9050}{17919}$
 (3) $\frac{1261}{1649}, \frac{6435}{7293}, \frac{1236}{4764}, \frac{6006}{8008}, \frac{9504}{10692}, \frac{7447}{15729}, \frac{48510}{49005}$
 (4) $\frac{8991}{10589}, \frac{12540}{21945}, \frac{13478}{16701}, \frac{8398}{29393}, \frac{43365}{44688}, \frac{13667}{14186}, \frac{217800}{245025}$
 (5) $\frac{11050}{35581}, \frac{20301}{33633}, \frac{714285}{999999}, \frac{109375}{1000000}, \frac{135795}{222210}, \frac{99715}{113960}$
 (6) $\frac{95469}{359784}, \frac{180194}{1973594}, \frac{256417}{7006987}, \frac{1854432}{3171276}, \frac{1832051}{2592525}, \frac{496606401}{1006110363}$

3 Reduce (by *cancelling*) to their lowest terms —

- (1) $\frac{18 \times 32}{27 \times 52}, \frac{16 \times 45}{24 \times 75}, \frac{21 \times 24}{56 \times 84}, \frac{45 \times 70 \times 15}{81 \times 90 \times 100}, \frac{48 \times 64 \times 49}{96 \times 88 \times 63}$
 (2) $\frac{51 \times 39 \times 42}{68 \times 52 \times 70}, \frac{19 \times 23 \times 26 \times 56}{57 \times 92 \times 78 \times 98}, \frac{85 \times 84 \times 38}{102 \times 154 \times 95}, \frac{76 \times 87 \times 65}{114 \times 145 \times 143}$

254 To reduce two or more fractions having different denominators to equivalent fractions having a common denominator

RULE Multiply each numerator by all the denominators *except* the one placed under it, for the new numerator and multiply all the denominators together for the common denominator

Ex Express $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{3}{4}$ as equivalent fractions with a common denominator

$$\begin{array}{l} \text{Here, first: } \left. \begin{array}{l} 1 \times 5 \times 7 = 35 \\ 2 \times 2 \times 7 = 28 \\ 3 \times 2 \times 5 = 30 \end{array} \right\} \begin{array}{l} \text{the new} \\ \text{numerators} \end{array} \end{array} \quad \begin{array}{l} \text{For } \frac{1}{2} = \frac{1 \times 5 \times 7}{2 \times 5 \times 7} = \frac{35}{70}, \\ \frac{2}{3} = \frac{2 \times 2 \times 7}{5 \times 2 \times 7} = \frac{28}{70}, \\ \text{and } \frac{3}{4} = \frac{3 \times 2 \times 5}{7 \times 2 \times 5} = \frac{30}{70} \end{array}$$

and $2 \times 5 \times 7 = 70$, the com den

\therefore the equivalent fractions are $\frac{35}{70}$, $\frac{28}{70}$ and $\frac{30}{70}$ *Ans*

255 If two or more of the denominators have a common measure, the equivalent fractions may be expressed in simpler terms than obtainable by the above Rule, and having a least common denominator (L C D) by the following Rule

RULE Find the L C M of the denominators, this will be the least common denominator. Then divide the L C M so found by the denominator of each fraction, and multiply each quotient so found into the numerator of the fraction which belongs to it for the new numerator of that fraction

Note Before applying the above Rules, reduce mixed numbers to improper fractions, and compound fractions to simple ones, moreover, if the L C D be required, the given fractions should be reduced to their lowest terms

Ex Reduce $\frac{4}{5}$, $\frac{11}{12}$ and $\frac{3}{20}$ to equivalent fractions having the least common denominator

The L C M of 5, 12 and 20 is 60, which is here the L C D

$$60 \div 5 = 12, \quad 60 \div 12 = 5, \quad 60 \div 20 = 3$$

$$\therefore \frac{4}{5} = \frac{4 \times 12}{5 \times 12} = \frac{48}{60}, \quad \frac{11}{12} = \frac{11 \times 5}{12 \times 5} = \frac{55}{60}, \quad \frac{3}{20} = \frac{3 \times 3}{20 \times 3} = \frac{9}{60}$$

Hence, the equivalent fractions are $\frac{48}{60}$, $\frac{55}{60}$ and $\frac{9}{60}$ *Ans*

256 Similarly we can reduce fractions to equivalent ones having a least common numerator (L C N)

Ex Reduce $\frac{5}{6}$, $\frac{4}{9}$, $\frac{8}{16}$ and $\frac{16}{17}$ to fractions having a least common numerator

The L C M of 5, 4, 8 and 16 = 80, which is here the L C N

$$80 \div 5 = 16; \quad 80 \div 4 = 20, \quad 80 \div 8 = 10, \quad 80 \div 16 = 5$$

$$\therefore \frac{5}{6} = \frac{5 \times 16}{6 \times 16} = \frac{80}{96}, \quad \frac{4}{9} = \frac{4 \times 20}{9 \times 20} = \frac{80}{180},$$

$$\frac{8}{9} = \frac{8 \times 10}{9 \times 10} = \frac{80}{90}, \quad \frac{16}{17} = \frac{16 \times 5}{17 \times 5} = \frac{80}{85}$$

\therefore the fractions with a L C N are $\frac{80}{96}$, $\frac{80}{180}$, $\frac{80}{90}$, $\frac{80}{85}$ *Ans*

Examples LXVI

1 Reduce to equivalent fractions with a common denominator —

- (1) $\frac{2}{3}, \frac{4}{5}$ (2) $\frac{1}{4}, \frac{5}{7}$ (3) $\frac{3}{4}, \frac{9}{11}$ (4) $\frac{1}{2}, \frac{4}{5}, \frac{6}{7}$ (5) $\frac{2}{3}, \frac{5}{7}, \frac{1}{11}$
 (6) $\frac{1}{2}, \frac{3}{4}, \frac{1}{5}, \frac{9}{13}$ (7) $\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{8}, \frac{5}{11}$ (8) $1\frac{1}{3}, 2\frac{1}{5}, 3\frac{1}{7}$
 (9) $\frac{4}{5}, 2\frac{1}{6}, 3\frac{1}{11}$ (10) $7, \frac{8}{9}, 10\frac{1}{11}, 26\frac{1}{12}$ (11) $\frac{2}{3}$ of $\frac{9}{10}, \frac{1}{6}$ of $5\frac{1}{4}, \frac{1}{4}$ of $1\frac{1}{2}$

2 Reduce the fractions in each of the following sets to equivalent fractions, having the least common denominator —

- (1) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$ (2) $\frac{2}{3}, \frac{4}{5}, \frac{7}{8}$ (3) $\frac{19}{21}, \frac{32}{33}$ (4) $\frac{3}{10}, \frac{8}{11}, \frac{5}{12}$
 (5) $\frac{7}{15}, \frac{1}{11}, \frac{2}{13}$ (6) $\frac{5}{6}, \frac{7}{8}, \frac{9}{10}, \frac{1}{12}$ (7) $\frac{7}{8}, \frac{8}{9}, \frac{1}{11}$ (8) $\frac{3}{4}, \frac{1}{5}, \frac{2}{6}, \frac{5}{7}$
 (9) $\frac{1}{25}, \frac{3}{40}, \frac{1}{45}, \frac{1}{60}, \frac{1}{75}$ (10) $\frac{1}{35}, \frac{1}{40}, \frac{1}{45}, \frac{1}{60}, \frac{2}{75}, \frac{2}{99}$
 (11) $\frac{1}{2}, \frac{1}{3}$ of $\frac{1}{2}$ of $\frac{1}{3}, \frac{1}{2}$ of $\frac{2}{3}$ of $2\frac{1}{2}, \frac{4}{5}, \frac{5}{6}$ (12) $1\frac{1}{2}, 3\frac{1}{3}, 4\frac{1}{4}, 6\frac{1}{6}$
 (13) $3\frac{1}{5}, 4\frac{1}{6}, 1\frac{1}{8}, \frac{1}{9}, 13\frac{1}{12}$ (14) $\frac{2}{7}$ of $\frac{2}{10}, \frac{5}{6}$ of $2\frac{8}{9}, \frac{4}{5}$ of $3\frac{1}{4}$ of $3\frac{1}{2}$

3 Reduce the following fractions to equivalent ones with the least common numerator —

- (1) $\frac{1}{2}, \frac{3}{4}, \frac{4}{5}, \frac{2}{3}$ (2) $\frac{1}{6}, \frac{8}{9}, \frac{1}{12}, \frac{1}{15}$ (3) $\frac{1}{25}, \frac{2}{30}, \frac{1}{45}, \frac{7}{15}$
 (4) $\frac{3}{7}, \frac{7}{13}, \frac{9}{22}, \frac{2}{10}$ (5) $\frac{2}{27}, \frac{1}{34}, \frac{9}{15}, \frac{2}{31}$ (6) $\frac{4}{6}, \frac{2}{3}, \frac{1}{10}, \frac{1}{18}, \frac{1}{15}$

257 To compare the magnitudes of different fractions

- (1) RULE Reduce the fractions to equivalent ones with the least common denominator (L C D), and then compare the numerators so obtained. That fraction which has the greatest numerator is the *greatest*, and that which has the least is the *least*.

Ex 1 Find the *greatest* and *least* of the fractions $\frac{7}{9}, \frac{5}{8}$ and $1\frac{1}{4}$

The L C M of the denominators = 504

$$504 - 9 = 56, 504 - 8 = 63, 504 - 14 = 36$$

$$\frac{7}{9} = \frac{7 \times 56}{9 \times 56} = \frac{392}{504}, \frac{5}{8} = \frac{5 \times 63}{8 \times 63} = \frac{315}{504}, \frac{11}{14} = \frac{11 \times 36}{14 \times 36} = \frac{396}{504}$$

Hence $1\frac{1}{4}$ is the *greatest* and $\frac{5}{8}$ is the *least* Ans

Ex 2 Arrange $\frac{4}{5}, \frac{1}{12}, \frac{1}{15}$ and $\frac{8}{9}$ in order of magnitude

The L C M of the denominators = 180

$$180 - 5 = 36, 180 - 12 = 15, 180 - 15 = 12, 180 - 9 = 20$$

$$\frac{4}{5} = \frac{4 \times 36}{5 \times 36} = \frac{144}{180}, \frac{11}{12} = \frac{11 \times 15}{12 \times 15} = \frac{165}{180},$$

$$\frac{13}{15} = \frac{13 \times 12}{15 \times 12} = \frac{156}{180}, \frac{8}{9} = \frac{8 \times 20}{9 \times 20} = \frac{160}{180}$$

Hence the fractions arranged in order of magnitude stand thus —

$$1\frac{1}{4}, \frac{8}{9}, \frac{13}{15} \text{ and } \frac{4}{5} \text{ Ans}$$

- (2) Fractions may also be compared by reducing them to a least common numerator (L C N). In this case, the new fraction that has the least denominator is the *greatest*, and that which has the greatest denominator is the *least*.

Ex Find the *greatest* and the *least* of $\frac{2}{5}$, $\frac{3}{10}$, $\frac{7}{15}$ and $\frac{9}{16}$

The L C N of the numerators = 126

$$126 \div 2 = 63, 126 \div 3 = 42, 126 \div 7 = 18, 126 \div 9 = 14$$

$$\frac{2}{5} = \frac{2 \times 63}{5 \times 63} = \frac{126}{315}, \quad \frac{3}{10} = \frac{3 \times 42}{10 \times 42} = \frac{126}{420},$$

$$\frac{7}{15} = \frac{7 \times 18}{15 \times 18} = \frac{126}{270}, \quad \frac{9}{16} = \frac{9 \times 14}{16 \times 14} = \frac{126}{224}$$

Hence $\frac{9}{16}$ is the *greatest* and $\frac{3}{10}$ is the *least* Ans

258 The defect of a fraction from 1 is called its **complement**

Thus, $\frac{1}{4}$ and $\frac{3}{4}$ are respectively the *complements* of $\frac{3}{4}$ and $\frac{1}{4}$

- (3) Fractions may also be compared by taking their complements, provided that each of the complements has 1 for its numerator

The *greatest* and *least* fractions will be those that have the least and the greatest complement

Ex Find the *greatest* and the *least* of the fractions $\frac{1}{8}$, $\frac{2}{5}$, and $\frac{3}{4}$

The complements of these fractions are $\frac{7}{8}$, $\frac{3}{5}$ and $\frac{1}{4}$ respectively

Now, of these complements $\frac{3}{5}$ is the least and $\frac{7}{8}$ the greatest, $\frac{2}{5}$ is the *greatest* and $\frac{1}{8}$ is the *least* Ans

- (4) Fractions may also be compared by the method illustrated by the following example

Ex Arrange in order of magnitude $\frac{3}{7}$, $\frac{5}{21}$ and $\frac{6}{19}$

$$\frac{3}{7} = \frac{3-3}{7-3} = \frac{1}{2\frac{1}{2}}, \quad \frac{5}{21} = \frac{5-5}{21-5} = \frac{1}{4\frac{1}{2}}, \quad \frac{6}{19} = \frac{6-6}{19-6} = \frac{1}{13\frac{1}{2}} \quad (\text{Art. 242})$$

The given fractions = $\frac{1}{2\frac{1}{2}}$, $\frac{1}{4\frac{1}{2}}$, $\frac{1}{13\frac{1}{2}}$ respectively. Of these $\frac{1}{2\frac{1}{2}}$ is

the greatest and $\frac{1}{13\frac{1}{2}}$ is the least, for they have respectively the least and greatest denominators

the order of magnitude is $\frac{6}{19}$, $\frac{3}{7}$ and $\frac{5}{21}$ Ans

Examples LXVII

- 1 Which is the greater? (by the *first method*)

$$\frac{3}{8} \text{ or } \frac{4}{9}, \quad \frac{7}{11} \text{ or } \frac{9}{12}, \quad \frac{1}{28} \text{ or } \frac{3}{28}, \quad \frac{1}{24} \text{ or } \frac{1}{25}, \quad \frac{1}{4} \text{ or } \frac{1}{5}, \quad \frac{1}{10} \text{ or } \frac{1}{11}, \quad \frac{15+8}{19+8}$$

The sum should always be expressed in its lowest terms, and, if an improper fraction, should be reduced to a mixed number.

Ex. Find the sum of $\frac{2}{3}$, $\frac{1}{4}$, $\frac{5}{6}$ and $\frac{7}{8}$.

The L.C.M. of the denominators = 24

$$\frac{2}{3} = \frac{2 \times 8}{3 \times 8} = \frac{16}{24}, \quad \frac{1}{4} = \frac{1 \times 6}{4 \times 6} = \frac{6}{24},$$

$$\frac{5}{6} = \frac{5 \times 4}{6 \times 4} = \frac{20}{24}, \quad \frac{7}{8} = \frac{7 \times 3}{8 \times 3} = \frac{21}{24}$$

$$\therefore \text{the sum} = \frac{16}{24} + \frac{6}{24} + \frac{20}{24} + \frac{21}{24} = \frac{16+6+20+21}{24} \\ = \frac{63}{24} = \frac{21 \times 3}{8 \times 3} = \frac{21}{8} = 2\frac{5}{8} \quad \text{Ans}$$

260 All fractions should be reduced to their lowest terms, improper fractions to whole or mixed numbers, and compound fractions to simple ones before the application of the Rule.

261 If any one of the given numbers be whole or mixed numbers add together the whole numbers as in simple addition and the fractional parts by the Rule given above.

Ex. Add together $5\frac{1}{2}$, $1\frac{3}{4}$, $2\frac{1}{8}$ and $\frac{3}{4}$ of 3.

Here, $1\frac{3}{4} = 3\frac{6}{8}$, $2\frac{1}{8} = 2\frac{1}{8}$, $\frac{3}{4}$ of 3 = $\frac{3}{4}$ of 3 = $\frac{9}{4} = 2\frac{1}{4}$

$$\therefore \text{sum of the fractions} = 5\frac{1}{2} + 3\frac{6}{8} + 2\frac{1}{8} + 2\frac{2}{4} \\ = (5+3+2+2) + (\frac{4}{8} + \frac{6}{8} + \frac{1}{8} + \frac{2}{4}) \\ = 12 + \frac{20+8+2+8}{8} \\ = 12 + \frac{38}{8} = 12 + 4\frac{7}{8} = 16\frac{7}{8} \quad \text{Ans}$$

Examples LXVIII

1 Add together orally the following fractions --

(1) $\frac{1}{2} + \frac{1}{3}$, $\frac{1}{4} + \frac{1}{6}$, $\frac{1}{5} + \frac{1}{10}$, $\frac{1}{6} + \frac{1}{12}$, $\frac{1}{8} + \frac{1}{16}$, $\frac{1}{9} + \frac{1}{18}$, $\frac{1}{10} + \frac{1}{20}$, $\frac{1}{12} + \frac{1}{24}$

(2) $\frac{1}{2} + \frac{1}{4}$, $\frac{1}{3} + \frac{1}{6}$, $\frac{1}{4} + \frac{1}{8}$, $\frac{1}{5} + \frac{1}{10}$, $\frac{1}{6} + \frac{1}{12}$, $\frac{1}{8} + \frac{1}{16}$, $\frac{1}{9} + \frac{1}{18}$, $\frac{1}{10} + \frac{1}{20}$

(3) $\frac{1}{2} + \frac{1}{4} + \frac{1}{6}$, $\frac{1}{3} + \frac{1}{6} + \frac{1}{12}$, $\frac{1}{4} + \frac{1}{8} + \frac{1}{16}$, $\frac{1}{5} + \frac{1}{10} + \frac{1}{20}$, $\frac{1}{6} + \frac{1}{12} + \frac{1}{24}$, $\frac{1}{8} + \frac{1}{16} + \frac{1}{32}$

(4) $3\frac{1}{2} + 2\frac{1}{4}$, $4\frac{1}{3} + 3\frac{1}{6}$, $1\frac{1}{4} + 3\frac{1}{8} + 2\frac{1}{2}$, $3\frac{1}{6} + 4\frac{1}{3} + 7\frac{1}{2}$

2 Find the values of the following --

(1) $3\frac{1}{2} + 2\frac{1}{4}$, $4\frac{1}{3} + 3\frac{1}{6}$, $5\frac{1}{4} + 2\frac{1}{8}$, $6\frac{1}{5} + 3\frac{1}{10}$, $7\frac{1}{6} + 4\frac{1}{12}$, $8\frac{1}{8} + 5\frac{1}{16}$, $9\frac{1}{9} + 6\frac{1}{18}$, $10\frac{1}{10} + 7\frac{1}{20}$

(2) $1\frac{1}{2} + 7\frac{1}{3}$, $2\frac{1}{4} + 13\frac{1}{6}$, $5\frac{1}{8} + 12\frac{1}{4}$, $37\frac{1}{12} + 24\frac{1}{3}$, $7\frac{1}{5} + 4\frac{1}{10}$

(3) $2\frac{1}{3} + \frac{1}{6}$, $4\frac{1}{4} + \frac{1}{8}$, $5\frac{1}{5} + \frac{1}{10}$, $6\frac{1}{6} + \frac{1}{12}$, $7\frac{1}{7} + \frac{1}{14}$, $8\frac{1}{8} + \frac{1}{16}$, $9\frac{1}{9} + \frac{1}{18}$, $10\frac{1}{10} + \frac{1}{20}$

(4) $\frac{1}{2} + \frac{1}{4} + \frac{1}{6}$, $\frac{1}{3} + \frac{1}{6} + \frac{1}{12}$, $\frac{1}{4} + \frac{1}{8} + \frac{1}{16}$, $\frac{1}{5} + \frac{1}{10} + \frac{1}{20}$, $\frac{1}{6} + \frac{1}{12} + \frac{1}{24}$, $\frac{1}{8} + \frac{1}{16} + \frac{1}{32}$

(5) $1\frac{1}{2} + 4\frac{1}{3} + 5\frac{1}{6}$, $2\frac{1}{4} + 3\frac{1}{6} + 5\frac{1}{12}$, $8\frac{1}{8} + 13\frac{1}{4} + 27\frac{1}{8}$, $1\frac{1}{5} + 3\frac{1}{10} + 11 + \frac{1}{2}$

(6) $\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{12}$, $\frac{1}{3} + \frac{1}{6} + \frac{1}{12} + \frac{1}{24}$, $\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32}$, $\frac{1}{5} + \frac{1}{10} + \frac{1}{20} + \frac{1}{40}$

- (7) $\frac{1}{3}$ of $9\frac{5}{6} + \frac{4}{7}$ of $8\frac{1}{2}$, $14\frac{5}{6} + \frac{2}{3}$ of $\frac{5}{6}$ of 8, $\frac{7}{8} + 4\frac{1}{2} + \frac{1}{5}$ of 2
 (8) $1\frac{2}{3} + \frac{5}{6} + 1\frac{1}{2} + 3\frac{1}{6}$ (9) $3\frac{5}{6} + 2\frac{2}{3} + \frac{1}{11} + 7\frac{1}{6}$
 (10) $2\frac{1}{2} + 3\frac{1}{6} + 4\frac{1}{2} + 5\frac{1}{6}$ (11) $3\frac{1}{2} + 7\frac{1}{2} + 8\frac{2}{3} + 4\frac{1}{6}$
 (12) $1\frac{1}{2} + 2\frac{1}{3} + 3\frac{1}{6} + 4\frac{1}{6}$ (13) $4\frac{2}{3} + 8\frac{2}{3} + 3\frac{1}{3} + 8\frac{2}{3}$
 (14) $\frac{1}{11}$ of $\frac{5}{6}$ of $\frac{1}{2} + \frac{1}{3}$ of $\frac{1}{5}$ of $\frac{10}{3} + \frac{2}{3}$ of $1\frac{2}{3} + 2\frac{1}{2}$
 (15) $\frac{1}{2} + \frac{2}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$ of $11\frac{1}{2}$, $\frac{2}{3} + 2\frac{1}{2}$ of $\frac{1}{5} + 3\frac{1}{2}$ of $\frac{2}{3}$
 (16) $387\frac{1}{2} + 285\frac{1}{2} + 394\frac{1}{2} + \frac{2}{3}$ of 3704, $1\frac{1}{2} + 2\frac{1}{2} + 4\frac{1}{2} + 6\frac{1}{2}$
 (17) $275\frac{1}{2} + 62\frac{1}{2} + 103\frac{1}{2} + \frac{2}{3}$ of 41501
 (18) $\frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9}$ (19) $\frac{1}{2} + \frac{2}{3} + \frac{1}{4} + \frac{1}{5} + \frac{2}{6} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10}$
 (20) $\frac{2}{3}$ of 7 of $52\frac{1}{2} + \frac{2}{3}$ of $\frac{2}{3}$ of $506\frac{1}{2} + \frac{2}{3}$ of 8 of 1864
 (21) 1 of $5 + \frac{1}{2}$ of $\frac{1}{10} + \frac{1}{5}$ of $(\frac{1}{2} + \frac{1}{3}) + \frac{1}{7}$ of $(2 + \frac{1}{4})$

IV SUBTRACTION OF FRACTIONS

262 To subtract one fraction from another fraction

- (1) When the given fractions have the same denominator

RULE Find the difference of the numerators of the given fractions for the numerator of the remainder, and take their denominator for its denominator

Ex Subtract $\frac{4}{17}$ from $\frac{10}{17}$

$$\text{Here, } \frac{10}{17} - \frac{4}{17} = \frac{10-4}{17} \quad \text{For, } 10 \text{ seventeenths} - 4 \text{ seventeenths} \\ = \frac{6}{17} \quad = (10-4 \text{ seventeenths} = 6 \text{ seventeenths})$$

- (2) When the given fractions have different denominators

RULE Reduce the fractions to a least common denominator; subtract the less numerator from the greater, under the remainder place the least common denominator, and the result, properly reduced will be the required difference

Note Before applying the Rule reduce fractions to their lowest terms, improper fractions to whole or mixed numbers and compound fractions to simple ones

Ex Subtract $\frac{1}{8}$ from $1\frac{5}{8}$, and $\frac{2}{3}$ of $\frac{1}{8}$ from $\frac{7}{8}$ of 4

- (1) The L C M of 18 and 24 = 72

$$1\frac{5}{8} - \frac{1}{8} = \frac{57}{72} - \frac{1}{72} = \frac{56}{72} \quad \text{Ans}$$

- (2) Here, $\frac{7}{8}$ of 4 = $3\frac{1}{2}$ and $\frac{2}{3}$ of $\frac{1}{8}$ = $\frac{1}{12}$ Also L C D = 96

$$\therefore \text{their difference} = \frac{21}{32} - \frac{5}{24} = \frac{63-20}{96} = \frac{43}{96} \quad \text{Ans}$$

263 Additions and subtractions of fractions may be performed in any order

$$\text{Thus } 7\frac{1}{2} - 4\frac{1}{2} = (7 + \frac{1}{2}) - (4 + \frac{1}{2}) = 7 + \frac{1}{2} - 4 - \frac{1}{2} \quad (\text{Art } 107) \\ = (7-4) + (\frac{1}{2} - \frac{1}{2}) = 3 + \frac{0}{2} = 3$$

Hence, if either of the given fractions be a whole or mixed number, it is most convenient to take separately the difference of the integral parts and that of the fractional parts, and then add the two results together

Ex 1 From $3\frac{2}{3}$ take $2\frac{1}{3}$

$$\begin{aligned}\text{Here, } 3\frac{2}{3} - 2\frac{1}{3} &= (3 + \frac{2}{3}) - (2 + \frac{1}{3}) = (3 - 2) + (\frac{2}{3} - \frac{1}{3}) \\ &= 1 + (\frac{2}{3} - \frac{1}{3}) = 1\frac{1}{3} = 1\frac{1}{3} \quad \text{Ans}\end{aligned}$$

Ex 2 Find the difference between $2\frac{1}{2}$ and $5\frac{1}{3}$

$$\begin{aligned}\text{Here, } 5\frac{1}{3} - 2\frac{1}{2} &= (4 + 1 + \frac{1}{3}) - (2 + \frac{1}{2}) = (4 - 2) + (1 + \frac{1}{3} - \frac{1}{2}) \\ &= 2 + (\frac{4}{3} - \frac{1}{2}) = 2 + (\frac{8}{6} - \frac{3}{6}) = 2 + \frac{5}{6} = 2\frac{5}{6} \quad \text{Ans}\end{aligned}$$

264 The following peculiarities in *Subtraction of Fractions* should be carefully noticed

(1) *When both the fractions have a common numerator*

RULE Multiply the difference of the denominators by the common numerator for the *new numerator*, and take the product of the denominators for the *new denominator*. The resulting fraction is the required difference

$$\text{Ex 1} \quad \frac{8}{11} - \frac{8}{13} = \frac{(13 - 11) \times 8}{13 \times 11} = \frac{2 \times 8}{13 \times 11} = \frac{16}{143} \quad \text{Ans}$$

(2) *To subtract fractions when both have 1 for numerator*

RULE Find the difference between the denominators for a *new numerator* and multiply the denominators for a *new denominator*. The resulting fraction is the required difference

$$\text{Ex 2} \quad 1 - \frac{1}{9} = \frac{9 - 1}{9 \times 9} = \frac{8}{72}$$

(3) *To subtract a proper fraction from unity*

RULE Subtract the numerator from the denominator for the *new numerator*, and underneath place the given denominator. The resulting fraction is the required difference

$$\text{Ex 3} \quad 1 - \frac{5}{11} = \frac{11 - 5}{11} = \frac{6}{11} \quad \text{Ans}$$

(4) *To subtract a mixed number from an integer*

RULE Subtract the fractional part from unity as in (3) and the integral part from the integer diminished by unity

$$\text{Ex 4} \quad 7 - 3\frac{1}{2} = (6 - 3) + (1 - \frac{1}{2}) = 3\frac{1}{2} \quad \text{Ans}$$

(5) *To subtract a mixed number from another, when the fractional part of the subtrahend is greater than that of the minuend*

RULE Subtract the subtrahend (composed of the integral and fractional part) from the integral part of the minuend as in (4) and to this difference add the fractional part of the minuend

Ex 5 $15\frac{2}{3} - 7\frac{2}{3} = (15 - 7\frac{2}{3}) + \frac{2}{3} = 7\frac{1}{3} + \frac{2}{3} = 7\frac{1}{2}$ Ans

265 An expression made up of additions and subtractions of fractions may be made equal to the difference of two sums

Thus, $5\frac{1}{4} - 1\frac{1}{4} + 2\frac{5}{8} + \frac{3}{8} = (5\frac{1}{4} + 2\frac{5}{8} + \frac{3}{8}) - (1\frac{1}{4} + \frac{3}{8})$ (Art 107)

Examples LXIX

1 Perform orally the following subtractions -

- (1) $\frac{5}{8} - \frac{1}{8}$, $\frac{7}{8} - \frac{2}{8}$, $\frac{4}{8} - \frac{2}{8}$, $\frac{1}{2} - \frac{1}{3}$, $\frac{1}{3} - \frac{1}{6}$, $\frac{1}{2} - \frac{1}{6}$, $\frac{1}{3} - \frac{1}{6}$, $\frac{1}{4} - \frac{1}{8}$
 (2) $1 - \frac{7}{8}$, $2 - \frac{4}{8}$, $2 - \frac{1}{8}$, $1 - \frac{3}{8}$, $1 - \frac{3}{4}$, $\frac{3}{8} - \frac{1}{2}$, $\frac{3}{4} - \frac{5}{8}$, $\frac{4}{5} - \frac{1}{5}$ ~
 (3) $1 - \frac{7}{8}$, $3 - \frac{1}{4}$, $2\frac{1}{2} - 1\frac{1}{8}$, $4\frac{3}{4} - 3\frac{1}{2}$, $6\frac{1}{2} - 4\frac{1}{2}$, $7\frac{7}{8} - 4\frac{1}{4}$, $41\frac{9}{10} - 22\frac{9}{10}$

2 Perform the following subtractions -

- (1) $\frac{3}{8} - \frac{1}{8}$, $\frac{7}{8} - \frac{2}{8}$, $\frac{1}{2} - \frac{1}{8}$, $\frac{5}{8} - \frac{1}{8}$, $\frac{1}{2} - \frac{1}{4}$, $\frac{1}{4} - \frac{1}{8}$
 (2) $1\frac{3}{8} - 1\frac{1}{8}$, $\frac{3}{8} - \frac{1}{8}$, $8\frac{4}{8} - \frac{8}{8}$, $9\frac{1}{8} - 2\frac{1}{8}$, $3\frac{1}{2} - 2\frac{1}{2}$, $8\frac{7}{8} - 5\frac{3}{8}$
 (3) $19\frac{7}{8} - 13\frac{1}{8}$, $18\frac{1}{2} - 17\frac{1}{2}$, $1000 - 384\frac{7}{8}$, $279\frac{9}{10} - 168\frac{8}{10}$
 (4) $2\frac{7}{8}$ of $16\frac{3}{4}$ - $1\frac{1}{2}$ of $3\frac{1}{2}$, $\frac{3}{4}$ of $1\frac{1}{2}$ of 25 - $\frac{3}{8}$ of $\frac{5}{8}$, $7\frac{1}{2}$ of $10\frac{1}{2}$ - $2\frac{1}{2}$ of $4\frac{1}{2}$

3 Find the values of -

- (1) $\frac{5}{8} - \frac{3}{4} + \frac{2}{3} - \frac{1}{2}$ (2) $\frac{1}{8} - \frac{1}{8} + \frac{1}{4} - \frac{1}{2}$
 (3) $\frac{1}{2} + \frac{2}{3} - \frac{1}{6} + \frac{2}{3} - \frac{1}{2}$ (4) $\frac{1}{2} - \frac{3}{8} + \frac{1}{8} - \frac{1}{10} + \frac{1}{2} + \frac{2}{3} - \frac{1}{2} + \frac{1}{8}$
 (5) $13\frac{4}{8} - 9\frac{8}{8} - 1\frac{8}{8}$ (6) $3\frac{1}{8} - \frac{1}{3} - 1\frac{1}{6} + \frac{1}{6}$
 (7) $7\frac{3}{8} + 6\frac{3}{8} - 3\frac{1}{8} - 2\frac{1}{8} + \frac{1}{8}$ (8) $3\frac{1}{8} + 2\frac{1}{4} - (5\frac{1}{2} + 1\frac{3}{8}) + 2\frac{8}{8}$
 (9) $10\frac{1}{2} - (4\frac{7}{8} + 6\frac{1}{8}) + 7\frac{5}{8} + (8\frac{2}{8} - 6\frac{2}{8})$ (10) $6\frac{7}{8}$ of $2\frac{1}{8}$ ($6\frac{7}{8} - 2\frac{1}{8}$)
 (11) $2\frac{1}{2} - (4\frac{7}{8} + 10\frac{1}{8} + 3\frac{1}{8}) + 3\frac{1}{8} + 20\frac{7}{8}$ (12) $\frac{3}{4}$ of $\frac{4}{5} - \frac{1}{15}$ of $3\frac{1}{2} + \frac{5}{6}$ of $3\frac{1}{2}$
 (13) $22\frac{1}{4} - (9\frac{3}{4} - 7\frac{1}{4} + \frac{1}{2} \text{ of } \frac{1}{4}) + \frac{2}{3}$ of $3\frac{1}{2}$
 (14) $8\frac{3}{4} - 3\frac{1}{4} + 2\frac{7}{8}$ of $1\frac{3}{4}$ of $4\frac{1}{2} - (5\frac{1}{2} - 2\frac{1}{2})$
 (15) $47\frac{5}{8} - (3\frac{3}{8} + 31\frac{5}{8} + 2\frac{5}{8}) + 6\frac{5}{8} - (2\frac{8}{8} - 1\frac{8}{8})$

V MULTIPLICATION OF FRACTIONS

266 To multiply a fraction by a whole number

[We have already given an outline of this method in Arts 241 and 244 Now, we propose to treat it at length]

RULE Multiply the numerator by the whole number for the new numerator, and leave the denominator unchanged The resulting fraction should always be expressed in its lowest terms, by cancelling those factors that are common to the multiplier and to the denominator of the fraction

$$\text{Thus, } 8 \times \frac{5}{9} = \frac{8 \times 5}{9} = \frac{40}{9} = 4\frac{4}{9}$$

$$\text{For } 8 \times \frac{5}{9} = 8 \times 5 \text{ nths} = 40 \text{ nths} = \frac{40}{9} = \frac{8 \times 5}{9}$$

$$\text{Also, } 9 \times \frac{4}{15} = \frac{3 \times 3 \times 4}{3 \times 5} = \frac{3 \times 4}{5} = 1\frac{2}{5} = 2\frac{2}{5}$$

267 To multiply a mixed number by an integer

RULE Either reduce the mixed number to an improper fraction and multiply as above, or multiply the integral part and the fractional part separately, and add the two products

Thus (1) $6\frac{1}{2} \times 3 = 7\frac{1}{2} \times 3 = 19\frac{3}{2} = 20\frac{1}{2}$

(2) $6\frac{1}{2} \times 3 = 6 \times 3 + \frac{1}{2} \times 3 = 18 + 1\frac{1}{2} = 18 + 2\frac{1}{2} = 20\frac{1}{2}$

268 To multiply a proper fraction differing very little from 1, or a mixed number differing very little from the next superior integer by a whole number, we have recourse to such artifices as are explained in Art 264

Thus, (1) $1\frac{99}{100} \times 35 = (1 - \frac{1}{100}) \times 35 = 35 - \frac{35}{100} = 35 - \frac{7}{20} = 34\frac{13}{20}$

(2) $15\frac{7}{8} \times 12 = (16 - \frac{1}{8}) \times 12 = 192 - \frac{12}{8} = 192 - \frac{3}{2} = 191\frac{1}{2}$

(3) $99\frac{1}{2} \times 46 = (100 - \frac{1}{2}) \times 46 = 4600 - \frac{46}{2} = 4600 - 23 = 4577$

Examples LXX**1 Multiply orally —**

- (1) $\frac{1}{2}$ by 3, $1\frac{1}{2}$ by 2, $1\frac{2}{3}$ by 3, $\frac{4}{5}$ by 4, $2\frac{3}{4}$ by 5, $3\frac{5}{6}$ by 7, $\frac{2}{3}$ by 3, $\frac{8}{9}$ by 2
 (2) $1\frac{1}{2}$ by 7, $1\frac{3}{4}$ by 21, $1\frac{2}{3}$ by 100, $1\frac{3}{4}$ by 25, $2\frac{3}{4}$ by 28, $3\frac{1}{2}$ by 30

2 Multiply —

- (1) $1\frac{1}{2}$ separately by 55, 88, 90 (2) $1\frac{1}{2}$ separately by 12, 36, 48, 60
 (3) $1\frac{1}{2}$ 32, 128, 168 (4) $3\frac{1}{2}$ 11, 15, 21, 132
 (5) $3\frac{1}{2}$ 55, 77, 110 (6) $2\frac{3}{4}$ 13, 39, 42, 117
 (7) $159\frac{3}{4}$ by 12, $1625\frac{1}{2}$ by 23, $41\frac{3}{4}$ by 23, $1727\frac{1}{2}$ by 34, $3589\frac{1}{2}$ by 47

3 Find the product of —

- (1) $99\frac{9}{10}$ separately by 6, 8, 15, 18, 25 (2) $999\frac{7}{8}$ by 99, 550
 (3) $499\frac{3}{4}$ 25, 50, 75, 100, 150, 200, 250
 (4) $74\frac{1}{2}$ by 43, $99\frac{1}{2}$ by 324, $999\frac{1}{2}$ by 999

269 The meaning of Multiplication as given in Art 59 is not applicable when the multiplier is a fraction. Hence, to suit our purpose we make the following definition

"To multiply by a fraction is to take that fraction of the multiplicand

Thus, to multiply $\frac{7}{8}$ by $\frac{5}{6}$, we take $\frac{5}{6}$ of $\frac{7}{8}$ by the new definition

But $\frac{5}{6}$ of $\frac{7}{8} = \frac{5 \times 7}{6 \times 8}$ by Art 249, therefore $\frac{7}{8} \times \frac{5}{6} = \frac{35}{48}$

Hence the Rule

270 To multiply a fraction by a fraction

RULE Multiply together the respective numerators and denominators, reduced to fractional forms if necessary, and the fraction thence arising will be the product, which may be simplified by striking out any factor common to numerator and denominator

Ex 1 Multiply $\frac{2}{3}$ by $\frac{7}{8}$

$$\text{Here } \frac{2}{3} \times \frac{7}{8} = \frac{2 \times 7}{3 \times 8} = \frac{2 \times 7}{9 \times 2 \times 4} \\ = \frac{7}{9 \times 4} = \frac{7}{36} \text{ Ans}$$

For, if $\frac{2}{3}$ be multiplied by 7 the product will be $\frac{14}{3}$ (Art 241), but 7 being 8 times as great as $\frac{7}{8}$, the multiplier above used is 8 times too large, and the product $\frac{14}{3}$ will therefore be 8 times too large; also whence the product required must be $\frac{14}{3} \div 8 = \frac{14}{24} = \frac{7}{12}$ (Art 241) = $\frac{7}{36}$

Ex 2 Multiply $3\frac{2}{3}$ by $2\frac{3}{4}$, and $5\frac{1}{2}$ by $2\frac{3}{4}$ of $\frac{1}{5}$

$$(1) \text{ Product} = \frac{22}{3} \times \frac{14}{4} = \frac{99 \times 14}{25 \times 54} = \frac{11 \times 9 \times 5 \times 29}{5 \times 5 \times 6 \times 9} \\ = \frac{11 \times 29}{5 \times 6} = \frac{319}{30} = 10\frac{19}{30} \text{ Ans}$$

$$(2) \text{ Product} = \frac{11}{2} \times (\frac{1}{5} \times \frac{1}{5}) = \frac{49 \times 17 \times 15}{9 \times 7 \times 17} = \frac{49 \times 15}{9 \times 7} \\ = \frac{7 \times 7 \times 3 \times 5}{3 \times 3 \times 7} = \frac{7 \times 5}{3} = \frac{35}{3} = 11\frac{2}{3} \text{ Ans}$$

271 To find the continued product of three or more fractions

RULE Multiply all the numerators together for the numerator of the continued product, and all the denominators for its denominator, cancelling all the factors common to numerator and denominator before obtaining the final result

Ex 1 Find the continued product of $\frac{3}{4}$, $\frac{5}{7}$ and $\frac{8}{15}$

$$\text{Here, Product} = \frac{3 \times 5 \times 8}{4 \times 7 \times 15} = \frac{3 \times 5 \times 2 \times 4}{4 \times 7 \times 3 \times 5} = \frac{2}{7} \text{ Ans}$$

Ex 2 Multiply $\frac{5}{6}$, $3\frac{5}{11}$, $19\frac{1}{2}$ and $\frac{1}{10}$ together

$$\text{Product} = \frac{5}{6} \times \frac{35}{11} \times \frac{96}{5} \times \frac{11}{56} = \frac{5 \times (5 \times 7) \times (2 \times 2 \times 2 \times 2 \times 2 \times 3) \times 11}{(3 \times 2) \times 11 \times 5 \times (2 \times 2 \times 2 \times 7)} \\ = \frac{5 \times 2}{1} = \frac{10}{1} = 10 \text{ Ans}$$

Examples LXXI

1 Multiply orally —

(1) $\frac{1}{2}$ separately by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{4}$

(3) $\frac{2}{3}$ $\frac{4}{5}$, $\frac{5}{6}$, $\frac{2}{3}$, $\frac{1}{2}$

(2) $\frac{1}{4}$ separately by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{4}$

(4) $\frac{1}{11}$ $\frac{1}{11}$, $\frac{1}{11}$, $\frac{1}{11}$, $\frac{1}{11}$

2 Multiply —

- (1) $\frac{2}{5}$ by $\frac{3}{7}$, $\frac{2}{5}$ by $\frac{1}{10}$, $\frac{1}{4}$ by $\frac{3}{4}$, $\frac{1}{4}$ by $\frac{5}{8}$, $\frac{1}{5}$ by $\frac{1}{18}$, $\frac{1}{8}$ by $\frac{2}{30}$
 (2) $\frac{1}{4}$ by $\frac{1}{10}$, $\frac{1}{5}$ by $\frac{1}{10}$, $\frac{1}{10}$ by $\frac{1}{10}$, $\frac{1}{10}$ by $\frac{1}{10}$, $\frac{1}{10}$ by $\frac{1}{10}$, $\frac{1}{10}$ by $\frac{1}{10}$
 (3) $\frac{2}{3}$ by $\frac{7}{10}$, $\frac{8}{9}$ by $\frac{10}{11}$, $\frac{6}{7}$ by $\frac{14}{15}$, $\frac{15}{4}$ by $\frac{3}{8}$, $\frac{6}{1}$ by $\frac{2}{1}$
 (4) $\frac{2}{3}$ by $\frac{1}{2}$ of $\frac{1}{2}$ of $\frac{1}{2}$, $\frac{13}{4}$ of $\frac{7}{10}$ by $\frac{3}{4}$ of $\frac{1}{2}$ of $\frac{1}{2}$, $\frac{2}{3}$ of $\frac{4}{5}$ by $\frac{2}{3}$
 (5) $\frac{1}{2}$ of $\frac{15}{17}$ by $\frac{1}{17}$ of $\frac{3}{4}$, $\frac{3}{5}$ of $\frac{5}{8}$ by $\frac{6}{7}$ of $\frac{10}{11}$, $\frac{7}{8}$ of $\frac{19}{20}$ by $\frac{1}{10}$

3 Find the values of —

- (1) $\frac{1}{2} \times \frac{3}{5} \times \frac{1}{10}$, $\frac{1}{2} \times \frac{1}{10} \times \frac{1}{10}$, $\frac{3}{4} \times \frac{1}{5} \times \frac{1}{10}$, $\frac{1}{11} \times \frac{16}{1} \times \frac{1}{10}$, $\frac{4}{1} \times \frac{1}{1} \times \frac{1}{1}$
 (2) $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$, $\frac{3}{4} \times \frac{1}{5} \times \frac{1}{10} \times \frac{1}{10}$, $\frac{1}{15} \times \frac{2}{3} \times \frac{1}{10} \times \frac{1}{10}$, $\frac{1}{10}$ of $\frac{1}{10} \times \frac{3}{10} \times \frac{1}{10}$
 (3) $\frac{1}{10} \times \frac{2}{3} \times \frac{1}{10} \times \frac{1}{10}$, $\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}$, $\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}$, $\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}$
 (4) $\frac{1}{2}$ of $\frac{1}{5}$ of $\frac{1}{10}$ of $\frac{2}{3}$ of $8 \times \frac{1}{2}$ of $\frac{1}{10}$ of $\frac{1}{10}$ of $\frac{1}{10}$
 (5) $\frac{1}{2}$ of $\frac{3}{8}$ of $\frac{7}{10}$ of $\frac{1}{2}$ of $\frac{1}{10}$ of $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{2}{3}$ of $\frac{1}{10}$

4 Find the continued product of —

- (1) $\frac{49}{133}$, $\frac{76}{75}$ and $\frac{28}{98}$ (2) $\frac{428}{515}$, $\frac{5253}{1819}$ and $\frac{615}{492}$
 (3) $\frac{17}{24}$, $\frac{384}{591}$, $\frac{851}{864}$, and $\frac{1584}{1591}$ (4) $\frac{5687}{319}$, $\frac{667}{22011}$, $\frac{221}{629}$ and $\frac{72816}{8528}$
 (5) $\frac{324}{561}$, $\frac{1444}{1296}$, $\frac{441}{529}$ and $\frac{2116}{1764}$ (6) $\frac{36}{65}$, $\frac{35}{132}$, $\frac{39}{108}$ and $\frac{75}{144}$

5 Simplify —

- (1) $(\frac{1}{10} + \frac{2}{3}) \times \frac{10}{7}$, $\frac{3}{5} + \frac{2}{3} \times \frac{10}{7}$, $(\frac{1}{2} + \frac{1}{3} + \frac{1}{4}) \times (\frac{1}{2} \text{ of } \frac{1}{7})$
 (2) $(\frac{1}{2} \times \frac{1}{5} + \frac{1}{5} \times \frac{1}{2}) - (\frac{1}{7} \times \frac{1}{10} - \frac{1}{10} \times \frac{1}{7})$, $\frac{1}{2}$ of $(\frac{6}{10} + \frac{2}{3} - 3)$, $(\frac{1}{2} + \frac{1}{3}) \times (\frac{3}{2} - \frac{1}{2})$
 (3) $(\frac{19}{4} - \frac{3}{4}) \times (\frac{3}{4} - \frac{2}{4})$, $\frac{19}{4} - \frac{3}{4} \times \frac{3}{4} - \frac{2}{4}$, $\frac{19}{4} - \frac{3}{4} \times (\frac{3}{4} - \frac{2}{4})$
 (4) $\{(\frac{1}{2} + \frac{1}{3}) \text{ of } (\frac{1}{2} + \frac{1}{3})\} \times \{(2\frac{1}{3} - 1\frac{1}{3}) \text{ of } (3\frac{1}{3} - 2\frac{1}{3})\}$
 (5) $\{1\frac{1}{2} \text{ of } 26\frac{1}{2} \text{ of } (1 - \frac{1}{2})\} \times \{2\frac{1}{2} \text{ of } (4\frac{1}{2} - 3\frac{1}{2}) \text{ of } 1\frac{1}{2}\}$
 (6) $(\frac{1}{2} \text{ of } \frac{2}{3} - \frac{1}{3}) \times (\frac{5}{3} \text{ of } 4\frac{1}{2} - 3\frac{1}{2} \text{ of } \frac{3}{8}) \times \frac{2}{7} \text{ of } 1\frac{2}{3} \times \frac{1}{2}$

VI DIVISION OF FRACTIONS

272 To divide a fraction by a whole number

[We have already given an outline of this method in Arts 241 and 244. Now, we propose to treat it at length.]

RULE Multiply the denominator by the whole number and leave the numerator unaltered. The resulting fraction should always be reduced to its lowest terms by removing all factors common to numerator and denominator.

$$\text{Thus, } \frac{35}{36} \div 28 = \frac{35}{36 \times 28} = \frac{7 \times 5}{36 \times 7 \times 4} = \frac{5}{36 \times 4} = \frac{5}{144}$$

273 The meaning of Division as given in Art 87 is not applicable when the divisor is a fraction. Hence, *Division* may be extended to express the finding of the fraction, the product of which and the divisor is the dividend, and the *quotient* shows what *part* or *parts* the dividend is of the divisor.

Thus, to divide $\frac{3}{8}$ by $\frac{5}{7}$, we have, by definition,

$$\text{quotient} \times \frac{5}{7} = \frac{3}{8},$$

multiply each term of this equality by $\frac{7}{5}$,

$$\begin{aligned} \text{therefore } \text{quotient} \times \frac{7}{5} \times \frac{5}{7} &= \frac{3}{8} \times \frac{7}{5} \\ \text{or } \text{quotient} &= \frac{3}{8} \times \frac{7}{5}, \end{aligned}$$

that is, $\frac{3}{8} \div \frac{5}{7} = \frac{3}{8} \times \frac{7}{5}$. Hence the rule

274 *To divide a fraction by a fraction*

RULE Multiply the dividend by the divisor *inverted*, and the result will be the quotient, which may be reduced to its lowest terms by cancelling any factors common to numerator and denominator, or, which is the same thing, *invert* the divisor, and then proceed by the Rule for the Multiplication of Fractions.

Ex Divide $\frac{3}{4}$ by $\frac{4}{5}$. For, if $\frac{3}{4}$ be divided by 4, the quotient is $\frac{3}{16}$ (Art 241), but this quotient is 5 times

too *small*, because the divisor has been taken 5 times too *great*, whence the quotient will be $\frac{3}{16} \times 5 = \frac{15}{16}$ (Art 241).

275 If the dividend be a whole number, or if dividend or divisor or both be mixed numbers, reduce them to improper fractions, and compound fractions to simple ones before the application of the Rule.

Ex Divide $1\frac{1}{4}$ by $5\frac{5}{8}$, and $7\frac{7}{8}$ by $3\frac{3}{4}$ of $2\frac{1}{10}$

$$(1) \quad 1\frac{1}{4} \div 5\frac{5}{8} = \frac{15}{14} \div \frac{40}{7} = \frac{15}{14} \times \frac{7}{40} = \frac{3 \times 5 \times 7}{7 \times 2 \times 5 \times 8} = \frac{3}{2 \times 8} = \frac{3}{16} \quad \text{Ans}$$

$$\begin{aligned} (2) \quad 7\frac{7}{8} - 3\frac{3}{4} \text{ of } 2\frac{1}{10} &= \frac{63}{8} - \frac{45}{14} \text{ of } \frac{21}{10} = \frac{63}{8} - \frac{5 \times 9 \times 7 \times 3}{7 \times 2 \times 5 \times 2} \\ &= \frac{63}{8} - \frac{9 \times 3}{2 \times 2} = \frac{63}{8} \times \frac{2 \times 2}{9 \times 3} = \frac{7 \times 9 \times 2 \times 2}{2 \times 2 \times 2 \times 9 \times 3} \\ &= \frac{7}{2 \times 3} = \frac{7}{6} = 1\frac{1}{6} \quad \text{Ans} \end{aligned}$$

276 Numbers connected by *of* are considered a single number. The student should carefully notice the difference in meaning between $2\frac{1}{2} \div 1\frac{1}{2} \times \frac{3}{4}$ and $2\frac{1}{2} - 1\frac{1}{2}$ of $\frac{3}{4}$. In the former, the sign \div applies only to the next number $1\frac{1}{2}$, but in the latter, $1\frac{1}{2}$ of $\frac{3}{4}$ is a single number.

Thus, the former $= \frac{5}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{5}{4} = 1\frac{1}{4}$, the latter $= \frac{5}{2} \times \frac{3}{4} \times \frac{3}{4} = \frac{45}{16} = 2\frac{13}{16}$.

Examples LXXII

1 Divide orally —

- (1) $\frac{1}{2}$ separately by 2, 3, 4, 5, 6 (2) $\frac{3}{4}$ separately by 4, 5, 7, 10, 12
 (3) $\frac{5}{8}$ 12, 14, 15, 18, 20 (4) $\frac{15}{16}$ 3, 5, 30, 45

2 Divide —

- (1) $\frac{49}{72}$ separately by 8, 16, 24, 36 (2) $\frac{11}{12}$ separately by 25, 75, 87
 (3) $\frac{21}{24}$ 13, 65, 117 (4) $\frac{16}{17}$ 19, 25, 32

3 Divide orally —

- (1) $\frac{1}{2}$ separately by $\frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}$ (2) $\frac{1}{3}$ separately by $\frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{8}$
 (3) $\frac{1}{4}$ $\frac{1}{5}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}$ (4) $\frac{1}{5}$ $\frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \frac{1}{12}$

4 Divide —

- (1) $\frac{1}{5}$ separately by $\frac{1}{10}, \frac{1}{15}, \frac{1}{20}, \frac{1}{25}$ (2) $\frac{1}{4}$ separately by $\frac{1}{10}, \frac{1}{15}, \frac{1}{20}, \frac{1}{25}$
 (3) $\frac{1}{10}$ $\frac{1}{15}, \frac{1}{20}, \frac{1}{25}, \frac{1}{30}$ (4) $\frac{1}{15}$ $\frac{1}{20}, \frac{1}{25}, \frac{1}{30}, \frac{1}{40}$

Examples LXXIII

1 Divide —

- (1) $\frac{1}{2}$ by $\frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}$ by $\frac{1}{12}, \frac{1}{15}, \frac{1}{20}$ by $\frac{1}{24}, \frac{1}{30}, \frac{1}{36}$ by $\frac{1}{40}, \frac{1}{45}, \frac{1}{60}$ by $\frac{1}{72}, \frac{1}{80}, \frac{1}{90}$
 (2) $\frac{1}{3}$ by $\frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}$ by $\frac{1}{12}, \frac{1}{15}, \frac{1}{20}$ by $\frac{1}{24}, \frac{1}{30}, \frac{1}{36}$ by $\frac{1}{40}, \frac{1}{45}, \frac{1}{60}$ by $\frac{1}{72}, \frac{1}{80}, \frac{1}{90}$
 (3) $\frac{1}{4}$ by $\frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \frac{1}{12}$ by $\frac{1}{15}, \frac{1}{20}, \frac{1}{24}, \frac{1}{30}$ by $\frac{1}{36}, \frac{1}{40}, \frac{1}{45}, \frac{1}{60}$ by $\frac{1}{72}, \frac{1}{80}, \frac{1}{90}$
 (4) $\frac{1}{5}$ by $\frac{1}{10}, \frac{1}{15}, \frac{1}{20}, \frac{1}{25}$ by $\frac{1}{30}, \frac{1}{36}, \frac{1}{40}, \frac{1}{45}$ by $\frac{1}{60}, \frac{1}{72}, \frac{1}{80}, \frac{1}{90}$

2 Find the values of —

- (1) $(\frac{1}{2} \times \frac{2}{3} \times 13\frac{1}{2}) - (1 \times \frac{3}{4} + 54)$ (2) $(\frac{1}{2} + \frac{1}{3} \text{ of } \frac{1}{4}) - (\frac{1}{5} \text{ of } 2\frac{1}{2} - \frac{1}{6} \text{ of } 1)$
 (3) $6\frac{1}{2} + (1\frac{1}{2} + 3\frac{1}{2}) - 6\frac{1}{2}$ (4) $6\frac{1}{2} + 1\frac{1}{2} + 3\frac{1}{2} - 6\frac{1}{2}$
 (5) $10\frac{1}{2} \times 3\frac{1}{2} - 4\frac{1}{2} - (2\frac{1}{2} + 3\frac{1}{2}) - 4\frac{1}{2} + 7\frac{1}{2} - (3\frac{1}{2} + 4\frac{1}{2})$
 (6) $2\frac{1}{2} \times 1\frac{1}{2} - 1\frac{1}{2} \text{ of } 2\frac{1}{2}$ (7) $2\frac{1}{2} \text{ of } 1\frac{1}{2} - 1\frac{1}{2} \times 2\frac{1}{2}$ (8) $2\frac{1}{2} \text{ of } 1\frac{1}{2} - 1\frac{1}{2} \text{ of } 2\frac{1}{2}$
 (9) $1\frac{1}{2} \times 1\frac{1}{2} - 1\frac{1}{2} \times 1\frac{1}{2} - 1\frac{1}{2} \times 1\frac{1}{2}$ (10) $1 - [4 - 1 - \{2 - 1 - \frac{1}{1 - \frac{1}{1}}\}]$
 (11) $(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}) - (\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5})$
 (12) $(2 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}) - (1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5})$

VII COMPLEX AND CONTINUED FRACTIONS

277 A fraction having a fraction or mixed number in the numerator or denominator or in both is a **Complex Fraction**

Thus, $\frac{7}{\frac{1}{2}}$, $\frac{8\frac{1}{2}}{9}$, $\frac{7}{9\frac{1}{2}}$, $\frac{4\frac{1}{2}}{5\frac{1}{2}}$, $\frac{1\frac{2}{3} + \frac{1}{4}}{4\frac{1}{2} \text{ of } 3\frac{1}{2}}$ are *Complex Fractions*

278 A complex fraction is read by inserting the word *for* *divided by*, between the readings of numerator and denominator

Thus $\frac{3\frac{1}{2}}{4\frac{1}{2}}$ is read $3\frac{1}{2}$ *for* $4\frac{1}{2}$

279 In the *sum* of a whole number and a fraction, when the fraction is either complex or simple (Art 231), the sign is sometimes omitted, as in $5\frac{4\frac{1}{2}}{7}$ which means $5 + \frac{4\frac{1}{2}}{7}$, and in a *product* when one of the factors is enclosed in a bracket the sign is often omitted, as in $\frac{2}{3}(\frac{5}{6} - \frac{1}{2})$, which means $\frac{2}{3} \times (\frac{5}{6} - \frac{1}{2})$

280 Complex fractions are subject to the same rules as simple fractions, and can always be reduced to simple ones by treating them as the *quotient* of the numerator by the denominator (Art 235)

$$\text{Thus, } \frac{2\frac{1}{2}}{3\frac{5}{6}} = 2\frac{1}{5} - 3\frac{2}{9} = \frac{11}{5} - \frac{29}{9} = \frac{11}{5} \times \frac{9}{29} = \frac{99}{145}$$

281 To Reduce a complex fraction to a simple fraction

RULE Express the numerator and denominator of the complex fraction in the form of proper or improper fractions, and multiply the numerator by the denominator *inverted*, or more simply, multiply the numerator and denominator of the complex fraction by the L C M of the denominators of the simple fractions

Ex 1 Reduce $\frac{5\frac{7}{9}}{9\frac{5}{11}}$ and $\frac{13\frac{1}{3}}{20}$ to simple fractions

$$(1) \frac{5\frac{7}{9}}{9\frac{5}{11}} = 5\frac{7}{9} \div 9\frac{5}{11} = \frac{52}{9} \div \frac{104}{11} = \frac{52}{9} \times \frac{11}{104} = \frac{52 \times 11}{9 \times 2 \times 52} = \frac{11}{18} \text{ Ans}$$

$$\text{Or thus, } \frac{5\frac{7}{9}}{9\frac{5}{11}} = \frac{5\frac{7}{9} \times 99}{9\frac{5}{11} \times 99} = \frac{495 + 77}{891 + 45} = \frac{572}{936} = \frac{11 \times 52}{18 \times 52} = \frac{11}{18} \text{ Ans}$$

$$(2) \frac{13\frac{1}{3}}{20} = 13\frac{1}{3} \div 20 = \frac{40}{3} \div 20 = \frac{40}{3} \times \frac{1}{20} = \frac{20 \times 2}{3 \times 20} = \frac{2}{3} \text{ Ans}$$

$$\text{Or thus, } \frac{13\frac{1}{3}}{20} = \frac{13\frac{1}{3} \times 3}{20 \times 3} = \frac{40}{20 \times 3} = \frac{20 \times 2}{20 \times 3} = \frac{2}{3} \text{ Ans}$$

Ex 2 Simplify $\frac{12\frac{2}{3} \text{ of } 1\frac{8}{19}}{1\frac{5}{9} \text{ of } 3\frac{7}{7}}$ and $\frac{8\frac{5}{6} - 4\frac{2}{3}}{3\frac{1}{4} + 7\frac{1}{2}}$

$$\begin{aligned} (1) \frac{12\frac{2}{3} \text{ of } 1\frac{8}{19}}{1\frac{5}{9} \text{ of } 3\frac{7}{7}} &= 12\frac{2}{3} \text{ of } 1\frac{8}{19} \div 1\frac{5}{9} \text{ of } 3\frac{7}{7} = \frac{38}{3} \text{ of } \frac{27}{19} \div \frac{14}{9} \text{ of } \frac{24}{7} \\ &= \frac{19 \times 2 \times 3 \times 9}{3 \times 19} \div \frac{7 \times 2 \times 8 \times 3}{3 \times 3 \times 7} = 2 \times 9 \div \frac{2 \times 8}{3} \\ &= \frac{2 \times 9 \times 3}{2 \times 8} = \frac{27}{8} = 3\frac{3}{8} \text{ Ans} \end{aligned}$$

$$\begin{aligned} (2) \frac{8\frac{5}{6} - 4\frac{2}{3}}{3\frac{1}{4} + 7\frac{1}{2}} &= \frac{(96 + 10) - (48 + 8)}{(36 + 9) - (84 + 5)}, \quad \left\{ \begin{array}{l} \text{Multiplying Numr and Denr} \\ \text{by 12, the L C M of the Denrs} \end{array} \right. \\ &= \frac{106 - 56}{45 + 89} = \frac{50}{134} = \frac{2 \times 25}{2 \times 67} = \frac{25}{67} \text{ Ans} \end{aligned}$$

Examples LXXIV

1 Reduce to their simplest forms —

$$(1) \frac{47}{54}, \frac{81}{140}, \frac{3}{6}, \frac{18}{5^2}, \frac{9}{128}, \frac{25}{341}, \frac{7}{11}, \frac{45}{450}, \frac{2}{31+24}$$

$$(2) \frac{1}{13} \text{ of } \frac{1}{11}, \frac{2}{27} \text{ of } \frac{8}{11}, \frac{3}{32} \text{ of } \frac{9}{11}, \frac{5}{81} \text{ of } \frac{1}{17}, \frac{5}{71} - \frac{3}{11} \text{ of } \frac{1}{21}$$

$$(3) \frac{7}{9} \text{ of } \frac{144}{13^2}, \frac{6}{13 \frac{1}{4}} \text{ of } \frac{8}{23}, \frac{5}{51} \text{ of } \frac{6}{105} \text{ of } \frac{15}{124} \text{ of } \frac{8}{42}, \frac{5}{6} \text{ of } \frac{13}{47}$$

$$(4) \frac{3}{27} - \frac{2}{810}, \frac{7}{6+41} - \frac{5}{61-23}, \frac{5}{37+42} \times \frac{5}{3-25} - \frac{28}{1438} - \frac{22}{844}$$

$$(5) \frac{5}{21-14} \text{ of } \frac{21 \times 8}{41-(1-1)}, \frac{5}{34+10} \text{ of } \frac{41+51}{420}, \frac{3}{317} \times \frac{2}{32} \times \frac{3}{371} \text{ of } 2$$

2 Reduce $\frac{2}{7}$, $\frac{1}{51}$, $\frac{9}{9 \times 11}$ and $10\frac{1}{2}$ to equivalent fractions with the least common denominator, also reduce $\frac{1}{2}$ to a complex fraction having the denominator 5 and $\frac{1}{2}$ to a complex fraction having the numerator 10

3 Compare the quantities $2\frac{1}{2}$, $\frac{1}{2}$ of $9\frac{1}{2}$ and $\frac{7}{20}$

4 Find the values of —

$$(1) - \text{ of } 3\frac{1}{10} + \frac{1}{20} \text{ of } 17 + \frac{1}{2} \text{ of } 5\frac{1}{4} \text{ of } 8\frac{1}{2} \quad (2) \frac{1}{2} \text{ of } 2 + 9 + \frac{2}{7} + \frac{1}{27}$$

$$(3) \frac{1}{2} \text{ of } \frac{1}{2} \text{ of } 5 + \frac{1}{2} \text{ of } 7 \text{ of } 18 + \frac{1}{2} \text{ of } 1 \text{ of } \frac{28}{2} \quad (4) 1\frac{1}{2} + \frac{1}{2} \text{ of } \frac{1}{2} + \frac{1}{210}$$

5 Find the difference between —

$$(1) \frac{1}{2} \text{ of } \frac{4}{51} \text{ and } 2 \text{ of } 7\frac{1}{2} \quad (2) \frac{3}{46} \text{ and } \frac{6}{128} \quad (3) 2\frac{1}{2} \text{ of } \frac{5}{41} \text{ and } \frac{7}{11} \text{ of } 15$$

6 Find the values of —

$$(1) 2\frac{1}{2} \text{ of } \frac{6}{14} \times \frac{3}{57} \quad (2) 6\frac{1}{2} \text{ of } 9\frac{1}{2} \times 12\frac{1}{11} \text{ of } \frac{1}{112} \quad (3) \frac{7}{405} - \frac{17}{73}$$

$$(4) \frac{2}{318} \text{ of } \frac{6}{8} \times \frac{1}{3} \text{ of } 8\frac{1}{2} \text{ of } \frac{1}{2} \quad (5) \frac{2}{57} \text{ of } \frac{1}{2} \times \frac{1}{2} \text{ of } \frac{4}{71} \times \frac{7}{51}$$

$$(6) 2\frac{1}{11} \text{ of } 5\frac{1}{10} \text{ of } 133\frac{1}{2} \quad 3\frac{1}{2} \text{ of } \frac{4}{132} \text{ of } 202\frac{1}{2}$$

$$(7) \frac{11}{29} \text{ of } \frac{1}{13} \text{ of } \frac{3}{105} \times 5\frac{1}{2} \text{ of } 6 \times 20\frac{1}{2} \quad (8) 8\frac{1}{2} \text{ of } 7\frac{1}{2} + 2\frac{1}{2} \text{ of } \frac{4}{141} - 8\frac{1}{2}$$

$$(9) \frac{1\frac{1}{2} + \frac{1}{2} + \frac{8}{2} - \frac{1}{2}}{\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}} \text{ of } \frac{\frac{4}{1\frac{1}{2}} \text{ of } \frac{3}{1\frac{1}{2}} - \frac{4}{3\frac{1}{2}} \text{ of } \frac{2\frac{1}{2}}{1}}{\frac{2}{2\frac{1}{2}} \text{ of } 1\frac{1}{2} \text{ of } 3\frac{1}{2} + 2\frac{1}{2} \text{ of } 1\frac{1}{2} \text{ of } 1\frac{1}{2}}$$

282 To find the complete quotient in dividing a number by an integer

RULE Divide in the usual way, and to the integral quotient add the fraction whose numerator is the remainder and denominator the divisor

Ex Divide 4148 by 117, and $3136\frac{1}{2}$ by 95, giving the complete quotient in each case

<p>(1) $117 \overline{) 4148} (35$</p> $\begin{array}{r} 351 \\ 638 \\ 585 \\ 53 \end{array}$ <p>the complete quotient = $35\frac{53}{117}$</p>	<p>(2) $95 \overline{) 3136\frac{1}{2}} (33$</p> $\begin{array}{r} 285 \\ 286 \\ 285 \\ 1 \end{array}$ <p>In dividing by 95, the integral remainder is 1 and the full remainder is $1\frac{1}{2}$</p> <p>But $1\frac{1}{2} - 95 = \frac{1}{2} \times \frac{1}{95} = \frac{1}{190}$</p> <p>the complete quotient = $33\frac{1}{190}$</p>
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283 Fractions of the nature given below are **Continued Fractions**, and can be simplified by beginning at the bottom and working upwards

$$\begin{aligned} \text{Ex } \frac{1}{4 - \frac{1}{2 - \frac{1}{1 - \frac{1}{1}}}}} &= \frac{1}{4 - \frac{1}{2 - \frac{13}{13 - 5}}} = \frac{1}{4 - \frac{1}{2 - \frac{13}{8}}} = \frac{1}{4 - \frac{8}{16 - 13}} \\ &= \frac{1}{4 - 4} = \frac{3}{12 - 8} = \frac{3}{4} \quad \text{Ans} \end{aligned}$$

Examples LXXV

1 Find the complete quotient in dividing —

- (1) 3127 by 43 (2) 6556 by 401 (3) 2221 by 87
 (4) 8768 by 45 (4) $845\frac{1}{5}$ by 12 (6) $6739\frac{1}{3}$ by 37 and by 73
 (7) $4164\frac{1}{8}$ by 11, and by 132 (8) $56941\frac{2}{3}$ by 27

2 Simplify —

(1) $\frac{3}{1 + \frac{2}{5 + \frac{1}{4}}}$ (2) $\frac{1}{3}$ of $\frac{1}{1 + \frac{1}{3 + \frac{1}{4}}}$ (3) $\frac{2}{5 + \frac{3}{4 + \frac{1}{8}}}$ (4) $2 + \frac{1}{3 + \frac{4}{5 + \frac{1}{4}}}$

$$(5) 3 + \frac{1}{2 + \frac{3}{5 + \frac{1}{2}}} \quad (6) 2 + \frac{3}{1 + \frac{3}{6 + \frac{4}{5 + \frac{1}{4}}}} \quad (7) 6\frac{2}{3} + \frac{3\frac{1}{2}}{2}$$

$$(8) 2\frac{2}{3} + \frac{4}{5 + \frac{6}{7 + \frac{5}{6}}} \quad (9) \frac{3}{4 + \frac{3}{5 + \frac{3}{6 + \frac{2}{7}}}} + \frac{1}{2 + \frac{1}{3 + \frac{1}{4 + \frac{1}{5}}}}$$

$$(10) 2\frac{1}{2} \times \frac{1}{3\frac{1}{2} + 4\frac{1}{2}} + \frac{2 - \frac{1}{2} \times (\frac{1}{2} + \frac{1}{2})}{1 + \frac{1}{2 - \frac{1}{2}}} \quad (11) 1 + \frac{2}{3 + \frac{1}{4 + \frac{1}{5\frac{1}{2}}}} + \frac{18}{23}$$

$$(12) \left(3 - \frac{1}{2 - \frac{1}{6 + \frac{1}{8}}} \right) - \left(1 + \frac{1}{4 + \frac{1}{3 - \frac{1}{5 + \frac{1}{4}}}} \right) \quad (13) 3\frac{1}{2} + \frac{5\frac{1}{2}}{7\frac{1}{2} + \frac{8\frac{1}{2}}{10\frac{1}{2} + \frac{13}{9\frac{1}{2}}}}$$

$$(14) \frac{1}{3 + \frac{1}{5 + \frac{1}{2}}} + \frac{3}{8 - \frac{7}{2 - \frac{1}{2}}} + \frac{5}{6 - \frac{5}{2 - \frac{1}{2}}} \quad (15) \frac{\frac{1}{1 - \frac{1}{2 - \frac{1}{2}}} + \frac{1}{2} + (1 + \frac{1}{2})}{3 \left(1 + \frac{2}{3\frac{1}{2}} \right) - 4}$$

$$(16) 1\frac{1}{2} \times 12\frac{1}{2} \text{ of } \frac{2}{5} - \frac{6\frac{1}{2} + 7\frac{1}{2}}{12 - 6\frac{1}{2}} - \frac{4 + \frac{1}{4 - \frac{1}{4}}}{4}$$

$$(17) \frac{2}{4 + \frac{2}{3 + \frac{1}{2}}} + \frac{3}{3 - \frac{2}{3 - 2}} + \frac{4}{4 - \frac{2}{4 - \frac{1}{2}}} + \frac{5}{3 + \frac{1}{2 + \frac{1}{4}}} \quad (18) \frac{4\frac{1}{2}}{4\frac{3}{4} + \frac{4\frac{1}{2}}{4\frac{3}{4} + \frac{5}{4\frac{3}{4} + 2\frac{1}{2}}}}$$

$$(19) \frac{2}{2 + \frac{2}{2 + \frac{1}{2 + \frac{2}{5 + \frac{1}{2}}}}} + \frac{2}{5 + \frac{1}{2 + \frac{1}{3 + \frac{1}{2}}}} + \frac{1\frac{1}{3} \times \frac{2\frac{1}{2}}{3\frac{1}{2}} \left(3 - \frac{1}{4} \right) - 1\frac{1}{2}}{2 + \frac{1}{3 + \frac{1}{1 + \frac{4}{5 + \frac{1}{2}}}}}$$

$$(20) 1 - \frac{1}{2 + \frac{1}{1 - \frac{1}{2 + \frac{1}{1 - \frac{1}{\frac{1}{2} + \frac{3}{5}}}}}} \times 1 - \frac{1}{2 + \frac{1}{2 \times \frac{1}{1 - \frac{1}{1\frac{1}{2}}}}} \times 2 + \frac{1}{1 - \frac{1}{1 + \frac{1}{1 - \frac{1}{1 + \frac{1}{\frac{1}{2}}}}}}$$

3 Simplify —

- (1) $15\frac{1}{2} + 1\frac{1}{4} - (7\frac{3}{4} - 6\frac{2}{3})$ (2) $(15\frac{1}{2} + 1\frac{1}{3}) - 13(7\frac{3}{4} - 6\frac{2}{3})$
 (3) $16 - \{2\frac{1}{2} - (4\frac{1}{2} + 1\frac{1}{4})\}$ (4) $16 + \{2\frac{1}{2} - (4\frac{1}{2} - 1\frac{1}{4})\}$
 (5) $5\frac{1}{2} - \{5\frac{1}{4} - \frac{2}{3}(3^2 + 2\frac{1}{2})\}$ (6) $3\frac{2}{5} \times \frac{1}{11} - 3\frac{1}{2}$ of $3\frac{3}{4}$ of $\frac{1}{4}$ of $1\frac{1}{2}$;
 (7) $(\frac{1}{2} + \frac{1}{3})$ of $(1\frac{1}{2} + 2\frac{3}{4})$ of $(2\frac{1}{2} - 1\frac{1}{2})$ of $(3\frac{1}{10} - 2)$
 (8) $(3\frac{1}{11} - 4\frac{2}{7})$ of $(10\frac{5}{7} - 7\frac{1}{2})$ of $\frac{1}{20}$ (9) $3\frac{1}{11} - (4\frac{2}{7} - 10\frac{5}{7}) - (7\frac{1}{2} - 8\frac{7}{10})$
 (10) $1\frac{1}{6}$ of $5\frac{1}{2} + (4\frac{1}{2} - 1\frac{1}{10}) - 1\frac{1}{15} - 6\frac{2}{3} - \frac{1}{2}$ of $2\frac{2}{3}$

4. Shew that the simple fraction equivalent to the value of $\frac{1}{3}$ of $\frac{1}{4} + \frac{1}{2}$ of $\frac{\frac{3}{4}}{\frac{9}{10} + 1} + \frac{1}{6}$ of $\frac{1}{5}$, is of the same magnitude as that expressed by $\frac{1}{10} + \frac{1}{2}$ of $\frac{1}{5}$ of $\frac{1}{10}$

5. Prove that $\frac{1}{2}$ of $(1 - \frac{2}{3}) + \frac{1}{4}$ of $\frac{1}{10} + \frac{3}{4}$ of $(\frac{1}{2} + \frac{1}{4}) + \frac{1}{10}$ of $(\frac{2}{3} + \frac{1}{4}) = 1$

VIII SIMPLIFICATION OF FRACTIONS

284 What has been proved in the adaptation of the fundamental operations to fractions, will furnish the means of simplifying arithmetical expressions formed by their combinations, and, in general, only very slight *mental* exertion will be required, if the attention of the *eye* be directed to the *composition* of the *terms* of the fractions concerned, and their *resolution* into the *factors* of which they are made up

Ex 1 Simplify $\frac{\frac{1}{2} + \frac{1}{3} + \frac{1}{4}}{\frac{1}{2} + \frac{1}{3} + \frac{1}{4}} - \frac{13}{24}$ of $\frac{576}{264}$

$$\begin{aligned} \text{The expression} &= \frac{\frac{1}{2} + \frac{1}{3} + \frac{1}{4}}{\frac{1}{2} + \frac{1}{3} + \frac{1}{4}} - \frac{13}{24} \text{ of } \frac{24 \times 24}{264} = \frac{6+4+3}{12} - \frac{13 \times 24}{264} \\ &= \frac{\frac{1}{2} + \frac{1}{3} + \frac{1}{4}}{\frac{1}{2} + \frac{1}{3} + \frac{1}{4}} - \frac{13 \times 24}{11 \times 24} = \frac{13}{12} \times \frac{315}{286} - \frac{13}{11} \\ &= \frac{13}{4 \times 3} \times \frac{105 \times 3}{13 \times 22} - \frac{13}{11} = \frac{105}{88} - \frac{13}{11} = \frac{105 - 104}{88} = \frac{1}{88} \quad \text{Ans} \end{aligned}$$

Ex 2 Simplify $\left\{ 2\frac{1}{4} + \frac{5}{2} \text{ of } \frac{7}{3\frac{1}{2}} - \frac{1\frac{2}{3}}{2\frac{1}{3}} \right\} - 1\frac{77}{228}$

$$\begin{aligned} \text{The expression} &= \left\{ \frac{11}{4} + \frac{5}{2} \text{ of } \frac{7 \times 5}{19} - \frac{\frac{5}{3}}{\frac{5}{2}} \right\} - \frac{305}{228} \\ &= \left\{ \frac{11}{4} + \frac{5 \times 7 \times 5}{2 \times 19} - \frac{5}{3} \times \frac{2}{5} \right\} \times \frac{2 \cdot 8}{305} \end{aligned}$$

$$\begin{aligned}
 &= \left\{ \frac{11}{4} + \frac{175}{36} - \frac{2}{3} \right\} \times \frac{228}{305} = \frac{627 + 1050 - 152}{228} \times \frac{228}{305} \\
 &= \frac{1677 - 152}{228} \times \frac{228}{305} = \frac{1525}{305} = 5 \quad \text{Ans}
 \end{aligned}$$

Examples LXXVI

Simplify the following —

- 1 $\frac{2}{3}$ of $\frac{4}{7} - \frac{5}{12}$ of $3\frac{1}{2} + \frac{1}{8}$ of $3\frac{2}{3}$
- 2 $(\frac{2}{3} + \frac{1}{4}$ of $\frac{1}{2}\frac{3}{4}) - 2\frac{1}{11}$ of $(1\frac{5}{4} - \frac{1}{3}\frac{2}{3})$
- 3 $\frac{4\frac{1}{2} \times 4\frac{1}{2} \times 4\frac{1}{2} - 1}{4\frac{1}{2} \times 4\frac{1}{2} - 1}$
- 4 $\frac{4\frac{1}{2} \times 4\frac{1}{2} - 3\frac{1}{2} \times 3\frac{1}{2}}{4\frac{1}{2} - 3\frac{1}{2}}$
- 5 $\frac{1 + 6\frac{1}{2} \times (1 + 6\frac{1}{2})}{1 + 5\frac{1}{2} \times (1 + 5\frac{1}{2})}$
- 6 $\frac{7\frac{1}{2}}{6\frac{1}{2}} + \frac{11\frac{1}{2} - 2\frac{3}{4}}{11\frac{1}{2} + 2\frac{3}{4}} \times 10\frac{9}{13} - 6\frac{4}{2}\frac{2}{3}$
- 7 $\frac{14\frac{1}{2} - 6\frac{1}{2}}{3\frac{1}{2} + 6\frac{1}{2}} - \frac{4\frac{1}{2} + 6\frac{1}{2}}{9\frac{1}{2} - 3\frac{1}{2}} + (30\frac{1}{18} - 22\frac{2}{9})$
- 8 $\frac{1 + 2 \times \frac{4}{5} + \frac{4}{5} \times \frac{4}{5}}{1 - \frac{4}{5} \times \frac{4}{5}}$
- 9 $\frac{1}{3\frac{1}{2}} - \frac{2\frac{1}{2}}{9} + \frac{3\frac{1}{2}}{2} + \frac{4}{4\frac{1}{2}}$
- 10 $(\frac{1}{2}\frac{1}{2}$ of $3\frac{1}{2}) + (\frac{2}{3} - \frac{2}{9}\frac{2}{3}) - (\frac{1}{1\frac{1}{2}} - \frac{7\frac{2}{3}}{9})$
- 11 $\frac{3\frac{1}{2} - \frac{1}{2}\frac{2}{3}}{7\frac{1}{2} - \frac{1}{2}\frac{2}{3}}$ of $\frac{11\frac{1}{2}}{17} \times 52\frac{1}{11}$
- 12 $\frac{3 + \frac{5}{3}$ of $\frac{21}{7\frac{1}{2}} - \frac{1}{4} - \frac{1\frac{2}{3}}{2\frac{1}{2}}}{10 - \frac{151}{228}}$ of 5
- 13 $\frac{1\frac{1}{2}}{3 + \frac{1}{3\frac{1}{2}}} + \frac{1\frac{1}{2}}$ of $4\frac{7}{8}$ + $\frac{5\frac{1}{2}}$ of $7\frac{7}{8}$
- 14 $\frac{1\frac{1}{2}}$ of $3\frac{1}{2}$ + $\frac{5\frac{1}{2}}$ of $7\frac{7}{8}$
- 15 $\frac{2\frac{1}{2} - \frac{1}{2}\frac{2}{3}}{2\frac{1}{2} + \frac{1}{2}\frac{2}{3}} + \frac{7}{12}$ of $\frac{9 \times 10}{14 \times 3} - \frac{22\frac{1}{2}}{30}$
- 16 $\frac{1\frac{1}{2}}{3\frac{1}{2}} - \frac{5\frac{1}{2}}{6\frac{1}{2}}$ of $(\frac{1}{5} - \frac{1}{4\frac{1}{2}} - \frac{1}{3\frac{1}{2}})$
- 17 $\frac{1}{26} (5\frac{1}{2} - 2\frac{1}{4}) + (5\frac{1}{3}$ of $\frac{9}{128} - \frac{9}{8}$ of $\frac{3}{8}) - (\frac{1}{1\frac{1}{2}} - \frac{1\frac{1}{2}}{3}) - \frac{8}{7} (2 - \frac{4}{9})$
- 18 $\frac{1 - \frac{2}{3}}$ of $2\frac{11}{26} - \frac{4}{13 - 3\frac{1}{2}} + 3\frac{11}{16} - \frac{3}{3 - 1\frac{1}{2}\frac{1}{2}}$
- 19 $\frac{1 + 2\frac{1}{2} + 3\frac{1}{2}}{\frac{1}{1\frac{1}{2}} + \frac{2}{2\frac{1}{2}} + \frac{3}{3\frac{1}{2}}}$ of $\frac{55\frac{1}{2} - 11}{1\frac{2}{11}}$ of $13\frac{2}{3}$
- 20 $\frac{1 - \frac{4}{5}}{3\frac{1}{2} + 1\frac{1}{2} + \frac{1\frac{1}{2}}{6\frac{2}{3}}}$ of $(\frac{1 + \frac{1}{2}}{1 - \frac{1}{2}})$
- 21 $\frac{\frac{1}{2} + \frac{1}{4} + \frac{1}{4}}{\frac{1}{2\frac{1}{2}} + \frac{1}{3\frac{1}{2}} + \frac{1}{4\frac{1}{2}}}$ of $\frac{1}{13}$ of $\frac{1}{1 + \frac{1}{3 + \frac{1}{4}}}$
- 22 $\left\{ \frac{11\frac{1}{2} - 10\frac{1}{2}}{11\frac{1}{2} + 10\frac{1}{2}} - \frac{10\frac{1}{2} + 11\frac{1}{2}}{10\frac{1}{2} - 9\frac{1}{2}} \right\} \times \frac{2 + \frac{1}{2}\frac{1}{2}}{2 - \frac{1}{2}\frac{1}{2}} \times \frac{8\frac{1}{2}}{2\frac{1}{2}\frac{1}{2}}$

- 23 $\frac{2\frac{1}{2}}{2\frac{3}{4}} + \frac{2\frac{1}{2} + 5\frac{1}{2}}{3\frac{1}{8} + 9\frac{1}{2}} + \frac{5}{9} \text{ of } \frac{9}{10} + \frac{3}{8} \text{ of } \frac{3}{20}$
- 24 $\left(6\frac{3}{7} \text{ of } \frac{5\frac{1}{2} - 4\frac{1}{2}}{12\frac{1}{2} - 7\frac{1}{2}}\right) - \frac{1 + \frac{1}{2\frac{1}{2}}}{2}$ 25 $\frac{4\frac{1}{2}}{1\frac{1}{2} \text{ of } 3\frac{1}{2}} - \left(\frac{2}{7} \text{ of } \frac{2\frac{1}{2} + 1\frac{1}{2}}{2 - \frac{1}{2}}\right)$
- 26 $\frac{1}{3} \times \frac{17}{3} \text{ of } \frac{27}{85} + \left(2\frac{1}{3} + \frac{1}{3 + \frac{1}{4}}\right) \times \frac{39}{103}$
- 27 $1\frac{1}{11} - \frac{1 - \frac{1}{2\frac{1}{2}}}{2 - \frac{1}{3}} + \frac{1\frac{1}{2}}{3\frac{1}{2}} - \frac{5\frac{1}{2}}{6\frac{1}{2}} \text{ of } \frac{2}{3} \left\{ \frac{1}{\frac{1}{2}} - \frac{1 - 1}{4\frac{1}{2} - 3\frac{1}{2}} \right\}$
- 28 $\left\{ \frac{7}{5 - \frac{1}{3}} - \frac{3 - \frac{2}{3 - \frac{1}{2}}}{4 - 1\frac{1}{2}} \right\} - \frac{5}{7} \text{ of } \frac{3}{4} \left\{ \frac{1}{1\frac{1}{2}} + \frac{6}{5} \text{ of } \frac{3\frac{1}{2} - 2\frac{1}{2}}{\frac{1}{3} - 2} \right\}$
- 29 $\frac{1 + 5\frac{1}{2}(1 + 5\frac{1}{2})}{1 + 2\frac{1}{2}(1 + 2\frac{1}{2})} \times \frac{4\frac{1}{2} + 2\frac{1}{2}}{13\frac{1}{2} - 3\frac{1}{2}} \text{ of } \frac{11}{111}$ 30 $\frac{1\frac{1}{2} + 7\frac{1}{2} \times 1\frac{1}{2} - 9\frac{1}{2}}{4\frac{1}{2} \text{ of } \left(5\frac{5}{7} - 3\frac{7}{10}\right)}$
- 31 $\frac{5\frac{1}{2} - \frac{1}{2}}{1\frac{1}{2} \text{ of } \frac{1}{2} - 10\frac{1}{2}} \times \frac{2}{5} \text{ of } \frac{1\frac{1}{2}}{13\frac{1}{2}} \text{ of } 4\frac{1}{2}$ 32 $\frac{1}{4\frac{1}{2}} \text{ of } 4\frac{1}{2} + \frac{2}{4\frac{1}{2}} \text{ of } \frac{1\frac{1}{2}}{1\frac{1}{2}} - 2\frac{1}{2}$
- 33 $\frac{\frac{1}{2} \text{ of } \left(\frac{1}{2} - \frac{1}{2}\right)}{\frac{1}{2} \text{ of } \left(\frac{1}{2} - \frac{1}{2}\right)} \text{ of } \left\{ \frac{\frac{1}{2} - \frac{1}{2}}{1\frac{1}{2} - 1\frac{1}{2}} - \frac{9\frac{1}{2} + 3\frac{1}{2}}{5\frac{1}{2} \text{ of } 7\frac{1}{2}} + \frac{7\frac{1}{2} - 4\frac{1}{2}}{2\frac{1}{2} - 1\frac{1}{2}} \right\}$
- 34 $7\frac{1}{2} \text{ of } \frac{1}{10 + \frac{1}{3 + \frac{1}{20}}}$ 35 $3\frac{1}{2} + \frac{2\frac{1}{2}}{3\frac{1}{2} + \frac{-2}{5\frac{1}{2} + \frac{1}{4\frac{1}{2}}}}$
- 36 $11 + \frac{1}{1 - \frac{1}{1 + \frac{1}{8 + \frac{1}{11}}}}$ 37 $\left(2 + \frac{1}{3 - \frac{1}{5 + \frac{1}{8}}}\right) - \{1\frac{1}{2} - (1\frac{1}{2} \times 14\frac{1}{2})\}$
- 38 $\frac{3\frac{1}{2}}{1\frac{1}{2} \text{ of } 2\frac{1}{2}} + \frac{8\frac{1}{2} \text{ of } 2\frac{1}{2}}{11} - \frac{9\frac{1}{2} \text{ of } 1\frac{1}{2}}{4(\frac{1}{2} + \frac{1}{2})} + \frac{2 - \frac{1}{2}}{1 - \frac{1}{2}}$
- 39 $\frac{4\frac{1}{2} \text{ of } 3 - 3\frac{1}{2} \text{ of } 3\frac{1}{2}}{4\frac{1}{2} - 3\frac{1}{2}} + \left\{ 6\frac{1}{2} \text{ of } \frac{\frac{1}{2} + \frac{1}{2}}{3\frac{1}{2} - 2\frac{1}{2}} \right\} \times 6\frac{35}{117}$
- 40 $\frac{5\frac{1}{2} - 2\frac{1}{2}}{3\frac{1}{2} + \frac{1}{2}} \text{ of } \frac{4\frac{1}{2} + 5\frac{1}{2}}{4\frac{1}{2}} \text{ of } \frac{2\frac{1}{2} + 1\frac{1}{2}}{7\frac{1}{2} - 2\frac{1}{2}} \text{ of } \frac{6\frac{1}{2}}{7\frac{1}{2}}$
- 41 $\left\{ \left(3\frac{1}{2} - \frac{2}{3} \text{ of } \frac{8\frac{1}{2}}{7} + \frac{2\frac{1}{2}}{5\frac{1}{2}}\right) - \frac{2\frac{1}{2} + 9\frac{1}{2}}{6\frac{1}{2} - 4\frac{1}{2}} \right\} \text{ of } 34\frac{16}{23}$
- 42 $\frac{6\frac{1}{2} - 1\frac{1}{2}}{2\frac{1}{2} + 1\frac{1}{2}} \text{ of } \frac{(3\frac{1}{2} + 5\frac{1}{2} - 1\frac{1}{2})(4\frac{1}{2} - 3\frac{1}{2})}{11\frac{1}{2} + 2\frac{1}{2} - (2\frac{1}{2} - \frac{1}{2} - \frac{1}{2})}$ 43 $8 - 8 \times \frac{2\frac{1}{2} - 1\frac{1}{2}}{2 - \frac{1}{6 - \frac{1}{8}}}$

- 44 $\left\{ \frac{3\frac{1}{2} + 2\frac{1}{4}}{\frac{1}{2} \text{ of } 9\frac{1}{6}} - \frac{2\frac{4}{5} - 1\frac{1}{3} + 0\frac{1}{11}}{4\frac{1}{5} - 2\frac{1}{4} + 13\frac{7}{11}} \right\} \text{ of } \frac{5\frac{1}{2}}{2\frac{1}{5}}$
- 45 $\frac{1\frac{1}{2} - \frac{2}{3} \text{ of } 1\frac{5}{6} + 1\frac{1}{6}}{1\frac{4}{5} - \frac{1}{2} \text{ of } 1\frac{1}{2} + 1\frac{1}{6}} \text{ of } 1\frac{1}{2} \text{ of } \frac{6\frac{1}{2} - 1\frac{5}{6}}{2\frac{1}{6} + 1\frac{1}{6}}$
- 46 $\left\{ \frac{2}{3 - \frac{1}{1 - \frac{1}{2}}} - \frac{1}{3} \text{ of } \left(5 - \frac{2}{\frac{1}{2} - \frac{1}{1}} \right) \right\} - \frac{\frac{1}{2} + \frac{2}{3}}{1\frac{1}{2}}$
- 47 $\frac{1\frac{1}{2}}{7 + \frac{3}{4 - 2\frac{1}{4}}} \times \frac{2021}{2193} - \left(1\frac{37}{48} - \frac{15}{16} \right)$ 48 $\left(\frac{1 + \frac{1}{2}}{1 - \frac{1}{2}} \right)^2 - \left(\frac{1 + \frac{5}{13}}{1 - \frac{6}{13}} \right)^2$
- 49 $\frac{8\frac{1}{2} - 7\frac{1}{2} + 5\frac{1}{2} - 4\frac{1}{2}}{9\frac{1}{6} - 8\frac{1}{6} + 7\frac{1}{6} - 6\frac{1}{6}} - \left\{ \frac{1}{2} \text{ of } 2\frac{1}{2} - \frac{1\frac{2}{3}}{2\frac{1}{3}} \right\}$
- 50 $\left(\frac{2}{3 - \frac{4}{5}} + \frac{3}{4 - \frac{5}{6}} \right) - \left(\frac{3}{2 - \frac{1}{4}} - \frac{1}{3 - \frac{4}{5}} \right) \times \left(\frac{1}{\frac{1}{2} - \frac{1}{3}} - \frac{1}{1\frac{1}{2} - 1\frac{2}{3}} \right)$
 $\div \left(\frac{1}{1\frac{1}{2} + 1} - \frac{2}{6 - 2\frac{1}{2}} \right), \frac{\frac{1}{2} + \frac{2}{3}}{4 - \frac{1}{6}} \text{ of } 5\frac{1}{2} - \frac{\frac{2}{3} + \frac{1}{4}}{1\frac{1}{6} \text{ of } 4\frac{1}{2} - 2\frac{1}{2}}$
- 51 $\left\{ \left(\frac{1}{\frac{1}{2} - \frac{1}{3}} - \frac{1}{1\frac{1}{2} - 1\frac{2}{3}} \right) - \left(\frac{1}{1\frac{1}{2} - \frac{1}{6}} - \frac{2}{6\frac{1}{2} - 2\frac{1}{6}} \right) \right\} \text{ of } \left\{ \left(\frac{2}{3 - \frac{4}{5}} + \frac{3}{4 - \frac{5}{6}} \right) \right.$
 $\left. \div \left(\frac{3}{2 - \frac{1}{4}} - \frac{1}{3 - \frac{4}{5}} \right) \right\}, \frac{3\frac{7}{8} - 2\frac{1}{4}}{\frac{1}{2} \text{ of } (1\frac{1}{2} - 2\frac{1}{2})} - \frac{3\frac{2}{3} \text{ of } 5\frac{2}{3}}{5\frac{1}{6} + \frac{1}{2} \text{ of } 4\frac{3}{8}}$
- 52 $1\frac{1}{2} \text{ of } \frac{\frac{1}{2} + \frac{1}{3} + \frac{1}{4}}{2\frac{1}{2} - 3\frac{1}{3} + 4\frac{1}{4}} \times \left(\frac{2\frac{1}{2}}{3\frac{1}{2}} + \frac{1\frac{1}{2}}{1\frac{1}{6}} \right) - \left(\frac{3}{4\frac{1}{2}} + \frac{4\frac{1}{2}}{3} \right)$
- 53 $\frac{11\frac{1}{4} - 2\frac{1}{4}}{6\frac{1}{4} - 3\frac{1}{4}} - \frac{3\frac{1}{2} + 1\frac{1}{2}}{2\frac{1}{2} + 1\frac{1}{2}} \times \frac{3\frac{1}{2} - 1\frac{1}{2}}{2\frac{1}{2} - 1\frac{1}{2}}$ 54 $\frac{7\frac{1}{2} + 1\frac{1}{2}}{8\frac{1}{2} + 3\frac{1}{2}} - \frac{3\frac{1}{2} + \frac{2}{3}}{3\frac{1}{2} + 14\frac{1}{8}}$
- 55 $\left(\frac{1}{5\frac{1}{2}} + \frac{4}{4\frac{1}{2}} + \frac{3\frac{1}{2} - 8\frac{1}{6}}{\frac{1}{2\frac{1}{2}} + \frac{1}{1\frac{1}{2}}} \text{ of } \frac{1}{\frac{1}{6}} \right) \times \left(\frac{1\frac{1}{2}}{1\frac{1}{2}} - 1 + \frac{1}{2\frac{1}{2}} - \frac{1}{2\frac{1}{2}} + 7 \right) \text{ of } \frac{1\frac{1}{2} + \frac{1}{2}}{\frac{1}{2} + \frac{1}{2}},$
 $3 - \left(\frac{3\frac{1}{2} - \frac{1}{6}}{3\frac{1}{2} + \frac{1}{6}} - 2\frac{1}{2} \text{ of } \frac{4}{19} \right)$
- 56 $\left\{ \frac{\frac{2}{3} \text{ of } \frac{1}{8} \text{ of } 6\frac{1}{2} + 7\frac{1}{2} + 19\frac{1}{6} + 8\frac{1}{6}}{3\frac{1}{5} + \frac{1}{7} + 4\frac{1}{6} - 1\frac{1}{4}} \text{ of } 1\frac{1}{4}} - \frac{221}{680} \right\} - \left\{ \frac{1\frac{3}{8} + \frac{6}{8}}{3\frac{1}{8}} + 39\frac{1}{2} - 24\frac{3}{5} \right\},$

IX G C M AND L C M OF FRACTIONS

285 The definitions that we have already given of the G C M and L C M of two or more whole numbers will also be applicable when the given numbers are fractions, provided that we understand by *exactly*, that the complete quotients must be *integers*

286 To find the G C M of two or more fractions

RULE Express the fractions in their lowest terms, if they be not already so. Then take the G C M of the numerators for numerator and the L C M of the denominators for denominator. The fraction so formed is the G C M of the given fractions.

Ex Find the G C M of $\frac{8}{9}$, $\frac{12}{14}$, $\frac{48}{33}$

Here, the fractions reduced to their lowest terms are $\frac{8}{9}$, $\frac{6}{7}$, $\frac{16}{11}$

The G C M of the numerators 8, 6, 16 is 2, and the L C M of the denominators 9, 7, 21 is 63

Thus the required G C M = $\frac{2}{63}$ Ans

287 To find the L C M of two or more fractions

RULE Express the fractions in their lowest terms. Then take the L C M of the numerators as numerator and the G C M of the denominators as denominator. The fraction so formed is the L C M of the given fractions.

Ex Find the L C M of $\frac{8}{25}$, $\frac{4}{7}$, $\frac{9}{20}$

Here, the fractions reduced to their lowest terms are $\frac{8}{25}$, $\frac{4}{7}$, $\frac{9}{20}$

The L C M of the numerators 8, 2, 9 is 72, and the G C M of the denominators 25, 7, 20 is 1

Thus, the required L C M = $\frac{72}{1}$ Ans

Note Before applying the Rules given above, reduce mixed numbers to improper fractions and compound fractions to simple ones.

Examples LXXVII

1 Find the G C M and the L C M of —

- (1) $\frac{7}{8}$, $\frac{9}{10}$ (2) $1\frac{1}{2}$, $1\frac{1}{5}$ (3) $\frac{1}{9}$, $1\frac{1}{2}$, $1\frac{1}{8}$ (4) $\frac{7}{9}$, $6\frac{5}{12}$, $\frac{1}{3}$
 (5) $\frac{2}{3}$, $\frac{4}{5}$, $\frac{6}{7}$, $\frac{8}{9}$ (6) $\frac{7}{8}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{8}{9}$ (7) $1\frac{1}{28}$, $2\frac{2}{31}$, $3\frac{1}{7}$ (8) $1\frac{5}{6}$, $2\frac{1}{3}$, 4 , $5\frac{1}{2}$
 (9) $1\frac{1}{5}$, $5\frac{1}{10}$, $14\frac{1}{15}$, $6\frac{1}{18}$ (10) $1\frac{2}{5}$, $2\frac{2}{7}$, $3\frac{1}{3}$, $8\frac{1}{2}$
 (11) $7\frac{1}{2}$, $1\frac{1}{3}$, $1\frac{1}{6}$, $4\frac{1}{5}$, $\frac{1}{2}$ (12) $3\frac{2}{3}$, $1\frac{1}{3}$, $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$

2 What is the greatest length that is contained a whole number of times exactly in $26\frac{1}{2}$ ft, $28\frac{7}{8}$ ft, and $29\frac{1}{6}$ ft?

3 A man gives away to each of five people $\frac{1}{12}$, $\frac{1}{15}$, $\frac{1}{18}$, $\frac{1}{20}$, $\frac{1}{25}$ of a basket of apples, how many has he left, supposing he has only just enough apples to do the above operation without dividing an apple?

4 What is the least number which when divided by each of the fractions $\frac{1}{5}$, $\frac{1}{10}$, $1\frac{1}{5}$, and $\frac{8}{15}$, gives a whole number as quotient in each case?

5 Three lines of paling run side by side for a distance of 150 yds. The upright posts are respectively $2\frac{1}{2}$, $3\frac{1}{3}$, $4\frac{1}{6}$ ft apart.

How often will a person walking outside be able, on looking across, to see 3 posts in a line ?

6 Eight bells commence to toll simultaneously They toll at intervals of $1\frac{1}{2}$, $2\frac{1}{2}$, 3, $4\frac{1}{2}$, 5, 6, 8 and 9 seconds respectively, after what interval will they again toll together ?

7 Three wheels are respectively $10\frac{1}{2}$ ft, $6\frac{1}{2}$ ft and $4\frac{1}{2}$ ft round Find the least distance travelled when they will make complete revolutions

8 A man gives away to each of four people $\frac{1}{2}$, $\frac{3}{10}$, $\frac{1}{5}$ and $\frac{7}{10}$ of a basket of apples, and has only just enough apples to be able to do this without dividing an apple, how many apples had he ?

X MISCELLANEOUS EXAMPLES IN FRACTIONS.

288 The following Solutions, we hope, will be of service to students in acquiring a thorough knowledge of the principles of Vulgar Fractions

Examples worked out

Ex 1 What fraction added to the sum of $\frac{1}{7}$, $\frac{1}{5}$, $\frac{1}{11}$ and $2\frac{1}{2}$ will make the sum equal to 5 ?

$$\text{Here, } \frac{1}{7} + \frac{1}{5} + \frac{1}{11} + 2\frac{1}{2} = 2 + 2\frac{1}{11} = 4\frac{1}{11}$$

$$\therefore \text{the required fraction} = 5 - 4\frac{1}{11} = \frac{1}{11} \quad \text{Ans}$$

Ex 2 What fraction is that from which if there be taken $\frac{2}{3}$ and to the remainder be added $\frac{1}{10}$ of $\frac{1}{10}$, the sum will be 10 ?

$$\text{Here, } \frac{2}{3} \text{ of } \frac{1}{10} = \frac{2}{30} \text{ and } \frac{1}{10} \text{ of } \frac{1}{10} = \frac{1}{100}$$

$$\therefore \text{the required number} = 10 - \frac{2}{30} + \frac{1}{100} = 9\frac{1}{30} + \frac{1}{100} = 9\frac{11}{150} \quad \text{Ans}$$

Ex 3 Find what fraction multiplied by the sum of $2\frac{3}{5}$, $1\frac{1}{10}$ and $\frac{1}{10}$ will make the product equal to 17

$$\text{Here, } 2\frac{3}{5} + 1\frac{1}{10} + \frac{1}{10} = 3 + \frac{3}{10} = 3\frac{3}{10}$$

$$\therefore \text{the required fraction} = 17 \div 3\frac{3}{10} = 17 \times \frac{10}{33} = \frac{170}{33} = 5\frac{5}{33}$$

Ex 4 Find what least fraction added to the sum of $\frac{3}{4}$, $1\frac{1}{8}$ and $2\frac{1}{8}$ will make the result an integer

$$\text{Here, } \frac{3}{4} + 1\frac{1}{8} + 2\frac{1}{8} = 3 + 2\frac{1}{4} = 5\frac{1}{4}$$

$$\therefore \text{the required fraction} = 1 - \frac{1}{4} = \frac{3}{4} \quad \text{Ans}$$

Ex 5 What number divided by $2\frac{1}{8}$ will produce $\frac{1}{16}$?

$$\text{The required number} = \frac{1}{16} \times 2\frac{1}{8} = \frac{1}{16} \times \frac{17}{8} = \frac{17}{128} \quad \text{Ans}$$

Ex 6 A man has $\frac{1}{2}$ of an estate, he gives his son $\frac{1}{4}$ of his share, what portion of the estate has he then left ?

$$\frac{1}{4} \text{ of his share being given away there remains } (1 - \frac{1}{4}) \text{ or } \frac{3}{4}$$

$$\text{But his share} = \frac{1}{2} \text{ of the estate, } \therefore \text{he remains } \frac{3}{4} \text{ of } \frac{1}{2} = \frac{3}{8} \quad \text{Ans.}$$

Examples LXXVIII

- 1 What number added to $\frac{1}{2}$ makes $1\frac{1}{2}$? and what taken from $1\frac{1}{2}$ leaves $\frac{1}{2}$?
- 2 What number added to $1\frac{0}{1}$, $1\frac{1}{2}$, $3\frac{3}{4}$, $\frac{1}{2}$, will make the sum total equal to 3?
- 3 Multiply the sum of $3\frac{2}{3}$, $4\frac{2}{3}$ and $4\frac{4}{5}$ by the difference of $7\frac{0}{4}$ and $5\frac{5}{8}$, and divide the product by the sum of $94\frac{1}{8}$ and $93\frac{1}{8}$.
- 4 Prove that the sum of $5\frac{1}{2}$ and $3\frac{1}{2}$ is equal to four times their difference.
- 5 Compare the product and quotient of $\frac{7}{9}$ by $1\frac{0}{1}$.
- 6 Find what quantity multiplied by $\frac{2}{3}$ of $\frac{1}{5}$ of $3\frac{1}{2}$, gives a result equal to $\frac{1}{2}$ of $2\frac{7}{8}$ of $6\frac{1}{4}$.
- 7 What number is that, whereof the part expressed by $1 + \frac{1}{2} + \frac{1}{4}$ is 45 ? What number must be added to $\frac{1}{3}$ of $2\frac{1}{2}$ to give $3\frac{2}{3}$?
- 8 Find the least fraction which, added to the sum of $\frac{7}{8}$, $1\frac{0}{10}$ and $\frac{1}{2}$, will make the result an integer.
- 9 To 479 add $1\frac{0}{10}\frac{4}{10}$ and repeat the addition 6 times.
- 10 From $11\frac{0}{10}$ take the sum of $2\frac{1}{2}$, $3\frac{3}{4}$ and $4\frac{1}{2}$, and multiply the difference by $2\frac{1}{3}$ of $\frac{5}{8}$ of $6\frac{1}{2}$.
- 11 Multiply $49\frac{1}{8}$ by $50\frac{1}{8}$ and add $2\frac{1}{8}$ to the result.
- 12 How many times does $\frac{2}{3} + \frac{2}{3} - 1\frac{1}{8}$ contain $\frac{2}{7} + \frac{1}{3} - 2\frac{1}{4}$?
- 13 Multiply the sum of 1, $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{3}{4}$ by the difference of $\frac{4}{5}$ and $\frac{3}{10}$ and divide the product by the double of $21\frac{7}{10}$.
- 14 Of the fractions $1\frac{0}{10}$, $\frac{5}{12}$, $\frac{7}{10}$, $\frac{1}{2}$, find how much the sum of the greatest and least exceeds the difference of the other two.
- 15 From 1 take its half, third, and twenty fourth parts add the product of those parts to the remainder, and multiply this sum by $7\frac{1}{4}$. What must $\frac{3}{4}$ be divided by to produce 2?
- 16 To $\frac{1}{8}$ of a dozen add $\frac{1}{2}$ of three hundred, and divide this sum by the difference of $3\frac{3}{4}$ of a hundred and $43\frac{3}{8}$.
- 17 Find the sum of the greatest and least of the fractions $\frac{3}{8}$, $\frac{1}{12}$, $\frac{4}{9}$ and $\frac{7}{10}$, the sum of the other two, and the difference of these sums.
- 18 Multiply the sum of $\frac{1}{2}$, $1\frac{3}{4}$ and $\frac{5}{8}$ by the difference of $\frac{4}{5}$ and $\frac{3}{10}$, and divide the product by $\frac{1}{18}$ of $11\frac{1}{8}$.
- 19 What fraction is that from which if $\frac{2}{3}$ of $\frac{3-1\frac{1}{2}}{28}$ be subtracted and the remainder be divided by $5\frac{1}{5} - 16\frac{2}{5}$, the result will be $\frac{1}{3}$?
- 20 Divide the sum of $2\frac{2}{3}$, $3\frac{3}{4}$ and $5\frac{5}{8}$ by the sum of $4\frac{4}{5}$ and $8\frac{8}{10}$, and to the quotient add the difference of $10\frac{0}{10}$ and $5\frac{5}{5}$.

21 To the sum of $3\frac{1}{2}$ and $4\frac{1}{6}$ add the difference between $4\frac{1}{2}$ and $5\frac{1}{8}$ and multiply the result by $11\frac{1}{11}$

22 A merchant owned $\frac{1}{10}$ of a ship and sold $\frac{3}{8}$ of his share, what share has he remaining?

23 If I pay away $\frac{1}{2}$ of my money, then $\frac{1}{3}$ of what remains, then $\frac{1}{4}$ of what then remains and then $\frac{1}{5}$ of what still remains, what fraction of the whole will be left?

24 What is the least fraction which must be added to the sum of 4 and $\frac{1}{2}$ divided by their difference to make the result an integer?

25 The difference of two numbers is $15\frac{1}{3}$, the smaller number is $5\frac{1}{2}$, find the greater number

26 Multiply $3\frac{3}{10}$ by $15\frac{1}{5}$, and divide $\frac{2}{3\frac{1}{2}}$ by $\frac{2\frac{3}{4}}{3}$, and add together the sum and difference of these results

27 Divide 2 by the sum of $2\frac{2}{3}$, $\frac{4}{5}$ and 4, add $1\frac{2}{3} - \frac{7}{9}$ to the quotient, and multiply the result by the difference of $5\frac{1}{5}$ and $4\frac{1}{2}$

28 If I pay away $\frac{1}{2}$ of my money, then $\frac{2}{3}$ of the remainder, then $\frac{1}{2}$ of what then remains and then $\frac{1}{20}$ of the original sum, what fractional part of my money have I left after the second, and also after the final payment?

29 What must be taken from

$$8\frac{1}{7} \text{ of } \frac{5\frac{1}{2} - 2\frac{1}{2}}{3\frac{1}{2} + 2\frac{1}{10}} - \frac{5\frac{1}{2} - \frac{2}{3}}{1\frac{1}{2} \text{ of } \frac{5}{6}} \text{ of } \frac{\frac{4}{5}}{1 - \frac{1}{7}} \text{ to reduce its value to } \frac{1}{11}?$$

30 A has a certain sum of money in his pocket of which he loses $\frac{1}{3}$ th, he gives $\frac{1}{12}$ th of what remains to B, and then $\frac{1}{4}$ th of $(\frac{1}{2} - \frac{1}{3})$ of what then remains to C, find what fractional part of A's original money B and C respectively receive, and compare these sums with the amount A has after his loss

$$31 \text{ A man having } \frac{17\frac{1}{2} - \left(\frac{3\frac{1}{2}}{4\frac{1}{2}} + \frac{\frac{1}{10}}{8 - 5\frac{1}{2}}\right)}{7\frac{1}{2} \text{ of } \frac{\frac{1}{2}}{7} - \frac{\frac{4}{7}}{\frac{5}{6}}} - \frac{6\frac{2}{3} \text{ of } 7}{5\frac{1}{2}} \text{ of an estate,}$$

gives $\frac{1}{3}$ of his share to his son, and $\frac{2}{3}$ of the remainder to his daughter, what fraction of the estate has he still remaining?

32 If I cut half a cake into 5 equal parts, and the remainder into 7 equal parts, and then cut one of the 5 equal parts into 6 equal parts, and one of the 7 equal parts into 4 equal parts and then give 2 children each one of each of these small slices, what fractional part of the whole cake will they receive, and what part of the cake will be left?

XI APPLICATION OF FRACTIONS TO COMPOUND QUANTITIES

289 In the Fundamental Operations of Compound Quantities, if the lowest denominations of the given compound quantities be mixed numbers, we shall treat separately, first the fractional parts by the ordinary method for Fractions and then the integral parts

Ex 1 Add together £16 2s $1\frac{1}{2}d$, £4 18s $1\frac{1}{2}d$ and £1 or $9\frac{1}{2}d$

£	s	d	
16	2	$1\frac{1}{2}$	
4	18	$1\frac{1}{2}$	
1	0	$9\frac{1}{2}$	
22	1	$0\frac{1}{2}$	

Now $(\frac{1}{2} + \frac{1}{2} + \frac{1}{2})d = \frac{3}{2}d = 1\frac{1}{2}d$, we therefore put down $1\frac{1}{2}d$, carry on 1d to the column of pence, and proceed in the usual way

Ex 2 Subtract Rs 32 14s $9\frac{1}{2}p$ from Rs 87 8s $6\frac{3}{4}p$

Rs	s	p	
87	8	$6\frac{3}{4}$	
32	14	$9\frac{1}{2}$	
54	9	$8\frac{1}{4}$	

Here $\frac{3}{4}$ is greater than $\frac{1}{2}$, therefore we add 1 to $\frac{1}{2}$, which makes it $\frac{3}{4}$

Now $1\frac{3}{4} - \frac{1}{2} = \frac{3}{4} = 1\frac{1}{4}$ We must add 1p to 9p, and proceed in the usual way

Ex 3 Multiply £6 12s $8\frac{5}{8}d$ by 57, and divide Rs 21 14s $5\frac{1}{2}p$ by 21

(1) £ s d

6	12	$8\frac{5}{8}$	57 = 11 × 5 + 2
<hr/>			
73	0	$1\frac{1}{8}$	for $\frac{5}{8}d \times 11 = \frac{55}{8}d$
<hr/>			
365	0	$5\frac{5}{8}$	for $\frac{1}{8}d \times 5 = \frac{5}{8}d$
<hr/>			
13	5	$5\frac{2}{8}$	for $\frac{5}{8}d \times 2 = \frac{10}{8}d$
<hr/>			
378	5	$11\frac{1}{2}$	

(2) Rs s p

21	14	$5\frac{1}{2}$	21 = 3 × 7
<hr/>			
7	4	$9\frac{1}{2}$	for $2\frac{1}{2} - 3 = 1$
<hr/>			
1	0	$8\frac{1}{2}$	for $1\frac{1}{2} - 7 = 1$

Examples LXXIX

1 Add together —

(1)	Rs	a	p	(2)	Rs	a	p	(3)	£	s	d	(4)	£	s	d
	3	15	$7\frac{1}{2}$		17	13	$5\frac{1}{4}$		7	13	$1\frac{1}{4}$		23	2	$6\frac{3}{4}$
	5	14	$2\frac{1}{2}$		32	6	$11\frac{1}{2}$		2	17	$4\frac{3}{4}$		14	1	$5\frac{1}{8}$
	7	6	$10\frac{5}{8}$		12	10	$9\frac{5}{8}$		5	2	$8\frac{1}{8}$		7	8	$11\frac{1}{4}$
	8	1	$11\frac{1}{2}$		7	0	$8\frac{1}{4}$		6	11	$2\frac{1}{2}$		4	9	$5\frac{1}{2}$
	2	4	$6\frac{3}{8}$		11	5	$4\frac{1}{8}$		4	5	$0\frac{1}{2}$		16	4	$2\frac{1}{8}$
	1	4	$5\frac{7}{8}$		6	10	$5\frac{7}{8}$		6	3	$4\frac{1}{2}$		5	4	$3\frac{1}{2}$

(5)	oz	dwt	grs	(6)	cwt	qrs	lbs	oz	(7)	poles	yds	ft	in
	5	16	$15\frac{3}{4}$		13	0	21	$13\frac{5}{8}$		25	4	2	8
	1	14	$23\frac{3}{4}$			3	18	$9\frac{7}{8}$		17	2	0	$6\frac{1}{2}$
		17	$0\frac{1}{2}$				25	$15\frac{1}{4}$		2	0	1	$7\frac{1}{8}$
	2	4	$21\frac{1}{8}$				13	$3\frac{1}{2}$		15	5	1	$11\frac{1}{4}$
	3	19	$8\frac{1}{2}$				2	$12\frac{1}{2}$		6	4	2	10
	6	18	$20\frac{1}{2}$			4	0	$15\frac{3}{8}$		20	3	0	$9\frac{1}{2}$

2 Perform the following subtractions —

- (1) $\begin{array}{r} Rs \quad a \quad p \\ 15 \quad 0 \quad 3\frac{5}{8} \\ 9 \quad 14 \quad 9\frac{1}{2} \end{array}$ (2) $\begin{array}{r} Rs \quad a \quad p \\ 17 \quad 15 \quad 7\frac{5}{8} \\ 6 \quad 15 \quad 9\frac{1}{8} \end{array}$ (3) $\begin{array}{r} £ \quad s \quad d \\ 46 \quad 13 \quad 6\frac{3}{4} \\ 34 \quad 19 \quad 9\frac{3}{4} \end{array}$ (4) $\begin{array}{r} £ \quad s \quad d \\ 163 \quad 1 \quad 7\frac{5}{8} \\ 64 \quad 2 \quad 5\frac{1}{8} \end{array}$
- (5) $\begin{array}{r} cwt \quad qrs \quad lbs \\ 15 \quad 1 \quad 16\frac{7}{8} \\ 8 \quad 3 \quad 25\frac{1}{2} \end{array}$ (6) $\begin{array}{r} cwt \quad qrs \quad lbs \\ 23 \quad 1 \quad 7\frac{7}{8} \\ 14 \quad 0 \quad 24\frac{3}{8} \end{array}$ (7) $\begin{array}{r} fur \quad po \quad yds \quad in \\ 5 \quad 15 \quad 0 \quad 0 \\ 2 \quad 4 \quad 3 \quad 8\frac{1}{2} \end{array}$ (8) $\begin{array}{r} hrs \quad min \quad sec \\ 23 \quad 45 \quad 35\frac{5}{8} \\ 15 \quad 50 \quad 48\frac{1}{2} \end{array}$

3 Multiply —

- (1) $Rs \ 9 \ 4a \ 2\frac{3}{4}p$ separately by 8, 11, 45 and 139
 (2) $£75 \ 13s \ 9\frac{1}{2}d$ separately by 4, 15, 88 and 96
 (3) 14 cwt 3 qrs 25 lbs $13\frac{3}{4}oz$ or separately by 12, 24 and 96
 (4) 45 mds 14 sr $7\frac{7}{8}ch$ separately by 9, 24 and 35
 (5) 3 fur 34 po 4 yds 1 ft $8\frac{1}{8}in$ separately by 45 and 99

4 Divide —

- (1) $Rs \ 246 \ 13a \ 8\frac{1}{2}p$ separately by 12, 14, 26 and 58
 (2) $£997 \ 18s \ 10\frac{1}{2}d$ separately by 26, 53, 84 and 145
 (3) 789 lbs 12 oz $14\frac{3}{4}drs$ separately by 7, 15 and 67
 (4) 1994 mds $20\frac{3}{4}sr$ separately by 729 and 1521
 (5) $Rs \ 7 \ 8a \ 11\frac{5}{8}p$ separately by $Rs \ 3 \ 2a \ 7\frac{5}{8}p$, and $15a \ 9\frac{3}{8}b$
 (6) $£282 \ 18s \ 7\frac{1}{2}d$ separately by $£6 \ 18s \ 0\frac{5}{8}d$, and $£27 \ 15s \ 9\frac{3}{4}d$

XII REDUCTION OF FRACTIONS

290 Our attention has hitherto been confined to fractions considered *generally*, without regard to the particular value of their *units*, and it remains to apply what has been said to such *concrete* quantities as constitute the principal subjects of practical computation.

291 We shall notice here, that while *times* denotes the multiplication of a quantity by an integer, *of* denotes its multiplication by a fraction, and either *times* or *of* its multiplication by a mixed number.

Thus, each of the expressions *5 times Rs 7*, $\frac{1}{5}$ *of Rs 7*, and either $3\frac{1}{5}$ *times Rs 7* or $3\frac{1}{5}$ *of Rs 7* denotes the multiplication of *Rs 7* by 5, by $\frac{1}{5}$ and by $3\frac{1}{5}$ respectively. Also the notation for *5 times Rs 7* is either $5 \times Rs \ 7$ or $Rs \ (5 \times 7)$.

292 Reduction of Fractions can conveniently be divided into the two following cases —

(1) To reduce a fraction of one denomination to a lower denomination, and conversely

(2) To reduce a quantity of one denomination to a fraction of a higher denomination

293 Case I To reduce a fraction of one denomination to a lower denomination (Descending Reduction)

RULE Multiply the fraction of the given denomination by the number which connects the lower denomination with one (or unit) of the given denomination

Ex Reduce $\pounds \frac{2}{7}$ to pence, and $\frac{8}{27}$ of a day to seconds

$$(1) \pounds \frac{2}{7} = \frac{2}{7} \times (20 \times 12)d = \frac{2 \times 20 \times 12}{7}d = \frac{480}{7}d = 68\frac{4}{7}d \quad \text{Ans}$$

$$(2) \frac{8}{27} \text{ of a day} = \frac{8}{27} \times (24 \times 60 \times 60) \text{ sec} = \frac{8 \times 24 \times 60 \times 60}{27} \text{ sec} \\ = 25600 \text{ sec} \quad \text{Ans}$$

294. Case II To reduce a quantity of one denomination to a fraction of a higher denomination (Ascending Reduction)

RULE Divide the number of the given denomination by the number which connects that denomination with one (or unit) of the higher denomination

Ex Reduce $5\frac{1}{4}d$ to the fraction of a pound, and $18\frac{3}{4}$ grs to the fraction of an oz Troy

$$(1) 5\frac{1}{4}d = \pounds \frac{5\frac{1}{4}}{12 \times 20} = \pounds \frac{21}{4} \times \frac{1}{12 \times 20} = \pounds \frac{7}{320} \quad \text{Ans}$$

$$(2) 18\frac{3}{4} \text{ grs} = \frac{18\frac{3}{4}}{24 \times 20} \text{ oz} = \frac{75}{4} \times \frac{1}{24 \times 20} \text{ oz} = \frac{5}{128} \text{ oz} \quad \text{Ans}$$

295 Sometimes we employ both the descending and the ascending process in reducing a fraction of one denomination to a fraction of another denomination

Ex Reduce $\frac{2}{5}$ of a guinea to the fraction of $\pounds 1$

$$\frac{2}{5} \text{ of a guinea} = \frac{3 \times 21}{5} s = \frac{63}{5} s = \pounds \frac{63}{5 \times 20} = \pounds \frac{63}{100}$$

Examples LXXX

1 Reduce $\frac{1}{8}$, $\frac{2}{9}$, $\frac{5}{16}$, $\frac{4}{15}$ and $\frac{1}{5}$ of a rupee to annas, and $\frac{3}{8}$ of Re 1 to gandas

2 Reduce $\frac{1}{8}$, $\frac{1}{16}$, $\frac{3}{16}$, and $\frac{5}{8}\frac{1}{2}$ of a pound to pence

3 Express $\frac{2}{5}$ of a shilling, $\frac{1}{4}$ of a penny, and $\frac{1}{11}\frac{6}{10}$ of a farthing as fractions of a pound

4 Reduce $\frac{2}{3}$ of a guinea, $\frac{1}{4}$ of a half-guinea, and $6\frac{6}{7}$ of a crown to fractions of $\pounds 1$

5 Reduce $3\frac{3}{5}$ of a cwt to the fraction of 1 lb, $\frac{4}{7}$ of an ounce to that of 1 cwt, and $\frac{1}{2}\frac{9}{10}$ of an ounce (Avoir) to that of 1 grain

6 Express $\frac{1}{32}$ of a yard as the fraction of an *inch*, and $\frac{1}{16}$ of an inch as that of a *foot*.

7 Find the fraction of a *year* which expresses $\frac{1}{4}$ of an *ell* of 5 quarters, and that of a *day* which is equal to $\frac{1}{12}$ of a year of 365 days.

8 Reduce $\frac{1}{56}$ of a maund to the fraction of a *seer*, $\frac{1}{8}$ lb to the fraction of 1 lb Troy, and $\frac{1}{6}$ of a maund to *chhatals*.

9 Reduce $\frac{1}{24}$ of a barrel of beer to the fraction of a *quart*; and $\frac{1}{4}$ of a pint of wine to the fraction of a *hogshead*.

10 Reduce $\frac{1}{1600}$ of a mile to *poles*, $\frac{1}{64}$ of an acre to *sq yards*.

11 Express $\frac{1}{2}$ of a guinea, $\frac{1}{4}$ of a shilling and $\frac{1}{8}$ of a farthing as fractions of £10.

12 Reduce 492 $\frac{1}{2}$ hours to the fraction of a *year* of 365 $\frac{1}{2}$ days.

13 Express $\frac{1}{4}$ of 2151 $\frac{1}{2}$ sq yards in *acres*, 31 $\frac{1}{2}$ miles in *yards*, and $\frac{1}{2}$ cubit as the fraction of an *amut*.

14 What fraction expresses $\frac{51}{71}$ of 5940 seconds in *weeks*?

295 The preceding two cases in Art. 292 enable us

(1) To find the value of a given fraction of any **concrete** quantity in terms of its own or lower denominations, and

(2) To reduce a **compound** quantity to a fraction of a higher denomination.

297 **Case I** To find the value of a given fraction of any **concrete** quantity in terms of its own or lower denominations.

(1) When the quantity is *simple* or can be easily reduced to a *simple* quantity.

RULE. Multiply the given quantity by the numerator of the fraction, and divide the product (if possible) by the denominator, the quotient (if any) is the required number of parts of that denomination. If there be a remainder, multiply the numerator of the fraction which remains by the number of units of the next inferior denomination which are equivalent in value to the given denomination and divide the product by the denominator, the quotient will be the number of parts of that denomination. Proceed in the same way with the remainder (if any), and the parts of the next denomination will be found; repeat this process till the lowest denomination, to which the given quantity is capable of being reduced, is obtained.

Ex 1 Find the value of $\frac{1}{4}$ of £1.

$$\frac{1}{4} \text{ of } £1 = \frac{3 \times 20}{8} s = \frac{3 \times 5}{2} s = \frac{15}{2} s = 7 \frac{1}{2} s, \frac{1}{2} s = \frac{1 \times 12}{2} d = 6d$$

∴ the required value = 7s 6d

Ex 2 Find the value of $\frac{5}{8}$ of Rs 4

$$\frac{5}{8} \text{ of Rs } 4 = \text{Rs } \frac{5 \times 4}{8} = \text{Rs } 1\frac{1}{2} = \text{Rs } 3\frac{1}{2}, \text{ Re } \frac{1}{2} = \frac{1 \times 16}{3}a = 1\frac{1}{3}a = 5\frac{1}{3}a ;$$

$$\frac{1}{3}a = \frac{1 \times 12}{3}p = 4p \quad \text{the required value} = \underline{\text{Rs } 3 \quad 5a \quad 4p}$$

Ex 3 Find the value of $\frac{1}{5}$ of 13s 4d

$$\frac{1}{5} \text{ of } 13s \quad 4d = \frac{1}{5} \text{ of } 160d = \frac{3 \times 160}{5}d = (3 \times 32)d = 96d = \underline{8s} \quad \text{Ans}$$

298 When the given fraction is a mixed number, — (1) multiply separately by the integer and by the fraction and add the products so obtained or (2) reduce the mixed number to a fraction and proceed as in Art 297, Case 1

Ex Find the value of $3\frac{1}{2}$ of Re 1 4a

$$\begin{aligned} \text{The required value} &= \text{Re } 1 \quad 4a \times 3 + \text{Re } 1 \quad 4a \times \frac{1}{2} \\ &= \text{Rs } 3 \quad 12a + 20a \times \frac{1}{2} = \text{Rs } 3 \quad 12a + 10a \\ &= \text{Rs } 3 \quad 12a + 1a \quad 8p = \underline{\text{Rs } 3, \quad 13a \quad 8p} \end{aligned}$$

299 Before applying the Rule reduce compound and complex fractions to simple ones

Ex 1 Find the value of $2\frac{1}{2}$ of $\frac{3}{4}$ of 10a 9p

$$\text{Here, } 2\frac{1}{2} \text{ of } \frac{3}{4} = \frac{3}{2} \text{ of } \frac{1}{4} = 1\frac{1}{2}$$

$$\begin{aligned} \text{the required value} &= 1\frac{1}{2} \text{ of } 129p = \frac{10 \times 129}{3}p = (10 \times 43)p \\ &= 430p = \underline{\text{Rs } 2 \quad 3a \quad 10p} \end{aligned}$$

Ex 2 Find the value of $\frac{7}{8}$ of $7\frac{1}{2}$ of $\frac{8\frac{1}{2}}{4}$ of 3 maunds

$$\text{Here, } \frac{7}{8} \text{ of } 7\frac{1}{2} \text{ of } \frac{8\frac{1}{2}}{4} = \frac{7}{8} \text{ of } \frac{22}{3} \text{ of } \frac{17}{2} \times \frac{1}{4} = \frac{1309}{96}$$

$$1\frac{1309}{96} \text{ of } 3 \text{ mds} = 1\frac{1309}{32} \text{ mds} = 40\frac{29}{32} \text{ mds},$$

$$\frac{29}{32} \text{ mds} = \frac{29}{32} \times 40 \text{ sr} = \frac{29 \times 40}{32} \text{ sr} = \frac{145}{4} \text{ sr} = 36\frac{1}{4} \text{ sr},$$

$$\frac{1}{4} \text{ sr} = \frac{1}{4} \times 16 \text{ ch} = 4 \text{ ch}$$

$$\therefore \text{the required value} = \underline{40 \text{ mds } 36 \text{ sr } 4 \text{ ch}}$$

300 The preceding Articles enable us to find the value of the sum or difference of fractional parts of magnitudes of the same kind.

Ex 1 Find the value of $\frac{3}{4}$ of £1 + $\frac{4}{5}$ of a guinea — $\frac{1}{7}$ of 3s 6d

$$\left. \begin{aligned} \frac{3}{4} \text{ of } £1 &= \frac{3}{4} \times 20s = 15s = 13s \quad 4d \\ \frac{4}{5} \text{ of } 1 \text{ gu} &= \frac{4}{5} \times 21s = 16s = 9s \quad 4d \end{aligned} \right\} = £1 \quad 2s \quad 8d$$

$$\frac{1}{7} \text{ of } 3s \quad 6d = \frac{1}{7} \text{ of } 42d = (3 \times 6d) = 18d = 1s \quad 6d$$

$$\therefore \text{the required value} = \underline{£1 \quad 1s \quad 2d}$$

301. The following table, if carefully committed to memory, will greatly help a student in his calculations,

$Re \frac{1}{16} = 1a$	$Re \frac{1}{8} = 5a$	$4p$	$\mathcal{L} \frac{1}{20} = 1s$	$\mathcal{L} \frac{1}{4} = 5s$
$Re \frac{1}{12} = 1a$	$4p$	$Re \frac{1}{4} = 8a$	$\mathcal{L} \frac{1}{12} = 1s$	$8d$
$Re \frac{1}{6} = 2a$	$Re \frac{1}{3} = 10a$	$8p$	$\mathcal{L} \frac{1}{10} = 2s$	$\mathcal{L} \frac{1}{2} = 10s$
$Re \frac{1}{5} = 2a$	$8p$	$Re \frac{1}{2} = 12a$	$\mathcal{L} \frac{1}{5} = 2s$	$6a$
$Re \frac{1}{4} = 4a$			$\mathcal{L} \frac{1}{2} = 4s$	$\mathcal{L} \frac{1}{1} = 15s$

Examples LXXXI

1 Find the respective values of —

- (1) $\frac{1}{16}$ of Re 1, $\frac{1}{12}$ of Re 1, $\frac{1}{10}$ of Rs 30, $\frac{1}{12}$ of Rs 9, $\frac{1}{10}$ of Rs 8
- (2) $7\frac{1}{2}$ of Rs 50, — of Rs 2, 8- of $\frac{1}{4}$ of 10a 9p, $\frac{1}{12}$ of $\frac{1}{2}$ of 5a
- (3) — of \mathcal{L} 1, $\frac{1}{7}$ of 1s, $\frac{1}{12}$ of a guinea, $\frac{1}{6}$ of \mathcal{L} 1, $\frac{1}{12}$ of \mathcal{L} 1, $\frac{1}{12}$ of \mathcal{L} 1
- (4) $\frac{1}{10}$ of \mathcal{L} 5, $\frac{1}{2}$ of 6s 8d, $3\frac{1}{4}$ of 2s 6d, $2\frac{1}{2}$ of a guinea, $\mathcal{L} 3\frac{1}{4}$
- (5) — of $\frac{1}{7}$ of $\frac{1}{10}$ of 5 guineas, $\frac{1}{12}$ of a moidore, $\frac{1}{5}$ of 13s 4d, $\frac{1}{12}$ of \mathcal{L} 9
- (6) $\frac{1}{2}$ of 1 cwt, $\frac{1}{12}$ of 1 qr, $\frac{1}{7}$ of 1 lb, $\frac{1}{12}$ of a cwt, $2\frac{1}{2}$ of 8 cwt
- (7) $\frac{1}{2}$ of a ton, $2\frac{1}{2}$ of 6s 8d, $\frac{1}{12}$ of 5s 3d, $\frac{1}{12}$ of a mile
- (8) $\frac{1}{2}$ of a lb Troy, $\frac{1}{4}$ of a lb Avoir, $\frac{1}{7}$ of a lb Apoth, $2\frac{1}{2}$ lbs Troy
- (9) $\frac{1}{10}$ of $2\frac{1}{2}$ vds, $\frac{1}{12}$ of $\frac{1}{10}$ of 2c fur, $\frac{1}{12}$ of an acre, $\frac{1}{12}$ of an acre
- (10) $2\frac{1}{2}$ of $\frac{1}{12}$ of a cwt, $\frac{1}{12}$ of a week, $\frac{1}{12}$ of 1 mo of 28 days
- (11) $\frac{1}{12}$ of 1 qr, $\frac{1}{7}$ of 1 bus, — of a peck, $\frac{1}{12}$ of $\frac{1}{10}$ of $2\frac{1}{2}$ of $2\frac{1}{2}$ loads
- (12) $8\frac{1}{2}$ of 17 cub vds, $\frac{1}{2}$ of $3\frac{1}{2}$ of $4\frac{1}{2}$ of 2 mds, $\frac{1}{10}$ of 175 tons
- (13) $\frac{1}{7}$ of $\frac{1}{2}$ of 10 $\frac{1}{2}$ hrs, $\frac{1}{12}$ of a day, $2\frac{1}{2}$ of a pipe of wine
- (14) $\frac{6\frac{1}{2}}{3-}$ of \mathcal{L} 1, $\frac{3\frac{1}{2}}{12\frac{1}{2}}$ of 365 days, $\frac{3\frac{1}{2}}{4-}$ of $\frac{10\frac{1}{2}}{7\frac{1}{2}}$ of $\frac{77}{540}$ of a moidore
- (15) $\frac{3\frac{1}{2}}{20}$ of a ton, $\frac{6\frac{11\frac{1}{2}}{12}}{14}$ of 1 week, $\frac{2\frac{7\frac{1}{2}}{20}}{25}$ of \mathcal{L} 50

2 Find the respective values of —

- (1) $\frac{1}{2}$ of 5 guineas + $\frac{1}{2}$ of $\frac{1}{12}$ of \mathcal{L} 1, $\frac{1}{2}$ of 5 guineas - $\frac{1}{2}$ of $\frac{1}{12}$ of \mathcal{L} 1
- (2) $\frac{1}{2}$ of a guinea + $\frac{1}{12}$ of \mathcal{L} 1 + $\frac{1}{10}$ of a crown + $\frac{1}{10}$ of 1s
- (3) $\frac{1}{2}$ of a guinea + $\frac{1}{12}$ of a crown + $\frac{1}{10}$ of 7s 6d - $\frac{1}{2}$ of 2d
- (4) $\frac{1}{2}$ of a ton + $\frac{1}{4}$ of a cwt + $\frac{1}{12}$ of a lb, $\frac{1}{2}$ cwt + $8\frac{1}{2}$ lbs + $3\frac{1}{10}$ or
- (5) $\frac{1}{2}$ of a week + $\frac{1}{4}$ of a day + $\frac{1}{7}$ of an hour + $\frac{1}{10}$ of a minute
- (6) $3\frac{1}{2}$ miles - $7\frac{1}{2}$ fur + $35\frac{1}{2}$ ppo, $\frac{1}{2}$ of 28 mds + $\frac{1}{5}$ of $1\frac{1}{2}$ mds + $\frac{1}{10}$ of 8ch
- (7) $\frac{1}{2}$ of Rs 10 $\frac{1}{2}$ + $\frac{1}{12}$ of $\frac{1}{2}$ of Rs 10 - $\frac{1}{12}$ of $\frac{1}{2}$ of Rs 2 $\frac{1}{2}$ + $\frac{1}{12}$ of $\frac{1}{2}$ of 8a.
- (8) $\frac{15\frac{1}{2}}{7\frac{1}{2}}$ of \mathcal{L} 1 + $\frac{1}{12}$ of \mathcal{L} 140 $\frac{1}{12}$ + $1\frac{13}{126}$ guineas

(2) When the quantity is a *compound* one

RULE Multiply the quantity by the numerator and divide the product by the denominator of the fraction

Ex Find the value of $\frac{5}{8}$ of Rs 3 9a 4p

$$\begin{aligned}\text{The required value} &= (\text{Rs } 3 \text{ } 9a \text{ } 4p \times 5) \div 8 \\ &= \text{Rs } 17 \text{ } 14a \text{ } 8p \div 8 = \underline{\text{Rs } 2 \text{ } 3a \text{ } 10p}\end{aligned}$$

302 To multiply a compound quantity by a mixed number, multiply separately by the integer and by the fraction and add the two products thus obtained

Ex Multiply £13 15s 4d by $4\frac{5}{8}$

$\begin{array}{r} \text{£} \quad \text{s} \quad \text{d} \\ 13 \quad 15 \quad 4 \\ 8 \overline{) 68 \quad 16 \quad 8} \\ \underline{ 8 \quad 12 \quad 1} \end{array}$	$\begin{array}{r} \text{£} \quad \text{s} \quad \text{d} \\ 13 \quad 15 \quad 4 \\ \underline{ 55 \quad 1 \quad 4} \end{array}$	<p>∴ the required value</p> $= \text{£} 55 \text{ } 1s \text{ } 4d + \text{£} 8 \text{ } 12s \text{ } 1d$ $= \underline{\text{£} 63 \text{ } 13s \text{ } 5d}$
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303 To divide a compound quantity by a fraction, multiply by the denominator and divide the product by the numerator

Ex Divide Rs 600 13a 4p by $\frac{7}{9}$

$$\begin{aligned}\text{The required value} &= (\text{Rs } 600 \text{ } 13a \text{ } 4p \times 9) \div 7 \\ &= \text{Rs } 5407 \text{ } 8a \div 7 = \underline{\text{Rs } 772 \text{ } 8a}\end{aligned}$$

304 To divide a compound quantity by a mixed number, reduce the mixed number to an improper fraction and then proceed as in Art 303

Ex Divide £5 4s 6½d by $1\frac{2}{3}$

$\begin{array}{r} \text{£} \quad \text{s} \quad \text{d} \\ 5 \quad 4 \quad 6\frac{1}{2} \\ 5 \overline{) 15 \quad 13 \quad 6\frac{1}{2}} \\ \underline{ 3 \quad 2 \quad 8\frac{1}{2}} \end{array}$	<p>for $1\frac{2}{3} = \frac{5}{3}$</p> <p>for $2\frac{3}{4} - 5 = \frac{1}{4}$</p>	<p>Hence the required value</p> $= \underline{\text{£} 3 \text{ } 2s \text{ } 8\frac{1}{2}d}$
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Note Before applying the above Rules of both Multiplication and Division, the compound and complex fractions must first be reduced to simple ones

Examples LXXXII

1 Multiply -

- (1) £3 16s 8½d by $\frac{3}{4}$, £6 18s 7½d by $\frac{9}{10}$, £10 11s 2½d by $3\frac{1}{2}$
- (2) Rs 50 5a 6p separately by $9\frac{2}{3}$, $18\frac{1}{10}$, $53\frac{1}{3}$ and $156\frac{2}{3}$
- (3) £12 5s 7½d by $6\frac{2}{3}$, £13 5s 7½d by $7\frac{1}{2}$, £34 12s 5½d by $11\frac{1}{3}$
- (4) 5 tons 3 cwt 6 lbs separately by $4\frac{2}{3}$, $20\frac{1}{4}$, $46\frac{2}{3}$ and $213\frac{1}{4}$
- (5) 19 hrs 43m 56½s by $12\frac{7}{10}$, 10ac 3ro 37po $15\frac{8}{9}$ yds by $10\frac{2}{3}$

2 Divide —

- (1) Rs 307 4a 4p by $\frac{1}{3}$, Rs 76 10a 8p by $\frac{1}{2}$, £5 4s 6d by $\frac{3}{4}$
 (2) £25 8s 4d by $\frac{1}{2}$, £4 7s 3d by $\frac{1}{12}$, £34 16s 9d by $\frac{1}{6}$
 (3) Rs 173 5a 4p separately by $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$ and $\frac{1}{10}$
 (4) 13 cwt 3 qrs 26 lbs 15 oz by $\frac{1}{2}$, 15 ac 3 ro 25 po by $\frac{1}{4}$
 (5) 8 days 15 hrs 48 m 57 sec by $\frac{1}{4}$; 12 cub yds 20 c ft 100 in by $\frac{1}{2}$
 (6) 1 mi 5 fur 91 yds 2 ft by $\frac{1}{2}$ of $\frac{1}{4}$, 7 mds 35 sr by $\frac{1}{2}$

3 Find the respective values of —

- (1) $\frac{1}{10}$ of Rs 10 8a, $\frac{1}{2}$ of Rs 2 6a, $\frac{1}{5}$ of Rs 31 8a, $\frac{1}{7}$ of Rs 1 12a
 (2) $\frac{1}{10}$ of Rs 7 5a 4p, $\frac{1}{2}$ of Rs 51 4a, $\frac{1}{3}$ of $\frac{1}{2}$ of Rs 173 12a
 (3) $\frac{1}{10}$ of 6s 11d, $\frac{1}{8}$ of $\frac{1}{2}$ of 2s 9d, $\frac{1}{3}$ of $\frac{1}{4}$ of 16s 6d
 (4) $\frac{1}{12}$ of £4 14s 6d, $\frac{1}{12}$ of £8 8s 5d, $\frac{1}{12}$ of Rs 15 12a
 (5) $\frac{1}{12}$ of 3 mds 10 sr 8 ch, $\frac{1}{12}$ of 3 cwt 3 qrs 20 lbs
 (6) $\frac{1}{12}$ of 10 ft 6 in, $\frac{1}{8}$ of $\frac{1}{4}$ of $\frac{1}{2}$ sq yds, $\frac{1}{7}$ of $\frac{1}{4}$ of $\frac{1}{2}$ cub ft
 (7) $\frac{1}{12}$ of $\frac{8}{14}$ of 5 mi 3 fur 37 po 4 yds, $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{1}{2}$ of £6304
 (8) $\frac{1}{3}$ of $(3\frac{1}{2} - \frac{1}{2})$ of 5 cwt 2 qrs 10 lbs 7 oz, $\frac{1}{10}$ of $\frac{7}{9}$ of 1 mile
 (9) $\frac{1}{3}$ of $(3\frac{1}{2} + 1\frac{1}{2})$ of 5 days 17 hrs, $\frac{1}{3}$ of $\frac{1}{2} - \frac{1}{10}$ of Rs 10 8a
 (10) $\frac{7\frac{1}{2} - 3\frac{1}{2}}{18\frac{1}{2} - \frac{1}{2}}$ of 3 ac 1 ro 35 po, $\frac{\frac{1}{2} + \frac{1}{2}(\frac{1}{2} - \frac{1}{2}) - \frac{1}{2}(\frac{1}{2} + \frac{1}{2})}{\frac{1}{2}(\frac{1}{2} - \frac{1}{2}) - \frac{1}{2}(\frac{1}{2} - \frac{1}{2})}$ of £44 17s

4 Find the values of —

- (1) $\frac{1}{2}$ of Rs 3 5a 4p + $\frac{1}{4}$ of Rs 21 14a + $\frac{1}{10}$ of Rs 47 3a 4p
 (2) $\frac{1}{2}$ of $\frac{1}{2}$ of Rs 13 8a + $\frac{1}{10}$ of Rs 6 10s 8p - $\frac{1}{2}$ of $\frac{1}{4}$ of Rs 3 5a 4p.
 (3) $\frac{1}{2}$ of 6s 8d + $\frac{1}{4}$ of £2 3s 9d + $\frac{1}{10}$ of £4 14s 5d
 (4) $\frac{1}{2}$ of £15 + $\frac{1}{2}$ of $\frac{1}{2}$ of $\frac{1}{2}$ of £1 2s + $\frac{1}{2}$ of 3d
 (5) $\frac{15\frac{1}{2}}{7\frac{1}{2}}$ of £1 + $\frac{1}{2}$ of £140 10s 6d + $2\frac{1}{2}$ of half a guinea
 (6) $\frac{1}{2}$ of £5 10s 6d - $\frac{1}{2}$ of 2 guineas + $\frac{2}{3}$ of $\frac{4}{5}$ of £6
 (7) $\frac{11\frac{1}{2}}{1\frac{1}{2}}$ of Rs 84 3a 6p - $\frac{31\frac{1}{2}}{4\frac{1}{2}}$ of $\frac{10\frac{1}{2}}{7\frac{1}{2}}$ of Rs 20 4a
 (8) $\frac{1}{2}$ of 3 mds 34 sr + $\frac{1}{2}$ of 8 mds 9 sr + $\frac{1}{2}$ of 3 sr 12 ch
 (9) $\frac{1}{2}$ of a year of 365 days + $3\frac{1}{10}$ of $\frac{1}{2}$ of a week + $\frac{1}{2}$ of 5 hrs
 (10) 5 yds 2 ft 5 in \times 77 - 9 yds 2 ft 7 in - 15 yds 1 ft 9 in
 - 3 lbs + 3 po 3 yds 2 ft 3 in - 2 lbs

305 Case II *To reduce a compound quantity to a fraction of a higher denomination*

Proceed as in the following Examples

Ex 1 Reduce $8p$, $6a$ $10p$ and $14a$ $4p$ to the fraction of a rupee

$$8p = \frac{8}{12}a = \frac{2}{3} \times Re \frac{1}{12} = Re \frac{1}{9}$$

$$6a \quad 10p = 6a + \frac{1}{2}a = 6\frac{1}{2}a = \frac{13}{2} \times Re \frac{1}{12} = Re \frac{13}{24}$$

$$14a \quad 4p = 14a + \frac{1}{3}a = 14\frac{1}{3}a = \frac{43}{3} \times Re \frac{1}{12} = Re \frac{43}{36}$$

Ex 2 Express $9d$, $2s$ $4d$ and $18s$ $11\frac{1}{2}d$ in pounds

$$9d = \frac{9}{12}s = \frac{3}{4}s = \frac{3}{4} \times \pounds \frac{1}{20} = \pounds \frac{3}{80}$$

$$2s \quad 4d = 2s + \frac{1}{3}s = 2\frac{1}{3}s = \frac{7}{3} \times \pounds \frac{1}{20} = \pounds \frac{7}{60}$$

$$18s \quad 11\frac{1}{2}d = 18s + \frac{1}{4}s = 18\frac{1}{4}s = \frac{73}{4} \times \pounds \frac{1}{20} = \pounds \frac{73}{80}$$

Ex 3 Reduce $\pounds 4$ $9s$ $2\frac{1}{2}d$ to pounds

$$9s \quad 2\frac{1}{2}d = 9s + \frac{1}{8}s = 9\frac{1}{8}s = \frac{73}{8} \times \pounds \frac{1}{20} = \pounds \frac{73}{160}$$

$$\pounds 4 \quad 9s \quad 2\frac{1}{2}d = \pounds 4\frac{73}{160} \quad \text{Ans}$$

Ex 4 Reduce 5 cwt 3 qrs 24 lbs to the fraction of a ton

$$3 \text{ qrs} \quad 24 \text{ lbs} = 3 \text{ qrs} + \frac{1}{8} \text{ qrs} = 3\frac{1}{8} \text{ qrs} = \frac{1}{4} \times 3\frac{1}{8} \text{ cwt}$$

$$= \frac{1}{4} \times \frac{27}{8} \text{ cwt} = \frac{27}{32} \text{ cwt}$$

$$5 \text{ cwt} + \frac{27}{32} \text{ cwt} = 5\frac{27}{32} \text{ cwt} = \frac{1}{20} \times 5\frac{27}{32} \text{ ton}$$

$$\text{Hence } 5 \text{ cwt } 3 \text{ qrs } 24 \text{ lbs} = \frac{1}{20} \times \frac{171}{8} \text{ ton} = \frac{171}{160} \text{ ton} \quad \text{Ans}$$

Examples LXXXIII

1 Reduce $3a$ $6p$, $5a$, $5a$ $10p$, $6a$ $10p$, $7a$ $8p$, $13a$ $7\frac{1}{2}p$; $15a$ $7\frac{3}{8}p$, each to the fraction of a rupee

2 Express $4s$ $11d$, $17s$ $11\frac{1}{2}d$, $19s$ $10\frac{1}{2}d$, $6s$ $11\frac{1}{2}d$, $14s$ $4\frac{1}{2}d$ $16s$ $9\frac{1}{2}d$, each as the fraction of a pound

3 Express $\pounds 1$ $13s$ $11\frac{1}{2}d$ $\frac{10}{11}q$, $\pounds 3$ $19s$ $8\frac{1}{2}d$, $\pounds 37$ $16s$ $6\frac{1}{2}d$, $\pounds 5$ $16s$ $11\frac{1}{2}d$ $\frac{7}{8}q$, each in pounds

4 Reduce Rs 3 $10a$ $8p$, Rs 8 $5a$ $4p$, Rs 15 $10a$ $7p$, Rs 81 $7a$ $3\frac{1}{2}p$ to rupees

5 Reduce 2 cwt 1 qr 16 lbs to the fraction of a ton, 3 qrs 27 lbs 9 oz $12\frac{1}{2}$ drs to the fraction of a cwt, 2 sr 15 ch 2 kan to the fraction of a maund

6 What fraction is 2 ft 9 in of a pole, 23 po 4 yds of a mile; and 3 ro 26 po of an acre?

7 Reduce 3 fur 29 po 4 yds 1 ft 9 in to the fraction of a mile, and 1 sq ft $\frac{5}{8}$ sq in to the fraction of a sq vd

8 Reduce 4 mds 37 sr 8 ch to maunds, 2 bi 15 kat 5 ch to bighas, and 2 qts $1\frac{1}{2}$ pt to the fraction of a barrel

9 Express 5 bus 3 pks 1 gal as the fraction of a quarter

10 Express 2 wks 5 days 18 hrs as the fraction of a year of 365 days, and 3 ro 27½ po as the fraction of an acre

11 Reduce 72 days 6 hrs 56 m 15 sec to the fraction of a year of 365¼ days, and 1 sc 13 grs to the fraction of a lb

12 Express 1 day 1 hr 4 dan 30 pals and 3 mo 12 days as fractions of a year

306 To find what fraction one concrete quantity is of any other, of the same kind

RULE Reduce both the quantities to the same denomination, then the fraction whose numerator is the first and denominator the second of these results, will be the one required

Ex 1 Reduce Rs 2 8a 2p to the fraction of Rs 3 12a

Rs 2 8a 2p = 482p, and Rs 3 12a = 720p

∴ the required fraction = $\frac{482p}{720p} = \frac{241}{360}$ Ans

Ex 2 What part of 4½ of £1 is 3¼ of a guinea?

3¼ of a gui = $\frac{1}{4} \times 21s = 5\frac{1}{4}s$, and 4½ of £1 = $\frac{9}{2}$ of 20s = $17\frac{1}{2}s$

∴ the required fraction = $\frac{5\frac{1}{4}s}{17\frac{1}{2}s} = \frac{9}{16}$ Ans

Ex 3 What fraction is 1 md 4 sr of 2 mds 32 sr?

1 md 4 sr = 44 sr, and 2 mds 32 sr = 112 sr

∴ the required fraction = $\frac{44 \text{ sr}}{112 \text{ sr}} = \frac{11}{28}$ Ans

307 By means of the preceding Articles, magnitudes of the same kind, consisting of fractions of simple or compound quantities, and connected by the operations of Addition or Subtraction, may be reduced to simple fractions of a given denomination

Ex 1 Express $\frac{3}{5}$ of a £ guinea - $\frac{1}{4}$ of a shilling - $\frac{1}{8}$ of 7s 6d as the fraction of £2 19s 6d

Here, $\frac{3}{5}$ of a guinea = $\frac{3}{5} \times £2\frac{1}{2} = £1\frac{3}{5}$, $\frac{1}{4}s = £\frac{1}{4} \times \frac{1}{20} = £\frac{1}{80}$,

and $\frac{1}{8}$ of 7s 6d = $\frac{1}{8}$ of $7\frac{1}{2}s = £\frac{1}{8} \times \frac{15}{2} \times \frac{1}{20} = £\frac{1}{16}$

∴ the exp = $£(1\frac{3}{5} - \frac{1}{80} - \frac{1}{16}) = £1\frac{11}{16}$, also £2 19s 6d = $£2\frac{39}{8} = £4\frac{19}{8}$

∴ the required fraction = $£1\frac{11}{16} \div £4\frac{19}{8} = \frac{11}{16} \times \frac{8}{19} = \frac{11}{38}$ Ans

Ex 2 Reduce $\frac{7}{10}$ of Rs 10 - $\frac{2}{5}$ of Rs 10 8a to the fraction of Rs 5 4a

$\frac{7}{10}$ of Rs 10 = Rs 7, $\frac{2}{5}$ of Rs 10 8a = $\frac{2}{5}$ of Rs 10½ = Rs 4½

∴ the difference = Rs (7 - 4½) = Rs 2½, also Rs 5 4a = Rs 5½

the required fraction = $Rs 2\frac{1}{2} \div Rs 5\frac{1}{2} = \frac{5}{11}$ Ans

Ex 3 What fraction of Rs 10 8a together with Rs 2 4a is equivalent to Rs 7 8a ?

Here, Rs 7 8a - Rs 2 4a = Rs 5 4a

Now, the question reduces itself to finding—What fraction of Rs 10 8a is Rs 5 4a ?

Rs 5 4a = 84a, and Rs 10 8a = 168a

∴ the fraction required = $\frac{84a}{168a} = \frac{1}{2}$ Ans

Ex 4 What fraction of £2 10s is the sum which being diminished by 10s 6d is equal to £2 2s ?

The meaning is—What fraction of £2 10s is £2 2s + 10s 6d ?

£2 2s + 10s 6d = £2 12s 6d = £2 $\frac{5}{8}$ = £2 $\frac{5}{8}$,

also £2 10s = £2 $\frac{5}{2}$ = £2 $\frac{5}{2}$

∴ the required fraction = £2 $\frac{5}{8}$ ÷ £2 $\frac{5}{2}$ = $\frac{5}{8} \times \frac{2}{5} = \frac{2}{8} = \frac{1}{4}$ Ans

Ex 5 Compare the values of $\frac{7}{8}$ of £1, $\frac{5}{8}$ of a guinea, and $\frac{2}{3}$ of 15s 7 $\frac{1}{2}$ d

$\frac{7}{8}$ of £1 = £ $\frac{7}{8}$, $\frac{5}{8}$ of a guinea = $\frac{5}{8} \times £1 \frac{1}{4}$ = £ $\frac{5}{4}$, $\frac{2}{3}$ of 15s 7 $\frac{1}{2}$ d = £ $\frac{25}{12}$

Now, to compare £ $\frac{7}{8}$, £ $\frac{5}{4}$, £ $\frac{25}{12}$, reduce them to equivalent fractions with the same denominator and proceed as in Art 257

The L C M of the denominators 8, 12, 48 is 48,

∴ £ $\frac{7}{8}$ = £ $\frac{7 \times 6}{8 \times 6}$ = £ $\frac{42}{48}$, £ $\frac{5}{4}$ = £ $\frac{5 \times 12}{4 \times 12}$ = £ $\frac{15}{12}$, £ $\frac{25}{12}$ = £ $\frac{25 \times 4}{12 \times 4}$ = £ $\frac{100}{48}$

∴ $\frac{7}{8}$ of £1 is the greatest and $\frac{2}{3}$ of 15s 7 $\frac{1}{2}$ d is the least Ans

Ex 6 What sum is that $\frac{2}{3}$ of which is Rs 2 10a 8p ?

Rs 2 10a 8p = Rs 2 $\frac{2}{3}$ = Rs $\frac{2}{3}$

the required sum = Rs $\frac{2}{3} \div \frac{2}{3}$ = Rs $\frac{2}{3} \times \frac{3}{2}$ = Rs 6 10a 8p Ans

Examples LXXXIV

1 Express —

- (1) Rs 4 15a 4p as the fraction of Rs 6 11a 4p, and Rs 7 7a 2p as the fraction of Rs 31 4a 6p
- (2) Rs 25 0a 6p as the fraction of Rs 29 8a 6p
- (3) £1 7s 8 $\frac{1}{2}$ d as the fraction of £2 7s 6d, and £3 4s 0 $\frac{1}{2}$ d as the fraction of £7 7s 11d
- (4) 13s 10 $\frac{1}{2}$ d $\frac{2}{3}$ g as the fraction of £2 9s 7d
- (5) $\frac{1}{11}$ of £5 17s 4d and $\frac{2}{3}$ of $\frac{6\frac{2}{3}}{9\frac{1}{10}}$ of £1 12s 1 $\frac{1}{2}$ d as fractions of £10
- (6) 2 fur 29 po 2 ft 10 in as the fraction of 1 mi 5 fur 16 $\frac{2}{3}$ po
- (7) £22 13s 8 $\frac{1}{4}$ d as the fraction of 3 $\frac{1}{2}$ guineas

(8) $\frac{7\frac{1}{2}-3\frac{1}{2}}{18\frac{1}{2}-\frac{1}{2}}$ of £33 14s 5½d as the fraction of £157 17s 8½d

2 Reduce —

- (1) $\frac{2}{3}$ of 2s 4½d to the fraction of a half-crown, and 9s 10½d to the fraction of 13s 2½d
 - (2) 6½d of Rs 15 3a 10p to the fraction of Rs 31 8a 2p
 - (3) 33½ of 1 md 2 sr to the fraction of 3½ of 28 mds, and 32 seers to the fraction of 3 mds 22 sr 2 ch
 - (4) 2½ of 2 bi 7 kat 4 ch to the fraction of 4 bi 11 kat
 - (5) 1 md 11 si 8 ch to the fraction of 28 mds, and 12½ of 15 sr 12 ch to the fraction of 30 mds 32 sr
 - (6) 3 qrs 1 pt 2¼ gills to the fraction of 5 gals 2 qts 1 pt
 - (7) 2 sq yds 2 ft 120 in to the fraction of 3 sq po 13½ yds 1 ft 72 in
 - (8) 12 oz 12½ drs Avoir to the fraction of 1 lb Troy, and 35 lbs 8½ oz Troy to the fraction of a cwt
 - (9) 2½ half guinea is to the fraction of 10s 11½d
 - (10) 7½ of 10 oz 18 dwts 11 grs to the fraction of 8 lbs 8½ oz Avoir
- 3 What part of $\frac{3}{4}$ of $\frac{5}{7}$ of 3 guineas is $\frac{1}{2}$ of $\frac{8}{9}$ of 15r 9d ?
- 4 What part of 13 cwt 2qrs 21lbs is 11cwt 1qr 14 lbs 15 oz ?
- 5 What part 6 ft 3½ in of 13 ft 8½ in ?
- 6 What part of 1 maund is 10 sr 13 ch 2 kan ?
- 7 What fraction of $\frac{1}{2}$ of Rs 2 5a 8p is $\frac{3}{4}$ of $\frac{3}{4}$ of Rs 12 9a 3p, and of 7 guineas is $\frac{1}{3}$ of 1 moidore ?
- 8 What fraction of 3 cwt 2 qrs 14 lbs is 3 cwt 19 lbs 2 oz ?
- 9 What fraction of a year of 365½ days is 27 days 16 hrs 29 min 4 sec and of 1 oz Avoir is 1 oz Troy ?
- 10 What fraction of 19½ of 4 cub yds 18 ft 1127 in is $\frac{7}{15}$ of 200 cub vds, and of 2½ miles is 3½ furlongs ?
- 11 What fraction of $\frac{7\frac{1}{2}}{4s}$ of Rs 306 9a 10p is $(8\frac{1}{2}-3\frac{1}{2})$ of Rs 54 15a 8p, and of 2½ tons is $\frac{5}{8}$ of 2 lbs ?
- 12 What fraction of 8 lbs 12½ oz is 3 lbs 9 oz 62½ grs ?
- 13 How many times is —
- (1) Rs 9 12a 4½p contained in Rs 7 9a 7½p ?
 - (2) £24 16s 4½d contained in £335 1s 0½d ?
 - (3) 2 tons 2 cwt 2 qrs contained in 3 cwt 14 lbs ?
 - (4) 7 kathis 9 ch contained in a biga ?
- 14 Express $\frac{3\frac{5}{8}}{7\frac{1}{2}}$ of $\frac{4}{7\frac{1}{2}}$ of Rs 33 11a 6p, and $\frac{5}{2}$ of $\frac{7}{3\frac{1}{2}}$ of Rs 19 1a 7p in terms of Rs 70 5a 10p as unit

15 What is the measure of $7\frac{3}{8}$ of $\frac{3\frac{1}{2}}{3\frac{1}{4}}$ of 5 cwt 3 qrs $3\frac{1}{2}$ lbs, when the unit is $(5\frac{2}{3} - 3\frac{1}{4})$ of 3 tons 16 cwt 3 qrs $22\frac{2}{3}$ lbs ?

16 Express —

- (1) $\frac{3}{5}$ of $\frac{1}{10}$ of 13s 4d + $\frac{1}{2}$ of $\frac{7}{8}$ of 10s 6d as the fraction of £1
- (2) $\frac{1}{10}$ of a guinea + $\frac{1}{8}$ of £1 + $\frac{1}{4}$ of 1s + $\frac{1}{2}$ of 1d as the fraction of a guinea, and of £24 3s
- (3) $\frac{3}{8}$ of Rs 2 8a + $\frac{1}{4}$ of 8a as the fraction of Rs 10 8a
- (4) $\frac{2}{5}$ of Rs 3 8a + $\frac{3}{4}$ of Rs 5 4a - $\frac{1}{10}$ of Rs 10 8a as the fraction of Rs 13 8a, and of Rs 39 8a
- (5) $\frac{1}{11}$ of £13 10s 10½d - $\frac{5}{7}$ of £1 2s 9d as the fraction of £6
- (6) Rs 7½ - $\frac{3}{4}$ of Rs 7 as the fraction of Rs 103 5a 4p
- (7) $\frac{3\frac{1}{2}}{1\frac{1}{3}}$ of $\left\{ \frac{19}{120} \text{ of } £1 - \frac{17}{48} \text{ of } 1s \right\}$ as the fraction of 27s

17 Compare the values of —

- (i) $\frac{1}{10}$ of £1, $\frac{1}{10}$ of a guinea and $\frac{1}{8}$ of a crown
- (2) $\frac{1}{11}$ of £1, $\frac{1}{12}$ of £1 1s and $\frac{1}{4}$ of 3s 9½d
- (3) $\frac{7}{8}$ of Rs 10, $\frac{5}{8}$ of Rs 10 8a and $\frac{3}{4}$ of Rs 7 13a
- (4) $\frac{1}{2}$ of a maund, $\frac{1}{6}$ of 14 sr and $\frac{5}{8}$ of 3 sr 6 ch
- (5) $\frac{2}{5}$ of 5 days, $\frac{1}{7}$ of 20 hours, and $\frac{1}{2}$ of 59 min

18 What fraction of Rs 100 together with Rs 36 12a is equivalent to Rs 52 8a ?

19 What fraction of 3 mds 20 sr together with 1 md 9 sr will give 42 mds ?

20 What fraction of a ton added to $\frac{11}{12}$ of 2 cwt will make it equal to 1 cwt 2 qrs 11 lbs ?

21 What fraction of 2 tons 12 lbs is the weight which being diminished by 1 cwt 20 lbs is equal to 1 cwt 1 qr 8 lbs ?

22 What fraction of Rs 29 12a must be added to $\frac{3\frac{1}{2}}{4\frac{1}{2}}$ of $(3\frac{3}{4} + 1\frac{2}{3})$ of Rs 6 9a to make the sum equal to Rs 32 8a ?

23 What fraction of a mile diminished by 39 yds 1 ft 9 in is equal to 87 yds 9 in ?

24 What fraction of 2 lbs 10 oz Avóir must be added to 1 lb 8 oz Troy to give 3 lbs 7 oz 10 dwts ?

25 What sum is that $\frac{2}{3}$ of $\frac{1}{2}$ of which is $\frac{1}{4}$ of $\frac{2}{5}$ of Rs 5 10a ?

26 What length is that $\frac{2}{5}$ of which is $\frac{3}{4}$ of $7\frac{1}{2}$ of $16\frac{1}{2}$ yards ?

27 What is the sum $\frac{1\frac{3}{4}}{1\frac{3}{5}}$ of which is $(4\frac{1}{4} - 10\frac{1}{8} + 9\frac{1}{5} - 1\frac{5}{7})$ of 8p, and what fraction is it of $\frac{2}{5}$ of Rs 6 8a ?

28 What weight is the same fraction of 15 cwt 2 qrs 13 lbs that £1 11s 10½d is of £3 10s 10½d?

XIII SIMPLIFICATION OF CONCRETE FRACTIONS

308 It should be borne in mind what has already been said that when a concrete number is divided by another concrete number of the same kind, the quotient is an abstract number

Ex 1 Simplify $\frac{\frac{9}{7} \text{ of } 1\frac{3}{4} \text{ of Rs } 37 \text{ } 6s \text{ } 6p}{Rs \text{ } 65 \text{ } 6s \text{ } 6p} \times \frac{1\frac{9}{8} \text{ of } 16 \text{ cwt } 3 \text{ qrs}}{41 \text{ cwt } 3\frac{1}{2} \text{ qrs}}$

$$\begin{aligned} \text{The result} &= \frac{\frac{9}{7} \text{ of } \frac{7}{4} \text{ of Rs } 37\frac{3}{4}}{Rs \text{ } 65\frac{3}{4}} \times \frac{1\frac{9}{8} \text{ of } 67 \text{ qrs}}{167\frac{1}{2} \text{ qrs}} = \frac{\frac{9}{7} \text{ of } \frac{7}{4} \text{ of } \frac{299}{2}}{299\frac{1}{2}} \\ &= \frac{\frac{9}{8} \text{ of } 67}{1\frac{1}{2}} = \left(\frac{6 \times 7 \times 299}{7 \times 4 \times 8} \times \frac{32}{2093} \right) \times \left(\frac{25 \times 67}{16} \times \frac{2}{335} \right) \\ &= \frac{9}{7} \times \frac{8}{5} = 1\frac{5}{8} \text{ Ans} \end{aligned}$$

Ex 2 Simplify $\frac{\frac{£1 \text{ } 11s \text{ } 8d}{£2 \text{ } 17s \text{ } 0d} \text{ of } \frac{142 \text{ yds } 0\frac{4}{5} \text{ ft}}{2 \text{ yds } 1\frac{7}{10} \text{ ft}} \text{ of } 13 \text{ days } 3\text{ hrs}}{}$

$$\begin{aligned} \text{The result} &= \frac{31\frac{1}{2}s}{57s} \text{ of } \frac{426\frac{4}{5} \text{ ft}}{7\frac{7}{10} \text{ ft}} \text{ of } 13\frac{3}{8} \text{ days} = \left(\frac{95}{3} \times \frac{1}{57} \right) \text{ of} \\ &\quad \left(\frac{2134}{5} \times \frac{10}{77} \right) \text{ of } \frac{105}{8} \text{ days} = \frac{95}{3 \times 57} \times \frac{2134 \times 10}{5 \times 77} \times \frac{105}{8} \text{ days} \\ &= \frac{5}{9} \times \frac{388}{7} \times \frac{105}{8} \text{ days} = \frac{2425}{6} \text{ days} = 404\frac{1}{6} \text{ days} = \underline{404 \text{ days } 4 \text{ hrs}} \text{ Ans} \end{aligned}$$

Examples LXXXV

Simplify the following —

- 1 $1\frac{1}{2} \text{ of } \frac{4\frac{1}{2}}{5\frac{1}{4}} \text{ of } \frac{18s \text{ } 6\frac{3}{4}d}{£1} \text{ of } 3 \text{ days } 2 \text{ hrs}$ 2 $\frac{£3 \text{ } 9\frac{1}{2}}{8s} \text{ of } 2 \text{ mds } 32 \text{ sr}$
- 3 $\frac{14 \text{ lbs } 8 \text{ oz } 18 \text{ dwts}}{1 \text{ lb } 0 \text{ oz } 10 \text{ dwts}} \text{ of } 5s \text{ } 2\frac{1}{2}d$ 4 $\frac{7 \text{ lbs } 3 \text{ oz}}{11 \text{ lbs } 8 \text{ oz}} \text{ of } 8 \text{ po } 1 \text{ ft } 4\text{ in}$
- 5 $\frac{15s \text{ } 10d}{£1 \text{ } 18s \text{ } 6d} \text{ of } \frac{71 \text{ yds } 0\frac{3}{8} \text{ ft}}{5 \text{ yds } 0\frac{2}{5} \text{ ft}} \text{ of } 2021\frac{1}{4} \text{ days}$
- 6 $\frac{\frac{1}{2} - \frac{2}{3}}{6\frac{1}{6} - 1\frac{1}{3} \text{ of } 4\frac{1}{2}} \text{ of } \frac{£1 \text{ } 11s \text{ } 8d}{£2 \text{ } 17s} \text{ of } \frac{142 \text{ yds } 0\frac{4}{5} \text{ ft}}{2 \text{ yds } 1\frac{7}{10} \text{ ft}} \text{ of } 13 \text{ days } 3$
- 7 $\frac{7 \text{ tons } 13 \text{ cwt } 1\frac{1}{2} \text{ qrs } 14 \text{ lbs}}{15 \text{ cwt } 3 \text{ qrs } 7 \text{ lbs } 12 \text{ oz}} \text{ of } \frac{£15 \text{ } 10s \text{ } 6\frac{3}{4}d}{8s \text{ } 9d}$
- 8 $\frac{£10 \text{ } 17s \text{ } 6d}{£19 \text{ } 6s \text{ } 8d} + \frac{5 \text{ years } 73 \text{ days}}{18 \text{ hrs } 40 \text{ min}} - \frac{Rs \text{ } 4 \text{ } 6s \text{ } 8p}{Rs \text{ } 5 \text{ } 8s}$

9 $\left(\frac{3\frac{1}{2} \text{ of } 5\frac{1}{2}}{2\frac{1}{2} \text{ of } 3\frac{1}{2}} - \frac{2\frac{1}{2} \text{ of } 1\frac{1}{2}}{3\frac{1}{2} \text{ of } 7\frac{1}{2}} \right)$ of $\frac{1s \ 5d}{4s \ 7d}$ of $\frac{2 \text{ ft } 3 \text{ in}}{5 \text{ ft } 5 \text{ in}}$ of 24 weeks.
4 days 19 hrs

10 Reduce $\frac{\pounds 2 \ 3s \ 4d}{\pounds 5 \ 6s \ 8d}$ of $\frac{2 \text{ tons } 4 \text{ cwt}}{5 \text{ tons } 10 \text{ cwt}}$ to a complex fraction having $12\frac{1}{2}$ for its numerator, and also to a complex fraction having $5\frac{1}{2}$ for its denominator

XIV MISCELLANEOUS PROPOSITIONS

(ON VULGAR FRACTIONS)

309 The Unitary Method We have in Art 171 given an outline of this method and treated it in the case of *integers*. We now propose to extend the method to fractional quantities. The following solutions, we hope, will serve as a guide to the students

If the value, weight, length, &c of **one** thing be given, the value, weight, length, &c of **any number** of them (whether *integral* or *fractional* or *mixed*) may always be found by Multiplication, and *conversely*, if the value, weight, length, &c of **any number** of things (whether *integral* or *fractional* or *mixed*) be given, the value, weight, length, &c of **one** of them may always be found by Division

Ex 1 If a yard of lace cost *Re 1 6a 6p* what will 7 yds. 4 in cost?

Here, *Re 1 6a 6p* = *Rs 1\frac{1}{2}*, and 7 yds 4 in = $7\frac{1}{6}$ yds

The cost of 1 yard = *Rs 1\frac{1}{2}*,

\therefore the cost of $7\frac{1}{6}$ yds = *Rs 1\frac{1}{2} \times 7\frac{1}{6}* = *Rs 10\frac{1}{2}* = *Rs 10* Ans

Ex 2 If the cost of $20\frac{1}{2}$ yds of cloth be *Rs 173 5a 4p*, find the cost per yard of the same quality

The cost per yard = *Rs 173 5a 4p* $\div 20\frac{1}{2}$ = $\frac{\text{Rs } 173 \ 5a \ 4p \times 2}{41}$

= *Rs 1 10a 8p* $\times 2$ = *Rs 8 5a 4p* Ans

Ex 3 If $3\frac{3}{8}$ lbs of tea cost *Rs 7 10a*, how much can I buy for *Rs 41 15a*?

Here, *Rs 7 10a* = *Rs 7\frac{5}{8}*, and *Rs 41 15a* = *Rs 41\frac{3}{8}*

The cost of $3\frac{3}{8}$ lbs = *Rs 7\frac{5}{8}*,

\therefore the cost of 1 lb = *Rs 7\frac{5}{8} \div 3\frac{3}{8}* = *Rs 1\frac{1}{8} \times \frac{8}{11}*,

the reqd no of lbs = *Rs 41\frac{3}{8} \div Rs 1\frac{1}{8} \times \frac{8}{11}*

= $\frac{67\frac{1}{2}}{10} \times \frac{8}{11} \times \frac{1}{8} = \frac{187}{10} = 18\frac{7}{10}$ Ans

Ex 4 If $\frac{2}{3}$ of an estate be worth *Rs 2200*, find the value of $\frac{3}{4}$ of it

$\frac{2}{3}$ of the estate is worth *Rs 2200*,

Ex 10 If the cost of maintaining a family be Rs 50 a month, when rice is 12 seers a rupee, and Rs 48 when rice is 14 seers a rupee, what will be the cost when rice is 16 seers a rupee?

The price of 1 sr is first reduced from Re $\frac{1}{2}$ to Re $\frac{1}{4}$ and lastly to Re $\frac{1}{6}$

Now, $\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$ and $\frac{1}{2} - \frac{1}{6} = \frac{1}{3}$, also $50 - 48 = 2$

Since a reduction of Re $\frac{1}{4}$ in price causes a diff of Rs 2 in expenses

of Re 1 Rs (2 × 84)

of Re $\frac{1}{8}$ Rs $\frac{2 \times 84}{48}$

or Rs 3 8a

Hence the required expenses = (Rs 50 - Rs 3 8a) = Rs 46 8a Ans

Examples LXXXVI

1 Find the value of $5\frac{1}{2}$ yds of silk, when $3\frac{1}{2}$ yds cost Rs 21 14a

2 If $12\frac{1}{2}$ articles cost Rs 26 3a 4p, how many can be bought for Rs 117 3a 8p?

3 If 3 cwt 3 qrs 21 lbs $12\frac{1}{2}$ oz cost £4 8s 9d, what is the price per cwt?

4 If a silver cup weighing 20 oz 19 dwts $2\frac{1}{4}$ grs cost Rs 57 10a, what is the price per oz?

5 If $4\frac{1}{2}$ oz of tea cost 8s, what will $30\frac{1}{2}$ lbs cost?

6 If $\frac{1}{8}$ of a lottery ticket cost £4 10s, what is the price of $\frac{3}{4}$ of a ticket?

7 The owner of $\frac{1}{4}$ of a ship sold $\frac{1}{4}$ of $\frac{2}{3}$ of his share for £12 4s, what would $\frac{2\frac{1}{2}}{4\frac{1}{2}}$ of $\frac{2}{3}$ of it cost, at the same rate?

8 Express a degree of $69\frac{1}{2}$ miles in metres, where 32 metres are equal to 35 yards

9 If the sum paid for 247 bottles of wine amount, together with the duty, to Rs 774 7a 2p, and the duty on each bottle be $\frac{1}{4}$ th part of its original cost, what is the duty per bottle?

10 If the rent of 39 ac 2 ro 20 po be Rs 1485 15a, what is the rent of 6 acres?

11 If $\frac{1}{5}$ of a ship be worth Rs 365 5a, what share of it will cost Rs 1252 8a?

12 A ship is worth Rs 160000 and a person possessed of $\frac{1}{8}$ of it, sells $\frac{3}{8}$ of his share, what share has he remaining, and what is it worth?

13 A party having a bill to pay of Rs 123 9a, one of them

pays for himself and three friends the sum of Rs 54 14a 8p , how many were there ?

14 If 7 men or 11 women can finish a piece of work in 17 days, how many days will it take 11 men and 7 women to finish it ?

15 If 74 men had provisions for 35 days, and if after 5 days, 20 men were sent away , how long will the provisions last the remaining men ?

16 If 6 men or 10 women can do a piece of work in 12 days, in what time will 5 men and 7 women do a piece of work twice as great ?

17 If $3\frac{1}{2}$ tons of goods are carried 49 miles for Rs 19 6a , how far ought 26 tons 5 cwt to be carried for the same money ?

18 If $22\frac{1}{2}$ cwt be carried 20 miles for Rs 5 7a , what weight can be carried the same distance for Rs 14 8a ?

19 The four-penny loaf weighs 1 lb $15\frac{1}{2}$ oz , when wheat is at 7s 11d per bushel , find what its weight should be when wheat is at 7s $1\frac{1}{2}$ d per bushel

20 A fortress is provisioned for 3 weeks at the rate of 15 ch a day for each man , if only $10\frac{1}{2}$ ch be served out daily to each man, how long can the place hold out ?

21 A borrowed of B Rs 1752 8a for 102 days, and afterwards would return the favour by lending B the sum of Rs 2103 , for how long should he lend it ?

22 A besieged town, containing 22400 inhabitants, has provisions to last 3 weeks , how many must be sent away that they may be able to hold out 7 weeks ?

23 If the two anna loaf weighs 4 ch , when wheat is Rs 3 6a a maund , what would be the price of wheat per maund when the same loaf weighs 3 ch ?

24 When rice is Rs 3 a maund, how many people can be fed for the same sum that would feed 90 people when rice is Rs 2 8a a maund ?

25 If 2000 men have provisions for 95 days, and if after 15 days 400 men go away, find how long the remaining provisions will serve the number left

26 The monthly expenditure of a shop in oil is Rs 40 8a when oil is sold at $3\frac{1}{2}$ seers a rupee , what will it amount to when the price of oil has risen to 4a 10p per seer ?

27 A piece of cloth, measured with a yard measure which is $\frac{2}{3}$ of an inch too short, appears to be $10\frac{1}{2}$ yards long , what is its true length ?

28 The expenses of a family when rice is sold at 20 seers a rupee are Rs 50 a month , when rice is sold at 25 seers a rupee the

expenses are Rs 48 a month, what will they be when rice is sold at 30 seers a rupee?

310 Bankruptcy or Insolvency

A tradesman becomes bankrupt or insolvent, when the money that he owes is more than that which he has in his possession. What he owes is called his liabilities or debts, his property or what he possesses is called his effects or assets. He is the debtor, those to whom he owes anything are his creditors. The amount paid by bankrupts is generally reckoned at so much in the rupee or pound called a dividend, and each creditor receives the same fraction of the assets that the money due to him is of the bankrupt's whole debts.

Thus if the assets amount to $\frac{3}{4}$ of the debts, each creditor receives $\frac{3}{4}$ of a rupee for each rupee due to him, and the bankrupt is said to pay a dividend of 10a 8p in the rupee.

Book debts are moneys which other men owe to the bankrupt, they are, therefore, considered a part of his assets. Book-debts may be good or bad, as the whole or part can be recovered or realized.

Ex 1 A bankrupt's estates amount to Rs 3780 and his debts to Rs 5040, how much can he pay in the rupee?

On Rs 5040 he can pay Rs 3780,
in one rupee he can pay $Rs \frac{3780}{5040}$ or Re $\frac{3}{4}$,
Hence he can pay Re $\frac{3}{4}$ or 12a Ans

Ex 2 A bankrupt's debts amount to Rs 3240, and he can pay 5a 4p in the rupee, find the amount of his assets.

On every rupee of debts he can pay 5 $\frac{1}{2}$ a or Re $\frac{1}{2}$,
on Rs 3240 of debts $\frac{1}{2} \times Rs 3240$,
Hence assets = $\frac{1}{2} \times Rs 3240 = Rs 1620$ Ans

Ex 3 A bankrupt can pay 10a 8p in the rupee, had he Rs 4250 more he could have paid 12a in the rupee. Find the amount of his debts and assets.

Here, 12a - 10a 8p = 1a 4p = Re $\frac{1}{2}$

He could have paid Re $\frac{1}{2}$ more on Re 1 of his debts

∴ he could have paid Re 1 more on Rs 12 of his debts

∴ Rs 4250 Rs 12 × 4250 of his debts.

Hence his debts = Rs 12 × 4250 = Rs 51000,

also his assets = 10a 8p × 51000 = Rs 34000 } Ans

Ex 4 A creditor receives on a debt of £296 a dividend of 12s 4d in the £, and he receives a further dividend of 3s 9d in the £ upon the deficiency, find how much the creditor receives in all.

The first payment = £(12½ - 20) or £ $\frac{37}{8}$ on £1 of debt,
the deficiency = £(1 - $\frac{37}{8}$) or £ $\frac{5}{8}$ on £1 of debt

Also the second payment = £(3½ - 20) or £ $\frac{17}{4}$ on £1 of deficiency,

∴ the second payment = £ $\frac{17}{8}$ × $\frac{5}{8}$ or £ $\frac{85}{64}$ on £1 of debt

∴ first payment + second payment = £($\frac{37}{8}$ + $\frac{85}{64}$) on £1 of debt
= £ $\frac{313}{64}$ on £1 of debt

Now, in £1 of debt the creditor receives £ $\frac{313}{64}$,

∴ in £296 of debt $\frac{313}{64} \times 296$

Hence the creditor receives £ $\frac{313}{64} \times 296 = \underline{\underline{£203\ 16s\ 2d}}$ Ans

Ex 5 A bankrupt has book debts equal in amount to his liabilities, but on £3000 of them he can only recover 6s 8d in the £, and the expenses of the bankruptcy are £5 for every £100 of the book-debts, if he pay 15s in the £, what is the amount of his liabilities?

As he can recover 6s 8d or £½ in the £, he recovers £½ × 3000 or £1500 out of £3000, therefore his *loss* amounts to £2000. Again, he pays £5 for £100, or 1s in the £ for expenses. Therefore he recovers (15 + 1)s or 16s in the £, and his *loss* per £ = 4s or £½

Now, £½ is the loss on £1 of liabilities

£1	.	£5
£2000		£5 × 2000

Hence liabilities = £5 × 2000 = £10000 Ans

Examples LXXXVII

1 A bankrupt's estates amount to Rs 050 and his debts to Rs 1200, how much can he pay in the rupee?

2 A bankrupt's debts amount to £5069 10s, and he can pay 14s 11½d in the £, find the value of his assets

3 A bankrupt's debts amount to Rs 35000, and his assets to Rs 13708 5a 4p, find how much his estate will pay in the rupee

4 A bankrupt's effects amount to Rs 1980, and he pays his creditors 13a 4p in the rupee, what do his debts amount to?

5 A bankrupt's debts amount to Rs 53422 8a and his creditors lose Rs 17362 ½s, find how much in the rupee the bankrupt pays

6 A bankrupt owes A Rs 5156 1a, B Rs 4070 and C Rs 2933 5a 4p, his estate is worth Rs 9119 11a how much can he pay in the rupee, and what will A, B and C each receive?

7 A bankrupt owes Rs 9000 to his three creditors, and his whole property amounts to Rs 6750, the claims of two of his creditors are Rs 1250 and Rs 3750 respectively, what sum will the remaining creditor receive for his dividend?

8 A creditor received 16s 3d in the £, and thereby lost £135 10s, how much was due to him?

9 A bankrupt's debts amount to £1700, and his assets to £900 15s, after paying costs his creditors receive 5s 9d in the £, find the amount of the costs

10 A bankrupt has good debts to the amount of £456 18s 1d, and the following bad debts, £360 7s 10d, £120 13s and £19 18s for which he receives respectively 4, 5 and 9 shillings in the £, his own liabilities amount to £3408 12s, how much can he pay in the £?

11 A creditor received on a debt of Rs 3600 a dividend of 9s 10p in the rupee, and a further dividend of 6s 8p upon the remainder. What did he receive altogether?

12 A bankrupt can pay 12s 4d in the £, if his assets were £4205 more, he could pay 15s 8d in the £. Find his debts and assets

13 A bankrupt has book debts equal in amount to his liabilities, but on Rs 8640 of such debts he can recover only 8s 6p in the rupee, and on Rs 6300 only 5s 3p in the rupee. After allowing Rs 1054 11s for the expenses of bankruptcy, he finds he can pay his creditors 12s in the rupee. Find the total amount of his debts

14 A bankrupt pays £5850 on the whole liabilities, at the rate of 13s 6d in the £ on half his debts and 15s 9d in the £ on the other half, find the amount of his debts

15 A bankrupt can pay 11s in the rupee, had he Rs 2550 more, he could have paid 14s in the rupee. Find the amount of his debts and assets

16 A bankrupt has book debts equal in amount to his liabilities, but on £6000 of them he can only recover 13s 4d in the pound, and the expenses of the bankruptcy are £5 on every £100 on the book debts, if he pays 13s in the pound, what is the amount of his liabilities?

311 Incomes, Taxes and Rates

Proceed as in the following Examples

Ex 1 If the income tax be at the rate of 4p in the rupee, and a man has to pay Rs 13 6s 8p, what is the amount of his income?

Here, Rs 13 6s 8p = Rs 13 $\frac{5}{8}$

He pays Re $\frac{1}{16}$ or Re $\frac{1}{8}$ income-tax on every Re 1 of income,

he pays Re 1 income-tax on every Rs 48 of income,

Rs 13 $\frac{5}{8}$

Rs 48 \times 13 $\frac{5}{8}$

Hence income required = Rs 48 \times $\frac{101}{8}$ = Rs 644 Ans

Ex 2 After paying an income-tax of 8p in the rupee a man has Rs 7283 5a 1p left, find his gross income

Here, $Re\ 1 - 8p = 15a\ 1p = Re\ \frac{1}{2}$; and $Rs\ 7283\ 5a\ 1p = Rs\ 7283\frac{1}{2}$,

Since $Re\ \frac{1}{2}$ is left out of $Re\ 1$ of income

$$\begin{array}{rcl} \therefore Re\ 1 & . & . & . & Rs\ \frac{2}{1} & \dots \\ \therefore Rs\ 7283\frac{1}{2} & .. & & & Rs\ \frac{2}{1} \times 7283\frac{1}{2} \end{array}$$

Hence income required = $Rs\ \frac{2}{1} \times 7283\frac{1}{2} = Rs\ 14567$ Ans

Ex 3 Find a man's gross rental if after paying an income-tax of 6d in the £ on the whole, and 3s 6d in the £ on $\frac{1}{4}$ of his rental, his net income is £2700

Tax on $\frac{1}{4}$ at 3s 6d = $\frac{1}{4} \times 42d = 31\frac{1}{2}d$

\therefore total amount paid in taxes = $(6 + 31\frac{1}{2})d$ or $37\frac{1}{2}d$ in the £

\therefore he has $(240 - 37\frac{1}{2})$ or $202\frac{1}{2}d$, or $\frac{2}{3}$ left out of £1

Since $\frac{2}{3}$ is left out of £1 of gross income,

$$\begin{array}{rcl} \therefore \text{£}1 & & \text{£} \frac{3}{2} \\ \therefore \text{£}2700 & . & \text{£} \frac{3}{2} \times 2700 \end{array}$$

Hence gross rental = $\text{£} \frac{3}{2} \times 2700 = \text{£}4050$ Ans

Ex 4 When the income-tax is 7d in the £, a person has to pay £63 less than when the tax is 11d in the £, find his income

On the diminution of tax from 11d to 7d in the £, the man has to pay 4d or $\frac{1}{6}$ less on £1

In every $\frac{1}{6}$ less of income-tax the man has £1,

$$\begin{array}{rcl} \therefore & \text{£}1 & . & . & \text{£}60 \\ \therefore & \text{£}63 & . & .. & \text{£}60 \times 63 = \text{£}3780 \end{array} \text{ Ans}$$

Ex 5 The rent of a man's house is £120 per annum. It is assessed to the rates at $\frac{2}{3}$ of this, the poor-rate is 7s 6d in the £, the paving rate is 1s 9d and the church rate 4d, how much does he pay altogether for his residence?

Assessed value = $\frac{2}{3}$ of £120 = £80

Amount of rates on £1 is (7s 6d + 1s 9d + 4d) = 9s 7d

\therefore rates on £80 is 9s 7d \times 80 = £38 6s 8d

Hence the annual cost of the house = £120 + £38 6s 8d
= £158 6s 8d Ans

Examples LXXXVIII

1 A man pays an income-tax of Rs 63 15a 5p at the rate of 7p in the rupee, find his income

2 How much will a poor rate of $2s\ 8d$ in the £ produce in a parish in which the whole property is rated at £4736 $5s\ 2$

3 A person after paying $7p$ in the rupee for income-tax has Rs 346 $14a$ left What was his gross income ?

4 After paying an income-tax of $3d$ in the £, a person has a net income of £590 $10s\ 6d$, find his gross income

5 Find a man's gross rental if after paying an income tax of $8d$ in the £ on the whole, and $2s\ 6d$ in the £ on two thirds of his rental, he has a net income of £398 $16s\ 6d$

6 After deducting $4p$ in the rupee for income-tax and $\frac{1}{2}\%$ of the value of the whole estate for collecting expenses, the value of the remainder is Rs 11270, what is the value of the whole estate ?

7 The net rental of an estate, after deducting $7d$ in the £ for income tax and $\frac{1}{2}\%$ of the remainder for cost of collecting, is £959 $3s\ 8d$, find the gross rental

8 A reduction in the income tax diminishes a tax which is Rs 15 when the tax is 8 pies in the rupee by Rs 3 $12a$, what is the diminished rate of tax in the rupee ?

9 I hire a house at £90 a year, which is assessed in the rate book at $\frac{4}{10}$ ths of its rent, I agree to pay the rates upon it, viz, 3 poor-rates of $9d\ 10d$ and $1s\ 2d$ respectively in the £, a church rate of $8d$ in the £, and a paving rate of $1s\ 7d$ in the £, what is the whole annual cost of the house ?

10 A man allows his agent $\frac{1}{8}$ of one anna in the rupee on his gross income for the expense of collecting his rents. He spends $\frac{1}{4}$ of his net income in assuring his own life, and this part of his income is in consequence exempt from income tax. The income tax being $8p$ in the rupee, and his income tax amounting to Rs 389 $8a$ find his gross income

11 A man pays a house-rate of $1s\ 6d$ per £ on his rental, a water-rate of $1s$ per £, a poor-rate of $1s\ 10\frac{1}{2}d$ per £. If the rent and rates amount to £85 $6s\ 3d$, what is the rent ?

12 An occupier pays house-rate of $3a$, police-rate of $9p$, water rate of $2a\ 6p$ and a lighting rate of $1a\ 9p$ in the rupee. If the rent and rates amount to Rs 1440, what is the assessed annual value of the house ?

13 The income-tax having been raised to $10d$ in the pound a man has to pay £45 $10s\ 6d$ more than when it was $7d$ in the pound. Find his income

14 If a person's net income after paying an income-tax of $7d$ in the £ be £291 $5s$, find his net income after paying an income-tax of $1s\ 4d$ in the £

Proceed as follows —

Ex 1 A post has $\frac{1}{4}$ of its length in the mud, $\frac{1}{3}$ in water and 10 ft above the water Find its whole length

Let unity or 1 represent the length of the post

Then the part in the mud = $\frac{1}{4}$, } Now, $\frac{1}{4} + \frac{1}{3} = \frac{7}{12}$

and . . . water = $\frac{1}{3}$, } \therefore the part above water = $1 - \frac{7}{12} = \frac{5}{12}$

Hence, by question, $\frac{5}{12}$ of the post = 10 ft

\therefore the length of the post = 10 ft $\times \frac{12}{5} = \underline{24 \text{ ft}}$ Ans

Ex 2 One half of the trees in an orchard are apple trees, one-fourth are pear trees, one-sixth plum trees, and there are 50 cherry trees, what number of trees does it contain?

Representing the number of trees in the orchard by the unit or 1, we have

$\frac{1}{2}$ = number of apple trees, }
 $\frac{1}{4}$ = number of pear trees, } Now $\frac{1}{2} + \frac{1}{4} + \frac{1}{6} = \frac{11}{12}$,
 $\frac{1}{6}$ = number of plum trees } \therefore the no of other trees = $1 - \frac{11}{12} = \frac{1}{12}$

Hence, by question, $\frac{1}{12}$ of the whole no of trees = 50,

the whole no of trees = $50 \times 12 = \underline{600}$ Ans

Ex 3 After paying away one half of a sum of money, and then $\frac{2}{3}$ of what was left, Rs 5 4a remained, what was the sum?

Let 1 represent the sum of money

Then $\frac{1}{2}$ of the sum = $\frac{1}{2}$, the first paid-up part,

$\therefore 1 - \frac{1}{2} = \frac{1}{2}$, remaining,

Again $\frac{2}{3}$ of $\frac{1}{2} = \frac{1}{3}$, the second paid up,

$\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$, remaining.

Hence, by question, $\frac{1}{6}$ of the sum = Rs 5 4a

the whole sum = Rs 5 4a $\times 6 = \underline{\text{Rs } 26 \text{ 4a}}$ Ans

Ex 4 A met two beggars, B and C, and having $\frac{37}{48}$ of $\frac{107}{72}$ of $\frac{77}{540}$ of a moidore of 27s in his pocket, gave $\frac{1}{4}$ of $\frac{1}{4}$ of it to B and $\frac{2}{3}$ of the remainder to C, what did each receive?

A had at first $\frac{40}{11}$ of $\frac{75}{12}$ of $\frac{77}{540}$ of 27s

= $\frac{40 \times 7}{30 \times 11}$ of $\frac{75 \times 2}{7 \times 15}$ of $\frac{77 \times 27}{540}$ s = $1\frac{4}{5}$ s

\therefore B received $\frac{1}{4}$ of $\frac{1}{4}$ of $1\frac{4}{5}$ s = $\frac{1}{5}$ s = 6d.

and A had afterwards left $(1\frac{4}{5} - \frac{1}{5})$ s = $2\frac{3}{5}$ s

. C received $\frac{2}{3}$ of $2\frac{3}{5}$ s = $\frac{8}{5}$ s = 2s 6d

Ex 5 A person left $\frac{1}{8}$ of his property to his eldest son and $\frac{1}{8}$ of the remainder to his younger son and the rest to his widow. The elder son received £1029 16s 4d more than the younger, how much did the widow receive?

Let 1 represent the whole property

Then the elder son received $\frac{1}{8}$, and the part left is $(1 - \frac{1}{8}) = \frac{7}{8}$

The younger son received $\frac{1}{8}$ of $\frac{7}{8} = \frac{7}{64}$, and the part left is $\frac{7}{8} - \frac{7}{64} = \frac{57}{64}$

∴ the widow's share is $\frac{57}{64}$ of the property

The sons' shares differ by $\frac{1}{8} - \frac{7}{64} = \frac{9}{64}$ of the whole

Hence, by question, $\frac{9}{64}$ of the whole estate = £1029 16s 4d

$$\begin{aligned} \therefore \text{the whole estate} &= \text{£}1029 \ 16s \ 4d \times \frac{64}{9} \\ &= \text{£}21 \ 0s \ 4d \times 324 = \text{£}6809 \ 8s \end{aligned}$$

$$\begin{aligned} \text{the widow's share} &= \text{£}6809 \ 8s \times \frac{57}{64} = \text{£}21 \ 0s \ 4d \times 121 \\ &= \underline{\underline{\text{£}2543 \ 0s \ 4d}} \quad \text{Ans} \end{aligned}$$

Ex 6 Gunpowder being composed of nitre 15 parts, charcoal 3 parts, and sulphur 2 parts, find how much of each is required for 18 mounds of powder

The whole number of parts = $(15 + 3 + 2) = 20$

of every 20 parts, $\frac{15}{20}$ or $\frac{3}{4}$ is nitre, $\frac{3}{20}$ is charcoal,
 $\frac{2}{20}$ or $\frac{1}{10}$ is sulphur

$$\begin{aligned} \text{Hence, required nitre} &= \frac{3}{4} \text{ of } 18 \text{ mds} = \frac{13 \text{ mds } 20 \text{ sr}}{} \\ \text{charcoal} &= \frac{3}{20} \text{ of } 18 \text{ mds} = \frac{2 \text{ mds } 28 \text{ sr}}{} \\ \text{sulphur} &= \frac{1}{10} \text{ of } 18 \text{ mds} = \frac{1 \text{ md } 32 \text{ sr}}{} \end{aligned} \quad \left. \vphantom{\begin{aligned} \text{Hence, required nitre} \\ \text{charcoal} \\ \text{sulphur} \end{aligned}} \right\} \text{Ans}$$

Examples LXXXIX

1 After detaching $\frac{1}{4}$ and $\frac{1}{8}$ of a company of soldiers, the general had 1110 left, required his original force

2 If a person lay out $\frac{1}{5}$ of his income in board and lodging, $\frac{1}{5}$ in clothes and save Rs 600 a year, what is his income?

3 What is the capacity of a vessel, out of which when a third of it is empty, 35 gallons being drawn, there remains $\frac{2}{3}$ of the whole content?

4 In an orchard, $\frac{1}{4}$ are apple trees, $\frac{1}{5}$ peach trees, $\frac{2}{5}$ pear trees, and the remainder which is 38, cherry trees. How many trees are there in the orchard?

5 After taking out of a purse $\frac{2}{5}$ of its contents, $\frac{2}{5}$ of the remainder was found to be Rs 6 11s 8p, what sum did it contain at first?

6 If $\frac{1}{3}$ of an estate be left to the elder and the remainder to the younger of two children, and the difference of their legacies be Rs 2250, find the value of the estate

7 Of a field $\frac{1}{4}$ is meadow, $\frac{3}{8}$ is arable and the remainder is 1 ac 3 ro 26 po, find the quantities of meadow and arable land

8 A had at first £1 8s, and B, when he had paid $2\frac{3}{4}$ of £1 11s 6d to A, found that he had remaining $\frac{1}{3}$ of what A then had, what had B at first?

9 A man pays away $\frac{1}{4}$ of his money, then $\frac{3}{5}$ of what remains, and then $\frac{2}{5}$ of the second remainder, after which he has 7s 6d left, how much had he at first?

10 A post is divided into 4 parts, the first part is $\frac{3}{7}$ of the whole length, the second part is $\frac{2}{7}$ of the first, the third $\frac{1}{3}$ of the second, and the fourth is 2 yds 1 ft 4 in, find the length of the post

11 Out of Rs 43 12a, $\frac{1}{4}$ is paid to A and $\frac{1}{7}$ to B, after this $\frac{4}{11}$ of the remainder is paid to A and the rest to B, find the sums respectively received by A and B

12 A gentleman left his eldest son $\frac{6}{8}$ of his money, to the younger $\frac{1}{8}$ of the remainder, and the rest to his wife, upon dividing the money, it was found that the eldest son had Rs 7500 more than the younger, how much was left to each?

13 A and B have Rs 18 and Rs 12 respectively, and if A gives B $2\frac{3}{4}$ of the difference of $\frac{21\frac{1}{2}}{13\frac{5}{13}}$ of their respective sums, and $\frac{1}{4}$ of $2\frac{1}{2}$ of A's present sum be added to $\frac{1}{10}$ of $\frac{1}{2}$ of B's, C's money will be $\frac{1}{3}$ of this sum, find it

14 A person had a legacy left to him, which he thus divided amongst 3 charities To one he gave $\frac{1}{10}$, to the second $\frac{3}{7}$ of the remainder, and to the third $\frac{1}{2}$ of what now remained, and he then had Rs 1500 left Find the amount of the legacy, and how much was given to each charity

15 What number is that of which the fourth, fifth and sixth parts together exceed the half of the number by 112?

16 A person making his will, gave to one child $\frac{1}{10}$ of his estate, and the rest to another When these legacies came to be paid, the one turned out to be £1200 more than the other, what did the testator die worth?

17 A, B and C rent a pasture for Rs 400 A puts in 8 cattle, B 9 and C 11, how much should each pay for his share?

18 A person dies worth Rs 100000, and leaves $\frac{1}{3}$ of his property to his wife, $\frac{1}{4}$ to his son, and the rest to his daughter The wife at her death leaves $\frac{1}{2}$ of her legacy to the son, and the rest to the daughter, but the son adds his fortune to his sister's and gives her $\frac{1}{3}$ of the whole How much will the sister gain by this, and what fraction will her gain be of the whole?

313 Pipes and Cisterns

If one or more pipes fill or empty a cistern in 8 min, they fill or empty $\frac{1}{8}$ th of it per min, and *conversely*, if they fill or empty $\frac{1}{8}$ th of it per min, they fill or empty the whole in 8 min. Similarly, if they fill or empty a cistern in $5\frac{1}{2}$ hours, they fill or empty $\frac{1}{5\frac{1}{2}}$ or $\frac{2}{11}$ of it in 1 hr, and *conversely*, if they fill or empty $\frac{2}{11}$ of it per hour, they will fill or empty the cistern in $(1 + \frac{1}{11})$ or $5\frac{1}{2}$ hrs.

Ex 1 Two pipes can separately fill a cistern in 10 and 15 minutes. If both the pipes are opened, how soon will the cistern be filled?

The first pipe fills $\frac{1}{10}$ of the cistern in 1 min
second $\frac{1}{15}$

\therefore both the pipes fill $(\frac{1}{10} + \frac{1}{15})$ or $\frac{1}{6}$ of the cistern in 1 min

Hence they fill the cistern in $(1 + \frac{1}{5})$ min = 6 min. *Ans*

Ex 2 Pipes A and B can fill a cistern in 3 min and 5 min respectively, and C can empty it in $7\frac{1}{2}$ min. In what time will the cistern be filled when A, B and C are all turned on?

The first pipe fills in 1 min $\frac{1}{3}$ of the cistern,
second. $\frac{1}{5}$

The third pipe empties in 1 min $(1 - \frac{1}{7\frac{1}{2}})$ or $\frac{1}{15}$ of the cistern,
with all open, $(\frac{1}{3} + \frac{1}{5} - \frac{1}{15})$ or $\frac{1}{2}$ of the cistern is filled in 1 min

Hence the cistern will be filled in $(1 + \frac{1}{1})$ min = 2 min. *Ans*

Ex 3 Two taps take 4 hours and 6 hours respectively to fill a cistern. When the waste pipe is left open along with the two taps, the cistern is filled in 24 hours. In what time does the waste pipe empty the cistern?

The first tap fills $\frac{1}{4}$ of the cistern in 1 hour
second $\frac{1}{6}$

The three together fill $\frac{1}{24}$

the waste pipe empties $(\frac{1}{4} + \frac{1}{6} - \frac{1}{24})$ or $\frac{1}{8}$ of the cistern in 1 hour

Hence, the waste pipe will empty the cistern in $(1 - \frac{1}{8}) = \frac{7}{8}$ hrs. *Ans*

Ex 4 A cistern which would be filled in 8 hours requires 2 hours more to be filled, owing to a leak in the bottom. If the cistern is full, in what time will the leak empty it?

Had there been no leak, $\frac{1}{8}$ of the cistern would have been filled in 1 hr, but the leak allows only $\frac{1}{10}$ to be filled in 1 hour

$(\frac{1}{8} - \frac{1}{10})$ or $\frac{1}{40}$ of the cistern is emptied by the leak in 1 hour

Therefore the leak requires $(1 - \frac{1}{40})$ or 40 hours to empty the cistern. *Ans*

Examples XC

1 Two taps, *A* and *B*, fill a cistern in 10 and 20 hours respectively. In what time will they fill it together?

2 A cistern is filled by two taps in 10 and 15 hours respectively, and is emptied by a tap, *C*, in 8 hours. If all the three taps are open, in what time will the cistern be filled?

3 A cistern is fed by a spout which can fill it in 3 hrs. How long would it take to fill it, if the cistern has a leak which would empty, when full, in 17 hrs?

4 Two pipes together can fill a cistern in 8 min., and one of them alone in 24 min. How long would the other alone take?

5 A cistern has three pipes connected with it, two to supply and one to draw off. The first alone can fill $\frac{2}{3}$ of the cistern in 3 hours, and the second $\frac{1}{3}$ in 4 hours, the third can empty $\frac{1}{4}$ of the cistern in 5 hours. If all the pipes be opened together at once, when will the cistern be full?

6 A cistern is filled by two spouts in 20 and 24 minutes respectively and emptied by a tap in 30 minutes, what portion of it will be filled in 15 minutes when they are all left open together?

7 A cistern has three pipes *A*, *B* and *C*, *A* and *B* can fill it in 3 and 4 hours respectively, and *C* can empty it in 1 hour, if these pipes be opened in order at 1, 2 and 3 o'clock, when will the cistern be empty?

8 A cistern is provided with three spouts *A*, *B* and *C*. *A* can fill it in 30 minutes, *B* in 40 and *C* can empty it in 2 hours. If *A*, *B* and *C* be opened successively for 1 minute each, in what time will the cistern be filled, and how much of the content of the cistern will have passed out by *C*?

9 A cistern can be filled by three pipes, by the first in 10 hours, by the second in 9, and by the third in 8 hours. It is supplied by the first pipe till $\frac{1}{4}$ of it is full, then the second is also turned on till it becomes half full, and then all three begin to run. How long would it take to fill the cistern?

10 A tank can be filled by one pipe in 6 min., and by a second in 5 min., there is also a tap by which the tank can be emptied. If the tank be empty at first, and the pipes and tap be all left open, the tank is filled in 3 min. If the pipes are then closed, in what time will the tank be emptied by the tap?

11 A cistern can be filled by two pipes, *A*, *B* in 4 and 5 min. respectively and emptied by *C* in 144 seconds. *B* is opened 2 min. after *A*, *C* is opened 1 min. after *B*. The cistern contains 361 gallons just before *C* is opened. In what time will it be filled or emptied after the opening of *C* and how many gallons will go out by *C*?

12 Three taps, *A*, *B* and *C* can fill a cistern, *A* by itself in

24 min, B in 10 min, and C in 27 min. They are all turned on at once, but after $4\frac{1}{2}$ min B and C are turned off. How much longer will A by itself take then to fill the cistern?

314 Time and work The following points (if remembered) will greatly help students in solving problems concerning *Time and Work*

- (1) If 1 man can do a piece of work in a certain time, then in the same time 2 men will do *twice* as much, 3 men *thrice* as much, and so on. *Conversely*, if 3 men can do a piece of work in a certain time, 1 man will do $\frac{1}{3}$ of the work in the same time.
- (2) If one or more men can do a piece of work in 6 days, they can do $\frac{1}{6}$ of the work in 1 day, so, if a piece of work can be done in $6\frac{1}{2}$ days, $(1 - \frac{1}{6})$ or $\frac{5}{6}$ ths of the work can be done in 1 day, and *conversely*, if $\frac{1}{6}$ th of a piece of work is done in 1 day, the whole work can be done in 6 days, so, also if $\frac{5}{6}$ ths of a piece of work can be done in 1 day, the whole work will be done in $(1 - \frac{5}{6})$ or $6\frac{1}{5}$ days.
- (3) If 5 men can do a piece of work in 7 days, then they will do $\frac{5}{7}$ th of the work in one day, therefore 1 man will do $\frac{1}{5} \times \frac{5}{7} = \frac{1}{7}$ of the work in 1 day, or the whole work in 35 or (5×7) days.
- (4) If 1 man can do a piece of work in 5 days, then he will do $\frac{1}{5}$ th of the work in 1 day, therefore 3 men will do $\frac{3}{5}$ ths of the work in 1 day, and therefore the whole work in $(1 - \frac{3}{5})$ or $(5 - 3)$ days.
- (5) If 1 man can do $\frac{2}{5}$ of a piece of work in 7 days, he can do $\frac{2}{5} \times \frac{1}{7}$ of it in one day, and therefore the whole work in $1 - (\frac{2}{5} \times \frac{1}{7})$ or $(7 - \frac{2}{5})$ days.
- (6) If A can do $\frac{1}{5}$ of a work in 1 day, and B $\frac{1}{10}$ in 1 day, then A and B together will do $(\frac{1}{5} + \frac{1}{10})$ or $\frac{3}{10}$ of the work in 1 day, and therefore both will finish the work in $(1 - \frac{3}{10})$ or $2\frac{2}{3}$ days.

Ex 1 A can do a piece of work in 5 days, B in 6 and C in 7, how much of it can they jointly do in 2 days, and how long will they take to do the whole work?

A can do $\frac{1}{5}$ of the work in 1 day, B can do $\frac{1}{6}$ of the work in 1 day, C can do $\frac{1}{7}$ of the work in 1 day

$\therefore A, B$ and C can jointly do $(\frac{1}{5} + \frac{1}{6} + \frac{1}{7})$ or $\frac{107}{210}$ of the work in 1 day

Hence, in 2 days they will do $\frac{107}{210} \times 2$ or $\frac{214}{210}$ of the work. *Ans*

Also, they can jointly do the whole work in $(1 - \frac{214}{210})$ or $1\frac{1}{105}$ days

Ex 2 If A and B together can perform a piece of work in 10 days, and A himself can do it in 18 days, what time will it take B to do it alone?

A and B can do the work in 10 days ,

. they can do $\frac{1}{10}$ of the work in 1 day ,

A can do the work in 18 days ,

he can do $\frac{1}{18}$ of the work in 1 day ,

. B can do $(\frac{1}{10} - \frac{1}{18})$ or $\frac{2}{45}$ of the work in 1 day

Hence B can do the whole work in $(1 - \frac{1}{18})$ or $\underline{22\frac{1}{2}}$ days *Ans*

Ex 3 A does $\frac{1}{10}$ of a piece of work in 14 days , he then calls in B , and they finish the work in 2 days , how long would B take to do the whole work by himself ?

A does $\frac{1}{10}$ of the work in 14 days ,

he does $\frac{1}{140}$ or $\frac{1}{20}$ of the work in 1 day ,

in 2 days, A does $\frac{2}{20}$ or $\frac{1}{10}$ of the work

But $(1 - \frac{1}{10})$ or $\frac{9}{10}$ of the work remains to be done ,

. B does $(\frac{9}{10} - \frac{1}{10})$ or $\frac{8}{10}$ of the work in 2 days ,

B can do $\frac{1}{10}$ of the work daily, and

. B can do the whole work in $(1 - \frac{1}{10})$ or $\underline{10}$ days *Ans*

Ex 4 If A and B can do a piece of work in 18 days, A and C in 12 days, and B and C in 9 days, find the time in which A , B and C can together finish it, and also each working singly

A and B can do $\frac{1}{18}$ of the work in 1 day ,

A and C $\frac{1}{12}$. ,

B and C $\frac{1}{9}$,

. \therefore 2 men like A + 2 men like B + 2 men like C can do $(\frac{1}{18} + \frac{1}{12} + \frac{1}{9})$ or $\frac{1}{4}$ of the work in 1 day ,

. A , B and C can do $\frac{1}{8}$ of the work in 1 day ,

Hence they can jointly do the whole work in $(1 - \frac{1}{8})$ or $\underline{8}$ days *Ans.*

Also A can do $(\frac{1}{8} - \frac{1}{9})$ or $\frac{1}{72}$ of the work in 1 day, and

. \therefore the whole work in $\underline{72}$ days

B . $(\frac{1}{8} - \frac{1}{12})$ or $\frac{1}{24}$ of the work in 1 day, and
the whole work in $\underline{24}$ days

C . $(\frac{1}{8} - \frac{1}{18})$ or $\frac{5}{72}$ of the work in 1 day, and
. the whole work in $\underline{14\frac{2}{5}}$ days *Ans*

Ex 5 5 men or 10 women or 15 boys can do a piece of work in 16 days In how many days will 2 men, 3 women and 4 boys do it ?

Since 5 men can do the work in 16 days ,

1 man will do the work in (16×5) days ,

. \therefore 1 man in one day will do $\frac{1}{16 \times 5}$ of the work ,

• 2 men in one day will do $\frac{2}{16 \times 5}$ or $\frac{1}{40}$ of the work

Similarly, 3 women in one day will do $\frac{3}{160}$ of the work ,

and 4 boys $\frac{4}{80}$ of the work

2 men + 3 women + 4 boys in one day will do $(\frac{1}{40} + \frac{3}{160} + \frac{4}{80})$ or $\frac{29}{80}$ of the work

Hence, they will take $(1 - \frac{29}{80})$ or $\frac{51}{80} = 16\frac{1}{8}$ days *Ans*

Ex 6 A and B can do a piece of work in 10 days, B and C in 15 days, and A and C in 25 days, they all work at it together for 4 days, A then leaves, and B and C go on together for 5 days, and then B leaves, in how many days will C complete the work?

A and B can do $\frac{1}{10}$ of the work daily, B and C $\frac{1}{15}$ daily, and A and C $\frac{1}{25}$ daily, 2 men like A + 2 men like B + 2 men like C can together do $(\frac{1}{10} + \frac{1}{15} + \frac{1}{25})$ or $\frac{37}{300}$ of the work daily, and A + B + C can do $\frac{1}{300}$ of the work daily. Hence in 4 days, they do $(\frac{37}{300} \times 4) = \frac{37}{75}$ of the work

when A leaves, $(1 - \frac{37}{75})$ or $\frac{38}{75}$ of the work remains to be done.

Now, B and C together in 5 days do $(\frac{1}{15} \times 5)$ or $\frac{1}{3}$ of the work

when B leaves, $(\frac{38}{75} - \frac{1}{3})$ or $\frac{17}{75}$ of the work remains to be done and this work C finishes by himself

Again, C in one day can do $(\frac{1}{15} - \frac{1}{25})$ or $\frac{1}{375}$ of the work

Hence, C finishes the work in $(\frac{17}{75} \div \frac{1}{375})$ or $\frac{17}{75} \times 375 = 82\frac{1}{2}$ days *Ans*

Ex 7 If 10 excavators can dig 12 loads of earth in 16 hours, whilst 12 others can dig 9 loads in 15 hours, find the time in which they will jointly dig 100 loads

The first set of men can dig $\frac{12}{160}$ or $\frac{3}{40}$ load in 1 hour, the second set $\frac{9}{150}$ or $\frac{3}{50}$ load in 1 hour

they can jointly dig $(\frac{3}{40} + \frac{3}{50})$ or $\frac{27}{200}$ loads in 1 hour,

• they can dig 1 load in $(1 - \frac{27}{200})$ or $\frac{173}{200}$ hour

Hence they can dig 100 loads in $(\frac{173}{200} \times 100)$ or $86\frac{1}{2}$ hours *Ans*

Ex 8 A can do a piece of work in 10 days, B in 9 days and C in 12 days. All begin together, but A leaves after 4 days and B 2 days before the work is done. How long did the work last?

A can do $\frac{1}{10}$ of the work in 1 day, B $\frac{1}{9}$ in 1 day, and C $\frac{1}{12}$ in 1 day

A in 4 days does $\frac{4}{10}$ or $\frac{2}{5}$ of the work. Now, C worked 2 days, more than B, and during that time did $\frac{2}{12}$ or $\frac{1}{6}$ of the work.

Therefore the work done by B and C together is $(1 - \frac{2}{5} - \frac{1}{6})$ or $\frac{17}{30}$ of the work. Now, B and C in 1 day can do $(\frac{1}{9} + \frac{1}{12})$ or $\frac{7}{36}$ of the work, therefore they took $(\frac{17}{30} \div \frac{7}{36})$ or $\frac{38}{5}$ days = $7\frac{4}{5}$ days

Hence the whole time occupied = $(4 + 7\frac{4}{5} + 2)$ or $13\frac{4}{5}$ days *Ans*

Ex 9 If A can do as much work in 5 hours as B can do in

6 hours, or as C can do in 9 hours, how long will it take C to complete a piece of work, one-half of which has been done by A working 12 hours and B 24 hours ?

Since 5 hrs work of $A = 9$ hrs work of C ,

∴ 1 hr of $A = \frac{9}{5}$ hrs of C , or 12 hrs of $A = \frac{9}{5} \times 12$ or $21\frac{3}{5}$ hrs of C

Since 6 hrs work of $B = 9$ hrs work of C ,

∴ 24 hrs work of $B = 9 \times 4$ or 36 hrs of C

Hence 12 hrs of $A + 24$ hrs of $B = (21\frac{3}{5} + 36)$ or $57\frac{3}{5}$ hrs of C

But 12 hrs work of $A + 24$ hrs work of $B = \frac{1}{2}$ of the work ;

∴ C can finish the remaining half in $57\frac{3}{5}$ hrs *Ans*

Ex 10 A is thrice as good a workman as B , and together they finish $\frac{2}{3}$ of a work in 9 days. In how many days will it be done by each separately ?

Since 3 days' work of $B = 1$ day's work of A ,

9 days' work of $B = 3$ days' work of A

∴ 9 days' work of $B + 9$ days' work of $A = 12$ days' work of A

But 9 days' work $B + 9$ days' work of $A = \frac{2}{3}$ of the work,

∴ 12 days' work of $A = \frac{2}{3}$ of the work, ∴

A can do $\frac{1}{2}$ of the work in 12 days

Hence A does the whole work in $(12 \div \frac{1}{2})$ or 24 days, } *Ans*
and therefore B does the whole work in 3×24 or 72 days

Examples XCI

1 A alone can do a piece of work in 11 days, and B alone can do it in 17 days. Find how long they would take to do it together.

2 A , B and C can complete a piece of work in 10, 12 and 15 days respectively. How long would it take them if they work together ?

3 A can finish a piece of work in $2\frac{1}{2}$ days and B in $3\frac{1}{2}$ days, if they work together what part of the work will they finish in 14 days ?

4 A and B can do a piece of work in 12 days, when C joins them they can do it in 9 days, in what time can C do it working alone ?

5 A man alone can do a piece of work in 10 days which, if his son helps him, he can do in 6 days, in what time would his son working alone do the work ?

6 A can reap $\frac{1}{4}$ of a field in $2\frac{1}{2}$ days, and B can reap $\frac{1}{5}$ of it in $4\frac{1}{2}$ days, in what time could A and B working together reap the field ?

7 If A and B can do a piece of work in 24 days, A and C in

16 days, and B and C in 12 days, find the time in which A , B and C can together finish it

8 A and B can do a piece of work in 6 days which B and C can do in 4 days, and A and C in 3 days Find the time in which each can separately do it

9 A and B can do a piece of work in 8 days, A and C in $10\frac{2}{3}$ days, and B and C in $9\frac{1}{2}$ days, in how many days can A alone do it?

10 A , B and C can finish a piece of work in 12 hours, also A and B can do it in 16 hours, and A and C in 18 hours, what part of the whole work can B and C do in $9\frac{1}{2}$ hours?

11 A , B and C can do a piece of work together in 20 days, A alone can do it in 40 days, and B alone in 60 days In what time can C alone do it?

12 A performs $\frac{3}{4}$ of a piece of work in 13 days, and with the help of B finishes it in 6 days In what time could each of them do the piece of work separately?

13 A can do $\frac{2}{3}$ of a piece of work in 4 hours, B can do $\frac{1}{3}$ of the remainder in 1 hour, and C can then finish it in 20 minutes, in what time can A , B and C together do it?

14 A certain number of men mow 4 acres of grass in 3 hours, and a certain number of others mow 8 acres in 5 hours, how long will they be in mowing 11 acres, if all work together?

15 A can mow $2\frac{1}{2}$ acres in $6\frac{2}{3}$ days, and B $2\frac{1}{2}$ acres in $5\frac{1}{2}$ days, they mow together a field of 10 acres, how long will it take them to do it, and how many acres will each mow?

16 A and B can do a piece of work in 4 days, working 6 hours a day, B and C can do it in 4 days, working 5 hours a day, and A and C can do it in 4 days, working 4 hours a day In how many days of 8 hours will each do it separately?

17 A can do a piece of work in 27 days, A and B can do it in 15 days, A works alone for 12 days, and A and C together for 5 days, and B finishes it in 7 days, find in what time B and C together could do it

18 A can do a piece of work in 27 days and B in 15 days, A works at it alone for 12 days, B then works 5 days and afterwards C finishes it in 4 days, in what time could C have done the whole work?

19 A and B can do a piece of work, each, in 24 days, A and B work together for 6 days, when B goes away and C works with A for 3 days, then B rejoins them, and the work is finished in 2 days more How long would it have taken A , B and C to do the piece of work, if they had all worked together?

20 A can do a piece of work in 6 days and B in 9 days They begin together But 2 days before the completion of the work, A leaves off In how many days is the work finished?

21 A is twice as good a workman as B , and together they finish a work in 8 days. In how many days can it be done by each separately?

22 8 men or 12 women or 16 children can do a piece of work in 15 days. In how many days will 3 men, 4 women and 5 children do it?

23 A is thrice as good a workman as B . If the time taken by B to do a piece of work exceed that taken by A by 8 days, find in how many days each can do it.

24 A is twice as good a workman as B and thrice as good as C . Working together for 10 days they can finish a work. They all begin together. But after working for 3 days A leaves off. After 5 days more B also leaves off. In how many days more will C finish the work?

25 A can do a piece of work in 10 days, B in 9 days and C in 12 days. All begin together, A leaves after $1\frac{1}{2}$ days, B leaves $\frac{1}{2}$ day before the work is done. How long did the work last?

26 A man can do as much work in 3 days as a boy can do in 5. How long will a man take to finish a work, $\frac{1}{3}$ of which has been done by a boy in 8 days?

27 If A in 2 days can do as much work as C in 3 days and B in 5 days as much as C in 4 days, what time will B require to execute a piece of work which A can accomplish in 6 weeks?

28 If A can do as much work in 5 hours as B can do in 6 hours, or as C can do in 9 hours, how long will it take A to complete a piece of work, one half of which has been done by B working 12 hours, and C working 24 hours?

315 Equations A statement of the equality of two arithmetical expressions is called an **arithmetical equation**.

Thus, $8 = 5 + 3$ is an *arithmetical equation*, for it asserts that 8 is equal to the sum of 5 and 3. The numbers 8, 5 and 3 are called **terms** of the equation. If one of the terms be unknown, it can be easily found from the above statement.

316 Although equation is an instrument of great power in all mathematical calculations, yet it is surprising to see that in no text-book of Arithmetic the method of solution by equations has received due favour. The following simple results are very useful in solving equations.

(i) *If equals be added to equals the sums are equal*

Thus, $15 - 3 = 12$, $\therefore 15 = 12 + 3$, (adding 3 to each side of the equation)

(ii) *If equals be taken from equals the remainders are equal*

Thus, $15 = 12 + 3$, $\therefore 15 - 3 = 12$, (taking 3 from each side of the equation)

(iii) *If equals be multiplied by equals the products are equal*

Thus, $15 = 12 + 3$, $15 \times 4 = (12 + 3) \times 4$, (multiplying each side by 4)

(iv) *If equals be divided by equals the quotients are equal*

Thus, $15 = 12 + 3$, $\therefore 15 \div 3 = (12 + 3) \div 3$, (dividing each side by 3)

Hence from (i) and (ii) we see that *any term of an equation may be transferred from one side of the equation to the other, if its sign be changed, plus becoming minus and minus becoming plus*

317 In a problem, the number to be found is called the **unknown quantity** or **unknown term**, and the numbers given are called the **known quantities** or **known terms**. To combine them and thus reduce their number, we transpose all the terms into which the unknown quantity enters to one side of the equation and the known terms to the other side, changing the sign of each term so transposed

Ex 1 If to the sum of $\frac{1}{3}$ and $\frac{1}{4}$ of a number 5 be added, the sum is 19, find the number

$(\frac{1}{3} + \frac{1}{4})$ of the number $+ 5 = 19$, $\therefore \frac{7}{12}$ of the number $+ 5 = 19$

Transposing the terms, we have

$\frac{7}{12}$ of the number $= 19 - 5 = 14$, number $= 14 \times \frac{12}{7} = \underline{24}$ Ans

Ex 2 What is the number from which if you take away 15, the remainder is $\frac{2}{3}$ of the original number?

The number $- 15 = \frac{2}{3}$ of the number

Transposing the terms, we have

the number $- \frac{2}{3}$ of the number $= 15$,

$\therefore \frac{1}{3}$ of the number $= 15$, \therefore the number $= 3 \times 15 = \underline{45}$ Ans

Ex 3 A boy loses $\frac{1}{4}$ of his money, and then gains 6ps, he then loses $\frac{1}{3}$ of what he has, and then gains 4ps, he afterwards loses $\frac{1}{5}$ of what he has, and then finds that he has 6s 1ps left. How much had he at first?

$\frac{1}{4}$ of the money is lost, $\frac{3}{4}$ of it remains, 6ps is then gained;

\therefore money now remaining $= \frac{3}{4}$ of original money $+ 6ps$, of this $\frac{1}{3}$ is lost,

$\therefore \frac{2}{3}$ of $(\frac{3}{4}$ of original money $+ 6ps)$ remains, 4ps is then gained

\therefore money now remaining $= \frac{2}{3}$ of $(\frac{3}{4}$ of original money $+ 6ps) + 4ps$, of this amount $\frac{1}{5}$ is lost,

$\therefore \frac{4}{5}$ of $[\frac{2}{3}$ of $(\frac{3}{4}$ of original money $+ 6ps) + 4ps]$ remains

$= \frac{4}{5}$ of $(\frac{1}{2}$ of original money $+ 4ps + 4ps)$,

$= \frac{1}{5}$ of original money $+ 5ps$,

$\therefore \frac{1}{5}$ of original money $+ 5ps = 6s 1ps$,

$\therefore \frac{1}{5}$ of original money $= 25ps - 5ps = 20ps$

\therefore original money $= 20ps \times \frac{1}{5} = 4ps = \underline{Re 1}$ Ans

Ex 4 From a tank $\frac{4}{5}$ ths full of water 12 gals are drawn, and the tank is then found to be $10\frac{1}{2}$ gals more than half full, find how many gals it will hold

After drawing 12 gals the quantity of water remaining = $\frac{4}{5}$ of tank - 12 gals, and it is then found that the tank is $10\frac{1}{2}$ gals more than half full,

$$\therefore \frac{4}{5} \text{ of tank} - 12 \text{ gals} = \frac{1}{2} \text{ of tank} + 10\frac{1}{2} \text{ gals}$$

$$\frac{4}{5} \text{ of tank} - \frac{1}{2} \text{ of tank} = 10\frac{1}{2} \text{ gals} + 12 \text{ gals} = 22\frac{1}{2} \text{ gals},$$

$$\therefore \frac{1}{10} \text{ of tank} = 22\frac{1}{2} \text{ gals}, \therefore \text{ tank holds } 22\frac{1}{2} \times 10 \text{ or } \underline{225} \text{ gals Ans}$$

Examples XCII

1 If to $\frac{3}{4}$ of a number 18 be added the sum is 42, find the number

2 If to the sum of $\frac{3}{4}$ and $\frac{1}{8}$ of a number 34 be added the sum is 128, find the number

3 If from the sum of $\frac{5}{6}$ and $\frac{1}{10}$ of a number 41 be taken the remainder is 97, find the number

4 What is the number to which if you add 60 the sum is 5 times the original number?

5 There is a number, to which 3 is added and $\frac{1}{10}$ of the result taken, to this 5 is added and $\frac{1}{8}$ of the result taken, then the result is $1\frac{1}{2}$, what is the number?

6 The sum of two numbers is 5760, and their difference is equal to one-third of the greater. What are the numbers?

7 The sum of four fractions is $2\frac{1}{3}$, and one common result is obtained by adding the fraction $\frac{3}{8}$ to the first, subtracting $\frac{1}{4}$ from the second, multiplying the third by $\frac{2}{3}$ and dividing the fourth by $\frac{1}{11}$. Find the four fractions

8 A person after paying away one-third of his money together with Rs 10, finds that he has remaining Rs 15 more than its half, what money had he?

9 A spends $\frac{3}{8}$ of his money and then earns Rs 5, he afterwards spends $\frac{1}{4}$ of what he then has, and has then Rs 10 8a left, find how much he had at first

10 Out of $\frac{3}{4}$ of my income I pay to one person Rs 100 and to another Rs 150, and then find that I have Rs 50 less than $\frac{1}{2}$ of my income left, find my income

11 Out of a cask two thirds full of wine 8 gals are drawn, and it is then found to be 2 gals less than half full, how many gals is the cask able to hold?

12 An army in a defeat loses $\frac{1}{4}$ of its number and 8000 prisoners, after being reinforced by 6000 men it again loses $\frac{1}{4}$ of its number in retreat, and 36000 are then left, what was the original force?

318 Irregular Distributions

Again means a second time *As much again* means as much once and as much a second time, (*i.e.*) twice as much *Half as much again* means as much once and half as much a second time, *i.e.*, $1\frac{1}{2}$ times as much

Proceed as in the following Examples

Ex 1 Divide Rs 11875 among *A*, *B* and *C* so that as often as *A* gets Rs 4, *B* shall get Rs 3, and as often as *B* gets Rs 6, *C* shall get Rs 5

As often as *A* gets Rs 4, *B* gets Rs 3, \therefore *B*'s share = $\frac{3}{4}$ of *A*'s

As often as *B* gets Rs 6, *C* gets Rs 5, \therefore *C*'s share = $\frac{5}{6}$ of *B*'s

\therefore *C*'s share = $\frac{5}{6}$ of $\frac{3}{4}$ of *A*'s = $\frac{5}{8}$ of *A*'s,

\therefore *A*'s share + *B*'s + *C*'s = $(1 + \frac{3}{4} + \frac{5}{8})$ of *A*'s = $2\frac{3}{8}$ times *A*'s share

Hence $2\frac{3}{8}$ times *A*'s share = Rs 11875,

A's share = Rs $11875 - 2\frac{3}{8} =$ Rs 5000

B's share = $\frac{3}{4}$ of Rs 5000 = Rs 3750

and *C*'s share = $\frac{5}{8}$ of Rs 5000 = Rs 3125

Otherwise thus If *A* gets Rs 8, *B* gets Rs 6 and *C* gets Rs 5

Now, $8 + 6 + 5 = 19$, and $11875 - 19 = 625$

A gets $\frac{8}{19}$ of Rs 11875 = Rs $8 \times 625 =$ Rs 5000, &c

Ex 2 Divide Rs 640 among *A*, *B* and *C*, so that *A* may have 3 times as much as *B*, and *C* $\frac{1}{2}$ of what *A* and *B* together have

A's share = 3 times *B*'s share, *C*'s share = $\frac{1}{2}(\textit{A's} + \textit{B's})$

C's share = $\frac{1}{2}(3 \textit{B's} + \textit{B's}) = \frac{4}{2} \textit{B's}$

A's share + *B*'s + *C*'s = $(3 + 1 + \frac{4}{2})$ of *B*'s = $5\frac{1}{2}$ of *B*'s share

Hence $5\frac{1}{2}$ of *B*'s share = Rs 640, \therefore *B*'s share = Rs $640 - 5\frac{1}{2} =$ Rs 120

A's share = Rs $120 \times 3 =$ Rs 360 and *C*'s = $\frac{1}{2} \times$ Rs 120 = Rs 160

Ex 3 The sum of Rs 155 is to be divided amongst 3 men, 5 women and 8 boys, so that for every 3*a* a man gets, a woman gets 2*a*, and a boy 1*a* 6*p*, find the share of each

A woman's share = $\frac{2}{3}$ of a man's, a boy's share = $\frac{1}{3}$ of a man's,

a man's share + a woman's + a boy's = $(1 + \frac{2}{3} + \frac{1}{3})$ of a man's,

3 men's shares + 5 women's + 8 boys' = $(3 + \frac{10}{3} + 4)$ of a man's
= $10\frac{1}{3}$ times a man's share

Hence $10\frac{1}{3}$ times a man's share = Rs 155,

a man's share = Rs $155 - 10\frac{1}{3} =$ Rs 15, a woman's share = $\frac{2}{3}$ of Rs 15 = Rs 10 and a boy's share = $\frac{1}{3}$ of Rs 15 = Rs 5

Ex 4 Divide Rs 8424 among *A*, *B* and *C*, so that *A* shall receive $\frac{4}{5}$ as much as *B* and *C* together, and *B* $\frac{1}{5}$ of what *A* and *C* together receive

$$\begin{aligned}
 A's \text{ share} &= \frac{4}{5} \text{ of } (B's + C's), \text{ and } B's \text{ share} = \frac{4}{5} \text{ of } (A's + C's) \\
 \therefore A's \text{ share} &= \frac{4}{5} B's + \frac{4}{5} C's = \frac{4}{5} \text{ of } \frac{4}{5} \text{ of } (A's + C's) + \frac{4}{5} C's = \frac{16}{25} \text{ of } \\
 (A's + C's) + \frac{4}{5} C's &= \frac{16}{25} A's + \frac{16}{25} C's + \frac{4}{5} C's = \frac{16}{25} A's + \frac{24}{25} C's, \\
 A's - \frac{16}{25} A's &= \frac{9}{25} C's, \text{ or } \frac{24}{25} A's = \frac{9}{25} C's, \quad A's = \frac{4}{5} \times \frac{4}{5} C's = \frac{16}{25} C's \\
 \therefore B's &= \frac{4}{5} A's + \frac{4}{5} C's = \frac{4}{5} \times \frac{16}{25} C's + \frac{4}{5} C's = \frac{64}{125} C's + \frac{100}{125} C's = \frac{164}{125} C's \\
 \therefore A's \text{ share} + B's + C's &= (\frac{16}{25} + \frac{164}{125} + 1) \text{ of } C's = \frac{241}{125} \text{ of } C's \text{ share}, \\
 \text{hence } \frac{1}{\frac{241}{125}} \text{ of } C's \text{ share} &= Rs \ 8424 \\
 \therefore C's \text{ share} &= Rs \ 8424 \times \frac{125}{241} = Rs \ 2088, \\
 \therefore A's \text{ share} &= \frac{16}{25} \text{ of } Rs \ 2088 = \underline{Rs \ 3592}, \\
 \text{and } B's \text{ share} &= \frac{164}{125} \text{ of } Rs \ 2088 = \underline{Rs \ 3744} \quad \left. \vphantom{\begin{aligned} \therefore C's \text{ share} \\ \therefore A's \text{ share} \\ \text{and } B's \text{ share} \end{aligned}} \right\} \text{ Ans}
 \end{aligned}$$

Examples XCIII

1 Divide Rs 6488 7a 10p amongst three persons *A*, *B* and *C* so that $\frac{1}{7}$ of *A*'s share = $\frac{1}{2}$ of *B*'s = $\frac{1}{8}$ of *C*'s

2 Divide Rs 75 8a between *A*, *B* and *C* giving *B* half as much again as *A* less Re 1, and *C* as much as *A* and *B* together

3 Divide Rs 1400 among *A*, *B* and *C* in such a manner that as often as *A* gets Rs 5, *B* shall get Rs 4, and as often as *B* gets Rs 3, *C* shall get Rs 2

4 Divide Rs 352 9a among *A*, *B* and *C*, so that *B* may get twice, and *C* 3 times as much as *A*

5 Divide Rs 1800 among *A*, *B* and *C*, so that *A* shall receive 3 times as much as *B*, and *B* and *C* together $\frac{1}{2}$ as much as *A*

6 Divide Rs 12540 among *A*, *B* and *C*, so that *A* shall receive as much as *B* and *C* together, and *B* $\frac{2}{3}$ of what *A* and *C* together receive

7 Divide Rs 2000 among *A*, *B* and *C*, so that *B*'s share may be $\frac{1}{2}$ of *A*'s share, and *C*'s share $\frac{2}{3}$ of *B*'s

8 Divide Rs 95 10s 8p among 10 men, 6 women and 4 children, giving a woman 3 times as much as a child and a man twice as much as a woman

9 Divide £1650 among *A*, *B*, *C* and *D*, so that *A* may have half as much as *B*, *B* a third as much as *C* and *C* a fourth as much as *D*

10 If $\frac{3}{4}$ of *A*'s money = $\frac{2}{3}$ of *B*'s = $\frac{1}{5}$ of *C*'s and *A*, *B* and *C*'s money together amount to Rs 8260, how much has each?

11 If $\frac{1}{6}$ of *A*'s money = $\frac{1}{7}$ of *B*'s = $\frac{1}{8}$ of *C*'s = $\frac{1}{10}$ of *D*'s and *A*, *B*, *C* and *D* together have Rs 23078, determine how much money each has

12 If $\frac{1}{10}$ of *A*'s money = $\frac{1}{11}$ of *B*'s, and *C*'s money = $\frac{2}{3}$ ($\frac{1}{7}$ of

A 's + $\frac{8}{9}$ of B 's), and C 's money - A 's money = Rs 667, find how much A , B and C each has

319 Travelling round a Circle

When two or more persons start simultaneously from the same place to travel round a circular course either in the same direction or in opposite directions, (i) they should first be together again at an interval of time which is the L C M of the times during which one of the walkers gains one complete round over each of the others, for each pair will be together after this time, (ii) they should first be together at the starting post again at an interval of time which is the L C M of the times during which each makes one complete round, for in that interval each shall make an integral number of rounds

Ex 1 A can go round a circular course in 18 min, B can go round it in 24 min, and C in 32 min. If they start simultaneously from the same point and travel in the same direction, in what time will they come together again?

Take 1 for the length of the course,

then A travels $\frac{1}{18}$, B $\frac{1}{24}$ and C $\frac{1}{32}$ of the course in 1 min

A gains on B ($\frac{1}{18} - \frac{1}{24}$) or $\frac{1}{72}$ of the course in 1 min

$\therefore A$ gains on B one complete round in $(1 - \frac{1}{72})$, or 72 min

Hence A and B will be together after 72 min

Again, A gains on C ($\frac{1}{18} - \frac{1}{32}$) or $\frac{7}{720}$ of the course in 1 min

A gains on C one complete round in $(1 - \frac{7}{720})$ or $\frac{288}{7}$ min

Hence A and C will be together after $\frac{288}{7}$ min

Therefore A , B and C will be together after a time which is the

L C M of 72 and $\frac{288}{7}$, but the L C M of 72 and $\frac{288}{7}$ is 288

$\therefore A$, B and C are first together after 288 min Ans

Ex 2 In the above question, if A and B travel in the same direction but C in the opposite direction, when will they meet again? As in the above question,

A and B will be together at the end of 72 min,

Again, A and C together pass over ($\frac{1}{18} + \frac{1}{32}$) or $\frac{10}{288}$ of the course in 1 min

\therefore they come together at the end of $(1 - \frac{10}{288})$ or $\frac{288}{19}$ min

Now, the L C M of 72 and $\frac{288}{19}$ is 288,

A , B and C will be together at the end of 288 min Ans

Ex 3 A , B and C start from the same point and travel in the same direction round an island 6 miles in circumference, A at the rate of 3, B at the rate of $2\frac{1}{2}$ and C at the rate of $1\frac{1}{4}$ miles an hour in how many hours will they come together again?

A gains on B ($3 - 2\frac{1}{2}$) or $\frac{1}{2}$ mile in 1 hour, \therefore he gains 6 miles or a complete round in $(6 - \frac{1}{2})$ or 12 hours

Hence A and B are together at the end of every 12 hours

Again, A gains on C $(3 - 1\frac{1}{4})$ or $1\frac{3}{4}$ miles in 1 hour, . he gains 5 miles or a complete round in $(6 - 1\frac{1}{4})$ or $\frac{5}{\frac{3}{4}}$ hours

Hence, A and C are together at the end of every $\frac{5}{\frac{3}{4}}$ hours
Therefore A , B and C are together at the end of any number of hours which a common multiple of 12 and $\frac{5}{\frac{3}{4}}$,

but the L C M of 12 and $\frac{5}{\frac{3}{4}}$ is 24,

hence A , B and C are first together at the end of 24 hrs *Ans*

Ex 4 In the above question, when will they be together again at the starting point?

Here, A takes $\frac{5}{3}$ or 2 hrs, B $(6 - 2\frac{1}{2})$ or $1\frac{1}{2}$ hrs and C $(6 - 1\frac{1}{4})$ or $\frac{5}{\frac{3}{4}}$ hrs to make one round

Now, the L C M of 2, $1\frac{1}{2}$ and $\frac{5}{\frac{3}{4}}$ is 24,

they will be together again at the starting point 24 hrs after

Examples XCIV

1 Two persons A and B start from the same point to walk round a circular course in the same direction A takes 9 min and B takes 24 min to complete one round, in what time will they be together again?

2 Three persons, A , B and C can respectively go round a circular path in 8, 18 and 30 min. If they start simultaneously from the same point and travel in the same direction, when will they meet again?

3 A , B and C start from the same point and travel in the same direction round an island 73 miles in circumference A at the rate of 10, B at the rate of 14 and C at the rate of 26 miles a day, in how many days will they come together again?

4 There is a park $1\frac{3}{4}$ miles in circumference. Five persons start from the same point to travel round it in the same direction at the respective rates of 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$ and 5 miles per hour. When will they be together again at the starting point?

5 A , B and C start from the same point and travel in the same direction round an island 36 miles in circumference, A at the rate of 3 miles, B at the rate of $3\frac{1}{2}$ miles and C at the rate of 4 miles an hour, when will they be together again?

6 An island is 43 miles in circumference. Three men A , B and C start from the same place to walk round it, at the rates of 4, $4\frac{1}{2}$ and $5\frac{1}{2}$ miles per hour respectively. In how many hours will they come together again, supposing them to travel in the same direction?

7 In the above question, if A and B travel in the same direction and C in the opposite direction, when will they come together again for the first time?

8 An island is 120 miles in circumference. Three persons A , B and C start from the same place to walk round at the respective

rates of 10, 12 and 15 miles per hour. When will they next meet, supposing (i) that they walk in the same direction, (ii) that A walks in one direction and B and C in opposite directions?

320 Chain Rule If we wish to express one quantity A in terms of another quantity R , and have data from which we can form the following series of relations *viz*,

$$a A = m M \quad (1)$$

$$b M = n N \quad (2)$$

$$c N = p P \quad (3)$$

$$d P = q Q \quad (4)$$

$$e Q = r R \quad (5)$$

which may be as numerous as we choose, then will

$$A = \frac{mnpqr}{abcde} R$$

Hence we see that the quantity required is found by dividing the product of the numbers on the right-hand side of these equations by the numbers on the left-hand side

Ex 1 If 3 lbs of tea be worth 4 lbs of coffee, and 6 lbs of coffee be worth 20 lbs of sugar, and 15 lbs of sugar be worth 24 lbs of rice, how many lbs of rice are equal to 18 lbs of tea?

$$\text{lbs reqd rice} = 18 \text{ lbs tea,}$$

$$3 \text{ lbs tea} = 4 \text{ lbs coffee,}$$

$$6 \text{ lbs coffee} = 20 \text{ lbs sugar,}$$

$$15 \text{ lbs Sugar} = 24 \text{ lbs rice,}$$

$$\therefore \text{lbs reqd rice} = \frac{18 \times 4 \times 20 \times 24}{3 \times 6 \times 15} = \underline{128}$$

321 In the preceding equations the quantity on the right-hand side of one equation is of the *same kind* as that on the left-hand side of the next equation, and thus the Chain of quantities from one kind to another is unbroken. And not only must they be of the *same kind* but also of the *same denomination*, for if not, the one or more missing links must be supplied

Ex 2 If 3 lbs of rice be worth 5 oz of tea, and 4 lbs of tea be worth 9 lbs of coffee, how many lbs of coffee are worth 48 lbs of rice?

Here, we must either supply the missing link 16 oz tea = 1 lb tea, or we must express 5 oz tea as $\frac{5}{16}$ lb tea, so that we have

$$\text{lbs reqd coffee} = 48 \text{ lbs rice,}$$

$$3 \text{ lbs rice} = 5 \text{ oz tea,}$$

$$16 \text{ oz tea} = 1 \text{ lb tea,}$$

$$4 \text{ lbs tea} = 9 \text{ lbs coffee,}$$

$$\text{lbs reqd coffee} = \frac{48 \times 5 \times 1 \times 9}{3 \times 16 \times 4}$$

$$= \underline{11\frac{1}{4}}$$

$$\text{lbs reqd coffee} = 48 \text{ lbs rice,}$$

$$3 \text{ lbs rice} = \frac{5}{16} \text{ lb tea,}$$

$$4 \text{ lbs tea} = 9 \text{ lbs coffee,}$$

$$\text{lbs reqd coffee} = \frac{48 \times \frac{5}{16} \times 9}{3 \times 4}$$

$$= \frac{48 \times 5 \times 9}{16 \times 3 \times 4} = \underline{11\frac{1}{4}}$$

322 It is unnecessary to name the quantity on the *left hand* side of any equation, for it must be the same as the quantity on the right-hand side of the preceding equation

Ex 3 If $\frac{1}{8}$ of a sheep be worth £ $\frac{2}{3}$, and $\frac{3}{7}$ of a sheep be worth $\frac{1}{14}$ of an ox, what must be given for 100 oxen?

$$£s \text{ reqd} = 100 \text{ oxen,}$$

$$\frac{1}{14} = \frac{3}{7} \text{ sheep,}$$

$$\frac{1}{8} = £\frac{2}{3},$$

$$£s \text{ reqd} = \frac{100 \times \frac{3}{7} \times \frac{2}{3}}{\frac{1}{14} \times \frac{1}{8}} = \frac{100 \times 3 \times 2 \times 14 \times 5}{7 \times 3} = \underline{2000}$$

Ex 4 If 1 lb of standard gold, of which 11 parts out of 12 are fine gold, be worth £46 14s 6d, find the value of 595 gold rupees of Bombay, each weighing 7 dwts 10½ grs of which 187 parts are fine gold and 13 alloy

£46 14s 6d = £46 $\frac{29}{40}$ = £ $\frac{1869}{40}$, . 40 lbs standard = £1869,
7 dwts 10½ grs = 7 $\frac{7}{8}$ dwts = $\frac{119}{8}$ dwts, 16 Bombay rupees = 119 dwts,
187 + 13 = 200, . 187 parts out of 200 are fine, hence

$$£s \text{ reqd} = 595 \text{ Bombay rupees,}$$

$$16 = 119 \text{ dwts Bombay standard,}$$

$$20 \times 12 = 1 \text{ lb}$$

$$200 = 187 \text{ lbs fine,}$$

$$11 = 12 \text{ lbs English standard,}$$

$$40 = £1869,$$

$$\therefore £s \text{ reqd} = \frac{595 \times 119 \times 187 \times 12 \times 1869}{16 \times 20 \times 12 \times 200 \times 11 \times 40} = £8784\frac{91453}{1000}$$

$$= \underline{£878 \ 15s \ 8\frac{1}{2}d}$$

Examples XCV

1 When 25 yards of muslin are equal to 16 yds of calico, 21 yds of calico to 13 yds of flannel, 40 yds of flannel to 27 yds of linen, 58½ yds of linen to 28 yds of silk, 47 yds of silk to 35 yds of velvet, find how many yards of velvet are equal in value to 60 yds of muslin

2 If 16 mangoes be equal in price to 25 apples, and 18 oranges, equal to 12 mangoes, and 20 lemons equal to 27 oranges, and lemons cost 9s a dozen, what is the cost of 15 apples?

3 If 12 of *A* count for 13 of *B*, 6 of *B* for 18 of *C*, and 13 of *C* for 2 of *D*, how many of *A* count for 100 of *D*?

4 If £3 = 20 thalers, 25 thalers = 93 francs, 27 francs = 5 scudi, and 62 scudi = 135 gulden, how many gulden = £1?

5 If 16 darics make 17 guineas, 19 guineas make 24 pistoles, 31 pistoles make 38 sequins, how many sequins are there in 1581 darics?

6 If 72 carlini be worth 25 shillings, 4 shillings worth 5 francs and 8 scudi worth 45 francs, how many carlini are equal to 100 scudi?

7 If 35 metres=39 yards, and 17 metres=9 toises, and 5 plethera=124 toises, how many yards are there in 1575 plethera?

8 If 6 horses cost as much as 24 cows, 10 cows as much as 8 buffaloes, 4 buffaloes as much as 15 asses, 8 asses as much as 32 sheep, and if the price of 9 sheep be Rs 25, find the cost of 8 horses

9 If $\frac{1}{4}$ of a sheep be worth £ $\frac{3}{8}$, and $\frac{3}{8}$ of a sheep worth $\frac{1}{4}$ of an ox, how much must be given for 300 oxen?

10 If 40 lbs of standard gold, of which 11 parts out of 12 are fine, be coined into 1869 sovereigns, how many grains of pure gold are there in 1 sovereign?

11 If 1 lb of standard gold, of which 11 parts out of 12 are fine, be worth £46 14s 6d, find the value of 550 Madras gold rupees, each weighing 7 dwts 12 grs, of which 916 parts out of 1000 are fine

12 If 1 lb of standard silver, of which 37 parts out of 40 are fine, be worth 66s, find the value of an Arcot Rupee, weighing 7 dwts 9 grs, of which 941 parts out of 1000 are fine

Examples worked out

Ex 1 What least number must be added to $8\frac{1}{9}$, that the result being divided by $1\frac{2}{3}$, the quotient shall be an integer?

$$8\frac{1}{9} - 1\frac{2}{3} = \frac{73}{9} - \frac{5}{3} = \frac{73}{9} \times \frac{3}{3} = \frac{73}{3} = 41\frac{2}{3}$$

Now, the least number that should be added to $41\frac{2}{3}$ to make it an integer is $\frac{1}{3}$, for $\frac{1}{3} + \frac{2}{3} = 1$

Then the question reduces to "What number divided by $1\frac{2}{3}$ will give $\frac{1}{3}$ as quotient?"

$$\text{Hence the required number} = \frac{1}{3} \times 1\frac{2}{3} = \frac{1}{3} \times \frac{5}{3} = \frac{5}{9} \quad \text{Ans}$$

Ex 2 Find two least integers such that $\frac{5}{8}$ of the first shall be equal to $\frac{7}{8}$ of the second

If $\frac{5}{8}$ of 1st number be = 1, then also $\frac{7}{8}$ of 2nd number = 1

$$\therefore \text{1st number} = (1 - \frac{5}{8}) = \frac{3}{8}, \text{ and 2nd number} = (1 - \frac{7}{8}) = \frac{1}{8}$$

Now to transform these fractions to least integers, multiply each of them by the L C M of their denominators, and divide the numbers thus found by their G C M

The L C M of 8 and 8 is 35, \therefore from 1st we have $\frac{3}{8} \times 35 = 42$, and from 2nd $\frac{1}{8} \times 35 = 40$ Now the G C M of 42 and 40 is 2

$$\text{Hence the numbers are } \frac{42}{2} \text{ and } \frac{40}{2}, \text{ or } 21 \text{ and } 20 \quad \text{Ans}$$

Ex 3 By selling an article for £12 7s 6d, I cleared $\frac{1}{5}$ of what it cost me, what was the original cost?

Taking 1 for the original cost, the gain is $\frac{1}{5}$, and the selling price $(1 + \frac{1}{5})$ or $\frac{6}{5}$

$$\text{of the original cost} = \text{£}12 \text{ 7s } 6d,$$

$$\text{the original cost} = \text{£}12 \text{ 7s } 6d \div \frac{6}{5} = \text{£}12 \text{ 7s } 6d \times \frac{5}{6}$$

$$= \text{£}10 \text{ 7s } 6d \times 5 = \text{£}6 \text{ 17s } 6d \quad \text{Ans}$$

Ex 4 By selling 15 seers of tea at Rs 5 4a per seer, a grocer clears $\frac{1}{8}$ of his outlay. He then raises the price to Rs 6 per seer and sells 50 seers more. What does he gain on the whole outlay for 65 seers?

Taking 1 for the original cost, the selling price is $(1 + \frac{1}{8})$ or $\frac{9}{8}$
 \therefore the original cost = Rs 5 4a $-\frac{1}{8}$ = Rs 4 10a 8p
 \therefore in the 1st case gain per seer = Rs 5 4a $-$ Rs 4 10a 8p = 9a 4p
 in the 2nd case = Rs 6 $-$ Rs 4 10a 8p = Re 1 5a 4p
 Now, gain on 15 seers = 9a 4p \times 15 = Rs 8 12a
 and gain on 50 seers = Re 1 5a 4p \times 50 = Rs 66 10a 8p
 \therefore his whole gain = Rs 75 6a 8p Ans

Ex 5 Find the least number of sovereigns that contains an exact number of 20-franc pieces of 15s 11 $\frac{1}{2}$ d each

Here, 15s 11 $\frac{1}{2}$ d = 191 $\frac{1}{2}$ d = $2^4 \times 11^2 \times 5$ d and a sovereign = 240d
 $\therefore 2^4 \times 11^2 \times 5$ d \times no. of 20-franc pieces = 240d \times no. of sovereigns,
 \therefore no. of 20-franc pieces = $240 \times \frac{1}{7 \times 11^2 \times 5} \times$ no. of sovereigns,
 = $\frac{1}{5 \times 11^2} \times$ no. of sovereigns

Hence the least no. of sovereigns that will make an exact number of 20-franc pieces is 51 Ans

Ex 6 A man brought 4 sorts of rice at an average price of Rs 6 a maund. If the prices increase by a common difference of 5a per maund, find the cost of each sort per maund

The price of 4 sorts at Rs 6 per maund = Rs 6 \times 4 = Rs 24

Each maund of second sort cost 5a more than a md of 1st sort,
 third 10a
 fourth 15a

these 3 maunds cost Re 1 14a more

Now, leaving out this sum, the cost of 4 maunds is Rs 24 $-$ Re 1 14a or Rs 22 2a, the cost of 1 md = Rs 22 2a $-$ 4 = Rs 5 8a 6p

Hence the cost of 1 md of 1st sort = $\left. \begin{array}{l} \text{2nd sort} = \text{Rs } 5 \quad 8a \quad 6p + 5a = \text{Rs } 5 \quad 13a \quad 6p \\ \text{3rd sort} = \text{Rs } 5 \quad 13a \quad 6p + 5a = \text{Rs } 6 \quad 2a \quad 6p \\ \text{4th sort} = \text{Rs } 6 \quad 2a \quad 6p + 5a = \text{Rs } 6 \quad 7a \quad 6p \end{array} \right\} \text{Ans}$

Ex 7 A and B undertake to do a piece of work for Rs 12 8a, A can do the work alone in 20 days and B in 15 days. They work together for three days, and then with the assistance of C finish it in 5 days more. How should the sum be divided?

Here, A and B each worked for (5+3) or 8 days, and C for 5 days
 As A can do $\frac{1}{20}$ of the work in 1 day, he did $\frac{8}{20}$ or $\frac{2}{5}$ of the work in 8 days
 \therefore B $\therefore \frac{1}{15}$ he did $\frac{8}{15}$
 \therefore A and B did in 8 days ($\frac{2}{5} + \frac{8}{15}$) or $\frac{14}{15}$ of the work

Hence the work done by C in 5 days $= (1 - \frac{1}{2})$ or $\frac{1}{2}$ of the work

$$\left. \begin{array}{l} A \text{ received } \frac{2}{3} \text{ of Rs } 12 \text{ } 8a = \text{Rs } 5 \\ B \quad \frac{8}{15} \text{ of Rs } 12 \text{ } 8a = \text{Rs } 6 \text{ } 10a \text{ } 8p \\ \text{and } C \quad \frac{1}{15} \text{ of Rs } 12 \text{ } 8a = \text{Rs } 4 \end{array} \right\} \text{Ans}$$

Ex 8 A man's income from Government Securities is $\frac{3}{4}$ of what he receives from his landed property. An income-tax of $5p$ in the rupee is charged on the first and of $4p$ in the rupee on the second, and he has to pay altogether Rs 31 as income tax. Find his total income

Suppose his income from landed property to be Rs 4,
then Government Securities is Rs 3,
income-tax on 1st $= (4 \times 4)$ or $16p$ and on second $= (3 \times 5) = 15p$,
and $16p + 15p = 31p = \text{Rs } \frac{31}{100}$

• he has to pay Rs $\frac{31}{100}$ as tax on every Rs 7 of income

$$\begin{array}{ll} \text{Rs } 1 \text{ as tax} & \text{Rs } 7 \times \frac{1}{31} \\ \text{Rs } 31 \text{ as tax} & \text{Rs } 7 \times \frac{1}{31} \times 31 \end{array}$$

Hence, required income $= \text{Rs } 7 \times \frac{1}{31} \times 31 = \text{Rs } 1344$ Ans

Ex 9 A can do as much work in one day as B can do in 2 days, or as C can do in 3 days or as D can do in 4 days. They together finish a piece of work in 8 days. How many days would each take to do it singly?

Suppose A 's one day's work to be 1, then B 's one day's work $\frac{1}{2}$, C 's $\frac{1}{3}$ and D 's $\frac{1}{4}$

• A, B, C and D 's one day's work

$= (1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4})$ or $\frac{25}{12}$ times A 's work per day,

A, B, C and D 's 8 days' work $= 8 \times \frac{25}{12}$ or $\frac{50}{3}$ times A 's work per day

But A, B, C and D 's 8 days' work = whole work,

$\therefore \frac{50}{3}$ times A 's work per day = whole work,

or A 's work per day $= \frac{3}{50}$ of the whole work.

Hence A can do the whole work in $(1 - \frac{3}{50})$ or $16\frac{2}{5}$ days

Therefore B 's time $= (2 \times 16\frac{2}{5})$ or $33\frac{1}{5}$ days, C 's time $= (3 \times 16\frac{2}{5})$ or 50 days, and D 's time $= (4 \times 16\frac{2}{5})$ or $66\frac{2}{5}$ days

Examples XCVI

1 Determine the least number which must be added to $3\frac{1}{2}$ that when the result is divided by $\frac{1}{2}$, the quotient shall be an integer

2 What least number must be subtracted from $8\frac{1}{2}$, that when the difference is divided by $\frac{3}{7}$, the quotient shall be an integer?

3 If a pound weight of standard gold is worth £61 18s 9d, find the least integral number of pounds of gold that can be coined into an integral number of sovereigns

4 If the rupee is worth 1s 9d and the mohur 30s, find the least number of pounds which can be paid exactly in rupees or mohurs

5 By selling an article for Rs 460, I cleared $\frac{3}{10}$ of the prime cost Find the cost price

6 By selling a horse for Rs 2520, a man lost $\frac{1}{5}$ of what it cost him What did it cost him?

7 Find the least number of sovereigns that contains an exact number of thalers and of dollars, 48 thalers being worth £7 3s and 8 dollars £1 13s

8 A has twice as much money as B They play together, and at the end of the first game B wins from A one-third of A's money, what fraction of the sum which B now has must A win back in the second game that they may have exactly equal sums?

9 How many maunds of rice at Rs 4 per maund must a corn-merchant mix with 1 maund of rice at Rs 5 per maund, that by selling the mixed rice at Rs 4 8a per maund, he may gain $\frac{1}{10}$ of his outlay?

10 Find the least number that must be added to 75 $\frac{1}{2}$, that the sum being severally divided by $\frac{1}{2}$, $1\frac{1}{2}$, $\frac{7}{8}$ or $1\frac{9}{10}$, the quotient in each case shall be an integer

11 A and B undertake to do a piece of work for Rs 7 8a A can do it alone in 8 days and B in 6 days With the assistance of C it is finished in 3 days How should the money be divided?

12 A and B engage to do a piece of work for Rs 40 A can do it alone in 16 days and B can do it in 12 days After working together for 4 days, A leaves off, when C, who can alone finish the work in 8 days, joins How should the sum be distributed after the work is completed?

13 A man derives his income from three sources His income from Government Securities is $\frac{1}{2}$ of his income from trade, and his landed property yields an income equal to $\frac{2}{3}$ of the sum of both The rate of tax on income from trade is 6p per rupee, on Securities 5p per rupee, and on landed property 4p per rupee If his total income tax amounts to Rs 260, find his gross income

14 By selling tea at Rs 2 10a 6p per lb, a grocer clears $\frac{1}{8}$ of his outlay, he then raises the price to Rs 3 What does he clear on every Rs 200 of his outlay by this price?

15 A tradesman buys 5 mds 24 sr of goods for Rs 150 intending to gain $\frac{3}{7}$ of his outlay by the sale, but Rs 10 8a worth at this calculation being damaged, at what price shall he sell the remainder per maund, to gain as much upon the whole outlay as he intended?

16 A can do as much work in 1 day as B in 3 days C in

5 and D in 7 days They together complete a piece of work in 8 days In how many days will each do it singly ?

17 A cloth-merchant bought a bale of cloth containing 150 pieces each, of cloths $2\frac{1}{2}$, 3, $3\frac{1}{2}$, 4 and $4\frac{1}{2}$ yds in length for Rs 626 9a If the prices increase by a common difference of 3a, at what price per piece must he sell them that he may gain Rs 100 by the transaction ?

18 By selling a horse for Rs 345, I lost $\frac{2}{5}$ of the prime cost What would have been my gain had I sold it for Rs 380 ?

19 A and B can finish a piece of work in $1\frac{1}{2}$ days, A and C in 2 days and B and C in 3 days If Rs 6 be paid for the piece of work, what are a day's wages of each workman ?

20 An elastic ball after striking the ground rises to $\frac{4}{5}$ of the height from which it fell After striking the ground the third time it rises $3\frac{1}{2}$ inches from what height did it fall at first ?

Miscellaneous Examples III

1 Divide the sum of 10 and $\frac{1}{10}$ by their difference, and also the difference by their sum, and find the sum and difference of the two quotients

2 Add together $1\frac{2}{3}$, $2\frac{2}{3}$, and $3\frac{1}{3}$, multiply this sum by the product of these fractions, subtract from the result the difference of $2\frac{2}{3}$ and $1\frac{1}{2}$, and divide the remainder by the sum of $5\frac{1}{2}$ and $1\frac{1}{2}$ of $3\frac{3}{4}$

3 Simplify $\frac{3}{4}$ of $\frac{3}{4}$ of $\frac{105}{45}$ — $\frac{4}{5}$ of $(\frac{1}{2} + \frac{2}{3})$ of $\frac{2 \text{ tons } 4 \text{ cwt}}{3 \text{ tons } 6 \text{ cwt}}$

4 Divide Rs 19000 among A , B , C and D , so that B may receive $\frac{3}{4}$ of A , C $\frac{4}{5}$ of B and D $\frac{5}{6}$ of C

5 If $\frac{5}{8}$ of $\frac{9}{10}$ of A 's money = $\frac{7}{8}$ of $\frac{1}{2}$ of B 's and the difference of their moneys be Rs 291, find A 's and B 's money

6 If 3 men and 2 boys can do a piece of work in 15 days, and 2 men and 3 boys can do the same in 18 days, in what time will a man and a boy jointly do the work ?

7 In an orchard, $\frac{1}{3}$ of the trees are apple trees, $\frac{1}{4}$ pear trees, $\frac{1}{5}$ cherry trees, $\frac{1}{6}$ filbert trees, and there are 12 walnut trees, what is the number of each sort ?

8 If A can do half a piece of work in 3 hours, which is twice as much as B can do, and A , B and C can together do the whole in $2\frac{1}{2}$ hours, shew that C can do in 5 hours as much as B can do in 9 hours

9 A 38 gallon cask of wine cost a wine merchant Rs 250, but he lost 8 gallons of it by leakage, how must he sell the remainder per gallon to gain $\frac{1}{10}$ th of the prime cost ?

10 A man owns $\frac{5}{10}$ of an estate He sells $\frac{1}{2}$ of his share and then finds that his remaining share is worth Rs 525 10a What is the value of the whole property ?

11 A can do a piece of work in 8 days, B in 12 days and C in 15 days. They all work together for 3 days at the same piece of work. In what time will B finish the remaining work?

12 A grocer mixes 5 maunds of rice at Rs 4 per md with $3\frac{1}{2}$ mds at Rs 4 8a per md. At how much per maund must he sell the mixed rice that he may gain $\frac{1}{2}\%$ of his outlay?

13 If A takes 8 days to complete a piece of work, B takes 9 days to do $\frac{1}{2}$ of the same, and if B takes 10 days to complete a piece of work, C takes 8 days to do $\frac{1}{3}$ of the same. In what time will B and C together finish a work which A alone can do in 25 days?

14 Reduce $\frac{1}{6}$ of Rs 3 12a + $\frac{1}{10}$ of Rs 7 6a - $\frac{1}{10}$ of Rs 8 4a 6p to the fraction of Rs, 20 10a

15 Reduce $\frac{1}{2}\%$ of £7 6s 8d + $\frac{1}{4}\%$ of £9 13s 4d - $\frac{1}{10}\%$ of £10 3s 4d to the fraction of £25 10s

16 If 2 men and 1 boy do a piece of work in 5 days, 1 man and 2 women do it in 6 days and 1 woman and 2 boys do the same in 8 days, in what time will a man, a woman and a boy do it jointly?

17 Of the population of a certain town $\frac{1}{3}$ can read, $\frac{1}{4}$ can write, $\frac{1}{5}$ can read and write and the remaining 130 can neither read, nor write. Find the total population of the town.

18 Simplify—

$$(a) \frac{\frac{1}{2} + \frac{1}{3}}{\frac{1}{2} - \frac{1}{3}} \text{ of } \frac{8}{9} - \frac{\frac{1}{2} + \frac{1}{3}}{\frac{1}{2} - \frac{1}{3}} \text{ of } \frac{13}{14} \times \frac{\frac{1}{2} + \frac{1}{3}}{\frac{1}{2} - \frac{1}{3}} \text{ of } \frac{2}{3} \left(\frac{3}{4} + \frac{1}{5} \right)$$

$$(b) \frac{9\frac{1}{2}}{11\frac{1}{2}} \text{ of } \frac{\text{Rs } 3 \text{ } 12a}{\text{Rs } 4 \text{ } 8a} - \frac{7\frac{1}{2}}{11\frac{1}{2}} \text{ of } \frac{\text{£ } 2 \text{ } 6s}{\text{£ } 3 \text{ } 9s}$$

19 The adult population of a country is 22815210, the adult females are $\frac{1}{5}$ of the whole population, and the adult males are $\frac{1}{6}$ of the adult females, find the whole population.

20 The wages of A and B together for $22\frac{1}{2}$ days amount to the same sum as the wages of A alone for $38\frac{1}{2}$ days. For how many days will this sum pay the wages of B alone?

21 A farmer paid a corn rent of 5 qrs of wheat and 3 qrs of barley, Winchester measure. What was the value of his rent when wheat was at 60s and barley 54s per quarter, Imperial measure, it being assumed that 32 Imperial gallons are equivalent to 33 Winchester gallons?

22 A man's debts amount to $\frac{1}{3}$ of his property, but before paying them he loses $\frac{1}{4}$ of his property. afterwards he recovers a portion equal to $\frac{1}{2}$ of what he has left, and then loses $\frac{1}{2}$ of what he has got. Can he pay his debts? What part of his property remains over?

23 A man can do 4 times a certain work in 9 hours, a woman 3 times the work in 10 hours, and a child twice the work in 11 hours,

if a man, a woman and 1 child work together, in what time can they do 7 times the work ?

24 Five brothers join in paying a sum of money, the eldest pays $\frac{1}{3}$ of it, and the others pay the remainder in equal shares, and thereby each of them pays Rs 840 less than the eldest brother. What is the sum of money ?

25 If 9 men or 16 women can do a piece of work in 144 days, in what time would 7 men and 9 women do it, working together ?

26 Out of a cistern, which is $\frac{2}{3}$ full, 20 gallons are drawn, the cistern is then found to be $\frac{1}{3}$ full. How much will the cistern hold ?

27 The product of three numbers is 340, the first is $7\frac{1}{2}$, the second is less than the first by $1\frac{1}{10}$. Find the third number.

28 A owned $\frac{1}{5}$ of a mine and sold $\frac{3}{4}$ of his share to B, who sold $\frac{1}{2}$ of his share to C, who sold $\frac{2}{3}$ of his share to D, D's share was worth Rs 20565. What was the worth of B's remaining share, and what the worth of the whole mine ?

29 There are two fractions whose sum is $1\frac{1}{3}$, and whose difference is $\frac{1}{18}$, find the fractions, and the quotient of the greater by the less.

30 If a turkey cost £ $\frac{7}{8}$ and a goose £ $\frac{3}{10}$, how many turkeys and geese, an equal number of each, can be bought for £14 4s ?

31 A boy, in flying his kite, lost $\frac{3}{4}$ of the string, he then added 65 ft, and then found that it was $\frac{1}{5}$ of the original length. What was the length at first ?

32 If $2\frac{1}{2}$ of $(A + \frac{2}{3} \text{ of } A) = \frac{1}{2}$ of $(B - \frac{1}{3}B)$, find the value of A in terms of B.

33 A man bequeathed $\frac{1}{2}$ of his estate to one son, $\frac{1}{3}$ of the remainder to another son, and the balance to his widow. The sons' shares differ by Rs 1320, find the widow's share.

34 A man gives away in charity $\frac{1}{3}$ of his income, and pays $\frac{1}{10}$ of it in rates and taxes, with these deductions he has Rs 4736 8a 8p left. What is his gross income ?

35 Find the whole annual cost of a house, of which the rent is Rs 360, the poor-rate being 2a 8p in the rupee, the gas-rate $\frac{2}{3}$ of the poor-rate, and the paving-rate $\frac{1}{4}$ of the gas-rate.

36 What sum must be added to or subtracted from £12 7s 6d, so that £5 3s 4d shall be the same fraction of the sum or difference that £3 6s 8d is of £8 6s 8d ?

37 Divide Rs 4200 among A, B, C and D so that A may get twice as much as B, A and C may get thrice as much as B and A, and D may get four times as much as B and C.

38 The sum of $\frac{2}{3}$ and $\frac{1}{4}$ of a man's debts amounts to Rs 198

7a 4p and his assets are Rs 45 1a 8p, how much in the rupee will his creditors lose?

39 One-third of A's money is equal to $\frac{1}{2}$ of C's and $\frac{1}{3}$ of C's is equal to $\frac{1}{4}$ of B's, B gives to A $\frac{1}{2}$ of his money and to C $\frac{1}{4}$ of the remainder, and has 2s 6d left. What amount had each at first?

40 Express $\frac{2}{3}$ of $\frac{1}{4}$ of £1 10s + $\frac{1}{4}$ of $\frac{1}{6}$ of 5s 4d - $8\frac{1}{2}$ of $\frac{1}{44}$ of 5s 3d as the fraction of 2s 1d

41 Find the value of

$$\frac{1}{4} \text{ of } \left(\frac{4\frac{1}{2} \text{ of } 6\frac{2}{3}}{7\frac{5}{7}} \right) \times \frac{3\frac{2}{3} - 3\frac{1}{8}}{3\frac{1}{8} + 2\frac{1}{6}} \text{ of Rs } 184 \text{ 11a } 5p$$

42 What sum must be added to or subtracted from Rs 8 12a 5p so that the sum or difference shall be the same fraction of Rs 20 10a that Rs 7 6a 6p is of Rs 18 8a 3p?

43 A can do in 6 days as much work as C can do in 4 days, and B in 10 days as much as C in 8 days. What time would B require to finish a piece of work which A can do in 12 weeks?

44 When rice is at 12 sr per rupee, the expenses of a family amount to Rs 140, but they amount to Rs 134 only, when the price falls to 16 sr per rupee. What will the expenses be, when rice is at 18 sr per rupee?

45 A can do in 2 days as much work as B in 3 days, and B in 5 days as much work as C in 4 days. What time will C require to finish a piece of work which A can do in 9 days?

46 A can by himself perform a certain quantity of work in 5 days, B twice as much in 7 days, and C four times as much in 11 days, in what time can A, B and C together perform three times the original work?

47 A was owner of $\frac{1}{4}$ of a privateer, and sold $\frac{1}{4}$ of $\frac{3}{4}$ of his share for £12 $\frac{1}{4}$, what was the value of $\frac{1}{4}$ of $\frac{1}{2}$ of the vessel at the same rate?

48 How much ore must be raised, that on losing $\frac{1}{6}$ in roasting, and $\frac{1}{8}$ of the residue in smelting, there may result 506 tons of pure metal?

49 Simplify—

$$\frac{\frac{1}{2} + \frac{1}{3} - \frac{1}{6}}{\frac{1}{2} - \frac{1}{3} + \frac{1}{6}} \text{ of } \frac{1 - (\frac{1}{2} + \frac{1}{3})}{\frac{1}{2} - (\frac{1}{2} - \frac{1}{3})} + \frac{\frac{1}{2} \times \frac{1}{3} - \frac{1}{6}}{\frac{1}{2} \times (\frac{1}{2} - \frac{1}{3})} \text{ of } \frac{\frac{1}{2} - \frac{1}{3} \times \frac{1}{6}}{(\frac{1}{2} - \frac{1}{3}) \times \frac{1}{6}}$$

50 A is $1\frac{1}{2}$ times as good a working person as B, and twice as good as C. They all three can do a piece of work together in 6 days. They begin together, but after working for 2 days A goes away. After 2 days more B goes away, and C then completes the work alone. In how many days from the commencement is the whole work finished?

51 *A* and *B* are engaged to do a piece of work, which can be done by each in 15 and 20 days respectively. If *A* leaves off 3 days before the completion of the work, how should a sum of Rs 12 8a. be distributed among them?

52 *A* and *B* undertake to do a piece of work in 15 days for Rs 22 8a. After working for 12 days they call *C* to their help, and finish it in time. *A* could have done the work alone in 25 days. If they give *C* Rs 2 4a, how many days would *B* take to finish the work?

53 A man is thrice as good a workman as a boy. If the time taken by a boy to do a piece of work exceed that taken by a man by $4\frac{1}{2}$ days, find the time in which a man can do it.

54 *A* and *B* can do a piece of work in 6 days, *B* and *C* in 7 days, and *A*, *B* and *C* can do it in 4 days. How long will *A* and *C* take to do it?

55 There is a leak in the bottom of a cistern. When the cistern was in thorough repair it could be filled in $\frac{3}{4}$ of an hour. It now takes 10 min longer. If the cistern were full, how long would it be in leaking itself to become empty?

56 10 men can do a piece of work in 30 days. After working for 10 days, a certain number of men are allowed to leave off, and then the work is finished in $43\frac{1}{4}$ days from the commencement. How many men are allowed to leave off?

57 The work which can be done by a certain number of men in 60 days, can be done by 15 men more in 40 days. Find the number of men required to do the work in 60 days.

58 There are two numbers of which the difference is 91. A third number is contained in them 13 and 20 times respectively. Find the numbers.

59 The number 483 divided by another gives 4 for the quotient and 7 for the remainder, find what number, when multiplied by the remainder, will give that divisor.

60 A boy was told to divide one-half of a certain number by 7, and the other half by 9, and then to add the two quotients. To save trouble he divided the number by 8, and his result was 6 wrong. What was the number?

61 At 3 o'clock I had completed $\frac{2}{3}$ of my journey, and at 5 o'clock $\frac{2}{3}$ of the same, when did I start and arrive?

62 40 men can do a piece of work in a certain number of days, if only 30 men be employed it requires 6 days more. Find the time in which 60 men can do it.

63 20 men do a piece of work in 24 days. After working for 6 days, an additional number of men is taken for assistance, and the work is finished in 21 days from the beginning. Required the additional number of men.

64. There are 4 casks of different sizes. The 1st is filled with water, the rest are empty. The 2nd cask is filled from the 1st and $\frac{1}{2}$ ths of the original water in the 1st remains. The third is then filled from the 2nd, and $\frac{1}{4}$ th of the water in the 2nd remains. The water in the 3rd is then poured into the 4th, and fills $\frac{2}{3}$ ths of it. Had the 3rd and 4th casks been filled from the contents of the 1st, 15 seers would still have remained in the 1st. Find the size of each cask.

CHAPTER VI

The Theory of Decimals

323 In the Notation of Integers, it has been seen that the figures in the units' place alone retain their *absolute* values, whilst the *local* values of figures in other situations increase tenfold for every figure we advance towards the left-hand from that place. Therefore, in beginning at the *left* hand figure of any number and proceeding towards the *right* hand it follows that the *local* value of every figure will be tenth part of that which immediately precedes it, and if we suppose figures to be situated to the right of the units' place, and this kind of tenfold *sub division* to be extended to them, it is manifest that the local values of such figures in order from the place of units, will be a *tenth*, a *hundredth*, a *thousandth* &c, parts of their absolute values.

Hence we are enabled to represent integers and fractions by one uniform system of notation, by merely marking the *place of units*, and whilst *Integers* are expressed by figures in the units, place and in places to the *left* of it, *Fractions* will be represented by figures situated in places on the *right* of the units, called the places of *tenths*, *hundredths*, *thousandths*, &c.

324 In this manner originates the system of Decimals, being merely an extension of the Notation of Integers, and though there are decimals of all denominations as *Decimal Integers*, yet from the circumstance of the system representing only *tenth*, *hundredth*, *thousandth*, &c, parts of the unit, all *fractions* belonging to it are termed *Decimal Fractions*, in contradistinction to *Vulgar Fractions*, whereof the denominations may be any parts whatever.

Whence, Decimals may be *defined* to be *Fractions* whose denominators are 10, 100, 1000, &c, these denominators not being *written* as in *Vulgar Fractions*, but *expressed* by the position of a *dot* or *point*, called the decimal point.

I. NOTATION AND NUMERATION OF DECIMALS

325 If we suppose the digit 1 to occupy the units' place, the following scheme will point out the denominations of the figures to

the left and right of it, and it may be extended so as to include both integers and fractions of all local values whatever

&c	&c	Ten thousands	Thousands	Hundreds	Tens	Units	Tenths	Hundredths	Thousandths	Ten-thousandths	&c	&c
		5	4	3	2	1	2	3	4	5		

A mixed quantity, formed of integers and fractions is separated into its *integral* and *fractional* portions by means of the *Decimal Point* placed on the right of the units' place towards the *top*, (to distinguish it from the sign of Multiplication), which dispenses with the description of the local denominations, given above

Thus, in 54321 2345, the figures 54321 on the *left* of the point denote so many integers, and the figures 2345 on the *right* of it, so many fractions, namely, 2 *tenths*, 3 *hundredths*, 4 *thousandths*, 5 *ten-thousandths*, and so on

326 A number thus expressed, composed of units and *decimals* parts of unity, or of *decimal* parts of unity only, is called a **decimal number**, or simply a **decimal**. The part to the left of the point is called the **integral**, and to the right the **decimal** parts of the given number.

Thus, 452 3678 is a *decimal*, 452 is the *integral* part and 3678 the *decimal* part

327 From what has been said above, it appears that the *expressing* and *reading* of *Decimals* will evidently be conducted upon the respective principles of the *Notation* and *Numeration* of integers, also, inasmuch as *Integers* denote assemblages of two or more *units*, *Decimals* will represent assemblages of two or more *tenth*, *hundredth*, &c, *parts* of a unit. Thus, to express,

- (1) 45 units 3 tenths 2 hundredths 6 thousandths 8 ten-thousandths we write 45 3268
 (2) 45 units 2 hundredths 8 ten thousandths we write 45 0208
 (3) 2 hundredths 8 ten-thousandths we write 0 0208 or simply 0208

328 In reading a decimal, we read off the decimal, part as an integer annexing the denomination of its *last* figure on the right hand. Thus,

- (1) 45 3268 is read 45 and 3268 *ten-thousandths*.

(2) 3 141596 is read 3 and 141596 *in thousandths*

(3) 00047 is read 47 *hundred thousandths*

329 In practice, however, we do not annex the decimal denomination, but saying (*decimal*) *point* read off the figures of the decimal *separately* in order. Thus,

(1) 45 3268 is read 45, point 3, 2, 6, 8

(2) 3 141596 is read 3 point 1, 4, 1, 5, 9, 6

(3) 00047 is read point 0, 0, 0, 4, 7

II RELATION OF DECIMALS TO VULGAR FRACTIONS

330 From the statements made in the preceding Articles, it is obvious that every magnitude made of one or more decimals is equivalent to, and may be expressed by, one or more vulgar fractions having 10, 100, 1000, &c. for their denominators, and that all mixed quantities expressed decimally may be represented by means of whole numbers and vulgar fractions of similar denominations.

Thus, $24\ 387 = 24 + \frac{3}{10} + \frac{8}{100} + \frac{7}{1000}$, $0\ 45 = \frac{4}{10} + \frac{5}{100} + \frac{0}{1000}$

331 *To convert a decimal into an equivalent vulgar fraction*

RULE. Write down the given number for the numerator (omitting the decimal point), and for the denominator write 1 followed by as many ciphers as there are figures in the decimal part.

Ex 1 $327 = \frac{327}{1000}$, for $327 = \frac{3}{10} + \frac{27}{100} + \frac{0}{1000} = \frac{327}{1000}$

Ex 2 $0\ 459 = \frac{459}{1000}$ for $0\ 459 = \frac{4}{10} + \frac{59}{100} + \frac{0}{1000} + \frac{0}{10000} + \frac{0}{100000}$.

Ex 3 $13\ 816 = \frac{13816}{1000}$, for $13\ 816 = 13 + \frac{8}{10} + \frac{16}{100} + \frac{0}{1000}$
 $= 13\frac{816}{1000} = \frac{13816}{1000}$

In these instances we see that the reduction to a common denominator, so tedious in vulgar fractions, is entirely dispensed with, and the immediate comparison of fractional quantities is one of the great advantages of the system.

332 *Conversely*, every vulgar fraction having 10, 100, 1000, &c. for its denominator, may be immediately represented by an equivalent decimal.

RULE. Write down the numerator and by beginning at the figure on the right hand, mark off by the decimal point as many figures as there are ciphers in the denominator. If the number of figures in the numerator be less than the number of ciphers in the denominator, prefix in the numerator the necessary number of ciphers.

Ex 1 $\frac{15243}{1000} = 15\ 243$, for there are 3 0's in the denominator

Ex 2 $\frac{5243}{10000} = 0\ 5243$, for there are 4 0's in the denominator

Ex 3 $\frac{00243}{100000} = 00243$, for there are 5 0's in the denominator

Examples XCVII

1 Express as decimals —

- (1) Four tenths, eighteen-hundredths, six-thousandths
- (2) Three and seven-tenths, one and fifty-four hundredths
- (3) Twenty-four and seventy-nine thousandths, one millionth
- (4) Five-tenths, seven hundredths and nine thousandths
- (5) Eight ten thousandths, seventy millionths, five ten-millionths
- (6) Three hundred and fifteen, eight thousandths and fifty millionths

2 Express the following decimals in words —

- (1) 5, 35, 326, 1, 01, 0001, 5 37, 0025
- (2) 43268, 003405, 0000456, 98 7654321, 100001

3 Transform the following decimals into vulgar fractions —

- (1) 7, 71, 751, 1461, 03, 0037, 00057, 0000457
- (2) 37, 52 19, 475 913, 276 00483, 1475 2045709

4 Convert the following decimals into equivalent vulgar fractions in their lowest terms —

- (1) 5, 25, 75, 625, 1875, 2 56, 432, 00625, 222464
- (2) 4 375, 8125, 47 256, 4 3125, 13 00085, 0006875
- (3) 1015625, 1 075, 3 01875, 7 0046875, 13 0005859375

5 Express as mixed numbers with the fractional parts in their lowest terms —

- (1) 5 2, 6 024, 41 015, 72 75, 9 4158, 307 251, 8 9125
- (2) 376 275, 970 00004, 7321 0505, 23 067138671875

6 Express the following as decimals —

- (1) $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, $\frac{1}{10000}$, $\frac{1}{100000}$, $\frac{1}{1000000}$, $\frac{1}{10000000}$, $\frac{1}{100000000}$
- (2) $\frac{1}{10000}$, $\frac{1}{100000}$, $\frac{1}{1000000}$, $\frac{1}{10000000}$, $\frac{1}{100000000}$, $\frac{1}{1000000000}$, $\frac{1}{10000000000}$
- (3) $\frac{1}{100}$, $\frac{1}{1000}$, $\frac{1}{10000}$, $\frac{1}{100000}$, $\frac{1}{1000000}$, $\frac{1}{10000000}$, $\frac{1}{100000000}$, $\frac{1}{1000000000}$
- (4) 16 tenths, 75 hundredths, 31 thousandths, 4 ten thousandths, 275 ten thousandths, 79 millionths, 1001 ten millionths, 79 hundred millionths, two hundred and sixty one hundred-thousandths, 328 billionths

333 *Ciphers annexed to the right hand of a decimal fraction have no effect upon its value*

Thus, $37 = \frac{37}{100}$, $370 = \frac{370}{1000} = \frac{37}{100}$, $3700 = \frac{3700}{10000} = \frac{37}{100}$, and so on, as appears also from the consideration, that there are *no* thousandths, &c, in addition to the tenths and hundredths expressed by 37

334. *Hence an integer or a whole number can be expressed as a decimal by writing ciphers in the decimal part*

Thus, $317\ 000 = 317 + \frac{0}{10} + \frac{0}{100} + \frac{0}{1000} = 317 + 0 + 0 + 0 = 317$,
 $\therefore 317 = 317\ 000$

Again, $31\ 72 = 31 + \frac{7}{10} + \frac{2}{100} = 31 + \frac{72}{100}$, and

$31\ 720 = 31 + \frac{7}{10} + \frac{2}{100} + \frac{0}{1000} = 31 + \frac{72}{100}$, $\therefore 31\ 72 = 31\ 720$

335 Every cipher affixed to the left hand of a decimal fraction after the point diminishes its value tenfold

Thus, $43 = \frac{43}{100}$, $043 = \frac{43}{1000}$, $0043 = \frac{43}{10000}$, &c, where each fraction is a tenth part of that which immediately precedes it, and indeed this is evident from the circumstance of every figure being reduced *one* denomination lower by means of each cipher

336 Hence, *Multiplication* and *Division* of a decimal by 10, 100, 1000, &c, are immediately effected, by shifting the decimal point *one, two, three*, &c, places towards the *right* and *left* respectively, adding ciphers, if necessary

Ex 1 $23\ 45 \times 10 = 234\ 5$, for $23\ 45 \times 10 = \frac{2345}{100} \times 10 = \frac{2345}{10} = 234\ 5$

Ex 2 $23\ 45 \times 10000 = 234500$, for $23\ 45 \times 10000 = \frac{2345}{100} \times 10000 = 2345 \times 100 = 234500$

Ex 3 $23\ 45 \div 10 = 2\ 345$, for $23\ 45 \div 10 = \frac{2345}{100} \times \frac{1}{10} = \frac{2345}{1000} = 2\ 345$

Ex 4 $23\ 45 \div 10000 = 002345$, for $23\ 45 \div 10000 = \frac{2345}{100} \times \frac{1}{10000} = \frac{2345}{1000000} = 002345$

Examples XCVIII

1 Multiply —

- (1) 8 separately by 10, 100, 1000, 100000, 10000000
- (2) 0053 separately by 100, 10000, 1000000, 100000000
- (3) 13 0014 separately by 10, 100, 1000, 10000, 1000000
- (4) 8 003056 separately by 100, 10000, 10000000

2 Divide —

- (1) 71 separately by 10, 100, 10000, 1000000
- (2) 73 58 separately by 1000, 10000, 1000000, 10000000
- (3) 007 separately by 100, 1000, 100000, a million
- (4) 1 by 100, 001 by 10000, 5742 6 by 10000000

337 The operation of Addition, Subtraction, Multiplication and Division of decimals are performed in the same way as in the case of whole numbers Hence it is an advantage to use decimals in preference to vulgar fractions

III ADDITION OF DECIMALS

338 RULE Place the numbers so that all the decimal points may be in the same vertical line, to insure the combination of those

of the *same* denominations, and add them together as in integers, taking care to place the decimal point in the sum, immediately under those of the numbers proposed

Ex Add together 25 61, 4 805, 0096, 653 27, 23

$$\begin{array}{r}
 25\ 61 \\
 4\ 805 \\
 0096 \\
 653\ 27 \\
 23 \\
 \hline
 706\ 6946
 \end{array}
 \quad
 \begin{array}{l}
 \text{For } 25\ 61 + 4\ 805 + 0096 + 653\ 27 + 23 \\
 = \frac{2561}{100} + \frac{4805}{1000} + \frac{96}{10000} + \frac{65327}{100} + 23 \\
 = \frac{256100 + 48050 + 96 + 6532700 + 230000}{10000} \\
 = \frac{7066946}{10000} = 706\ 6946
 \end{array}$$

339 Hence, decimals are said to be reduced to a *common denominator* when ciphers are supplied so that there is the *same* number of decimal places in each

Examples XCIX

1 Add together —

- (1) 295, 3 086, 12 87, 0051, 729 54, 7 419, 3 0256
- (2) 3608 26, 360 826, 36 0826, 3 60826, 360826, 22314
- (3) 36 053, 0079, 000952, 417, 85 5803, 0000501
- (4) 16, 12 2, 371 057, 8241, 9 1, 1 235, 23 000358
- (5) 17 215, 3 0567, 009, 2 07195, 365, 54 75
- (6) 231 8, 45 001, 2 7169, 4567 21, 00087, 6 05
- (7) 20 02, 576 89174, 1 0008159, 423564, 29, 7 21685
- (8) 61, 3 16004, 0478, 21 805, 1 00006, 12 9871
- (9) 00625, 30 698, 2 7535, 19 84, 1875, 8 096
- (10) 100, 1, 01, 001, 0001, 00001, 000001, 0000001

2 Find the values of —

- (1) 69 563 + 1307 2345 + 16 27 + 18 03 + 59 327 + 116 2491 + 3 0002
- (2) 15 063 + 002857 + 308 62 + 769 3276 + 58 739127 + 69325
- (3) 77 3 + 160 6734 + 26 345 + 46 + 31 1 + 117 154 + 0002 + 2343 008
+ 1 0000123 + 213 7 + 2 913 + 14 769 + 007871
- (4) R1159 217 + R387 61 + R71 316 + R91 204 + R74 031
- (5) £573 162 + £83 017 + £92 159 + £30 031 + £99 999
- (6) 1596 131 cwt + 702 021 cwt + 170038 cwt + 319 7 cwt + 5 93 cwt.
- (7) 97 316 yds + 1597 308 yds + 316 2917 yds + 03 yd + 159 1 yds
- (8) 27 tenths + 345 hundredths + 17 thousandths + 4256 millionths

IV SUBTRACTION OF DECIMALS

340 **RULE** Place the less number under the greater as in Addition, suppose ciphers to be supplied if necessary, in the upper line,

and the difference, found as in integers, will have as many decimal places as are contained in each, either expressed or understood

Ex 1 Subtract 34 917 from 41 62

$$\begin{array}{r} 41\ 62 \\ 34\ 917 \\ \hline 6\ 703 \end{array} \quad \text{For } 41\ 62 - 34\ 917 = \frac{4162}{100} - \frac{34917}{1000} = \frac{41620 - 34917}{1000} = \frac{6703}{1000} = 6\ 703$$

Ex 2 Is 90437532 more nearly represented by 90438 or by 90437?

$$90438 - 90437532 = 00000468, \quad 90437532 - 90437 = 00000532.$$

90438 is nearer to 90437532 than 90437

Examples C

1. Subtract —

- (1) 3806 from 57031, 7 998 from 19 201; 3 4796 from 56 036
- (2) 013096 from 13096, 21968 from 1 026103, 6 90086 from 7
- (3) 99999 from 9, 00071961 from 03107, 5968 from 1 11315
- (4) 01 from 1, 0009 from 001, 672163 from 1 29613

2 Find the difference between —

- (1) 27 903 and 054, 7295 06 and 254 738, 35 08989 and 3 508989
- (2) 2 057 and 1 0097, 3 025 and 003025, 7053 and 6729
- (3) 5 0009 and 089898, 136 159 and 136 0159, 13 and 5 90516

3 Find the values of —

- (1) 1500 5 - 714 286, 15 903 - 4 696843, 001 - 00001
- (2) R45 21 - R38 793, R8 264 - R6 03176, R5 71021 - R2 369684.
- (3) £83 6 - £83 47916, £70 151 - £15 8261, £70 107 - £69 89706
- (4) 64 mds - 000064 md, 23 5 tons - 9876 ton, 1 44 ft - 00144 ft.

4 What number subtracted from 13 007 leaves 3 594?

5 What number added to 13 265 makes up 100 0008?

6 Simplify —

- (1) 5 - 3 22 + 2 333 - 1 4444, 2 194 + 15 367 - 10 009 - 11 25 + 5 8
- (2) 227 9 - (420 315 + 27 291) + 865 21 - 1 057
- (3) 17 073 + 1 3591 - 10 84 - (11 03796 - 15 8 + 6 9)
- (4) 105 09 - 211 748 - 21 1748 - 15 73241 + 670 6 - 0053

7 Find the complement of 7781513, 000456, 98 654321; 9542 425, 998 899, and 00001 (See Art 58)

8 Whether is 3 1415926535 more accurately represented by 3 1415926 or by 3 1415927?

9 Express in the decimal notation, the value of 8 0625 - 6 04 - 00375 + 1 09236 - $\frac{25679}{10000}$

V MULTIPLICATION OF DECIMALS

341 RULE Multiply together the numbers proposed as if they were integers, and the product will contain as many places of decimals, as there are decimal places in the multiplicand and multiplier together. If there are not figures enough, prefix the necessary number of ciphers

Ex 1 Multiply 627 by 1.59

627 The number of decimal places in the multiplicand and
1.59 multiplier is 3 and 2 respectively, therefore the number
5643 in the product is $3+2=5$
3135 \therefore the required product = 99693
627 For $627 \times 1.59 = \frac{627}{1000} \times \frac{159}{100} = \frac{99693}{100000} = 99693$
99693

Ex 2 Multiply 7.5 by 0.00084

7.5 The number of decimal places in the multiplicand and
0.00084 multiplier is 1 and 6 respectively, therefore the number
300 in the product is $1+6=7$. But there are only 4 figures
600 in the product therefore prefix 3 ciphers
6300 the product = 0006300 = 00063

Examples CI

1 Multiply —

- (1) 718 by 57, 168 by 0024, 114 by 0625, 125 by 06216
- (2) 270.56 by 37025, 00579 by 37958, 36.2165 by 229
- (3) 421.619 by 547, 34.6875 by 119.808, 007853 by 00476
- (4) 384.759375 by 00032, 00082175 by 2.38645, 002 by 0004
- (5) 0000251472 by 0625, 148.7076 by 007087, 170.71 by 0325
- (6) 00015625 by 8192, 00025 by 0000625, 00711858 by 00024

2 Find the values of —

- (1) 3.51×0.75 , 0.167×0.08446 , 354178×0.05 , 3.12×2.0001
- (2) 3.005×40.23 , 1.279×0.006787 , $35.01 \times 0.008 \times 5.25$
- (3) $275 \times 2.75 \times 27.5$, $3.24 \times 0.026 \times 2.9375$, $11.01 \times 110 \times 1102$
- (4) $1.02 \times 102 \times 10.2 \times 102$, $5.107 \times 0.5107 \times 0.5 \times 700$
- (5) $4 \times 0.5 \times 0.06 \times 0.007 \times 800000$, $0.1 \times 0.4 \times 4 \times 0.004 \times 40000$
- (6) $0.1 \times 0.01 \times 0.001 \times 0.0001 \times 100000$, $8.45 \times 0.017 \times 7.4 \times 0.9 \times 10000$

3 Find the values of —

- (1) $7.94 \times 2.638 + 32.56 \times 0.0457 - 0.07853 \times 0.0476 - 0.00076 \times 18.4$
- (2) $592.9 \times 61.6 \div 0.064 + 1562.5 \times 0.625 \times 5 - 45.08 \times 64.4 \times 0.92$
- (3) $(37.1 - 19.08) \times 703$, $37.1 - 19.08 \times 703$, $(0.5)^2 + (0.25)^2 + 0.0025$
- (4) $(36.73)^2 - (25.894)^2$, $(888)^2 - (8008)^2$, $(3.025)^2 - 3.025 \times 0.03025$

4 Multiply 325 tenths by 547 millionths, 128 thousandths by 78125 ten millionths

VI DIVISION OF DECIMALS.

342 *When the divisor is an integer*

RULE Divide, as if dividend and divisor were whole numbers, and when, in the process of division, the decimal point of the dividend is arrived at, place a decimal point in the quotient. If the division do not terminate with the last digit of the dividend, annex ciphers to the dividend and continue the operation until it terminates or the required number of decimal places in the quotient is obtained

Ex Divide 187.5 by 25, 1770.89 by 4735 and 3217 by 625

(1) 25)187.5(7.5 (2) 4735)1770.89(374 (3) 625)3217.0000(5.1472

175

125

125

∴ the quotient = 7.5

14205

35039

33145

18940

18940

∴ the quotient = 374

3125

920

625

2950

2500

4500

4375

(1) For 187.5—25

$$= 187.5 \times \frac{1}{25} = 187.5 \times \frac{1}{10} \times \frac{1}{2.5}$$

$$= 75 \times \frac{1}{10} = 7.5$$

1250

1250

the quotient = 5.1472

343 *When the divisor does not exceed 20, or when it can easily be separated into factors none of which exceeds 20, the division should be performed by the method of short division*

Ex Divide 56787 by 12, and 121968 by 693

(1) 12)5678700

473225 Ans

(2) 693 { 7)121968

9) 17424

11) 01936

00176 Ans

344 *When the divisor is a decimal*

RULE Make the divisor a whole number by removing its decimal point altogether, and shift the decimal point of the dividend as many places to the right as there were decimal figures in the divisor, annexing for this purpose ciphers, if necessary, to the dividend. Then, divide as if the terms were integers, in the quotient, count off as many decimal places from the right as there are in the altered dividend, prefixing ciphers, if necessary

Ex Divide 10 836 by 5 16, 1875 by 2 5 and 62 5 by 025

$$\begin{array}{r}
 \text{(1) } 5 \overline{)10836} \\
 \underline{516} \\
 516 \\
 \underline{516} \\
 1032 \\
 \underline{1032} \\
 516 \\
 \underline{516} \\
 0
 \end{array}
 \quad
 \begin{array}{r}
 \text{(2) } 2 \overline{)1875} \\
 \underline{25} \\
 25 \\
 \underline{25} \\
 175 \\
 \underline{175} \\
 125 \\
 \underline{125} \\
 0
 \end{array}
 \quad
 \begin{array}{r}
 \text{(3) } 025 \overline{)625} \\
 \underline{25} \\
 25 \\
 \underline{25} \\
 50 \\
 \underline{50} \\
 125 \\
 \underline{125} \\
 0
 \end{array}$$

the quotient = 21

the quotient = 075

the quotient = 2500

345 In the course of the division, if there be any remainder after the last figure from the altered dividend has been brought down, add ciphers to the right of the dividend, and proceed as in Art 342

Ex Divide 01029 by 1 68

$$\begin{array}{r}
 1 \overline{)01029} \\
 \underline{168} \\
 1008 \\
 \underline{1008} \\
 210 \\
 \underline{168} \\
 420 \\
 \underline{336} \\
 840 \\
 \underline{840} \\
 0
 \end{array}$$

Here the altered dividend has 3 decimal figures, and we have added to it 3 ciphers, therefore in the quotient, we must count off 6 decimal places

the quotient = 006125

$$\begin{aligned}
 \text{For } 01029 - 1 \overline{)68} &= \frac{1029}{10000} - \frac{168}{10000} = \frac{1029 - 168}{10000} = \frac{861}{10000} \\
 &= 6 \frac{1}{1000} \times \frac{1}{1000} = (6 + \frac{1}{1000}) \times \frac{1}{1000} \\
 &= \frac{6125}{10000} = 006125
 \end{aligned}$$

346 In this case also the method of short division can advantageously be employed when the divisor has been made an integer, as in Art 343

Ex Divide 90 65 by 049, and 171 99 by 27 3

$$\begin{array}{r}
 \text{(1) } 049 \overline{)9065} \\
 \underline{49} \\
 49 \\
 \underline{49} \\
 1850 \text{ Ans}
 \end{array}
 \quad
 \begin{array}{r}
 \text{(2) } 273 \overline{)17199} \\
 \underline{317199} \\
 273 \\
 \underline{715733} \\
 13 \\
 \underline{13819} \\
 63 \text{ Ans}
 \end{array}$$

347 If the division do not terminate, the quotient may be required to a given number of decimal places, as in the following examples

Ex Divide 02 by 17, 1 by 013 and 1 by 007, each to 5 places of decimals

$$\begin{array}{r}
 \text{(1) } 17 \overline{)02} \\
 \underline{17} \\
 01176 \text{ Ans}
 \end{array}
 \quad
 \begin{array}{r}
 \text{(2) } 013 \overline{)1000} \\
 \underline{13} \\
 13 \\
 \underline{13} \\
 7692307 \text{ Ans}
 \end{array}
 \quad
 \begin{array}{r}
 \text{(3) } 007 \overline{)1000} \\
 \underline{7} \\
 7 \\
 \underline{7} \\
 14285714 \text{ Ans}
 \end{array}$$

348 An integral divisor ending with ciphers may be deprived of the ciphers, if we remove the decimal point of the dividend one place to the left for every cipher withdrawn

Thus, $78 - 60 = 078 - 6$, $78 - 600 = 0078 - 6$, and so on

Ex. Divide 1 5625 by 25000, and 7 by 796 3 to 5 places of decimals

$$\begin{array}{r} (1) \quad 25000 \overline{) 1\,5625} \\ 25 \left\{ \begin{array}{l} 5 \overline{) 0015625} \\ 5 \overline{) 0003125} \\ \underline{0000625} \end{array} \right. \quad \text{Ans} \end{array}$$

$$\begin{array}{r} (2) \quad 796\,3 \overline{) 7\,0} \quad - \\ 7963 \overline{) 70\,00000} \quad \underline{00879} \quad \text{Ans} \\ 63704 \\ \underline{62960} \\ 55741 \\ \underline{72190} \\ 71667 \end{array}$$

349 In the above divisions, it should be very carefully noticed that *for each digit in the decimal part of the dividend there is a digit in the decimal part of the quotient*

Examples CII

1 Divide —

- (1) 783 5 separately by 5, 25, 125, 625 and 6250
- (2) 773 682 separately by 6, 13, 78, 169, 507 and 1014
- (3) 00750116 separately by 677, 1354, 2708 and 10832
- (4) 35 9424 by 7 02, 278831 by 653, 11 444495 by 4 735
- (5) 1 68 by 024, 971 7 by 123, 142 025 by 0437, 84 375 by 00375
- (6) 020872522 by 08635, 0020925 by 000864, 39538 by 5300
- (7) 1 by 01, 01001 by 001, 92 7 by 06, 99 by 0009, 001 by 0001
- (8) 9864 1698175 by 35 0645, 124 59993 by 3194 87
- (9) 1 365 separately by 1 25, 12 5, 00125 and 12500
- (10) 7 835 separately by 5, 25, 12 5, 6 25, 625, 0625 and 625000
- (11) 0003738028 by 0476, 0064096 by 2 003, 614 50824 by 0010201
- (12) 2 and 22 hundredths by 74 ten-thousandths

2 Find the values of (to 5 places of decimals) —

- (1) $3 \div 876$, $0257 \div 0041$, $325\,46 \div 0187$, $0719 \div 27\,53$
- (2) $5 \div 76\,91342$, $11\,121 \div 3\,4571$, $16\,1 \div 63572\,45$, $25 \div 19$
- (3) $046 \div 00762089$, $32165 \div 0035216$, $314159\,26 \div 008597$

3 Find the quotient, by *short division*, of —

- (1) 3 6288 separately by 3, 7, 9, 63, 12 6, 189 and 024
- (2) 0255 separately by 03, 005, 3 4, 60, 0102 and 2 55

4 Divide, by *short division*, to 5 places of decimals —

- (1) 009384 separately by 7, 07, 007, 1 8, 0018 and 00063
- (2) 57982 6966 by 00000076, 346 72361 by 00016

5 Find the values of —

- (1) $01385 \times 61\,37 = 2\,77$, $399 \times 007 = 000019$, $24\,01 \times 0039 = 133\,77$

decimal fraction of an *inferior* denomination is added to the value already obtained

Ex Express $\frac{1}{2}$ as a decimal as far as the sixth place

$$22 \left\{ \begin{array}{r} 2)150 \\ 11)7500000 \\ \hline 6818181 \end{array} \right. \quad \text{the decimal reqd} = \underline{681818}$$

Note The following results are useful, and should be verified and remembered —

$$\begin{array}{lllll} \frac{1}{2} = 25, & \frac{1}{4} = 5, & \frac{1}{8} = 75, & \frac{1}{16} = 125, & \frac{1}{32} = 375, \\ \frac{1}{5} = 625, & \frac{1}{25} = 875, & \frac{1}{125} = 0625, & \frac{1}{1000} = 2, & \frac{1}{78125} = 03125 \end{array}$$

Examples CIII

1 Reduce to decimals —

- (1) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}, \frac{1}{128}, \frac{1}{256}, \frac{1}{512}, \frac{1}{1024}, \frac{1}{2048}, \frac{1}{4096}, \frac{1}{8192}, \frac{1}{16384}, \frac{1}{32768}, \frac{1}{65536}, \frac{1}{131072}, \frac{1}{262144}, \frac{1}{524288}, \frac{1}{1048576}, \frac{1}{2097152}, \frac{1}{4194304}, \frac{1}{8388608}, \frac{1}{16777216}, \frac{1}{33554432}, \frac{1}{67108864}, \frac{1}{134217728}, \frac{1}{268435456}, \frac{1}{536870912}, \frac{1}{1073741824}, \frac{1}{2147483648}, \frac{1}{4294967296}, \frac{1}{8589934592}, \frac{1}{17179869184}, \frac{1}{34359738368}, \frac{1}{68719476736}, \frac{1}{137438953472}, \frac{1}{274877906944}, \frac{1}{549755813888}, \frac{1}{1099511627776}, \frac{1}{2199023255552}, \frac{1}{4398046511104}, \frac{1}{8796093022208}, \frac{1}{17592186044416}, \frac{1}{35184372088832}, \frac{1}{70368744177664}, \frac{1}{140737488355328}, \frac{1}{281474976710656}, \frac{1}{562949953421312}, \frac{1}{1125899906842624}, \frac{1}{2251799813685248}, \frac{1}{4503599627370496}, \frac{1}{9007199254740992}, \frac{1}{18014398509481984}, \frac{1}{36028797018963968}, \frac{1}{72057594037927936}, \frac{1}{144115188075855872}, \frac{1}{288230376151711744}, \frac{1}{576460752303423488}, \frac{1}{1152921504606846976}, \frac{1}{2305843009213693952}, \frac{1}{4611686018427387904}, \frac{1}{9223372036854775808}, \frac{1}{18446744073709551616}, \frac{1}{36893488147419103232}, \frac{1}{73786976294838206464}, \frac{1}{147573952589676412928}, \frac{1}{295147905179352825856}, \frac{1}{590295810358705651712}, \frac{1}{1180591620717411303424}, \frac{1}{2361183241434822606848}, \frac{1}{4722366482869645213696}, \frac{1}{9444732965739290427392}, \frac{1}{18889465931478580854784}, \frac{1}{37778931862957161709568}, \frac{1}{75557863725914323419136}, \frac{1}{151115727451828646838272}, \frac{1}{302231454903657293676544}, \frac{1}{604462909807314587353088}, \frac{1}{1208925819614629174706176}, \frac{1}{2417851639229258349412352}, \frac{1}{4835703278458516698824704}, \frac{1}{9671406556917033397649408}, \frac{1}{19342813113834066795298816}, \frac{1}{38685626227668133590597632}, \frac{1}{77371252455336267181195264}, \frac{1}{154742504910672534362390528}, \frac{1}{309485009821345068724781056}, \frac{1}{618970019642690137449562112}, \frac{1}{1237940039285380274899124224}, \frac{1}{2475880078570760549798248448}, \frac{1}{4951760157141521099596496896}, \frac{1}{9903520314283042199192993792}, \frac{1}{19807040628566084398385987584}, \frac{1}{39614081257132168796771975168}, \frac{1}{79228162514264337593543950336}, \frac{1}{158456325028528675187087900672}, \frac{1}{316912650057057350374175801344}, \frac{1}{633825300114114700748351602688}, \frac{1}{1267650600228229401496703205376}, \frac{1}{2535301200456458802993406410752}, \frac{1}{5070602400912917605986812821504}, \frac{1}{10141204801825835211973625643008}, \frac{1}{20282409603651670423947251286016}, \frac{1}{40564819207303340847894502572032}, \frac{1}{81129638414606681695789005144064}, \frac{1}{162259276829213363391578010288128}, \frac{1}{324518553658426726783156020576256}, \frac{1}{649037107316853453566312041152512}, \frac{1}{1298074214633706907132624082305024}, \frac{1}{2596148429267413814265248164610048}, \frac{1}{5192296858534827628530496329220096}, \frac{1}{10384593717069655257060992658440192}, \frac{1}{20769187434139310514121985316880384}, \frac{1}{41538374868278621028243970633760768}, \frac{1}{83076749736557242056487941267521536}, \frac{1}{166153499473114484112975882535043072}, \frac{1}{332306998946228968225951765070086144}, \frac{1}{664613997892457936451903530140172288}, \frac{1}{1329227995784915872903807060280344576}, \frac{1}{2658455991569831745807614120560689152}, \frac{1}{5316911983139663491615228241121378304}, \frac{1}{10633823966279326983230456482242756608}, \frac{1}{21267647932558653966460912964485513216}, \frac{1}{42535295865117307932921825928971026432}, \frac{1}{85070591730234615865843651857942052864}, \frac{1}{170141183460469231731687303715884105728}, \frac{1}{340282366920938463463374607431768211456}, \frac{1}{680564733841876926926749214863536422912}, \frac{1}{1361129467683753853853498429727072845824}, \frac{1}{2722258935367507707706996859454145691648}, \frac{1}{5444517870735015415413993718908291383296}, \frac{1}{10889035741470030830827987437816582766592}, \frac{1}{21778071482940061661655974875633165533184}, \frac{1}{43556142965880123323311949751266331066368}, \frac{1}{87112285931760246646623899502532662132736}, \frac{1}{174224571863520493293247799005065324265472}, \frac{1}{348449143727040986586495598010130648530944}, \frac{1}{696898287454081973172991196020261297061888}, \frac{1}{1393796574908163946345982392040522594123776}, \frac{1}{2787593149816327892691964784081045188247552}, \frac{1}{5575186299632655785383929568162090376495104}, \frac{1}{11150372599265311570767859136324180752990208}, 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\frac{1}{12855504354071922204335696738729300820177623950262342682411008}, \frac{1}{25711008708143844408671393477458601640355247900524685364822016}, \frac{1}{51422017416287688817342786954917203280710495801049370729644032}, \frac{1}{102844034832575377634685573909834406561420991602098741459288064}, \frac{1}{205688069665150755269371147819668813122841983204197482918576128}, \frac{1}{411376139330301510538742295639337626245683966408394965837152256}, \frac{1}{822752278660603021077484591278675252491367932816789931674304512}, \frac{1}{1645504557321206042154969182557350504982735865633579863348609024}, \frac{1}{3291009114642412084309938365114701009965471731267159726697218048}, \frac{1}{6582018229284824168619876730229402019930943462534319453394436096}, \frac{1}{13164036458569648337239753460458804039861886925068638906788872192}, \frac{1}{26328072917139296674479506920917608079723773850137277813577744384}, \frac{1}{52656145834278593348959013841835216159447547700274555627155488768}, \frac{1}{105312291668557186697918027683670432318895095400549111254310977536}, \frac{1}{210624583337114373395836055367340864637790190801098222508621955072}, \frac{1}{421249166674228746791672110734681729275580381602196445017243910144}, \frac{1}{842498333348457493583344221469363458551160763204392890034487820288}, \frac{1}{1684996666696914987166688442938726917102321526408785780068975640576}, \frac{1}{3369993333393829974333376885877453834204643052817571560137951281152}, \frac{1}{6739986666787659948666753771754907668409286105635143120275902562304}, \frac{1}{13479973333575319897333507543509815336818572211270286240551805124608}, \frac{1}{26959946667150639794667015087019630673637144422540572481103610249216}, \frac{1}{53919893334301279589334030174039261347274288845081144962207220498432}, \frac{1}{107839786668602559178668060348078522694548577690162289924414440996864}, \frac{1}{215679573337205118357336120696157045389097155380324579848828881993728}, \frac{1}{431359146674410236714672241392314090778194310760649159697657763987456}, \frac{1}{862718293348820473429344482784628181556388621521298319395315527974912}, \frac{1}{1725436586697640946858688965569256363112777243042596638790631055949824}, \frac{1}{3450873173395281893717377931138512726225554486085193277581262111899648}, \frac{1}{6901746346790563787434755862277025452451108972170386555162524223799296}, \frac{1}{13803492693581127574869511724554050904902217944340773110325048447598592}, \frac{1}{27606985387162255149739023449108101809804435888681546220650096895197184}, \frac{1}{55213970774324510299478046898216203619608871777363092441300193790394368}, \frac{1}{110427941548649020598956093796432407239217743554726184882600387580788736}, \frac{1}{220855883097298041197912187592864814478435487109452369765200775161577472}, \frac{1}{441711766194596082395824375185729628956870974218904739530401550323154944}, \frac{1}{883423532389192164791648750371459257913741948437809479060803100646309888}, \frac{1}{1766847064778384329583297500742918515827483896875618958121606201292619776}, \frac{1}{3533694129556768659166595001485837031654967793751237916243212402585239552}, \frac{1}{7067388259113537318333190002971674063309935587502475832486424805170479104}, \frac{1}{14134776518227074636666380005943348126619871175004951664972849610340958208}, \frac{1}{28269553036454149273332760011886696253239742350009903329945699220681916416}, \frac{1}{56539106072908298546665520023773392506479484700019806659891398441363832832}, \frac{1}{113078212145816597093331040047546785012958969400039613319782796882727665664}, \frac{1}{226156424291633194186662080095093570025917938800079226639565593765455331328}, \frac{1}{452312848583266388373324160190187140051835877600158453279131187530910662656}, \frac{1}{904625697166532776746648320380374280103671755200316906558262375061821325312}, \frac{1}{1809251394333065553493296640760748560207343510400633813116524750123642650624}, \frac{1}{3618502788666131106986593281521497120414687020801267626233049500247285301248}, \frac{1}{7237005577332262213973186563042994240829374041602535252466099000494570602496}, \frac{1}{14474011154664524427946373126085988481658748083205070504932198000989141204992}, \frac{1}{28948022309329048855892746252171976963317496166410141009864396001978282409984}, \frac{1}{57896044618658097711785492504343953926634992332820282019728792003956564819968}, \frac{1}{115792089237316195423570985008687907853269984665640564039457584007913129639936}, \frac{1}{231584178474632390847141970017375815706539969331281128078915168015826259279872}, \frac{1}{4631683569492647816942839400347516314130$

were integers, and then mark off the said number of decimal places in the result, prefixing ciphers, if necessary

Ex Find the G C M and the L C M of 16, 24 and 14

Here, the numbers are equivalent to 160, 24 and 1400

The G C M of 160, 24 and 1400 = 8, their L C M = 16800

\therefore the G C M reqd = 08, and the L C M reqd = 16800 = 168

Examples CIV

1 Find the G C M of —

(1) 13536 and 23148 (2) 42237 and 75582 (3) 36795 and 5798

(4) 3761034 and 1081 (5) 14, 18 and 024 (6) 009, 18 and 24

(7) 24, 48, 64 and 192 (8) 016, 0024, 48 and 74

2 Find the L C M of —

(1) 15, 35, 063 and 72 (2) 63, 12, 084 and 0014

(3) 24, 39 and 376 (4) 312, 0124, 341 and 372

(5) 42237 and 75582 (6) 136652 and 2468642

IX RECURRING DECIMALS

353 In the conversion of a vulgar fraction into a decimal, we find that the division performed according to the Rule laid down in Art 350 terminates in some cases and does not terminate in others. Thus, $\frac{3}{4} = 625$, and here the division terminates, but $\frac{1}{3} = 272727$, and in this case the division does not terminate and can be extended to an unlimited length. The former is called a **terminating** or **finite** decimal, and the latter a **non-terminating** decimal.

354 It has already been shewn in Art 331 that to reduce a vulgar fraction in its lowest terms to a decimal is the same as reducing it to an equivalent one having 10 or some power of 10 for its denominator. Thus, it follows that no vulgar fraction can be reduced to a terminating decimal, unless it can be expressed as one having 10 or some power of 10 for its denominator. Now, no number can, by multiplication, be made a power of 10, unless it be composed of prime factors, each of which is 2 or 5. Hence, to find whether a vulgar fraction can be expressed as a terminating decimal or not, we have the following Rule

RULE Reduce the given vulgar fraction to its lowest terms, and resolve its denominator into its prime factors, if these prime factors be only 2 and 5, it can be expressed as an exact or terminating decimal otherwise, it cannot

Ex 1 Can $\frac{3}{8}$ and $\frac{61}{1250}$ be expressed as a terminating decimal?

(1) Yes, for $50 = 2 \times 5 \times 5$, and involves factors of 2 and 5 only

(2) Yes, for $1250 = 2 \times 5^4$, and involves factors of 2 and 5 only

Ex 2 Can $\frac{137}{10}$ be expressed as a finite decimal?

No, for $576 = 2^6 \times 3^2$, and involves other factors than 2 and 5

Examples CV

1 Which of the following fractions can be expressed as finite or terminating decimals —

$\frac{1}{12}, \frac{1}{18}, \frac{1}{45}, \frac{1}{60}, \frac{1}{90}, \frac{1}{120}, \frac{1}{180}, \frac{1}{270}, \frac{1}{360}, \frac{1}{540}, \frac{1}{720}$

2 Write down those numbers between 1 and 25, of which if any one be the denominator of a fraction in its lowest terms, that fraction can be reduced to a terminating decimal

355 In non-terminating decimals the figures of the quotient must recur over and over again

Take the fraction $\frac{2}{7}$. To reduce it to a decimal, we annex ciphers to 2 and divide by 7. Since the division does not terminate, we cannot have the remainder 0, and the only *possible* remainders that can arise are 1, 2, 3, 4, 5, and 6 and consequently after *six steps at most* (after as many divisions at least as there are units in the denominator) we must come to the given numerator or to one of the remainders that has occurred before, and therefore from that point we must have a recurrence of the remainders, and therefore of the quotient figures in the same order over and over again. Thus,

7)5 0/ 714285

$$\begin{array}{r} 49 \\ 10 \\ 7 \\ \hline 30 \\ 28 \end{array} \quad \begin{array}{r} 20 \\ 14 \\ \hline 60 \\ 56 \end{array} \quad \begin{array}{r} 40 \\ 35 \\ 5 \end{array}$$

Here, after 6 figures, we get 5 for remainder, and therefore the whole process will recur again from the beginning

$$\therefore \bar{y} = 7142857142857142$$

356 When, beginning from a certain point in the decimal part of a number, the figures repeat themselves indefinitely and in the same order, the number is called a recurring, circulating, repeating or periodic decimal, and the whole set of figures which recurs constantly in the same order is called the period or repetend.

357 The *period* is termed a *simple* or a *compound repetend* according as it consists of *one* or *more* figures, and the *extent* of the period is denoted by means of *dots* (\cdot) placed over the *first* and *last* of the figures which compose it

Thus, $n = 26666 = 2 \cdot 6$, and $f = 135135 = 135$

$$S_0, \dot{5}7 = 575757 \quad , \quad \dot{0}24 = 024024024 \quad , \quad 32\dot{6} = 3262626$$

The several *periods* in the above are 6, 135, 57, 024 and 26

358 Recurring Decimals are either Pure or Mixed.

(i) A pure circulating decimal is one which recurs from the first figure of the decimal part, as, $\frac{1}{3}$, $0\bar{3}$

(ii) A **mixed circulating decimal** is one which recurs after some figures and thus consists of a *non recurring* and a *recurring* part, as, 17, 2468

359 A vulgar fraction in its lowest terms, whose denominator contains neither of the prime factors 2 and 5, produces a *pure circulating decimal*, whereas one, whose denominator contains 2 or 5 and one or more other prime factors, produces a *mixed circulating decimal*

$$\text{Thus, } \frac{2}{3} = 6, \frac{1}{7} = 142857, \frac{7}{22} = \frac{7}{2 \times 11} = 318$$

Ex 1 Convert $\frac{1}{3}$ and $\frac{4}{7}$ into decimals

(1) 3)7(2 3

$$\begin{array}{r} 6 \\ 10 \\ 9 \\ 1 \end{array}$$

Here, a repetition of 1 in the remainder gives a repetition of the figure 3 in the quotient

$$\therefore \frac{1}{3} = \underline{23}$$

(2) 27)10(148

$$\begin{array}{r} 27 \\ 130 \\ 108 \\ 220 \\ 216 \\ 4 \end{array}$$

The figure 4 occurs again in the remainder after 3 steps, therefore the digits 1, 4 and 8 must recur in the quotient

$$\therefore \frac{10}{27} = \underline{148}$$

$$\text{For } \frac{1}{3} = 2\frac{1}{3} = 2 + \frac{1}{3} = 2 + \frac{10}{30} = 2 + \frac{31}{30} = 2 + \frac{1}{10} + \frac{2}{30} = 2 + \frac{1}{10} + \frac{1}{50} = 2 + \frac{10}{100} + \frac{2}{500} = 2 + \frac{1}{10} + \frac{1}{250} = 2.33$$

$$= 2 + \frac{1}{10} + \frac{1}{100} = 2 + \frac{1}{10} + \frac{1}{100} = 2 + \frac{1}{10} + \frac{1}{100} + \frac{1}{1000} = 2.33$$

Similarly, the second can be explained

Ex 2 Reduce $\frac{5}{8}$ to a decimal

36)50(138

$$\begin{array}{r} 36 \\ 140 \\ 108 \\ 320 \\ 288 \\ 32 \end{array}$$

Here, the remainder 32 which occurred after the second step occurs again in the third, and therefore the figure 8 will recur in the quotient

$$\frac{5}{8} = \underline{138}$$

Examples CVI

Reduce to recurring decimals —

$$1 \quad \frac{1}{6}, \frac{1}{11}, \frac{1}{12}, \frac{1}{14}, \frac{1}{15}, \frac{1}{16}, \frac{1}{17}, \frac{1}{18}, \frac{1}{19}, \frac{1}{20}, \frac{1}{21}, \frac{1}{22}, \frac{1}{23}, \frac{1}{24}, \frac{1}{25}, \frac{1}{26}, \frac{1}{27}, \frac{1}{28}, \frac{1}{29}, \frac{1}{30}, \frac{1}{31}, \frac{1}{32}, \frac{1}{33}, \frac{1}{34}, \frac{1}{35}, \frac{1}{36}, \frac{1}{37}, \frac{1}{38}, \frac{1}{39}, \frac{1}{40}, \frac{1}{41}, \frac{1}{42}, \frac{1}{43}, \frac{1}{44}, \frac{1}{45}, \frac{1}{46}, \frac{1}{47}, \frac{1}{48}, \frac{1}{49}, \frac{1}{50}, \frac{1}{51}, \frac{1}{52}, \frac{1}{53}, \frac{1}{54}, \frac{1}{55}, \frac{1}{56}, \frac{1}{57}, \frac{1}{58}, \frac{1}{59}, \frac{1}{60}, \frac{1}{61}, \frac{1}{62}, \frac{1}{63}, \frac{1}{64}, \frac{1}{65}, \frac{1}{66}, \frac{1}{67}, \frac{1}{68}, \frac{1}{69}, \frac{1}{70}, \frac{1}{71}, \frac{1}{72}, \frac{1}{73}, \frac{1}{74}, \frac{1}{75}, \frac{1}{76}, \frac{1}{77}, \frac{1}{78}, \frac{1}{79}, \frac{1}{80}, \frac{1}{81}, \frac{1}{82}, \frac{1}{83}, \frac{1}{84}, \frac{1}{85}, \frac{1}{86}, \frac{1}{87}, \frac{1}{88}, \frac{1}{89}, \frac{1}{90}, \frac{1}{91}, \frac{1}{92}, \frac{1}{93}, \frac{1}{94}, \frac{1}{95}, \frac{1}{96}, \frac{1}{97}, \frac{1}{98}, \frac{1}{99}, \frac{1}{100}$$

$$2 \quad \frac{1}{11}, \frac{1}{12}, \frac{1}{13}, \frac{1}{14}, \frac{1}{15}, \frac{1}{16}, \frac{1}{17}, \frac{1}{18}, \frac{1}{19}, \frac{1}{20}, \frac{1}{21}, \frac{1}{22}, \frac{1}{23}, \frac{1}{24}, \frac{1}{25}, \frac{1}{26}, \frac{1}{27}, \frac{1}{28}, \frac{1}{29}, \frac{1}{30}, \frac{1}{31}, \frac{1}{32}, \frac{1}{33}, \frac{1}{34}, \frac{1}{35}, \frac{1}{36}, \frac{1}{37}, \frac{1}{38}, \frac{1}{39}, \frac{1}{40}, \frac{1}{41}, \frac{1}{42}, \frac{1}{43}, \frac{1}{44}, \frac{1}{45}, \frac{1}{46}, \frac{1}{47}, \frac{1}{48}, \frac{1}{49}, \frac{1}{50}, \frac{1}{51}, \frac{1}{52}, \frac{1}{53}, \frac{1}{54}, \frac{1}{55}, \frac{1}{56}, \frac{1}{57}, \frac{1}{58}, \frac{1}{59}, \frac{1}{60}, \frac{1}{61}, \frac{1}{62}, \frac{1}{63}, \frac{1}{64}, \frac{1}{65}, \frac{1}{66}, \frac{1}{67}, \frac{1}{68}, \frac{1}{69}, \frac{1}{70}, \frac{1}{71}, \frac{1}{72}, \frac{1}{73}, \frac{1}{74}, \frac{1}{75}, \frac{1}{76}, \frac{1}{77}, \frac{1}{78}, \frac{1}{79}, \frac{1}{80}, \frac{1}{81}, \frac{1}{82}, \frac{1}{83}, \frac{1}{84}, \frac{1}{85}, \frac{1}{86}, \frac{1}{87}, \frac{1}{88}, \frac{1}{89}, \frac{1}{90}, \frac{1}{91}, \frac{1}{92}, \frac{1}{93}, \frac{1}{94}, \frac{1}{95}, \frac{1}{96}, \frac{1}{97}, \frac{1}{98}, \frac{1}{99}, \frac{1}{100}$$

$$\begin{array}{l}
 3 \quad \frac{7}{100}, \frac{214}{1000}, \frac{800}{1000}, \frac{625}{1000}, \frac{425}{1000}, \frac{212}{1000}, \frac{5542}{1000}, \frac{500}{1000} \\
 4 \quad \frac{1000}{1000}, \frac{20000}{10000}, \frac{7624}{10000}, \frac{28172}{10000}, \frac{887}{10000}, \frac{511}{10000}, \frac{661}{10000}
 \end{array}$$

360 (i) In a given recurring decimal, the period may be supposed to begin at any point we please after the first repeating figure

$$\text{Thus, } 15.45387387 = 15.45387 = 15.45387\dot{3} = 15.453873\dot{8} = \&c$$

(ii) Sometimes the period is made to commence in the *integral* part

$$\text{Thus, } 64.2\dot{5} = 64.25\dot{4} = 64.2542 = \&c$$

(iii) The number of digits in the period may be repeated as often as we please without altering the value of the decimal

$$\text{Thus, } 8.54\dot{6} = 8.54626 = 8.5464646 = \&c$$

(iv) In the conversion of a fraction to a recurring decimal, we may often shorten the work by expressing the remainder at some step as a fraction. Thus,

$$1 = 142\dot{6}, \quad \frac{6}{7} = 142\dot{6} \times 6 = 857\dot{1}, \quad \text{and} \quad .\dot{7} = 142857\dot{1} = 14285\dot{7}$$

361 When recurring decimals have the same number of non-recurring figures and also the same number of recurring figures, they are said to be *similar*

Thus, 34258 and 617863 are *similar* recurring decimals

362 *All recurring decimals can be made similar*

RULE. Extend each decimal as far as the farthest non-recurring figure in any of them, then find the I C M of the numbers of figures in each period, and extend each period so many places further

Ex. Make 4238, 1234 and 54023 similar

$$\begin{array}{rcl}
 4238 & = & 423888888 \\
 1234 & = & 123412342 \\
 54023 & = & 5402323232
 \end{array}$$

Here, we see that the first term has the largest number of non-recurring figures, i.e. 2 figures
So extend each decimal 2 places

The periods which consist of 1, 3, 2 figures respectively, are then extended 6 places, for 6 is the I C M of 1, 2 and 3

Examples CVII

1 In the following recurring decimals begin the period at the fifth decimal place —

$$325, 4\dot{7}, 2900\dot{2}, 36, 2114, 0352, 7065, 0463, 34\dot{5}$$

2 Extend 57, 234 and 0545 so that they may have the same number of figures in the period

3 Extend 123, 1234 and 1234 so that they may have the same number of recurring figures

4 Convert the following vulgar fractions into recurring decimals by the method of Art 360 (iv) —

$$\frac{11}{18}, \frac{1}{12}, \frac{1}{15}, \frac{2}{16}, \frac{3}{20}, \frac{1}{25}, \frac{1}{30}, \frac{1}{36}, \frac{1}{40}$$

5 Make the following recurring decimals similar —

$$(1) 3.076, 9.245, 2.03$$

$$(2) 8, 87, 876$$

$$(3) 414, 0.352, 6.1013$$

$$(4) 5507, 0.463, 1.413, 7065$$

$$(5) 7854, 59, 14.57, 0045$$

$$(6) 97012, 4.403, 108492137, 21865$$

363 To find the vulgar fraction which shall be equivalent to a pure recurring decimal

RULE. Make the period the *numerator* of a fraction whose *denominator* shall consist of as many *nines* as there are figures in the said period and this reduced to its simplest terms will be the vulgar fraction required

Ex Convert 6 and $\dot{9}6$ into equivalent vulgar fractions in their lowest terms

$$(1) 6 = \frac{6}{1} = \frac{2}{3}$$

$$(2) 96 = \frac{96}{1} = \frac{8}{9}$$

Proof For the sake of conciseness, let x and y represent their values respectively, then, we shall have

$$\begin{aligned} x &= 6666 \\ 10 \text{ times } x &= 66666 \end{aligned}$$

$$\begin{aligned} y &= 9696 \\ 100 \text{ times } y &= 969696 \end{aligned}$$

whence, subtracting in each case, the former from the latter, we obtain

$$\begin{array}{l|l} 9 \text{ times } x = 6, & 99 \text{ times } y = 96, \\ \text{and } x = \frac{6}{9} = \frac{2}{3} & \text{and } y = \frac{96}{99} = \frac{8}{9} \end{array}$$

364 To find the vulgar fraction which shall represent the value of a mixed recurring decimal

RULE Make the non-recurring and the recurring parts taken together, diminished by the non recurring part alone, the numerator of a fraction whose denominator shall consist of as many *nines* as there are recurring figures, followed by as many *ciphers* as there are non-recurring figures, and this reduced to its lowest terms will be the vulgar fraction required

Ex Convert 27, $245\dot{7}$ and $011\dot{3}6$ into equivalent vulgar fractions in their lowest terms

$$(1) 2\dot{7} = \frac{27-2}{90} = \frac{25}{90} = \frac{5}{18} \quad (2) 245\dot{7} = \frac{2457-24}{9900} = \frac{2433}{9900} = \frac{811}{3300}$$

$$(3) 011\dot{3}6 = \frac{1136-11}{99000} = \frac{1125}{99000} = \frac{1}{88}$$

Proof For the sake of conciseness, suppose x and y to represent the values of (1) and (2) respectively, then, we shall have

$$\begin{array}{l|l} x = 27777 & y = 2457575757 \\ 10x = 27777 & 100y = 2457575757 \\ 100x = 27777 & 10000y = 2457575757 \end{array}$$

whence, subtracting the second line from the third in each case, we find

$$\begin{array}{l|l} 90x = 27 - 2 = 25, & 9900y = 2457 - 24 = 2433, \\ \therefore x = \frac{27-2}{90} = \frac{25}{90} = \frac{5}{18} & y = \frac{2457-24}{9900} = \frac{2433}{9900} = \frac{811}{3300} \end{array}$$

365 The above method is also applicable if there should be some integral figures in the decimal, but the equivalent vulgar fraction is improper. If it is required as a mixed number, we may either reduce this to mixed number or apply the method given below and thus obtain it at once in that form

Ex Express 2.27 and 4.583 as vulgar fractions

$$(1) \ 2.27 = \frac{227-2}{99} = \frac{225}{99} = \frac{25}{11} = 2\frac{3}{11}, \text{ or } 2.27 = 2 + .27 = 2 + \frac{27}{99} = 2\frac{3}{11}$$

$$(2) \ 4.583 = \frac{4583-45}{990} = \frac{4538}{990} = \frac{2269}{495} = 4\frac{289}{495},$$

$$\text{or } 4.583 = 4 + .583 = 4 + \frac{583-5}{990} = 4 + \frac{578}{990} = 4\frac{289}{495}$$

366 It follows from the Rule that $9 = \frac{9}{9} = 1$, $0.9 = \frac{9}{10} = \frac{9}{10} = 1$. Similarly, $0.69 = 0.7$, $0.259 = 0.26$. Hence, whenever 9 occurs at the end of a decimal, it should be omitted, and the preceding figure increased by 1.

367 The following equivalent forms with their converses should be verified and committed to memory —

$$\frac{1}{3} = \dot{3}, \dot{3} = 6, \frac{1}{6} = 16, \frac{5}{6} = 83, \frac{1}{9} = 1, \dot{1} = 0\dot{9}$$

$$\frac{1}{7} = 142857, \dot{7} = 285714, \dot{2} = 428571, \dot{4} = 571428,$$

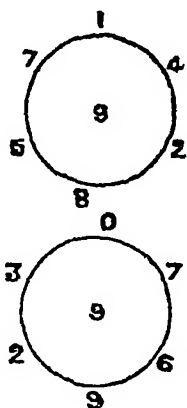
$$\dot{7} = 71428\dot{5}, \dot{5} = 857142$$

$$\text{Also } \dot{1}_3 = 07692\dot{3}, \dot{1}_3 = 23076\dot{9},$$

$$\dot{1}_3 = 30769\dot{2}, \dot{1}_3 = 69230\dot{7},$$

$$\dot{1}_3 = 76923\dot{0}, \dot{1}_3 = 92307\dot{6}$$

Students should carefully notice the decimals equivalent to vulgar fractions with denominators 7 and 13. All are pure circulating decimals, and the same digits 142857, 076923 and 153846 occur in all respectively. Now, if these digits be placed round a circle, and read off, beginning with 1, 2, 4, 5, 7, 8, 0, 2, 3, 6, 7, 9 and 1, 3, 4, 5, 6, 8, in turn, with the other digits in order as they stand round the circle, decimals equivalent respectively to $\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}, \frac{1}{13}, \frac{2}{13}, \frac{3}{13}, \frac{4}{13}, \frac{5}{13}, \frac{6}{13}, \frac{7}{13}, \frac{8}{13}, \frac{9}{13}$ will be obtained



Ex Express 382142857 as a vulgar fraction

$$382142857 = \frac{3821}{1000} = \frac{2675}{7000} = \frac{107}{280}$$

Examples CVIII

1 Convert the following recurring decimals into vulgar fractions in their lowest terms —

- (1) 5, $\dot{0}27$, $\dot{5}34$, $\dot{4}263$, 56, $\dot{2}59$, 7227, 620268
 (2) 3621, 47543, 05, 00495, 3545, $1\dot{9}6$, 16527, 5416
 (3) $043\dot{2}$, $219\dot{0}45$, $676190\dot{4}$, $00849713\dot{3}$, $8113\dot{6}$, 444108
 (4) 241254, 10428571, $\dot{2}6428571$, $38643\dot{0}18$, $1394\dot{2}3076\dot{9}$
 (5) $67692\dot{3}0$, 50230769, $415\dot{0}76923$, 012345679, $2784615\dot{3}$

2 Express the following as finite decimals —

$0\dot{9}$, $436\dot{9}$, 4579, $2599\dot{9}$, 1589, 37899, $5999\dot{9}$, 009

3 Required the least numbers of which 476190 is the recurring quotient, and find the error in the corresponding fraction when 47619 is taken to represent it

4 Prove that $\frac{1}{9} = \frac{1}{1} = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5} = \frac{6}{6} = \frac{7}{7} = \frac{8}{8} = \frac{9}{9}$

5 Prove that $\frac{1}{11} = \frac{45}{5} = \frac{54}{6} = \frac{63}{7} = \frac{72}{8} = \frac{81}{9} = \frac{90}{10}$

X ADDITION OF RECURRING DECIMALS

368 To find the accurate sum of several recurring decimals

RULE Write down the decimals under one another making them all similar (Art 362), and afterwards extend two places more to make sure that we are carrying the correct figure to the last place of the second extension. Add in the usual way. Then in the sum the first extension will give the *non recurring part*, and the second the *recurring part*.

Ex Add together 32 01011, 76 0914, 5 1375, 98 863

$$\begin{array}{r} 32\ 01\ 011\ 011\ 011\ 011\ 01 \\ 76\ 09\ 1491\ 491\ 491\ 49\ 14 \\ 5\ 13\ 7537\ 537\ 537\ 53\ 75 \\ 98\ 86\ 3333\ 3333\ 333\ 33 \\ \hline 212\ 10\ 247\ 337\ 346\ 347 \end{array}$$

Here, the greatest number of non-recurring figures is 2, so extend each decimal 2 places. The periods consist of 4, 3, 1 figures, of which the L. C. M. is 12, so, extend each to 12 places, and two places more to ensure accuracy of the last figure retained. In the sum, 10 is the non recurring

part and 247337346347 is the recurring part

369 To find the sum of several recurring decimals approximately correct to a given number of decimal places

RULE Set down the decimals under one another repeating the period of each 2 or 3 places more than what is required in the sum. Then add in the usual way, taking care that the last figure retained be increased by 1, if the succeeding figure be 5, or greater than 5.

Ex Find the sum of 13 5, 2 025, 111 0004, 3 14159, and 2 024 approximately correct to 6 decimal places

$$\begin{array}{r}
 13\ 555555|55 \\
 2\ 025252|52 \\
 111\ 000444|44 \\
 3\ 141591|59 \\
 2\ 024024|02 \\
 \hline
 131\ 746866|12
 \end{array}$$

Here, by carrying out the decimals to 8 places, we ensure the accuracy of the first 6 places. Also in the sum, as we stop at 8 and the succeeding figure is 1, we need not increase 8 by 1.

Examples CIX

1 Add together accurately —

- (1) $46 + 251 + 02514, 4187 + 304 + 125, 2001 + 1818 + 5$
- (2) $101 + 243183 + 1236 + 4529, 3090 + 4071 + 3513 + 765$
- (3) $2764235 + 92642037 + 54925 + 1498 + 603306$
- (4) $400358 + 08388994 + 361612 + 1006$
- (5) $31416 + 825142857 + 034 + 23257635 + 545627$

2 Find the values (app correct to 7 places of decimals) of —

- (1) $790 + 3416 + 3245 + 18, 6127 + 3801 + 10313 + 6$
- (2) $456701 + 41209 + 513317 + 67493 + 44567$
- (3) $7395 + 713 + 16284 + 1627354 + 1829 + 16 + 397$
- (4) $138 + 142857 + 2418 + 206 + 4263 + 008497135$

XI SUBTRACTION OF RECURRING DECIMALS

370 The RULES given for Addition are also applicable in the Subtraction of recurring decimals

Ex 1 Subtract 5 98765 from 28 035471

$$\begin{array}{r}
 28\ 035471717171 \\
 5\ 987657657657 \\
 \hline
 22\ 0478140595
 \end{array} \text{ Ans}$$

Here, the periods have 2 and 3 figures, their L C M is 6, therefore the recurring part in the difference contains 6 figures

Ex 2 Find (1) the difference of 2 02341 and 628 approximately correct to 6 decimal places, (2) the complement of 6142857

$$\begin{array}{r}
 (1) \quad 2\ 023413413 \\
 \quad 628888888 \\
 \hline
 1\ 394524525
 \end{array}$$

difference = 1 394525

$$\begin{array}{r}
 (2) \quad 1\ 00000000 \\
 \quad 614285714 \\
 \hline
 385714286
 \end{array}$$

complement = 3857143

Examples CX

1 Find the accurate difference of —

- (1) $17\ 2163 - 12\ 46$ (2) $30684 - 2346$ (3) $36801 - 2492$
 (4) $15\ 623 - 11\ 27$ (5) $365\ 27321 - 148\ 97$ (6) $25\ 47 - 16\ 8578$
 (7) $6\ 73459 - 3\ 0726$ (8) $714285 - 001136$ (9) $7\ 3214 - 1\ 207$

2 Find the values app correct to 6 places of decimals) of —

- (1) $0.4 - 0.0769238$ (2) $78\ 31 - 19\ 684$ (3) $142\ 345 - 109\ 32$
 (4) $314\ 2905 - 180\ 4162$ (5) $52\ 85 - 8\ 37235$ (6) $3\ 8564 - 2\ 0387$

3 Find the complements of 0.4563 , 0.789 , $25\ 6420370$

4 Find the values of —

- (1) $5\ 7892 - 2\ 368 + 17\ 54 + 2105 - 12\ 9761 - 3\ 215$
 (2) $14\ 8976 - 27\ 3150 - 49\ 81 + 15\ 763 + 183 + 21\ 05$
 (3) $18\ 7130 - 5\ 87 + 161\ 0235 + 21 - 8\ 004$
 (4) $7\ 5 + 12\ 30 - 59\ 7365 + 90\ 028 - 6\ 1257 - 20\ 71$

XII MULTIPLICATION OF RECURRING DECIMALS

371 To multiply a recurring decimal by an integer or by a terminating decimal

RULE Proceed in the usual way, extending the decimal 2 or 3 places beyond the end of the period, in order to ensure the correctness of the last digit retained, and in the product point off as many decimal places as there are decimal places in both the multiplicand and multiplier. The product will also be a recurring decimal of the same kind as the multiplicand, i.e., with a period containing the same number of digits

Ex 1 Multiply $37\ 83459$ by 7, and $37\ 8236$ by 11

- (1)
$$\begin{array}{r} 37\ 83459 \overline{) 459} \\ \underline{7} \\ 264\ 84216 \end{array}$$
 (2)
$$\begin{array}{r} 37\ 8236 \overline{) 36} \\ \underline{11} \\ 416\ 0599 = 416\ 06 \text{ (Art 366)} \end{array}$$

Ex 2 Multiply $6\ 3917825$ by $6\ 924$

$$\begin{array}{r} 6\ 3917825 \overline{) 917} \\ \underline{6\ 924} \\ 255671303 \\ 127835651 \\ 575260433 \\ 383506955 \end{array} \qquad \begin{array}{r} 255671303 \overline{) 671} \\ 127835651 \overline{) 356} \\ 575260433 \overline{) 043} \\ 383506955 \overline{) 095} \\ 44\ 256702665 \overline{) 025} \text{ Ans} \end{array}$$

372 To multiply one recurring decimal by another

RULE Convert the given decimals into equivalent vulgar

fractions, and multiply as in Art 270. Then reduce the resulting fraction to a decimal

Ex Multiply $0\dot{8}\dot{9}$ by $0\dot{2}\dot{8}$

$$0\dot{8}\dot{9} = \frac{89-8}{900} = \frac{81}{900} = \frac{9}{100}, \quad 0\dot{2}\dot{8} = \frac{28-2}{900} = \frac{26}{900} = \frac{13}{450}$$

$$\therefore \text{the product reqd} = \frac{9}{100} \times \frac{13}{450} = \frac{13}{5000} = \frac{26}{10000} = \underline{0026} \quad \text{Ans}$$

Examples CXI

1 Multiply —

- (1) $376\dot{4}\dot{2}$ by 9, $376\dot{4}\dot{2}$ by 11, $37\dot{6}\dot{4}\dot{2}$ by 37, $008\dot{3}76$ by 762
- (2) $432244\dot{3}18$ by 88, $785398\dot{1}$ by 3457, $63428\dot{7}$ by 501723
- (3) $3\dot{5}$ by 8, $3\dot{9}1$ by 022, $35426\dot{8}$ by 144, $15\dot{0}7\dot{3}$ by 24
- (4) $23\dot{8}5714\dot{2}$ by 56, $2738\dot{4}4\dot{3}$ by 267, $9385\dot{0}787$ by 7659

2 Find the values of —

- (1) $4\dot{8} \times 2\dot{4}$, $7\dot{6}\dot{3} \times 88\dot{3}$, $197\dot{2} \times 29\dot{4}\dot{5}$, $7\dot{2} \times 015\dot{9}\dot{0}$
- (2) $6\dot{3}\dot{6} \times 57142\dot{8}$, $118 \times 53846\dot{1}$, $5598924\dot{3} \times 8247$
- (3) $22\dot{7} \times 24\dot{9}$, $07\dot{3} \times 2\dot{7}\dot{2}$, $49\dot{3} \times 29\dot{9}5\dot{4}$, $12837 \times 252\dot{7}$
- (4) $0\dot{0}21 \times 4892\dot{6}$, $42857\dot{1} \times \frac{1}{3}$ of 38, $4420\dot{5}45 \times 1582370\dot{7}$

XIII DIVISION OF RECURRING DECIMALS

373 To divide a recurring decimal by a whole number or by a terminating decimal

RULE. Proceed as in ordinary division, bringing down the digits of the period in succession. The quotient will also be a recurring decimal

Ex Divide 89854 by 12 and 6559903 by 4876

(1) $12\overline{)898544444}$ ($7487870\dot{3}$)

$$\begin{array}{r} 84 \\ 58 \\ 48 \\ \hline 105 \\ 96 \\ \hline 94 \\ 84 \\ \hline 104 \\ 96 \end{array} \quad \begin{array}{r} 84 \\ 84 \\ \hline 44 \\ 36 \\ \hline 8 \end{array}$$

\therefore the quotient = $\underline{7487870\dot{3}}$

(2) $4876\overline{)655990399..}$ (134534)

$$\begin{array}{r} 4876 \\ 16839 \\ 14628 \\ \hline 22110 \\ 19504 \\ \hline 26063 \\ 24380 \\ \hline 16839 \end{array} \quad \begin{array}{r} 16839 \\ 14628 \\ \hline 22119 \\ 19504 \\ \hline 2615 \end{array}$$

\therefore the quotient = $\underline{0134534}$

374 To divide one recurring decimal by another

RULE Convert the given decimals into vulgar fractions, and divide as in Art 274 Then reduce the resulting fraction to a decimal

Ex Divide $1\dot{1}3$ by $00\dot{0}13\dot{2}$

$$1\dot{1}3 = \frac{113-11}{90} = \frac{102}{90} = \frac{17}{15}, \quad 00\dot{0}13\dot{2} = \frac{132}{999900} = \frac{1}{7575}$$

$$\therefore \text{the quotient reqd} = \frac{17}{15} \div \frac{1}{7575} = \frac{17}{15} \times \frac{7575}{1} = \underline{8585} \quad \text{Ans}$$

Examples CXII

1 Divide —

- (1) 3 by 5, by 7, $37\dot{0}87$ by 5, by 45, $332\dot{5}$ by 125, 461538 by 30
- (2) 3457954 by 8, 37635842 by 7, $5396343\dot{6}$ by 112
- (3) 23547 by 24×20 , 7476 by 07, 940 by 15, $3\dot{6}$ by 24
- (4) 028342012 by 14156 , 2013972 by 421, 10101 by 00036

2 Find the values of —

- (1) $38-273$, $10\dot{0}-583$, $6045-738$, $1183-24\dot{9}$, $37-14\dot{8}$
- (2) $40\dot{3}-1407$, $01236-051$, $953-32083$, $6891-1545$
- (3) $891-12\dot{9}$, $0057-213$, $125-251$, $73\dot{9}-079$
- (4) $411351\dot{9}-195881$, $1447619\dot{0}-2159\dot{0}$, $7767027-948\dot{6}$

XIV SIMPLIFICATION OF DECIMAL FRACTIONS

Ex 1 Simplify $\frac{13 \times 14 \times 01 - 12 \times 14 \times 02 + 12 \times 13 \times 01}{01 \times 2 \times 01}$

The given fraction = $\frac{000182 - 000336 + 000156}{00002} = \frac{000002}{00002} = \underline{1} \quad \text{Ans}$

Ex 2 Find the value of $\frac{28 \text{ of } 2\dot{2}7}{1136} + \frac{4\dot{4}-283}{16+262\dot{0}}$ of $\frac{68 \text{ of } 3}{225}$

$$\begin{aligned} \text{The value} &= \frac{2\dot{4} \times 21\dot{7}}{1136} + \frac{4444 - 283333}{16666 + 2629629} \text{ of } \frac{204}{225} \\ &= \frac{14}{5} \times \frac{25}{11} \times \frac{990}{1125} + \frac{161}{4296} \times \frac{2040}{225} = \frac{28}{5} + \frac{145}{498} \times \frac{136}{15} \\ &= \frac{28}{5} + \frac{145}{90} \times \frac{999}{4292} \times \frac{136}{15} = \frac{28}{5} + \frac{17}{5} = \frac{45}{5} = \underline{9} \quad \text{Ans} \end{aligned}$$

Examples CXIII

Simplify -

- 1 $1\frac{7}{2}$ of 276 of 15
- 2 $1\frac{8}{3}$ of 954 of 428571 of $2\frac{2}{5}$
- 3 65 of $4\frac{1}{2}$ of $\frac{3^2}{13}$ of $2\frac{4}{3}$
- 4 $\frac{2^3}{3^4}$ of 0006 of $\frac{4^5}{0024}$
- 5 $\frac{064+12\frac{2}{5}}{9375}$
- 6 $\frac{13\frac{5}{8}+078-003}{005}$
- 7 $\frac{011 \times 1331 - 723 \times 00723}{1\frac{1}{1377}}$
- 8 $\frac{5\frac{1183}{00705}}$ of $11\frac{1}{1}$ of 29 of 117
- 9 $\frac{12(02 \times 03 - 04 \times 01) + 16 \times 21}{1 \times 023 \times 01}$
- 10 $\frac{2\frac{5}{8}+1\frac{2}{5}-2\frac{1}{25}}{3\frac{7}{5}+2\frac{3}{4}-4\frac{2}{5}}$
- 11 $\frac{03-0\frac{3}{2}}{123}$
- 12 $\frac{1+5\frac{4}{3} \times 6\frac{4}{3}}{1+2\frac{3}{4} \times 3\frac{3}{4}}$
- 13 $\frac{005}{2}$ of $13\frac{1}{2}$ of $\frac{26\frac{2}{5}}{4}$ of $2\frac{7}{5}$
- 14 $(37 + \frac{37037}{100}) \times 54$
- 15 $\frac{428571}{01714285}$
- 16 $\frac{7\frac{1}{2} \times 3^2}{75 \times 366} + \frac{2\frac{5}{8} \text{ of } 15}{2\frac{1}{2} \text{ of } 3\frac{3}{4}} + \frac{7\frac{2}{5}}{11\frac{3}{8}}$
- 17 $\frac{04275}{305} \times \frac{4\frac{2}{3}16}{342} \times \frac{2\frac{7}{8}}{15318}$
- 18 $\frac{125}{100} - \frac{0625}{25} - 2\frac{2}{5} - \frac{005 \times 1\frac{2}{5}}{25} + 31 - \frac{5}{1000}$
- 19 $\frac{857142 + 142857}{571428 - 428571}$
- 20 $\frac{8571428 \times 17}{2\frac{1}{3} \text{ of } 1285714} \times \frac{216 \text{ of } 625}{48}$
- 21 $\frac{2\frac{6}{8} \text{ of } 2\frac{8}{3}}{6\frac{2}{3} \text{ of } 857142} + \frac{4\frac{5}{8} \text{ of } 4036}{3\frac{7}{5} \text{ of } 17}$
- 22 $\frac{2\frac{3}{16}}{3\frac{1}{16}} \text{ of } \frac{4\frac{4}{3}}{0625} - \frac{88}{7} \text{ of } \frac{16}{5625}$
- 23 $\frac{4\frac{2}{3}-3\frac{1}{4}}{1\frac{3}{4}+2\frac{1}{102}} \text{ of } \frac{1\frac{3}{4} \text{ of } 4}{37 \text{ of } 881}$
- 24 $\frac{044 \times 21}{000035} - \frac{3076923}{2\frac{3}{4} \times 56}$
- 25 $\frac{3302083}{16510416} + \frac{66 \times 375}{1\frac{1}{2} \text{ of } 538461 \text{ of } \frac{3}{4}} + \frac{2772}{1109}$
- 26 $\frac{1 \times 1 \times 1 + 01 \times 01 \times 01}{2 \times 2 \times 2 + 02 \times 02 \times 02}$
- 27 $\frac{375 \times 375 - 025 \times 025}{375 - 025}$
- 28 $\frac{02 \times 9 \times 15 - 14 \times 06 \times 03 + 13 \times 01 \times 04}{05 \times 04 \times 03}$
- 29 $\frac{6 \text{ of } 33 \text{ of } \frac{175}{2625} \text{ of } 17 + 4 \text{ of } 575}{2095238}$
- 30 $\frac{2\frac{3}{16}}{316} \text{ of } \frac{4\frac{4}{3}}{0625} - \frac{88}{58} \text{ of } 571428 - \left\{ \frac{28 \text{ of } 21\frac{1}{2}}{1136} - 41\frac{8}{5} \text{ of } \frac{4\frac{4}{3}-2\frac{8}{3}}{13+2629} \right\}$

XV. REDUCTION OF DECIMALS.

375 A general view having now been taken of decimals, we proceed to show how they may be made to change their denominations when they are considered as belonging to a particular unit, and in what ways they may be adapted to the particular computations in which they are most frequently employed

376 Reduction of Decimals can conveniently be classed under the two following heads —

- (1) To reduce a decimal of one denomination to a lower denomination and conversely,
- (2) To reduce a quantity of one denomination to a decimal of a higher denomination

377 Case I To reduce a decimal of one denomination to a lower denomination (Descending Reduction)

RULE Multiply the decimal of the given denomination by the number which connects the lower denomination with one (or unit) of the given denomination

Ex Reduce Rs 7 15 to pias, and 0.45 of £7 to farthings

- | | |
|---|---|
| <p>(1) $\text{Rs } 7 \text{ } 15$</p> $\begin{array}{r} 16 \\ \hline \text{Rs } 114 \text{ } 40 \\ 12 \\ \hline \text{p } 1372 \text{ } 8 \end{array}$ <p>the reqd result = <u>1372 8p</u></p> | <p>(2) $\text{£ } 7$</p> $\begin{array}{r} 0.45 \\ \hline \text{£ } 315 \\ 20 \\ \hline \text{s } 6300 \end{array}$ <p>∴ the reqd result = <u>302 4q</u></p> |
|---|---|

378 Case II To reduce a quantity of one denomination to a decimal of a higher denomination (Ascending Reduction)

RULE Divide the number of the given denomination by the number which connects that denomination with one (or unit) of the higher denomination

Ex Reduce 3333 pias to the decimal of a rupee, and 21½ grs to the decimal of an oz Troy

- | | |
|---|---|
| <p>(1) $12 \overline{) 3333 \text{ p}}$</p> $\begin{array}{r} 16 \overline{) 277 \text{ } 75 \text{ a}} \\ \hline \text{Rs } 17 \text{ } 359375 \end{array}$ <p>∴ the reqd decimal = <u>Rs 17 359375</u></p> | <p>(2) $24 \left\{ \begin{array}{l} 8 \overline{) 21 \text{ } 75 \text{ grs}} \\ 3 \overline{) 271875} \\ 20 \overline{) 90625 \text{ dwt}} \end{array} \right.$</p> $\begin{array}{r} 0.453125 \text{ oz} \\ \hline \end{array}$ <p>the reqd decimal = <u>0.453125 oz</u></p> |
|---|---|

379 Sometimes we employ both the *descending* and the *ascending* process in reducing a decimal of one denomination to a decimal of another denomination

Ex Reduce 78936 of a guinea to the decimal of £1
78936 gui

$$\begin{array}{r} 21 \\ 20 \overline{) 16\ 57656s} \\ \underline{ 828828} \end{array}$$

∴ the reqd decimal = £ 828828

Examples CXIV

Reduce —

- 1 £ 02375, £ 00375, £3 5478, £ 00625, £ 28125 to *pence*
- 2 03125s, £ 8947916, 001 guinea, £ 47083, £ 383 to *farthings*
- 3 Rs 5 00625, Rs 2 76543, 775625 of Rs 5, Rs 3 049 to *pies*
- 4 Re 972916, Re 40972, Re 68125, Re 634375, Re 3405 to *pice*
- 5 7859 cwt to *ounces*, 4 34954 miles to *yards*, 549675 days to *seconds*, 2 5384375 of a day to *seconds*
- 6 6197916 lb Troy to *grains*, 678571428 week to *minutes*
- 7 3 6874 acres to *sq yds*, 0475 gallon to *pints*, 2 274025 mds to *chataks*, 825 of a lea to *yards*
- 8 84d, 335s, 6 375d, 4068g to the decimal of £1
- 9 37 9872 sec to the dec of a *day*, 420 8138 sq yds to the decimal of an *acre*, 2 25 of 3 5 ac to *poles*
- 10 47 733 lbs to the dec of a *ton*, 1 oz Avoir to the dec of 1 oz Troy, 3 6 cwt to the dec of a *ton*
- 11 £ 625 to the decimal of a guinea, and of half a-guinea
- 12 527 3094 yds to the dec of a *mile*, 54375 lbs Troy to *ounces Avoir*, 1 oz to the dec of a *cwt*

380 The preceding two cases of Art 376 enable us

- (i) To reduce a decimal of one denomination to a compound quantity of lower denominations, and
- (ii) To reduce a compound quantity to a decimal of a higher denomination

381 Case I To reduce a decimal of one denomination to a compound quantity of lower denominations

RULE. Multiply the decimal by the numbers which connect the successive denominations in order, and the integral parts of the products *taken out*, as they occur, will be the value required

Ex 1 Find the values of Rs 3 46875 and £5 6125

$$\begin{array}{r} (1) \text{ Rs } 3\ 46875 \\ 16 \\ \hline 57\ 50000 \\ 12 \\ \hline 660 \end{array}$$

The reqd value = Rs 3 7a 6p

$$\begin{array}{r} (2) \text{ £ } 5\ 6125 \\ 20 \\ \hline 12\ 2500 \\ 12 \\ \hline 300 \end{array}$$

The reqd value = £5 12s 3d

Ex 2 Find the values of 4 215 of Rs 7 and 31 258½ of £2

$$\begin{array}{r}
 (1) \quad 4 \ 215 \\
 \underline{7} \\
 Rs \ 29 \ 505 \\
 \underline{16} \\
 a \ 8 \ 080 \\
 \underline{12} \\
 p \ 0 \ 96
 \end{array}$$

The reqd value = Rs 29 8a 0 96p

$$\begin{array}{r}
 (2) \quad 31 \ 258 \frac{1}{2} \\
 \underline{2} \\
 £62 \ 5166 \\
 \underline{20} \\
 s \ 10 \ 3333 \\
 \underline{12} \\
 d \ 3 \ 9999
 \end{array}$$

The reqd value = £62 10s 4d

382 Case II To reduce a compound quantity to a decimal of a higher denomination

RULE Divide the lowest denomination by the number which connects it with the next, and to the left of the quotient affix the number of this denomination, and continue the process till the required denomination is obtained

Ex 1 Express Rs 5 1a 6p as the decimal of Re 1, and £3 18s 11½d, as the decimal of £1

$$\begin{array}{r}
 (1) \quad 12)6p \\
 \underline{16)1 \ 5a} \\
 Rs \ 5 \ 09375
 \end{array}$$

The reqd decimal = Rs 5 09375

$$\begin{array}{r}
 (2) \quad 4) \quad 1q \\
 \underline{12)11 \ 25d} \\
 20)18 \ 9375s \\
 £3 \ 946875
 \end{array}$$

The reqd decimal = £3 946875

Ex 2 Reduce 7fur 25po to the decimal of a mile, and 14½ oz Avoir to the decimal of 1 oz Troy

$$\begin{array}{r}
 (1) \quad 4b)25 \ po \\
 \underline{8) \ 7625 \ fur} \\
 953125 \ mi
 \end{array}$$

$$\begin{array}{r}
 (2) \quad 5) \ 2 \\
 \underline{16)14 \ 4 \ oz} \\
 9 \ lb \\
 \underline{7000}
 \end{array}$$

$$\begin{array}{r}
 24 \left\{ \begin{array}{l} 8)6300 \ grs \\ 3) \ 787 \ 5 \\ 20) \ 262 \ 5 \ dwts \end{array} \right. \\
 13 \ 125 \ oz \ Troy
 \end{array}$$

The reqd decimal = 953125 mi

The reqd decimal = 13 125 oz Troy

Examples CXV

1 Find the values of —

- (1) Rs 5 07125, Rs 80 075, 016 of a rupee, 30 36 of 75 of Rs 10
- (2) 45 of £1, 16875 of £3, 2 36875 of £6, £ 5675, £ 0484, £ 7
- (3) 340625 of £1, 615 of 1s, 483½ of £1, £5 6125, 4375 of £1
- (4) 375 of a guinea, 1 025 of a guinea, 7635416 of £1, 4583 of 1s
- (5) 375 of a cwt, 6875 of a yard, 13 3375 acres, 655 of a day
- (6) Rs 5 7989583, 8716 of a ton, 2 5384375 days, 22 25 of 17 half crs.
- (7) 000035511363 mile, 10714285 of a cwt, 09375 of an acre
- (8) 00625 of 1 md, 0138 of 3 5 moidores, 3 23 of 1½ acres

2 Reduce —

- (1) $5\frac{1}{4}d$, $\frac{3}{4}d$, $8s$ $11\frac{1}{4}d$, $1s$ $3\frac{1}{4}d$, $\pounds 1$ $14s$ $10\frac{1}{2}d$ to the decimal of $1s$
 (2) $12s$ $6\frac{1}{4}d$, $15s$ $9\frac{3}{4}d$, $17s$ $0\frac{3}{4}d$, $1\frac{3}{4}d$, $\pounds 2$ $15s$ $9\frac{1}{4}d$ to the dec of $\pounds 1$
 (3) $7a$ $6p$, $8a$ $3p$, $13a$ $6\frac{3}{4}p$, Rs 53 $13a$ $18p$ to the decimal of $1Re$
 (4) $18s$ $11\frac{1}{2}d$ to the dec of a guinea, $4\frac{3}{4}$ guineas to the dec of $\pounds 50$
 (5) Rs 2 $13a$ $10p$ to the dec of Rs 5 , Rs 35 $14a$ $6p$ to the dec of Rs 25 , Rs 6 $6a$ $8p$ to the dec of Rs 10 $8a$
 (6) $12s$ $6\frac{1}{4}d$ to the decimal of $\pounds 1$ of $\pounds 100$ and of $\pounds 001$
 (7) 10 oz 11 dwts $21\frac{1}{2}$ grs to the dec of $1lb$ *Troy*, and of $1lb$ *Avoir*
 (8) 9 cwt $13lbs$ $4oz$ 3 84 drs to the dec of a *ton*, 4 cwt $1qr$ $10\frac{1}{2}lbs$ to the dec of $1cwt$, 17 cwt $3qrs$ $17lbs$ 8 7 oz to the dec of a *ton*
 (9) $12hrs$ 55 min $23\frac{1}{3}sec$ to the dec of a *day*, 5 days $12hrs$ $25min$ 37 92 sec to the dec of a *week*, 1 cwt 3 qrs $4lbs$ to the dec of a *ton*
 (10) $11yds$, $3fur$ $66yds$ and $6yds$ $2ft$ $7\frac{1}{2}in$ each to the dec of a *mile*
 (11) 002 of 2 75 pag to the dec of Rs 3 46 , 4 mds $8sr$ $1\frac{1}{2}ch$ to the dec of $14mds$, 3 sr 4 ch 2 to 11 m to the dec of $1md$
 (12) 6 fur 100 yds 2 ft $3in$ to the dec of a *mile*, 3 ro 31 po $16\frac{1}{2}yds$ to the dec of an *acre*, $13cub$ ft 1323 cub in to the dec of a *cub yard*

383 To multiply or divide a quantity by a decimal, or to find the value of a decimal of a quantity

RULE (1) Express the given quantity, when necessary, as a simple quantity, and perform the required operation, or (2) reduce the decimal to a fraction in its lowest terms, and proceed as in fractions (Arts 302 and 303)

Note When the decimal is *recurring* and the value is required to be *exact*, the second method is advantageous

Ex 1 Find the value of 432 of Rs 6 $10a$ $8p$

$$(1) Rs\ 6\ 10a\ 8p \times 432 = 1280p \times 432 = 552\ 96p = Rs\ 2\ \underline{14a\ 0\ 96p}$$

$$(2) 432\ \text{of}\ Rs\ 6\ 10a\ 8p = \frac{432}{1000}\ \text{of}\ Rs\ 6\ 10a\ 8p = \frac{54}{125}\ \text{of}\ Rs\ 6\ 10a\ 8p \\ = \frac{54}{125}\ \text{of}\ Rs\ 360 = Rs\ 2\frac{72}{125} = Rs\ 2\ \underline{14a\ 0\ 96p}$$

Ex 2 Find the value of $4\ 234\frac{5}{8}$ of $\pounds 2$ $15s$

$$4\ 234\frac{5}{8}\ \text{of}\ \pounds 2\ 15s = 4\frac{234\frac{5}{8}}{1000}\ \text{of}\ \pounds 2\ 15s = 4\frac{1875}{1000}\ \text{of}\ \pounds 2\ 15s \\ = \pounds 2\ 15s \times 4 + \pounds 2\ 15s \times \frac{1875}{1000} = \pounds 11 + 55s \times \frac{1875}{1000} \\ = \pounds 11 + 12\ 9s = \underline{\pounds 11\ 12s\ 10\ 8d}$$

Ex 3 Find the value of 33 of $\frac{44}{735}$ of 1 sq ft 3 sq in

$$\begin{aligned}\text{Value required} &= 33 \text{ of } \frac{44}{735} \text{ of } 1728 \text{ sq ft} = 33 \times 44 \times \frac{1728}{735} \times \frac{1}{144} \text{ sq ft} \\ &= 10 \times 44 \times \frac{1728}{735} \times \frac{1}{144} \text{ sq ft} = 310 \text{ sq ft} \\ &= 20 \frac{5}{7} \text{ sq ft} = \underline{20 \text{ sq ft } 80 \text{ sq in}}\end{aligned}$$

Ex 4 Find the value of 286805 of $Re\ 1\ 8a + 83$ of $Rs\ 2 - 18$ of $Rs\ 2\ 8a$

$$\begin{aligned}286805 \text{ of } Re\ 1\ 8a &= 286805 \text{ of } Re\ 1\ 8a = 21\frac{25}{24} \text{ of } Re\ 1\ 8a \\ &= Re\ 1\ 8a \times 2 + Rs\ \frac{5}{2} \times \frac{1}{4} = Rs\ 3 + Re\ 1\ 4a\ 10p \\ &= Rs\ 4\ 4a\ 10p \\ 83 \text{ of } Rs\ 2 &= \frac{7}{6} \text{ of } Rs\ 2 = \frac{5}{8} \text{ of } Rs\ 2 = Rs\ \frac{5}{8} = Re\ 1\ 10a\ 8p \\ 18 \text{ of } Rs\ 2\ 8a &= \frac{1}{6} \text{ of } Rs\ 2\frac{1}{2} = \frac{1}{6} \times Rs\ \frac{5}{2} = Rs\ \frac{5}{12} = Rs\ 4\ 8a \\ \therefore \text{value required} &= Rs\ 4\ 4a\ 10p + Re\ 1\ 10a\ 8p - Rs\ 4\ 8a \\ &= \underline{Re\ 1\ 7a\ 6p}\end{aligned}$$

Examples CXVI

1 Find the values of —

- (1) 185 of $Re\ 1\ 10a\ 8p$, 2375 of $Rs\ 6\ 10a\ 8p$, 775625 of $Rs\ 50$
- (2) 925 of $6s\ 8d$, 7365 of $6s\ 8d$, 59375 of $19s\ 4d$, 78125 of $\pounds 6$
- (3) 00390625 of $\pounds 1\ 12s$, 0474609375 of $\pounds 10\ 13s\ 4d$, 07 of $\pounds 2\ 10s$
- (4) 6156510416 of $Rs\ 40$, 001953125 of $Rs\ 400$, 146875 of 3 bighas
- (5) 046875 of 1 md 8 sr, 4106 of 4 mds 32 sr 8 ch, 045 of 4 miles
- (6) 7385 of $13s\ 4d$, 1625 of 2 tons 4 cwt, 27138 of 2 mi 450 yds
- (7) 3792 of $\pounds 3\ 18s\ 1\frac{1}{2}d$, 0013 of $\pounds 3\ 17s\ 10\frac{1}{2}d$, 365 of $\pounds 1\ 0s\ 10d$
- (8) $\pounds 3\ 14s\ 6\frac{1}{2}d \times 246875$, $\pounds 874\ 13s\ 4d \times 1875$
- (9) $\pounds 1205\ 6s\ 8d - 512$, $\pounds 503\ 12s\ 6\frac{1}{2}d - 26312$
- (10) $Rs\ 47\ 13a \times 245775$, $Rs\ 149\ 5a \times 34567$, $Rs\ 239\ 9a\ 6p - 1353$
- (11) 2775 of 1 sq yd 3 ft 72 in, 9765625 of 2 tons 18 cwt 3 qrs 14 lbs
- (12) 225 days 14 hrs 36 min $- 871846$, 27 lbs 13 oz 15 drs $\times 4352$

2 Find the values of —

- (1) $\frac{5}{3}$ of $Rs\ 2\ 6a\ 48p$, 306 of $Rs\ 2\ 1a$; $\frac{3}{5}$ of $Rs\ 3\ 8a\ 4p$
- (2) 714285 of $10s\ 6d$, 428 of $\pounds 3\ 8s$, 39583 of $Rs\ 8$
- (3) 3481 of $\pounds 4\ 18s\ 8d$, 40097 of $Rs\ 16\ 13a\ 4p$, 00015740 of $Rs\ 81$

- (4) 53571428 of 2 cwt 3 qrs 17½ lbs, 13 263798 of 3 m 7 fur 22½ po
 (5) 2083 of 3428571 of 2½ cwt, 1916 of 8s, 307 of 11s 3d,
 3242 of 7½ biggas, 36 of 4 qrs 4 bus
 (6) 846153 of 001 of Rs 6 8a, 017 101 of Rs 749 4a, 1 × 47 of
 Rs 3601 2a, 4694 of Rs 5 3 2p

3 What is the value of 234, when the unit is worth £20, and the worth of 3 of 3, when the unit is valued at Rs 108?

4. What is the value of 583, when the unit is 3 oz. 5 dwts?

5 Find the respective values of —

- (1) 45 of Rs 35 + 75 of Rs 2 5a 4p + 3245 of Re 1 10a 8p
 (2) 871875 of 5a 4p + 146875 of Rs 3 5a 4p - 0625 of Rs 10 8a
 (3) 375 of a guinea + 1875 of a crown + 3 of 7s 6d - 875 of 2d
 (4) 5s + 7 of a crown + £125, £6 + 5125s + 2 of a guinea
 (5) 1125 of Rs 13 8a + 44045 of 7a 6p - 0625 of Rs 3 12a
 + 1025 of 2 6p - 256 of Rs 5 7a 6p
 (6) 175 of 28 mds + 195 of 1 md 16 sr + 145 of 14 sr + 15 of 8 ch
 (7) 625 of £1 1s + 34 of 8s 3d + 027 of £2 15s
 (8) 7 of 7s 6d - 84 of 16s 6d + 927 of £2 10s 5d
 (9) 285714 of £30 + £6857142 + 0 of 714285 of £6 + 13 of 428571s
 (10) 857142 of 20625 tons + 571428 of 3375 cwt + 714285 of 125 qrs
 + 285714 of 105 lbs

384 To find what decimal one compound concrete quantity is of any other of the same kind

RULE. Express the first quantity as the fraction of the second, as in Art 306, and then reduce this fraction to a decimal

Ex 1 Reduce 3s 11½d to the decimal of £1 19s 4½d

3s 11½d = 47½d, and £1 19s 4½d = 472½d

47½d ÷ 472½d = 1¼ × 7⁄18 = 1⁄10, the reqd decimal = 1 Ans

Ex 2 Express 1 of Rs 3 12a + 625 of Rs 5 - 545 of Rs 4 9a as the decimal of Rs 100

1 of Rs 3 12a = 1⁄2 × 60a = 22½a = Re 1 6a 6p

625 of Rs 5 = Rs 3 125 = Rs 3 2a

545 of Rs 4 9a 4p = 545⁄100 of 73¹a = 1⁄1 × 220a = 40a = Rs 2 8a

∴ the first quantity = $Rs\ 1\ 6a\ 6p + Rs\ 3\ 2a - Rs\ 2\ 8a$
 $= Rs\ 2\ 0a\ 6p = Rs\ 2\frac{1}{2}$
 ∴ the reqd decimal = $2\frac{1}{2} - 100 = \underline{0203125}$ Ans

Examples CXVII

1 In the following Examples, reduce the first of the two given quantities to the decimal of the second —

- (1) $Rs\ 11\ 2a\ 2p$, $Rs\ 178\ 2a\ 8p$ (2) $Rs\ 12\ 0a\ 6p$, $Rs\ 25\ 7a$
 (3) $Re\ 1\ 11a$, $Rs\ 2\ 8a$ (4) $Rs\ 2\ 13a\ 10p$, $Rs\ 50$
 (5) $5s$, $13s\ 4d$ (6) $13s\ 6\frac{1}{2}d$, $15s\ 6d$
 (7) $£3\ 6s\ 8\frac{1}{2}d$, $£7\ 10s$ (8) $3\frac{1}{2}$ guineas, $£2\ 15s\ 5\frac{1}{2}d$
 (9) $1\frac{1}{2}d$, $7s\ 10\frac{1}{2}d$ (10) $7s\ 8\ 1942d$, $15s\ 9d$
 (11) $\frac{7}{8}$ of $10s\ 13s\ 4d$ (12) $\frac{1}{4}$ of $2s\ 6d$, $\frac{5}{8}$ of $1\frac{1}{2}$ guineas.
 (13) 345 of $10s\ 6d$, half-a crown (14) 0527 of $£1\ 7s\ 6d$, $13s\ 4d$
 (15) $3\ hrs\ 26\ min\ 37\ sec$, $13\ days\ 20\ hrs\ 23\ min$
 (16) $1\ cwt\ 2\ qrs\ 3\frac{1}{2}\ lbs$, $1\ ton\ 4\ cwt\ 1\ qr\ 24\ lbs$
 (17) $10\ lbs\ 11\ oz\ 12\ dwts\ 7\ grs$, $9\ lbs\ 8\ oz\ Avoir$
 (18) $5\ ac\ 3\ ro\ 15\ po$, $1\ ac\ 2\ ro\ 32\ po$
 (19) $3\frac{1}{2}$ of $£4\ 15s\ 4d$, 27 of $16s\ 3d$ (20) $2\ sr\ 4\ ch$, $1\ md\ 8\ sr$
 (21) $2\frac{1}{2}$ of $£2\ 6s\ 5\frac{1}{2}d$, $£18\ 17s\ 10\frac{1}{2}d$
 (22) 101 of $1\ lb\ 5\ oz$, $\frac{1}{8}$ of $1\ qr\ 22\ lbs\ 8\ oz$
 (23) $1\ bi\ 11\ k\ 8\ ch$, $16\ k\ 14\ ch$
 (24) $1\ md\ 3\ sr\ 8\frac{1}{8}\ ch$, $1\ md\ 16\ sr$

2 Express $3s\ 5\frac{1}{2}d$ as the decimal of a dollar of $1s\ 1\frac{1}{2}d$

3 Express $£5\ 456$ as the decimal of a rupee of $1s\ 10d$

4 Express 375 of a guinea + $\frac{1}{8}$ of a crown + 3 of $7s\ 6d - \frac{1}{8}$ of $2d$ as the decimal of $16s$

5 Find the value of $£\ 0375 + 625s + 75d + 3s\ 3\ 5d$ and reduce the result to the decimal of $7s\ 6d$

6 Find the value of $24\frac{1}{2}$ of $Rs\ 4\ 10a + 259$ of $Rs\ 12\ 8a + 02$ of $Rs\ 33\ 12a$ and reduce the result to the decimal of $Rs\ 30$

7 Find the value of $\frac{1}{4}$ of $\frac{1}{9\frac{1}{2}}$ of $£1\ 18s + \frac{1}{2}$ of 375 of $15s + \frac{1}{8}$ of 429 of $8s\ 3d$ and express the result as the decimal of $£5$

8 Express $\frac{5}{8}$ of $12s\ 6d + 625$ of $7s\ 6d - 505$ of $16s\ 6d$ as the decimal of $£1$

9 Express $£874\ 13s\ 4d \times 3\ 75$ as the decimal of $£1000$

10 What decimal of a crown is the difference between $6\frac{1}{2}$ half-guineas and £3 525 ?

11 Express the difference between $\text{£}378$ of 13s 10½d and 378 of 16s 6d as the decimal of $\text{£}1$ 17s 6d

12 Express $\text{£}9 + 27s + 36d$ as the decimal of $\text{£}(2 - 2) + (6 - 6)s + (8 - 8)d$

XVI APPROXIMATION

385 It has already been shewn in Art 351, that in converting a vulgar fraction to a decimal, where the division does not terminate (which is often denoted by dots () placed at the end of the quotient), an **approximation** to its true value can always be found to any degree of accuracy Thus $\frac{5}{7} = 29411764$ If we wish to *approximate* to the result by terminating the operation at the 5th place we write $\frac{5}{7} = 29412$ but if at the fourth place, we write $\frac{5}{7} = 2941$ and so on From this it is evident that we increase the last figure retained by 1, if the succeeding figure be 5, or greater than 5

386 The reason for the above is obvious from the following considerations If we take 29412 to represent 29411764 instead of 29411, it is clear that 29412 is greater, and 29411 less than the true value of the decimal, but 29412 is greater than the true value by 00000236, and 29411 is less than the true value by 00000764

Now 00000236 is less than 00000764

Therefore 29412 is nearer the true value than 29411

387 **Contracted Addition and Subtraction** These methods have already been explained in Art 369

388 **Contracted Multiplication** In multiplying one long decimal by another, it is generally required to get the product *approximately* correct, i.e., as far as a certain decimal place The following Rule enables us to shorten the work

RULE Mark off in the decimal part of the multiplicand as many figures as is one more than the number of decimal places we are required to retain in the product, under the last of these marked figures place the units' figure of the multiplier, writing the figures in a reverse order Omit decimal points of both the multiplicand and the multiplier and add 0's (if necessary) in the multiplicand, so that every figure of the multiplier shall have a figure above it. Begin the multiplication with the right-hand figure of the multiplier and multiply in succession by each of the others, in each case beginning the multiplication from the figure above the one we are multiplying by, but carrying to it the *nearest ten* from its product with the next figure on the right Place the units' figure of all these partial products in the same vertical line add as usual, and mark off the required number of decimal places in the result, striking out the last figure

Note In carrying the *nearest ten*, if the product is a number from 5 to 14 carry 1, from 15 to 24 carry 2, from 25 to 34 carry 3, from 35 to 44 carry 4, and so on. If the product is 4 or less than 4, reject it (Art 385)

Ex 1 Multiply 45063524 by 254637, retaining 3 places, 00040635 by 2416358, retaining 6 places, and 453 by 01694, retaining 4 places of decimals

$$\begin{array}{r}
 (1) \quad 4596352,4 \\
 \quad \quad 736452 \\
 \hline
 91927048 \\
 22981762 \\
 1838541 \\
 275781 \\
 13789 \\
 3217 \\
 \hline
 117040138
 \end{array}$$

$$\begin{array}{r}
 (2) \quad 4063,50 \\
 \quad \quad 8536142 \\
 \hline
 812700 \\
 162510 \\
 4064 \\
 2438 \\
 122 \\
 20 \\
 \hline
 3 \\
 0981887
 \end{array}$$

$$\begin{array}{r}
 (3) \quad 4530,0 \\
 \quad \quad 496100 \\
 \hline
 453 \\
 272 \\
 41 \\
 2 \\
 \hline
 00768
 \end{array}$$

for 01694 may be written as 0.01694

Ex 2 Multiply 32567834 by 42089542, retaining 7 places, and 182357 by 0785, retaining 6 places of decimals

$$\begin{array}{r}
 (1) \quad 325678340 \\
 \quad \quad 24598024 \\
 \hline
 1302713360 \\
 65135668 \\
 2605426 \\
 293109 \\
 16284 \\
 1303 \\
 65 \\
 \hline
 1370765218
 \end{array}$$

$$\begin{array}{r}
 (2) \quad 18235723,57 \\
 \quad \quad 758758700 \\
 \hline
 1276500 \\
 145886 \\
 9118 \\
 1276 \\
 146 \\
 9 \\
 1 \\
 \hline
 1432938
 \end{array}$$

389 Contracted Division In dividing one decimal by another where the quotient is required to be approximately correct only to a certain number of decimal places, we use the following Rule —

RULE Make the divisor a whole number, and determine first of all—by inspection or by taking one step in the ordinary way—the highest number of *integral* figures in the quotient, and then the whole number of figures in the quotient, from the left of the divisor cut off this number of figures, and one more for *approximation*, and strike out the rest. Proceed one step with this new divisor, but in multiplying its first figure by the quotient figure, carry the *nearest ten* from its product with the next figure on the right. Instead of bringing down a figure to the remainder, strike off another figure from the divisor, and proceed as before, until no figure is left in the divisor.

If the number of figures in the divisor be less than the number of figures to be cut off, proceed in the ordinary way until the number

of figures still to be found in the quotient is one less than the number of figures in the divisor, and then apply the Rule

Ex 1 Divide 2508 928065051 by 92 410357 approximately correct to 4 places of decimals

$$9,2,4,1,0,3,5)2508928065051(27\ 1\ 198$$

$$\begin{array}{r} 1848207 \\ \underline{660721} \\ 646872 \\ \underline{13849} \\ 9241 \\ \underline{4608} \\ 3696 \\ \underline{912} \\ 832 \\ \underline{80} \\ 74 \end{array}$$

Making the divisor a whole number, we find by inspection that there will be 2 figures in the *integral* part of the quotient, and 4 places of decimals are to be retained. Hence, 6 figures are retained in the divisor and 1 more for *approximation*, so that the divisor is 924103,5. In the next stage the divisor is 92410,3, 3 being retained for *approximation*, and so on.

Ex 2 Divide 257917 by 203458 approximately correct to 7 places of decimals

$$2,0,3,4,5,8)2579170(1267667$$

$$\begin{array}{r} 203458 \\ 544590 \\ \underline{406916} \\ 137674 \\ \underline{122075} \\ 15599 \\ \underline{14242} \\ 1357 \\ \underline{1220} \\ 137 \\ \underline{122} \\ 15 \\ \underline{14} \end{array}$$

Here, by inspection, we find that the quotient will contain no integral part, and as 7 places of decimals are to be retained, the divisor must consist of 8 figures, with 1 for *approximation*. But as there are only 6 figures in the divisor, proceed in the usual way of division for 2 figures in the quotient, when the number of figures still to be obtained will be one less than the number of figures in the divisor. Then apply the Rule.

Ex 3 Divide 549532676 by 9312167, retaining 7 places of decimals

$$9,3,1,2,1)549532676(0005901$$

$$\begin{array}{r} 46561 \\ 8392 \\ \underline{8381} \\ 11 \\ \underline{9} \end{array}$$

By inspection, we determine that there will be 3 ciphers after the decimal point in the quotient, hence only $(7 - 3)$ or 4 figures are required in the quotient. Therefore we retain 5 figures in the divisor, one for *approximation*.

380 Series The value of a *Series* is frequently required to be obtained correct to a certain number of decimal places. In such cases proceed as in the following Examples

Ex 1 Find the value, correct to 7 places of decimals, of

$$1 + \frac{1}{12} + \frac{1}{123} + \frac{1}{1234} + \&c$$

$\frac{1}{1}$	$= 1$	$= 1$
$\frac{1}{12}$	$= \frac{1}{2}$	$= 5$
$\frac{1}{123} = \frac{1}{3} \times \frac{1}{12}$	$= \frac{1}{3} \times 5$	$= 1666666$
$\frac{1}{1234} = \frac{1}{4} \times \frac{1}{123}$	$= \frac{1}{4} \times 166666667$	$= 041666667$
$\frac{1}{12345} = \frac{1}{5} \times \frac{1}{1234}$	$= \frac{1}{5} \times 041666667$	$= 008333333$
$\frac{1}{123456} = \frac{1}{6} \times \frac{1}{12345}$	$= \frac{1}{6} \times 008333333$	$= 001388889$
$\frac{1}{1234567}$	$= \frac{1}{7} \times 001388889$	$= 000198412$
$\frac{1}{12345678}$	$= \frac{1}{8} \times 000198412$	$= 000024801$
$\frac{1}{123456789}$	$= \frac{1}{9} \times 000024801$	$= 000002756$
$\frac{1}{12345678910}$	$= \frac{1}{10} \times 000002756$	$= 000000276$
$\frac{1}{1234567891011}$	$= \frac{1}{11} \times 000000276$	$= 000000025$
		<u>17182818</u>
		26

The next and the following terms need not be considered, as they will all give 0's only up to the 7th decimal place

Ex 2 Find the value, correct to 5 places of decimals, of

$$\frac{1}{18} + (\frac{1}{18})^2 + (\frac{1}{18})^3 + (\frac{1}{18})^4 + \text{to infinity}$$

Let s denote the sum of the given series

$$\text{Then } s = \frac{1}{18} + (\frac{1}{18})^2 + (\frac{1}{18})^3 + (\frac{1}{18})^4 +$$

$$\frac{1}{81}s = \frac{1}{18} + (\frac{1}{18})^2 + (\frac{1}{18})^3 + (\frac{1}{18})^4 +$$

Hence by subtraction, we get

$$(\frac{1}{81} - 1)s = 1, \text{ or } \frac{1}{81}s = 1, \quad \therefore s = \frac{81}{1} = \underline{23076}$$

Ex 3 Find the value, correct to 7 decimal places, of

$$\frac{1}{35} + \frac{2}{3^2 5^3} + \frac{3}{3^3 5^5} + \frac{4}{3^4 5^7} + \&c$$

Let s denote the sum of the series,

$$\begin{aligned}\text{then } s &= \frac{1}{3 \cdot 5} + \frac{2}{3^2 \cdot 5^1} + \frac{3}{3^1 \cdot 5^2} + \frac{4}{3^4 \cdot 5^1} + \&c \\ \therefore \frac{1}{3 \cdot 5^2} s &= \frac{1}{3^2 \cdot 5^1} + \frac{2}{3^1 \cdot 5^2} + \frac{3}{3^4 \cdot 5^1} + \&c\end{aligned}$$

By subtraction, we have

$$\begin{aligned}\left(1 - \frac{1}{3 \cdot 5^2}\right) s \text{ or } \frac{74}{75} s &= \frac{1}{3 \cdot 5} + \frac{1}{3^2 \cdot 5} + \frac{1}{3^1 \cdot 5^2} + \frac{1}{3^4 \cdot 5^1} + \&c \\ \therefore \frac{74}{75} s \times \frac{1}{3 \cdot 5^2} &= \frac{1}{3^2 \cdot 5^1} + \frac{1}{3^1 \cdot 5^2} + \frac{1}{3^4 \cdot 5^1} + \&c\end{aligned}$$

Again by subtraction, we get

$$\begin{aligned}\frac{74}{75} s \times \left(1 - \frac{1}{3 \cdot 5^2}\right) &= \frac{1}{3 \cdot 5}, \text{ or } \frac{74}{75} s \times \frac{74}{75} = \frac{1}{15} \\ \therefore s &= \frac{75 \times 75}{74 \times 74 \times 15} = \frac{375}{5476} = \underline{\underline{0.684806}}\end{aligned}$$

391 Abbreviated method of dividing a number by 9, 99, 999, &c

RULE Point off in the dividend as many decimal places (counting from the right) as there are *nines* in the divisor, then again twice as many decimal places, next three times as many, and so on. Then add these several numbers as in Addition of Decimals. The integral part will give the quotient and the recurring part the remainder.

Ex Divide 578921 by 99 by the abbreviated method

<div style="text-align: right;"> 5789 21 57 8921 578921 00578921 0000578921, &c <hr style="border: 0.5px solid black;"/> 5847 6868681021 </div>	$\begin{aligned}578921 - 99 &= 578921 \times \frac{1}{99} = 578921 \times 01 \\ &= 578921 \times \left\{ \frac{1}{100} + \frac{1}{(100)^2} + \frac{1}{(100)^3} + \&c \right\} \\ &= \frac{578921}{100} + \frac{578921}{10000} + \frac{578921}{1000000} + \&c \\ &= 5789 \ 21 + 57 \ 8921 + 578921 + 00578921 \\ &= 5847 \ 6868 \qquad \qquad \qquad + \&c\end{aligned}$
--	---

Hence the quotient is 5847 and remainder 68

Examples CXVIII

1 Multiply (by the *contracted method*) —

- | | |
|---------------------------|-------------------------------|
| (1) 43429448 by 6931472 | retaining 7 places of decimal |
| (2) 459 63524 by 25 4637 | 6 |
| (3) 583 26784 by 00985 | 2 |
| (4) 0008127 by 483 2716 | 6 |
| (5) 3670 257 by 12 61158 | 3 |
| (6) 86858896 by 1 0986123 | 5 |

- (7) 52 687640812 by 18 703216231 retaining 6 places of decimals
 (8) 1 050625 by itself 4
 (9) 27 5436 by 8 347 5
 (10) 012345 by 49 36 5

2 Divide (by the *contracted method*) —

- (1) 3789 436 by 265 5984 retaining 2 places of decimals
 (2) 742 876315 by 4967 358 4
 (3) 185 37612 by 08764032 4 ..
 (4) 154 362904 by 000541398 7
 (5) 10 926954 by 3547808034 3
 (6) 2 by 15 314865 5 ...
 (7) 1 by 3 1415926535 6 ..
 (8) 2 34721 by 3 27924 7 .
 (9) 176 80432 by 25 123456 . 3 .
 (10) 66 02037 by 248 722 5 ..

3 Find the respective values of —

- (1) $\frac{1}{5} + \frac{1}{5^2} + \frac{1}{5^3} + \frac{1}{5^4} + \&c$ to infinity
 (2) $\frac{1}{7} + \frac{1}{7^2} + \frac{1}{7^3} + \frac{1}{7^4} + \&c$ to
 (3) $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \&c$ to
 (4) $1 + \frac{1}{13} + \frac{1}{13^2} + \frac{1}{13^3} + \frac{1}{13^4} + \&c$ to infinity to 7 places of decimals
 (5) $\frac{1}{5} + \frac{1}{3} \times \frac{1}{5^2} + \frac{1}{5} \times \frac{1}{5^3} + \frac{1}{7} \times \frac{1}{5^4} + \&c$ to 6 . .
 (6) $\frac{1}{10} \times \left\{ 1 - \frac{3}{10} + \frac{34}{12} \times \frac{1}{10^4} - \frac{345}{123} \times \frac{1}{10^6} + \&c \right\}$ to 6 .
 (7) $16 \times \left\{ \frac{1}{5} - \frac{1}{3} \times \frac{1}{5^2} + \frac{1}{5} \times \frac{1}{5^3} - \frac{1}{7} \times \frac{1}{5^4} + \&c \right\} - \frac{4}{239}$ to 6 . . .
 (8) $\frac{1}{4} + \frac{3}{4^2} + \frac{1}{4^3} + \frac{3}{4^4} + \frac{1}{4^5} + \frac{3}{4^6} + \&c$ to infinity
 (9) $5 \times \left\{ 1 - \frac{1}{50} - \frac{1}{2 \times (50)^2} - \frac{1 \times 3}{6 \times (50)^3} - \frac{1 \times 3 \times 5}{24 \times (50)^4} \right.$
 $\left. - \frac{1 \times 3 \times 5 \times 7}{120 \times (50)^5} - \&c. \text{ to inf} \right\}$ to 5 places of decimals
 (10) $\frac{2}{5} + \frac{4}{5^2} + \frac{2}{5^3} + \frac{4}{5^4} + \frac{2}{5^5} + \frac{4}{5^6} + \&c$ to infinity

- 4 Divide (by the *abbreviated* method) —
 (1) 2916438 and 75061382 separately by 9
 (2) 51647901 and 7204561 separately by 99
 (3) 7204561 and 580844 separately by 999
 (4) 591608 and 7391684 separately by 9999
 (5) 236916 by 9999 and 720532876 by 99999

Examples worked out

Ex 1 A man owns $\frac{1}{8}$ of a house, and sells $\frac{1351}{10000}$ of his share ; what fraction of the house does he still own ?

He sells $\frac{1351}{10000}$ of $\frac{1}{8} = \frac{1351}{80000}$ of $\frac{1}{8} = \frac{1351}{80000}$ of $\frac{1}{8}$

he has left $(1 - \frac{1351}{80000})$ of $\frac{1}{8} = \frac{79649}{80000}$ of $\frac{1}{8} = \frac{79649}{640000}$ *Ans*

Ex 2 A vessel's cargo, $\frac{2}{3}$ of which is worth £6666 $\frac{6}{10}$, gets damaged, and the owner in consequence sells $\frac{8333 + 0.416}{1.05}$ of it for half the original value of the whole cargo. What is the value of the remainder at the same rate and what the loss on the whole cargo ?

The whole cargo is worth $\frac{2}{3}$ of £6666 $\frac{6}{10} = £9999 \frac{9}{10} = £10000$

He sells $\frac{8333 + 0.416}{1.05} = \frac{8749}{1.05} = \frac{875}{1.05} = \frac{875}{1050} = \frac{5}{6}$

he has remaining $(1 - \frac{5}{6})$ or $\frac{1}{6}$

Now since $\frac{2}{3}$ of the cargo sells for $\frac{1}{2}$ of £10000 = £5000 ,

∴ $\frac{1}{6}$ of the cargo must sell for $\frac{1}{2}$ of £5000 = £1000

Hence loss = £(10000 - 5000 - 1000) = £4000

Ex 3 A woman had a certain number of eggs , she sold $\frac{25}{100}$ of the number and 3 more to one person, $\frac{375}{1000}$ of the remainder to a second, and $\frac{6}{100}$ of what still remained to a third, when she had only 15 left. How many had she at first ?

After selling $\frac{25}{100}$ or $\frac{1}{4}$ of the second remainder, she had $(1 - \frac{1}{4})$ or $\frac{3}{4}$ of the eggs left. Therefore $\frac{3}{4}$ of the second remainder = 15 , the second remainder = $15 \times \frac{4}{3} = 45$

Again, $\frac{375}{1000}$ or $\frac{3}{8}$ of the first remainder being sold, $\frac{5}{8}$ remained , $\frac{5}{8}$ of the first remainder = 45 , ∴ the first remainder = $45 \times \frac{8}{5} = 72$

Next, after selling $\frac{25}{100}$ or $\frac{1}{4}$ of what she now had and 3 more, she had 72 left , $\frac{3}{4}$ of the number = $72 + 3 = 75$

∴ the whole number of eggs = $75 \times \frac{4}{3} = \underline{100}$

Ex 4 A owns $\frac{583}{1000}$ of an estate and B the rest. If $\frac{1}{5}$ of B's share is Rs 5000 less than A's, what is the worth of the whole estate ?

Since A 's share = $\frac{2}{3}$ or $\frac{1}{2}$ of the estate,

B 's share = $(1 - \frac{1}{2})$ or $\frac{1}{2}$ of the estate

$\therefore \frac{1}{2}$ of B 's share = $(\frac{1}{2} \times \frac{1}{2})$ or $\frac{1}{4}$ of the estate, and the difference of their shares = $(\frac{1}{2} - \frac{1}{4})$ or $\frac{1}{4}$ of the estate

Therefore $\frac{1}{4}$ of the estate = Rs 5000,

the whole estate = Rs 5000 \times 4 = Rs 15000

Miscellaneous Examples IV

1 Find the sum, difference, product and two quotients of 3033 and 0337, and find the sum of all the results

2 Reduce $(\frac{3}{4}$ of 245 - $\frac{1}{100}$ of 02) - 1000 to a decimal

3 Find the sum of 3102 + 00071 + 5876 + 12 + 31907 + 027 + 31068 + 0000743 + 38691 + 1041457

4 Which is the greater, 39 of a guinea, or 4099 of £1?

5 Divide the sum of 825 and 4125 by their difference

6 Divide the product of 1075 and 0101 by 13

7 Divide the difference between 31047 and 0731 by the sum of 127 and 11384

8 If 3 of an estate is sold for Rs 4504, find the value of 48 of it at the same rate

9 A man, who possesses $\frac{2}{3}$ of a ship, sells $\frac{1}{6}$ of his share for Rs 32400, what is the ship worth?

10 In a school of 200 children there are 4 classes, of which the first contains 24, the second 36, and the third 18 of the whole, of how many does the fourth class consist?

11 If 6 of the number of apples in a basket exceeds 6 of the number by 574, find the number of apples

12 Divide 8064 by 846 + $\frac{2}{3}$ of 2916

13 Divide $\frac{052}{13}$ of 156 by $\frac{0624}{144}$ of 2592

14 A butcher bought an equal number of calves and sheep for £265, for the calves he gave £375 a head, and for the sheep £2875 a head, how many did he buy of each kind?

15 A gentleman having given $\frac{1}{5}$ of the money in his purse for a horse, and $\frac{3}{7}$ of the remainder for a sheep, had £16875 still left, what sum had he at first?

16 Divide Rs 870 between A, B and C, so that 75 of C's share shall = 5 of A's = 6 of B's

17 A coal-dealer bought 198 mds of coal for Rs 325875, of which he sold 100 mds for Rs 2375 a maund. At what price per seer must he sell the remainder so as to gain Rs 21875 by his bargain?

18 *A* had Rs 2568 11a 4p, which was Rs 431 885416 less than 6 of $\frac{7}{7}$ of 25 times *B*'s money. How much money had *B*?

19 How many oranges at £ 084375 a dozen ought to be given for 378 eggs at 0625s each?

20 What number must be subtracted from the product of 927 and 80003 to give the sum of 19,279,652, 003, 50267 and 1709?

21 *A* has shares in an estate to the amount of 25 of it and of $\frac{36}{36}$ of it. *B* has shares in the same estate to the amount of $25\frac{7}{2}$ of it, find the difference in value between the properties of *A* and *B*, when $\frac{36}{36}$ of the estate is worth Rs 50000

22 Divide 9614 by 0000019 and $\frac{2\frac{1}{2}}{5\frac{1}{2}}$ by 0003 and multiply the sum of the quotients by 0005

23 Express the value of $\frac{133\frac{1}{2}}{83\frac{3}{7}} - \left(1 + \frac{2}{3 + \frac{4}{5 + \frac{1}{7}}}\right)$ of a rupee in decimals of £1, when the value of the rupee is Rs $5\frac{1}{2}d$

24 Simplify $0576 \times 197 + 142857 - 2\frac{1}{2} + 0454864$

25 Divide 1001 by 390625, 1001 by 000390625 and 1001 by 390625. Multiply 118 by 538461

26 Find the value (to three places of decimals) of

$$1 + \frac{1}{2} + \frac{1 \times 3}{1 \times 2} \left(\frac{1}{2}\right)^2 + \frac{1 \times 3 \times 5}{1 \times 2 \times 3} \left(\frac{1}{2}\right)^3 + \frac{1 \times 3 \times 5 \times 7}{1 \times 2 \times 3 \times 4} \left(\frac{1}{2}\right)^4 + \&c \text{ to infinity}$$

27 Simplify —

$$\frac{3\frac{3}{4}}{60625} \text{ of } \frac{97}{242} - \frac{25}{109} (725 + 275) \times \frac{£3 \ 6s \ 8d}{£10 \ 13s \ 4d}$$

28 Subtract 03 from $0\frac{3}{3}$ and divide the result by 102

29 Find the value of 016 of Rs 260 2a 6p + $\frac{3}{5}$ of Rs 13 14a + 100033 of Rs 7 14a 3p

30 Find how much more than $\frac{0338184}{03416}$ of 116 of 6 of $58\frac{7}{7}$ of Rs 52 1a 4p I need to pay a bill of Rs 21 4a

31 A person owns $\frac{7}{8}$ of an estate, and sells 3571428 of his share, what part of the whole estate has he still left?

32 *A* and *B* can do a piece of work in 1575 days, *B* and *C* can do it in 186 days and *A* and *C* in $16\frac{2}{3}$ days. In what time would *A*, *B* and *C* singly perform the whole work?

33 There is a number which, when multiplied by 4255 and divided by 0016, gives 851, find the number

34 Shew that, whether the value of $3\frac{1}{2} + 4\frac{2}{5} - 5\frac{1}{2} + 16\frac{3}{5} - \frac{1}{2} + 10 - 14\frac{3}{5}$ be found by vulgar fractions or by decimals, the results coincide

35 The owner of 375 of a mine sold $\frac{6}{5}$ of his share for Rs 25200, find the value of $\frac{8}{7}$ of the mine

36 A cistern of water lost 12 of its contents by leakage, then 26 gals were drawn off, and it was then $\frac{7}{5}$ full, how many gals. did it contain at first?

37 In a cricket match, one side of 11 men made a certain number of runs, one player obtained 25 of the number, each of three others 1, each of two others 0.625, and the rest 39 amongst them, find the whole number of runs

38 Reduce to their simplest forms -

$$(1) \frac{0.05}{\frac{1}{18} \text{ of } 11\frac{1}{2}} \text{ of } \frac{49\frac{1}{2}}{\frac{1}{4} \text{ of } 25} - \left(\frac{1}{21} + \frac{1}{27} \right) \quad (2) 70\frac{1}{2} \quad (3) \frac{4}{10} - \frac{1}{10}$$

39 Five bells which toll at intervals of 12, 15, 17.5, 18.25 seconds respectively, begin tolling simultaneously, how long after will they all toll simultaneously again?

40 Reduce £24 16s 4½d and £167 10s 6¼d 1q to decimals of the same denomination, so as to find how often the former is contained in the latter

41 Find the value of $\frac{0.9318}{5681}$ of $2\frac{1}{2}$ of 25 days

42 A woman has a certain number of eggs, she sells 3 of the number and one more to one person, 3 of the remainder to a second person, and 5 of the remainder to a third person, after these sales she has 15 eggs left. How many had she at first?

43 A clerk copied 55 of Rs 50 instead of 55 of Rs 50, what was the amount of the error?

44 From a rod 2.078 miles long, portions are cut off each equal to 0.037 of an inch how many such portions can be cut off and what will be the remainder?

45 Express the sum of 571428 of a vis, $\frac{2}{3}$ of $\frac{1}{3\pi}$ of $\frac{1}{164}$ of a maund and $\frac{3801}{10118}$ of a cwt as the decimal of 1 ton (a vis = 3lbs 2 oz, one maund = 82½lbs Avoir)

46 The difference in the values of the two shares into which a certain property is divided is Rs 48 575, and one share is $\frac{5}{1}$ of the whole. Find the value of the property and of each share

47 A has an income = $(6 \text{ of } 83 - 35)$ of B's income. If A after spending Rs 645 per annum, find that he has exceeded his income by 0.75 of it, find B's income

48 A can reap $\frac{1}{4}$ of a field in 26 days and B can reap $\frac{6}{5}$ of it in 45 days, A and B work together till they have reaped $\frac{7}{5}$ of

the field A then leaves, and B completes the work. If A earn Rs 2 8a a day, what ought the reaping of the field to cost?

49 Out of a bag of silver, I take Rs 25 more than $\frac{1}{5}$ of the whole sum which it contained, then Rs 15 more than $\frac{1}{2}$ of what then remained, and then Rs 10 more than $\frac{1}{3}$ of what then remained, after this Rs 5 remained. What did the bag contain at first?

50 A has shares in an estate to the amount of $15 - \frac{3}{8}$ of it, B has shares in the same estate to the amount of $\frac{4}{7}$ of it, find the difference in value between the properties of A and B , when $\frac{1}{16}$ of the estate is worth £373 3

CHAPTER VII

Rules of Practice and Invoices

392 We shall here shew how the primitive fractions, as defined in Art 228, may be applied to the *practical* calculation of prices, when the price of a unit of any denomination is supposed to be given, and the tediousness of the *enunciations* of the rules at length, will be a sufficient excuse for the mere *indications* of the processes to be employed, by means of examples

393 An aliquot part of a number is such that we may make up the number by taking the part a certain *integral* number of times. Its relation with the whole can therefore be expressed by a fraction which has unity for its numerator and an integer for its denominator

Thus, 5a 4p, being $\frac{1}{4}$ of Re 1, is an *aliquot* part of a rupee, 10s, being $\frac{1}{2}$ of £1, is an *aliquot* part of a pound

Table of Aliquot Parts

OF A Rupee		OF A £		OF A Maund	
8a	= $\frac{1}{2}$ Re	10s	= $\frac{1}{2}$ £	20 sr	= $\frac{1}{2}$ md
5a 4p	= $\frac{1}{4}$ Re	6s 8d	= $\frac{1}{3}$ £	10 sr	= $\frac{1}{3}$ md
4a	= $\frac{1}{3}$ Re	5s	= $\frac{1}{4}$ £	8 sr	= $\frac{1}{4}$ md
2a 8p	= $\frac{1}{5}$ Re	4s	= $\frac{1}{5}$ £	5 sr	= $\frac{1}{5}$ md
2a	= $\frac{1}{6}$ Re	3s 4d	= $\frac{1}{6}$ £	4 sr	= $\frac{1}{6}$ md
		2s 6d	= $\frac{1}{8}$ £	2 sr 8 ch	= $\frac{1}{8}$ md
		2s	= $\frac{1}{10}$ £	2 sr	= $\frac{1}{10}$ md
		1s 8d	= $\frac{1}{12}$ £	1 sr 4 ch	= $\frac{1}{12}$ md
1a 4p	= $\frac{1}{12}$ Re	1s 4d	= $\frac{1}{15}$ £	1 sr	= $\frac{1}{16}$ md
		1s 3d	= $\frac{1}{16}$ £		
1a	= $\frac{1}{16}$ Re	1s	= $\frac{1}{20}$ £		

OF AN Anna	OF A Shilling	OF A Seer
6p = $\frac{1}{2}a$	6d = $\frac{1}{2}s$	8 ch = $\frac{1}{2}sr$
4p = $\frac{1}{3}a$	4d = $\frac{1}{3}s$	4 ch = $\frac{1}{4}sr$
3p = $\frac{1}{4}a$	3d = $\frac{1}{4}s$	2 ch = $\frac{1}{8}sr$
2p = $\frac{1}{6}a$	2d = $\frac{1}{6}s$	1 ch = $\frac{1}{16}sr$
1p = $\frac{1}{12}a$	1½d = $\frac{1}{8}s$	
2ps = $\frac{1}{6}a$	1¼d = $\frac{1}{10}s$	
1ps = $\frac{1}{12}a$	1d = $\frac{1}{20}s$	
OF A Ton	OF A Cwt	OF A Quarter
10 cwt = $\frac{1}{10}$ ton	2 qrs = $\frac{1}{2}$ cwt	14 lb = $\frac{1}{2}$ qr
5 cwt = $\frac{1}{4}$ ton	1 qr = $\frac{1}{4}$ cwt	7 lb = $\frac{1}{4}$ qr
4 cwt = $\frac{1}{5}$ ton	16 lbs = $\frac{1}{4}$ cwt	4 lb = $\frac{1}{7}$ qr
2 cwt 2qr = $\frac{1}{8}$ ton	14 lbs = $\frac{1}{8}$ cwt	3 lb 8 oz = $\frac{1}{8}$ qr
2 cwt = $\frac{1}{10}$ ton		2 lb = $\frac{1}{14}$ qr
1 cwt 1qr = $\frac{1}{16}$ ton		1 lb 12 oz = $\frac{1}{16}$ qr
1 cwt = $\frac{1}{20}$ ton		1 lb = $\frac{1}{28}$ qr
OF A Bigha	OF A Katha	OF A lb AVOIR
10 kathas = $\frac{1}{2}$ big	8 ch = $\frac{1}{2}k$	8 oz = $\frac{1}{2}$ lb
5 k = $\frac{1}{4}$ big	4 ch = $\frac{1}{4}k$	4 oz = $\frac{1}{4}$ lb
4 k = $\frac{1}{5}$ big	2 ch = $\frac{1}{8}k$	2 oz = $\frac{1}{8}$ lb
2 k 8 ch = $\frac{1}{8}$ big	1 ch = $\frac{1}{16}k$	1 oz = $\frac{1}{16}$ lb
2 k = $\frac{1}{10}$ big		
1 k 4 ch = $\frac{1}{16}$ big		
1 k = $\frac{1}{20}$ big		
OF AN Acre	OF A Rood	OF AN Oz AVOIR
2 ro = $\frac{1}{2}$ ac	20 po = $\frac{1}{2}$ ro	8 dr = $\frac{1}{2}$ oz
1 ro = $\frac{1}{4}$ ac	10 po = $\frac{1}{4}$ ro	4 dr = $\frac{1}{4}$ oz
20 po = $\frac{1}{8}$ ac	8 po = $\frac{1}{8}$ ro	2 dr = $\frac{1}{8}$ oz
16 po = $\frac{1}{10}$ ac	5 po = $\frac{1}{20}$ ro	1 dr = $\frac{1}{16}$ oz
	4 po = $\frac{1}{10}$ ro	
	2 po = $\frac{1}{20}$ ro	
	1 po = $\frac{1}{40}$ ro	
OF A Week	OF A Month	OF A Mile
3½ da. = $\frac{1}{2}$ wk	1 wk = $\frac{1}{4}$ mo	4 fur = $\frac{1}{4}$ mi
1½ da. = $\frac{1}{4}$ wk	2 wk = $\frac{1}{2}$ mo	2 fur = $\frac{1}{2}$ mi
	15 da = $\frac{1}{2}$ mo	1 fur = $\frac{1}{8}$ mi
	10 da = $\frac{1}{3}$ mo	
		OF A Furlong
		110 yd = $\frac{1}{2}$ fur
		55 yd = $\frac{1}{4}$ fur

394 Practice is a short method of finding the value of any quantity by means of *aliquot parts*, when the value of a unit of any denomination is given. It is therefore another method of solving questions in *Compound Multiplication*.

395 Practice may be either Simple or Compound.

It is *Simple Practice*, when the value of one unit of a certain denomination is given, and the value of a number of these units is

required, but in *Compound Practice*, the given quantity is not wholly expressed in the same denomination as the unit whose value is given.

Thus, to find the value of 350 articles at 15s 8p each is *Simple Practice*, and to find the value of 14 mds 15 sr 7 ch at Rs 2 5a 8p per maund is *Compound Practice*

I. SIMPLE PRACTICE

396 The RULE for Simple Practice will be best understood by the following Examples

Ex 1 Find the value of 1298 things at Rs 8 14a 6p each

If the cost of a thing be Re 1, then the total cost is Rs 1298

	Rs	a	p	
8a = $\frac{1}{2}$ of Re 1	1298	0	0	= price @ Re 1 each
			8	
	10384	0	0	= price @ Rs 8
4a = $\frac{1}{2}$ of 8a	649	0	0	= price @ 8a
2a = $\frac{1}{2}$ of 4a	324	8	0	= price @ 4a
6p = $\frac{1}{4}$ of 2a	162	4	0	= price @ 2a
	40	9	0	= price @ 6p
	<u>Rs 11560</u>	5	0	= price @ Rs 8 14a 6p each

Note 1 It is generally most convenient, when possible, to use the *aliquot part* of the denomination next superior to the highest denomination of the price proposed

Here, Rs 8 14a 6p is less than Rs 9 by 1a 6p Hence the calculation may be shortened thus —

1a = $\frac{1}{2}$ of Re 1	Rs 1298	0a	0p	= price at Re 1 each
			9	
	Rs 11682	0	0	= price at Rs 9 each
6p = $\frac{1}{2}$ of 1a		Rs 81	2a	= price at 1a
		Rs 40	9a	= price at 6p
	121	11	0	= price at 1a 6p
	<u>Rs 11560</u>	5	0	= price at Rs 8 14a 6p

Ex 2 Find the cost of 345 things at £3 17s 10½d each

	£	s	d	
10s = $\frac{1}{2}$ of £1	345	0	0	= cost @ £1 each
			3	
	1025	0	0	= cost @ £3 each
5s = $\frac{1}{2}$ of 10s	172	10	0	= cost @ 10s
2s 6d = $\frac{1}{2}$ of 5s	86	5	0	= cost @ 5s
3d = $\frac{1}{16}$ of 2s 6d	43	2	6	= cost @ 2s 6d
1½d = $\frac{1}{2}$ of 3d	4	6	3	= cost @ 3d
	2	3	1½	= cost @ 1½d
	<u>£1343</u>	6	10½	= cost @ £3 17s 10½d

Otherwise thus — As £3 17s 10½d is the difference between £4 and 2s 1½d, we can simplify the process thus —

2s = $\frac{1}{10}$ of £1	£	s	d	£	s	d	
1½d = $\frac{1}{16}$ of 2s	345	0	0	345	0	0	= cost @ £1 each
	34	10	0			4	
	2	3	1½	1380	0	0	= cost @ £4 each
	£36	13	1½	36	13	1½	= cost @ 2s 1½d
				£1343	6	10½	= cost @ £3 17s 10½d

Note 2 Sometimes by introducing a *subsidiary* aliquot part we can easily find the required aliquot part, thus, taking the preceding example, we have

2s = $\frac{1}{10}$ of £1	£	s	d	
6d = $\frac{1}{4}$ of 2s	345	0	0	= cost at £1 each
1½d = $\frac{1}{4}$ of 6d	34	10	0	= cost at 2s
	8	12	0	
	2	3	1½	= cost at 1½d
	36	13	1½	= cost at 2s 1½d

Ex 3 Find the value of 456½ mds at Rs 8 5a 10p per maund

Since Re $\frac{1}{4}$ = 10a, the cost of 456½ mds at Re 1 is Rs 456 10a; we therefore proceed as before, thus —

4a = $\frac{1}{4}$ of Re 1	Rs	a	p	
	456	10	0	= value @ Re 1 each
			8	
1a = $\frac{1}{4}$ of 4a	3653	0	0	= value @ Rs 8
6p = $\frac{1}{4}$ of 1a	114	2	6	= value @ 4a
3p = $\frac{1}{4}$ of 6p	28	8	7½	= value @ 1a
1p = $\frac{1}{4}$ of 3p	14	4	3½	= value @ 6p
	7	2	1½	= value @ 3p
	2	6	0½	= value @ 1p
	Rs 3819	7	7½	= value @ Rs 8 5a 10p each

Ex 4 Find the cost of 2864½ cwt at 9s 10½d per cwt

Since £ $\frac{1}{2}$ would introduce a fraction of a farthing, it will be better to find separately the cost of 2864 cwt and $\frac{1}{2}$ of a cwt and then add

5s = $\frac{1}{4}$ of £1	£	s	d		s	d
4s = $\frac{1}{5}$ of £1	2864	0	0	= cost @ £1 each	9	10½
10d = $\frac{1}{8}$ of 5s	716	0	0	= cost @ 5s		3
6d = $\frac{1}{10}$ of 5s	572	16	0	= cost @ 4s	7)29	8½
	119	6	8	= cost @ 10d		4 2½
	77	12	0			
¾d = $\frac{1}{8}$ of 6d	8	19	0	= cost @ ¾d		
	£1417	1	8	= cost @ 9s 10½d each		
		4	2½	= cost of ½ of a cwt		
	£1417	5	10½	= cost @ 9s 10½d per cwt		

Ex 5. Find the price of 2108 cwt of sugar at £1 6s 2½d each

	£	s	d	
4s = $\frac{1}{2}$ of £1	2108	0	0	= price at £1 each
2s = $\frac{1}{4}$ of 4s	421	12	0	= price at 4s
2d = $\frac{1}{2}$ of 2s	210	16	0	= price at 2s
$\frac{1}{4}$ d = $\frac{1}{8}$ of 2d	17	11	4	= price at 2d
$\frac{1}{8}$ d = $\frac{1}{2}$ of $\frac{1}{4}$ d	2	3	11	= price at $\frac{1}{4}$ d
	1	1	11½	= price at $\frac{1}{8}$ d
	£2761	5	2½	= price at £1 6s 2½d each

Examples CXIX

Find by Practice the values of the following articles —

- | | |
|---------------------------|--------------------------|
| 1 3467 at 2a 6p | 2 659 at 13a 2p |
| 3 1448 at 10a 8p | 4 1281 at 5a 4p |
| 5 2370 at 13a 4p | 6 659 at 1s 7½d |
| 7 1250 at 2s 3½d | 8 328 at 8s 5½d |
| 9 7351 at 14s 9½d | 10 2345 at Rs 2 14a 8p |
| 11 1600 at Rs 2 5a 6p | 12 140321 at 13a 11½p |
| 13 632 at Rs 14 5a 6p | 14 7777 at 17s 8½d |
| 15 1298 at 17s 9½d | 16 537 at £1 7s 2½d |
| 17 2937 at £2 11s 10½d | 18 1684 at £8 5s 1½a |
| 19 412 at £5 14s 5½d | 20 6439 at Rs 16 15a 7½p |
| 21 295 at Rs 5 11a 7½p | 22 3655 at £9 16s 10½d |
| 23 3546 at £5 15s 7½d | 24 65437 at Rs 4 13a 2p |
| 25 1449½ at Rs 11 6a 6p | 26 237½ at 13a 8p |
| 27 1128¾ at Rs 2 15a 11p | 28 7432½ at Rs 6 12a 4p |
| 29 6147¾ at 17s 6½d | 30 2763¾ at 13s 6½d |
| 31 217½ at £2 17s 7½d | 32 769½ at Rs 16 4a |
| 33 674¾ at £3 19s 6½d | 34 22 6 at 5a 1p |
| 35 169 875 at £2 17s 10½d | 36 359 3125 at £1 6s 2d |
| 37 3764 6 at Rs 27 4a 10p | 38 178 6 at Rs 3 5a 2p |
| 39 821 5 at Rs 6 15a 2p | 40 861 at Rs 5 7a 5½p |
| 41 45656 at 6a 2½p | 42 2841 at 5s 10½d |
| 43 2731 at £4 8s 9¾d | 44 567384 at 5a 10½p |
| 45 30000 at Rs 4 2a 4¾p | 46 5109½ at £4 16s 4½d |

II COMPOUND PRACTICE

397 The RULE for Compound Practice will be easily shewn by the following Examples

Ex 1 Find the value of 8mds 6sr 12ch at Rs 5 6a 8p per md

5 sr = $\frac{1}{5}$ of 1 md	Rs	a	p	
	5	6	8	= value of 1 md
			8	
1 sr = $\frac{1}{5}$ of 5 sr	43	5	4	= value of 8 mds
8 ch = $\frac{1}{8}$ of 1 sr	10	10		= value of 5 sr
4 ch = $\frac{1}{2}$ of 8 ch	2	2		= value of 1 sr
	1	1		= value of 8 ch
			6 $\frac{1}{2}$	= value of 4 ch
	Rs 44	3	11 $\frac{1}{2}$	= value of 8 mds 6 sr 12 ch

Ex 2 What is the price of 3 cwt 2 qrs 16 lbs at £3 7s 8d per cwt ?

2 qrs = $\frac{1}{2}$ of 1 cwt	£	s	d	
	3	7	8	= price of 1 cwt
			3	
14 lbs = $\frac{1}{4}$ of 2 qrs	10	3	0	= price of 3 cwt
2 lbs = $\frac{1}{8}$ of 14 lbs	1	13	10	= price of 2 qrs
		8	5 $\frac{1}{2}$	= price of 14 lbs
		1	2 $\frac{1}{2}$	= price of 2 lbs
	£ 12	6	6	= price of 3 cwt 2 qrs 16 lbs

Ex 3 Find the value of 11 mds 4 sr 8 ch at Rs 1 14a 4p per seer

8 ch = $\frac{1}{8}$ of 1 sr	Rs	a	p	
4 sr = 1 sr × 4	1	14	4	= value of 1 seer
1 md = 4 sr × 10			4	
	7	9	4	= value of 4 sr
			10	
	75	13	4	= value of 1 md
			11	
	834	2	8	= value of 11 mds
	7	9	4	= value of 4 sr
		15	2	= value of 8 ch
	Rs 842	11	2	= value of 11 mds 4 sr 8 ch

Ex 4 Find the value of 365 mds 37 sr 8 ch at Rs 126 6a p per maund

20 sr = $\frac{1}{2}$ of 1 md

<i>Rs</i>	<i>a</i>	<i>p</i>	
126	6	8	=value of 1 md
<hr/>			
		10	
1264	2	8	=value of 10 mds
<hr/>			
		10	
12641	10	8	=value of 100 mds
<hr/>			
		3	
37925	0	0	=value of 300 mds
7585	0	0	=value of 60 mds
632	1	4	=value of 5 mds
63	3	4	=value of 20 sr
31	9	8	=value of 10 sr
15	12	10	=value of 5 sr
7	14	5	=value of 2 sr 8 ch
<hr/>			
<i>Rs</i> 46260	9	7	=value of 365 mds 37sr 8ch

10 sr = $\frac{1}{2}$ of 20 sr
 5 sr = $\frac{1}{2}$ of 10 sr
 2 sr 8 ch = $\frac{1}{2}$ of 5 sr

Ex 5 Find the rent of 71 bighas 6 kat 14 ch at *Rs* 8 12a per bigha

4 kat = $\frac{1}{2}$ of 1 big

<i>Rs</i>	<i>a</i>	<i>p</i>	
8	12	0	=rent of 1 bigha
<hr/>			
		10	
87	8	0	=rent of 10 bighas
<hr/>			
		7	
612	8	0	=rent of 70 bighas
8	12	0	=rent of 1 bigha
1	12	0	=rent of 4 kat
	14	0	=rent of 2 kat
	3	6	=rent of 8 ch
	1	9	=rent of 4 ch
<hr/>			
		101	=rent of 2 ch
<hr/>			
<i>Rs</i> 624	4	1 $\frac{1}{2}$	=rent of 71 big 6 kat 14 ch

2 kat = $\frac{1}{2}$ of 4 kat
 8 ch = $\frac{1}{2}$ of 2 kat
 4 ch = $\frac{1}{2}$ of 8 ch
 2 ch = $\frac{1}{2}$ of 4 ch

Examples CXX

Find by Practice the value, rent, &c (as the case may be) of —

- 15 mds 25 sr 11 ch at *Rs* 12 10a 8p per maund
- 8 mds 11 sr 7 ch at *Rs* 6 10a 8p per maund
- 18 mds 5 sr 6 ch at *Rs* 27 14a 8p per maund
- 777 mds 20 sr 12 ch at *Rs* 40 10a 8p per maund
- 373 mds 39 sr 7 ch at *Rs* 25 2a 4p per maund
- 3 cwt 2 qrs 17 lbs at £1 5s 8d per quarter
- 57 cwt 3 qrs 14 lbs at £5 9s 6d per cwt
- 45 oz 6 dwts 7 grs at 5s 10d per oz
- 37 cwt 3 qrs 2 lbs at £3 14s 7 $\frac{1}{2}$ d per cwt

- 10 72 cwt 3 qrs 17 lbs at 6s 1½d per quarter
 11 15 tons 11 cwt 3 qrs 18 lbs at £3 7s 6d per cwt
 12 6 tons 12 cwt 3 qrs 10½ lbs at £3 14s 8¼d per cwt
 13 5 ac 2 ro 4 po 4½ yds at Rs 10 per rood
 14 16 yds 2 ft 10 in at 2s 6½d per yard
 15 196 miles 3 fur 137½ yds at Rs 363 4a 8p per mile
 16 7 mds 2 sr 14 ch at 3a 6p per seer
 17 38 mds 25 sr 10 ch at 10a 6p per seer
 18 53 big 12 kat 2 ch at Rs 19 12a per bigha
 19 155 big 1 kat 4 ch at Rs 89 8a 4p per bigha
 20 44 ac 2 ro 25 po at £55 16s 7½d per acre
 21 35 qrs 7 bus 3¼ pls at 58s 6d per quarter
 22 9 cub vds 21 ft 432 in at £4 14s 6d per cub yard
 23 5 lbs 10 oz 12 dwts 6¾ grs at £3 17s 11d per oz
 24 17 tons 12 cwt 3 qrs 18 lbs at £6 15s 9d per cwt
 25 6231 cwt 2 qrs 11 lbs 15 oz at £3 14s 8d per cwt
 26 191 ac 3 ro 37 po at £42 3s 4d per acre
 27 18 gals 3 qts 1½ pts at 17s 10½d per gallon
 28 8 kan 4 mds 32 palm at Rs 3 7a 5p per md
 29 45 kan 14 mds 28 sr at Rs 33 13a 7p per kandi
 30 5 ac 2 ro 7 po 88 sq yds at £161 6s 8d per acre
 31 7 mo 2 wks 5 days at Rs 24 2a 8p per month
 32 9 mo 1 wk 6 days at Rs 11 6a per week
 33 48 sq yds 8 ft 114 in at 13s 7½d per sq yd
 34 28 yds 2 qrs 1¼ nl at £1 11s 1½d per yard
 35 7 mds 7 vis 39 palm at Rs 2 15a 6p per md

11

398 The method of Practice may conveniently be applied to such examples as the following —

Ex 1 Find the dividend on Rs 57201 12a at 5a 4½p in the Rupee

	Rs	a	p	
4a = ¼ of Re 1	57201	12	0	= amount of debts in full
1a = ¼ of 4a	14300	7	0	= amt at 4a in the Re
3p = ¼ of 1a	3575	1	9	= amt at 1a ..
1½p = ½ of 3p	893	12	5½	= amt at 3p
	446	14	2½	= amt at 1½p
	<u>Rs 19216</u>	3	4½	= amt at 5a 4½p in the Rupee

Ex 2 Find the rent for 3 mo 3 wks 4 days from January 1, at Rs 106 12a per month

The month of April for which rent is due for 3 wks 4 days or 25 days, contains 30 days

15 days = $\frac{1}{2}$ of 30 days	$\begin{array}{r} Rs \quad a \quad p \\ 106 \quad 12 \quad 0 \\ \hline \end{array}$	= rent of 1 month
10 days = $\frac{1}{3}$ of 30 days	$\begin{array}{r} 320 \quad 4 \quad 0 \\ 53 \quad 6 \quad 0 \\ 35 \quad 9 \quad 4 \\ \hline \end{array}$	= rent of 3 months = rent of 15 days = rent of 10 days
	$Rs \quad 409 \quad 3 \quad 4$	= rent of 3 mo 25 days, or 3 mo 3 wks 4 days

Ex 3 Find the value of 35 chests of tea, each containing 1 md 17 sr 9 ch at Rs 80 12a per maund

10 sr = $\frac{1}{4}$ of 1 md	$\begin{array}{r} Rs \quad a \quad p \\ 80 \quad 12 \quad 0 \\ \hline \end{array}$	= value of 1 md
5 sr = $\frac{1}{8}$ of 10 sr	$\begin{array}{r} 20 \quad 3 \quad 0 \\ \hline \end{array}$	= value of 10 sr
2 sr 8 ch = $\frac{1}{2}$ of 5 sr	$\begin{array}{r} 10 \quad 1 \quad 6 \\ \hline \end{array}$	= value of 5 sr
1 ch = $\frac{1}{16}$ of 2 sr 8 ch	$\begin{array}{r} 5 \quad 0 \quad 9 \\ 2 \quad 0 \quad 9 \\ \hline \end{array}$	= value of 2 sr 8 ch = value of 1 ch
	$Rs \quad 116 \quad 3 \quad 3\frac{9}{10}$	= value of 1 md 17 sr 9 ch or of 1 chest
35 = 5 \times 7	$\begin{array}{r} 35 \\ \hline \end{array}$	
	$Rs \quad 4067 \quad 2 \quad 4\frac{1}{2}$	= value of 35 chests

Ex 4 Find to the nearest pie the rent of 275 365 bighas at Rs 3 7a 9p per bigha

4a = $\frac{1}{4}$ of 1 Re	$\begin{array}{r} Rs \\ 275 \quad 365 \\ \hline \end{array}$	= rent at Re 1 per bigha
	$\begin{array}{r} 3 \\ \hline \end{array}$	
2a = $\frac{1}{2}$ of 4a	$\begin{array}{r} 826 \quad 095 \\ \hline \end{array}$	= rent at Rs 3
1a = $\frac{1}{4}$ of 2a	$\begin{array}{r} 68 \quad 841 \quad 25 \\ \hline \end{array}$	= rent at 4a
6p = $\frac{1}{4}$ of 1a	$\begin{array}{r} 34 \quad 420 \quad 625 \\ \hline \end{array}$	= rent at 2a
3p = $\frac{1}{8}$ of 6p	$\begin{array}{r} 17 \quad 210 \quad 3125 \\ \hline \end{array}$	= rent at 1a
	$\begin{array}{r} 8 \quad 605 \quad 15625 \\ \hline \end{array}$	= rent at 6p
	$\begin{array}{r} 4 \quad 302 \quad 578125 \\ \hline \end{array}$	= rent at 3p
	$Rs \quad 959 \quad 474 \quad 921875$	= rent at Rs 3 7a 9p per bigha
and Rs 959 475 = Rs 959 7a 7p		the required rent

Examples CXXI

- 1 A bankrupt pays 10a 6p in the rupee, find the dividend on a debt of Rs 3471
- 2 Find the price of 5222 yds at Rs 29 13a for a dozen yards
- 3 How much income tax must be paid on an income of £756 18s 6d at 1s 2d in the pound?
- 4 Find the price of 256479 articles at £4 12s 6 $\frac{1}{2}$ d per 100
- 5 Find the price of 265 sheep at £63 3s 1 $\frac{1}{2}$ d per score

6 How much will the carriage of 5 packages, each containing 4 cwt 3 qrs 21 lbs, come to, at Rs 6 4a per ton?

7 What is the dividend on Rs 57348 5a 4p at 7a 6p in the rupee?

8 What is the dividend on £1710 14s 6d at 13s 4½d per £?

9 Find the price of 111 things at £11 11s 11d per every 11

10 Find the weight of 2697 packages, each weighing 19 lbs 10 oz 18 dwts 22 grs

11 What distance will a train travel in 3 hours 39 min 22 sec at a speed of 49 miles 7 fur 52 yds per hour?

12 Find the rent for 11 mo 2 wks 6 days from March 1, 1889 at Rs 38 4a 6p per month

13 Find the produce of 14 bighas 18 k 2 ch at 12 mds 8 sr per bigha

14 Find the rent of 375 3675 bighas at Rs 29 15a per bigha

15 Find the value of 143 7526 gallons of spirit at Rs 11 14a per gallon

16 A bankrupt owes Rs 7953 75 and pays 12a 3p in the rupee, what is the value of his assets?

17 A bankrupt's debts amount to Rs 35483 5a 4p, find what his creditors will lose, if he pay 10a 3½p in the rupee

18 When exchange is at 2s 1½d per rupee, what is the value of Rs 4032 8a 8p in English money?

19 Find the rent for 7 mo 3 wks 4 days from Feb 1, at Rs 60 per month

20 If 1 lb Avoir is 1 lb 2 oz 11 dwts 16 grs Troy, what is the weight (Troy) of 1 cwt 2 qrs 25 lbs 10 oz 6 drs?

III INVOICES

399 Every tradesman sells his goods at two prices, cash and credit. When payment is made at the time of purchase, it is called *cash*, but otherwise *credit*. Both these sales are entered in a book called the **Day Book**, in the order in which they occur in the course of the day.

The *entries* in the *Day-Book* are posted at short intervals in the *Ledger*, the index of which contains a list of customers' names in alphabetical order. For facility of reference, opposite each name is the page of the *Ledger* in which is collected all the dealings which have taken place with that particular customer.

400 When a buyer has completed his purchases he is presented with a **Bill** containing in detail a written list of the goods bought with a statement of the cost of them attached. An **Invoice** is a copy of the *Bill* which is sent home with the goods or forwarded to a customer living at a distance. Each separate entry in an *Invoice* or a *Bill* is called an **Item**.

401 An Account is a statement sent by the seller to the buyer at the end of the term of a credit shewing the totals and dates of each *Invoice* and the sum total of the whole. In such a case the *account* is said to be rendered (*ie*) sent to the buyer. If the details of the goods are also given, it is called a Detailed Account or Bill of Parcels.

(i) SPECIMEN OF AN INVOICE

INVOICE		Calcutta, 4th April, 1897		
From S C AUDIN ESQ, 58, Wellington Street				
16 copies of Hall and Steven's Euclid	at Rs 3 1a 6p	Rs	a	p
14 copies of Todhunter's Euclid	at Rs 2 6a 6p	49	8	0
25 copies of Lock's Arithmetic	at Rs 3 1a 6p	33	11	0
10 copies of Dicken's Novels	at Rs 1 4a	77	5	6
		12	6	0
		173	0	6

(ii) SPECIMEN OF AN ACCOUNT

K C SETHI & CO		Calcutta, May 4th, 1897		
Bought of KHLIL MOHAN DEB & CO, 45, Radha Bazar Street, Calcutta				
1897		Rs	a	p
January 5	10 goods as per invoice	48	10	6
February 12	10 goods as per invoice	59	7	3
March 18	10 ditto	85	12	0
April 4	10 ditto	72	6	6
		266	4	3

(iii) SPECIMEN OF A DETAILED ACCOUNT

H BATHOUR, Esq		Calcutta, July 24th, 1897		
Bought of MOORE & CO, Dharmatollah Street				
1897		Rs	a	p
April 21	40yds Irish linen	at Rs 1 4a 8p	51	10 8
	1doz Dusters	" 4a 8p	3	8 0
	25yds Lovelling	" 7a 4p	11	7 4
May 4	23yds Flannel	" Rs 1 3a 4p	27	12 8
	15yds Brown Holland	" 7a 8p	7	3 0
June 26	14yds Calico	" 3a 8p	3	3 4
	23yds Brussels Carpet	" Rs 2 4a 8p	50	6 8
	2 Rugs, Rs 10 8a, Rs 18 8a		29	0 0
			82	100
			184	38

Examples CXXII

Make out invoices for the following —

1 10 sr of sugar at $3a\ 9p$ per sr , 6 sr of tea at $15a\ 3p$ per sr , 8 sr of coffee at $14a\ 3p$ per sr , 12 sr of wheat at $1a\ 2p$ per sr , 10 sr of rice at $1a\ 3p$ per sr , and 9 sr of cream at $11a$ per sr

2 $4\frac{1}{2}$ yds of long cloth at $2a\ 9p$ per yd , $7\frac{1}{4}$ yds of cambric at $4a\ 6p$ per yd , 6 pairs of socks at $1a\ 3p$ per pair , 3 pairs of hose at $4a\ 6p$ per pair , 1 doz pairs of socks at $2a\ 9\frac{1}{2}p$ per pair , and $5\frac{1}{2}$ yds of flannel at $8a\ 11p$ per yd

3 5 lbs of black tea at $Rs\ 1\ 5a\ 4p$ per lb , $2\frac{1}{4}$ lbs of green tea at $Rs\ 2\ 4a$ per lb , $15\frac{1}{2}$ lbs of lump sugar at $3a\ 8p$ per lb , 17 lbs of moist sugar at $2a\ 8p$ per lb , $7\frac{1}{2}$ lbs of raisins at $7a\ 4p$ per lb , and 4 lbs of currants at $4a\ 4p$ per lb

4 $15\frac{1}{2}$ yds flannel at $2s\ 3d$ per yd , 29 yds of calico at $8\frac{1}{2}d$ per yd , 25 yds of Irish linen at $2s\ 4d$ per yd , 17 yds of towelling at $1s\ 2d$ per yd , $12\frac{1}{2}$ yds of brown holland at $11\frac{1}{2}d$ per yd , and $3\frac{1}{2}$ doz handkerchiefs at $9s\ 10d$ a doz

5 $39\frac{1}{2}$ yds of Brussels carpet at $Rs\ 2\ 10a\ 8p$ per yd , $62\frac{1}{2}$ yds of Kidderminster carpet at $Rs\ 1\ 12a$ per yd , 27 yds of cocoa nut matting at $9a\ 4p$ per yd , $34\frac{1}{2}$ yds of drugget at $Rs\ 1\ 2a$ per yd , and $43\frac{1}{2}$ yds of India matting at $8a\ 8p$ per yd

6 $17\frac{1}{8}$ yds of calico at $6a\ 6p$ per yd , $35\frac{1}{4}$ yds of flannel at $14a\ 2p$ per yd , $96\frac{1}{4}$ yds of sheeting at $Rs\ 1\ 0a\ 4p$ per yd , $104\frac{1}{8}$ yds of holland at $8a\ 6p$ per yd , and $12\frac{3}{8}$ yds of ribbon at $5a\ 7p$ per yd

7 $17\frac{1}{2}$ mds of coal at $Rs\ 8\ 14a$ per md , carriage of ditto at $Rs\ 1\ 2a$ per md , 2 mds of coke at $Rs\ 14\ 9a\ 4p$ per md , 62 mds of gram at $Rs\ 2\ 2a$ per md , 23 sr of seed at $9a$ per seer , and 136 mds of grain at $Rs\ 3\ 11a$ per md

8 2425 yds of cloth at $Rs\ 5\ 4a$ per yd , 12 yds of flannel at $15a\ 4p$ per yd , 4375 yds of calico at $6a$ per yd , 1275 yds of drugget at $Rs\ 1\ 6a$ per yd , 37 yds of Brussels carpet at $Rs\ 1\ 13a\ 4p$ per yd , and 255 yds of Kidderminster do at $Rs\ 1\ 4a\ 8p$ per yd

9 $3\frac{1}{2}$ pharas of lime at $Rs\ 2\ 3a\ 4p$ per phara , 15 sr of ghee at $Rs\ 20\ 8a$ per md , $2\frac{3}{4}$ sr of tea at $Rs\ 1\ 0a\ 8p$ per seer , 20 sr of flour at $Rs\ 2\ 3a$ per md , $3\frac{1}{2}$ yds of flannel at $Rs\ 1\ 2a$ per yd , and 29 yds of calico at $9a\ 7p$ per yd

10 Calcutta, June 16th, 1885 — W Godfrey, Esq bought of Ghose and Co, 500 envelopes at $14s\ 8p$ per 100 , 3 boxes of elastic bands at $11a$ per box , $\frac{1}{2}$ a gross of penholders at $6a\ 4p$ per doz , $2\frac{1}{2}$ reams of Foolscap at $7a$ per quire , 4 dozen quill pens at $3a$ per doz , 13 note-books at $9a$ each , and 250 official envelopes at $Rs\ 1$ per 100 Make out a copy of the bill and find its amount

CHAPTER VIII

Involution and Evolution

402 A power of a number is the number which arises from successive multiplications by itself, the operation by which it is obtained is termed *involution*, and the degree or order of the power is denoted by the *number* of factors employed

Thus, taking the number 2, we shall have the *powers* of it as follows —

$2=2$, the first power of 2, $2 \times 2=4$, the second power of 2,

$2 \times 2 \times 2=8$, the third power of 2,

$2 \times 2 \times 2 \times 2=16$, the fourth power of 2,

$2 \times 2 \times 2 \times 2 \times 2=32$, the fifth power of 2,

$2 \times 2 \times 2 \times 2 \times 2 \times 2=64$, the sixth power of 2,

and so on, as far as we please,

but instead of expressing these multiplications at *length*, which would soon become inconvenient, we denote the same operations by means of indices or small figures placed a little above the line to the right of the quantities whose powers are intended to be exhibited, thus, what is put down above may be denoted by

$2^1=2$, $2^2=4$, $2^3=8$, $2^4=16$, $2^5=32$, $2^6=64$, &c.,

where the index sometimes called the *exponent* is equal to the number of *factors* and is greater by *one* than the number of *operations*

403 The *second* powers of the *nine* digits are expressed thus —

$1^2=1$, $2^2=4$, $3^2=9$, $4^2=16$, $5^2=25$, $6^2=36$,

$7^2=49$, $8^2=64$, $9^2=81$,

and their *third* power as follows —

$1^3=1$, $2^3=8$, $3^3=27$, $4^3=64$, $5^3=125$, $6^3=216$,

$7^3=343$, $8^3=512$, $9^3=729$

The *second* and *third* powers of numbers are styled their *squares* and *cubes* and the operations by which *all* powers are obtained are merely those of multiplication

404 A power of a fraction is equal to the fraction formed by *raising* both its numerator and denominator to the power, and the power of a quantity formed by factors is found by raising each factor to the power

Thus, $(\frac{2}{3})^2=\frac{2}{3} \times \frac{2}{3}=\frac{4}{9}$, $(\frac{2}{3})^3=\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}=\frac{8}{27}$, and so on

Also $(2 \times 7)^2=2^2 \times 7^2=4 \times 49=196$, $(2 \times 5)^3=2^3 \times 5^3=8 \times 125=1000$

Note A mixed quantity must be represented as a simple fraction or as a decimal, before the process can be applied

405 A root of a number is such a number as being multiplied into itself one or more times produces it, and the operation by which this root is obtained is called *evolution*

Thus, the second or square root of 16 is 4, because the *square* of 4 is 16, or $4^2=4 \times 4=16$. The third or cube root of 512 is 8, since the *cube* of 8 is 512, or $8^3=8 \times 8 \times 8=512$, and similarly of vulgar fractions and decimals

406 This operation is expressed by the sign $\sqrt{}$ which is called the *radical sign*, with a small figure placed on its left to *particularize* the root intended thus,

$$\sqrt[2]{16}=4, \sqrt[3]{512}=8, \text{ and } \sqrt[4]{32}=2,$$

but the *square root* is denoted by the sign $\sqrt{}$ *only*, without the small figure, as being of most frequent occurrence

These operations are also indicated by means of the primitive fractions $\frac{1}{2}, \frac{1}{3}, \&c.$, used as *indices*, so that the *indices* $\frac{1}{2}, \frac{1}{3}, \&c.$, denote operations exactly the reverse of those expressed by the *indices* 2, 3, &c., respectively thus,

$$4^2=16, 16^{\frac{1}{2}}=4 \text{ and } 8^3=512, 512^{\frac{1}{3}}=8$$

I EXTRACTION OF THE SQUARE ROOT

407 A perfect square is a number whose square root can be expressed exactly either by an integer or by a fraction

Thus, 16 is a *perfect square*, for its square root is 4

408 In squaring a number we see that its square has the same units' figure as the square of its units' figure, and if a number ends with 0, its square also ends with 0. Now as the squares of the simple numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, and also 10 are respectively 1, 4, 9, 16, 25, 36, 49, 64, 81 and 100, it follows that the square of every number (integral or decimal) must end with either 1, 4, 5, 6, 9, or an even number of 0's. Hence *no number ending with (1) the digits 2, 3, 7, 8, (11) an odd number of 0's, can be a perfect square*

409 By the help of the Multiplication Tables we can immediately obtain the square root of a number not exceeding 400

Thus, $9 \times 9=81$, and $15 \times 15=225$, $\sqrt{81}=9$, and $\sqrt{225}=15$

410 When a number can be easily resolved into its prime factors, its square root can be determined by inspection. In a perfect square, every prime factor that occurs must occur an *even* number of times. Thus,

Ex. 1 Find the square root of 1764

$$1764=2 \times 2 \times 3 \times 3 \times 7 \times 7=2^2 \times 3^2 \times 7^2=(2 \times 3 \times 7)^2$$

$$\therefore \sqrt{1764}=2 \times 3 \times 7=\underline{42} \text{ Ans}$$

Ex 2 Obtain the square root of 705600

$$705600 = 10 \times 10 \times 2 \times 2 \times 6 \times 6 \times 7 \times 7 = (10^2 \times 2^2 \times 6^2 \times 7^2) \\ = (10 \times 2 \times 6 \times 7)^2$$

$$\therefore \sqrt{705600} = 10 \times 2 \times 6 \times 7 = \underline{840} \text{ Ans}$$

Ex 3 What is the *least number* which, when multiplied into 51425, will make the product a perfect square?

$$51425 = 11 \times 11 \times 425 = 11 \times 11 \times 25 \times 17 = 11^2 \times 5^2 \times 17$$

$$\therefore \text{the required number} = \underline{17} \text{ Ans}$$

Examples CXXIII

1 Find the respective values of —

- | | | | |
|--|------------------------|-------------------------------|-----------------|
| (1) 31^3 | (2) $39^2 \times 48^1$ | (3) 925^2 | (4) $(31\ 5)^3$ |
| (5) $(806^2 + 31^2) \times 59$ | (6) $(2\frac{1}{2})^4$ | (7) $506^2 + 506^2 - 307^3$ | |
| (8) $(15^2 - 1\ 31^2) - 15$ | | (9) $502^3 + 18^1 - 1376^2$ | |
| (10) $(7\ 03^2 \times 19)^2 - (3\ 14 \times 02)^3$ | | (11) $1\ 03(4\ 07 + 3\ 16)^2$ | |

2 Find (*by inspection*) which of the following are square numbers —

- (1) 27, 96, 524, 9450, 7805, 9604, 12321, 494208
 (2) 4000, 75720, 388129, 582168, 12343225, 38812900

3 Find the square roots of (*using factors*) —

- (1) 49, 196, 289, 361, 324, 256, 121, 400, 144
 (2) 625, 529, 900, 1296, 17424, 63504, 99225
 (3) 680625, 48024900, 12446784, 2480625, 57153600

4 Find the *least numbers* which, when multiplied into the following numbers, will make the products perfect squares —

175, 693, 1456, 3465, 3456, 4536, 28413, 750750

5 What must be the least number of soldiers in a regiment, to admit of its being drawn up 2, 3, 4, 5, or 6 deep, and also of its being formed into a solid square?

411 From the number of figures in any proposed quantity, to find the number of figures in its square root

Since, the square root of 1 is 1,
 the square root of 100 is 10,
 the square root of 10000 is 100,
 the square root of 1000000 is 1000, &c,

we see immediately that the square root of a number of fewer than 3 figures must consist of only 1 figure, that of a number of more than 2 figures and fewer than 5, of 2 figures, that of a number of

more than 4 figures and fewer than 7, of 3 figures, and so on, whence it follows, that if a dot or full point be placed over every alternate figure, beginning at the *units'* place, the number of such points will be the same as the number of figures in the square root. This is called the **Rule of Pointing**

Thus, the square root of 198 consists of 2 figures in its integral part, the square root of 314256 consists of 3 figures in its integral part, and so on

412 The extraction of the square root of a number depends on the principle illustrated by the following example —

$$\text{Since } 28^2 = (20+8)^2 = 20^2 + 2 \times 20 \times 8 + 8^2,$$

$$28^2 - 20^2 = 2 \times 20 \times 8 + 8^2,$$

$$28^2 - 20^2 - (2 \times 20) = 8 + \text{a proper fraction}$$

413 *To extract the square root of a whole number*

RULE Point the alternate figures of the number proposed, beginning at the place of units, so as to form as many periods of two figures each as possible, and remember that each period consists of the figure over which the dot is placed and the figure to its left (The first period may consist of one figure only)

Find the greatest square number contained in the first period on the left hand, put down its root on the right as in division, and subtract it from that period. To the remainder bring down the next period for a dividend. double the root just found for a divisor (called the *trial divisor*), and find how often it is contained in this dividend exclusive of the figure on its right hand, annex this quotient to the figures in both the quotient and divisor. Multiply the divisor thus *completed* by the last figure of the quotient, and if the product be not greater than the dividend, subtract it from the dividend, but if the product be greater, use a *lower* number for the root figure until it becomes less, subtract the product as before. To this remainder bring down the period which comes next in order, take twice the number in the root, and see how often it is contained in this dividend with its last figure omitted, and proceed precisely as before. Repeat the process till every period in succession is disposed of, and the root will thus be obtained

Note If at any step the quotient figure is 0, set down 0 in the root, annex it to the trial divisor, bring down the next period and proceed as before

Ex 1 Find the square root of 8649

Place dots over 9 and 6, so that the number is divided into two periods, 86 and 49

The number whose square is immediately below 86 is 9 (for $9^2 = 81$ which is next below 86) Hence 9 is put in the root and 81 subtracted from 86

$$\begin{array}{r} 8649 \text{ (} \underline{93} \text{)} \\ 81 \\ \hline 183 \overline{) 549} \\ \underline{1549} \end{array}$$

To the remainder 5 is brought down the next period 49, thus the new dividend is 549. Now $2 \times 9 = 18$, is the *trial divisor*, which goes into 54 (549 with 9 omitted) 3 times. Hence 3 is put after 9 in the root and also annexed to 18. Multiply 183 by 3 and the product is 549, which subtracted from the dividend leaves nothing. Therefore 93 is the root obtained.

Ex 2 Extract the square roots of 804609, 12809241, and 21224449

$$\begin{array}{r} (1) \quad 804609 \overline{) 897} \\ \underline{64} \\ 1691646 \\ \underline{1521} \\ 178712509 \\ \underline{112509} \end{array}$$

$$\begin{array}{r} (2) \quad 12809241 \overline{) 3579} \\ \underline{9} \\ 65380 \\ \underline{325} \\ 7075592 \\ \underline{4049} \\ 714961341 \\ \underline{61341} \end{array}$$

$$\begin{array}{r} (3) \quad 21224449 \overline{) 4607} \\ \underline{16} \\ 86522 \\ \underline{516} \\ 920764449 \\ \underline{64449} \end{array}$$

414. When an integer (which is a perfect square) ends with an even number of ciphers, it would be sufficient to extract the square root of the significant figures and then to annex to the root *one* cipher for every *two* ciphers in the proposed number.

Ex Extract the square root of 841000000

$$\begin{array}{r} 841000000 \overline{) 290000} \\ \underline{4} \\ 49141 \\ \underline{141} \end{array}$$

Here are 6 ciphers in the given number, therefore we add 3 ciphers to 29 the square root of 841

415 Again, since the square root of 01 is 1,
the square root of 0001 is 01,
the square root of 000001 is 001, &c.,

we infer that the quantity proposed must first be made to have an *even* number of decimal places, and then the pointing must proceed from the place of *units* towards the right hand over every alternate figure as before, and the number of such points will be the same as the number of decimal places in the square root.

416 If there be no whole number or integral part in the proposed number, we must, in pointing, begin with the *second* figure from that which would be the *units* place, if there were a whole number, and place dots successively over every alternate figure to the right. If there be a whole number as well as a decimal fraction, it would be the safest method to begin at the *units* place and point over every alternate figure to the *right and left* of it. The number of dots over the whole numbers and decimals will shew the number of figures in the integral and decimal parts of the root respectively.

Ex Extract the square roots of 93 7024, 02819041 and 00822649.

<p>(1) $93\ 7024 \underline{968}$</p> $\begin{array}{r} 81 \\ 186 \overline{) 1270} \\ \underline{1116} \\ 1928 \overline{) 15424} \\ \underline{15424} \end{array}$	<p>(2) $02819041 \underline{1679}$</p> $\begin{array}{r} 1 \\ 36 \overline{) 181} \\ \underline{156} \\ 327 \overline{) 2590} \\ \underline{2289} \\ 3349 \overline{) 30141} \\ \underline{30141} \end{array}$	<p>(3) $00822649 \underline{0907}$</p> $\begin{array}{r} 81 \\ 1807 \overline{) 12649} \\ \underline{12649} \end{array}$
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Examples CXXIV

1 Find the square roots of —

- (1) 676, 1444, 16129, 21025, 288369, 998001, 71289
- (2) 2025, 692224, 54756, 822649, 97574884, 10004569
- (3) 33016516, 45859984, 5774409, 62805625, 4020025
- (4) 6512490000, 5777216064, 95481000000, 3915380329
- (5) 8260628544, 93870306991561, 787026841863680889

2 Extract the square roots of —

- (1) 22 09, 33 64, 1082 41, 22 8484, 187 4161, 128881
- (2) 0064, 005329, 00053361, 00038025, 3659 0401
- (3) 1164 1744, 136966 6081, 240168 6049, 236 144689
- (4) 41605 800625, 00501361708761, 00000049112064

3 A certain number of boys spent Rs 90 4a, each spending as many four-anna pieces as there were boys, what was the number of boys?

4 A square pavement contains 20736 square stones, all of the same size, what number composes one of its sides?

5 A society collected among themselves for certain purposes a fund of Rs 459 6a, each person paid twice as many pies as there were members in the whole society Find the number of members

6 A general, trying to mass his army of 15410 men into a square, found he had 34 men over, required the number of men in the front

417 If the number is not a perfect square, we can find an approximation to its square root to any required number of decimal places by affixing ciphers to the right hand of the proposed number and bringing down periods of 2 ciphers each

Ex Find the square roots of 11 and 4, each to 4 places of decimals

$$\begin{array}{r}
 (1) \quad 11 \ 00000000 \ (\underline{3 \ 3166}) \\
 \begin{array}{r}
 9 \\
 63 \overline{) 200} \\
 \underline{189} \\
 661 \overline{) 1100} \\
 \underline{661} \\
 6626 \overline{) 43900} \\
 \underline{39756} \\
 66326 \overline{) 414400} \\
 \underline{397956} \\
 16444
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 (2) \quad 40000000 \ (\underline{6324}) \\
 \begin{array}{r}
 36 \\
 123 \overline{) 400} \\
 \underline{369} \\
 1262 \overline{) 3100} \\
 \underline{2524} \\
 12644 \overline{) 57600} \\
 \underline{150576} \\
 7024
 \end{array}
 \end{array}$$

418 When the proposed number is a recurring decimal, extend the recurring part by a repetition of its period and then proceed as in decimals

Thus, to extract the square root of 4 3157 to four places of decimals, first extend the recurring part 157 and put 4 3157157157 for 4 3157, and then proceed as usual

419 When the number of figures to be found in the decimal part of the root is *large*, we may obtain in the usual way one more than half the required number of figures in the root, and then the remaining figures by dividing the last remainder by the last divisor, as in Art 389

Ex Extract the square root of 10 to 8 places of decimals

$$\begin{array}{r}
 10 \quad (\underline{3 \ 16227766}) \\
 \begin{array}{r}
 9 \\
 61 \overline{) 100} \\
 \underline{61} \\
 626 \overline{) 3900} \\
 \underline{3756} \\
 6322 \overline{) 14400} \\
 \underline{12644} \\
 63242 \overline{) 175600} \\
 \underline{126484} \\
 49116
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 6,3,2,4,4) 49116(7766 \\
 \underline{44271} \\
 4845 \\
 \underline{4427} \\
 418 \\
 \underline{379} \\
 39 \\
 \underline{38}
 \end{array}$$

The first 5 figures are obtained in the usual way, and the last 4 by Contracted Division

Examples CXXV

1 Find the square roots of (each to 4 places of decimals) —

(1) 2, 3, 5, 6, 7, 8, 12, 13, 18, 20, 32, 38

(2) 44, 51, 72, 80, 95, 638, 796, 801, 1000

(3) 5713, 363, 35120, 8837, 822646, 72471438, 7432

2 Extract the square roots of (each to 4 places of decimals) —

- (1) 1 2, 3, 4, 5, 6, 7, 8, 9, 12, 16
 (2) 05, 51, 49, 16, 016, 01, 51, 051, 403
 (3) 002, 225, 021, 3, 45, 3485, 32173025, 187
 (4) 314159, 175250564, 1256636, 2941275, 78946193
 (5) 2351, 50132, 402981, 4900521, 07, 5757, 0198

3 Find to 10 decimal places the square roots of —

- (1) 001728, 979, 0683, 3467, 44284, 157, 75347
 (2) 85, 07, 3, 979, 0003532, 27773, 0365

420 The square root of a fraction may be obtained by finding the square roots of its numerator and denominator separately

- (1) If the denominator of the given fraction, or of the fractional part of the mixed number, be a *perfect square*, we apply the Rule directly, whether the numerator be a perfect square or not

$$\text{Thus, } \sqrt{\frac{144}{169}} = \frac{\sqrt{144}}{\sqrt{169}} = \frac{12}{13}, \quad \sqrt{8\frac{17}{64}} = \sqrt{\frac{529}{64}} = \frac{\sqrt{529}}{\sqrt{64}} = \frac{23}{8} = 2\frac{7}{8}$$

$$\sqrt{\frac{29}{64}} = \frac{\sqrt{29}}{\sqrt{64}} = \frac{5\ 385164}{8} \dots = 673145$$

$$\sqrt{1278\frac{7}{25}} = \sqrt{\frac{31957}{25}} = \frac{\sqrt{31957}}{\sqrt{25}} = \frac{178\ 7652}{5} = 35\ 7530$$

- (2) But if the denominator of the given fraction or of the fractional part of the mixed number be *not a perfect square* we reduce the fraction or the mixed number either (i) to an equivalent fraction whose denominator is a perfect square and extract the square root of both numerator and denominator, or (ii) to a decimal, and proceed in the usual way

$$\text{Thus } \sqrt{\frac{5}{7}} = \sqrt{\frac{5 \times 7}{7 \times 7}} = \sqrt{\frac{35}{49}} = \frac{\sqrt{35}}{\sqrt{49}} = \frac{5\ 91607}{7} = 84515$$

$$\text{or} = \sqrt{(714285)} = 84515$$

$$\sqrt{25\frac{8}{11}} = \sqrt{\frac{283}{11}} = \sqrt{\frac{283 \times 11}{11 \times 11}} = \sqrt{\frac{3113}{121}} = \frac{\sqrt{3113}}{11} = \frac{55\ 794265}{11}$$

$$\text{or} = \sqrt{(25\ 72)} = 5\ 072205 \qquad \qquad \qquad = 5\ 072205$$

421 If a recurring decimal is a *perfect square*, it would be convenient to reduce it to a vulgar fraction and proceed as in Art 420 (1), above

$$\text{Thus, } \sqrt{(1\ \dot{7})} = \sqrt{\frac{16}{9}} = \frac{\sqrt{16}}{\sqrt{9}} = \frac{4}{3} = 1\frac{1}{3}$$

Examples CXXVI

1 Find the square roots of —

- (1) $\frac{4}{25}$, $\frac{16}{2500}$, $\frac{1361}{10000}$, $\frac{811}{10000}$, $\frac{304}{10000}$, $\frac{7071}{10000}$, $\frac{28642}{10000}$
 (2) $45\frac{5}{8}$, $10\frac{7}{8}$, $345\frac{1}{2}$, $32\frac{7}{8}$, $32\frac{1}{8}$, $41\frac{1}{2}$, $564\frac{2}{3}$
 (3) $3083\frac{1}{2}$, $72002\frac{1}{2}$, $30789\frac{1}{2}$, $15061\frac{1}{2}$, $154751\frac{1}{2}$

2 Find the square roots of (each to 4 places of decimals where the root does not come out exactly) —

- (1) $\frac{5}{8}$, $9\frac{1}{2}$, $6\frac{2}{5}$, $\frac{128}{125}$, $\frac{441}{64}$, $76\frac{14}{17}$, $\frac{504}{012}$, $\frac{00841}{1000}$
 (2) $\frac{2}{3}$, $21\frac{8}{11}$, $4\frac{5}{11}$, $1\frac{2}{3}$, $1\frac{1}{3}$, $21\frac{7}{11}$, $1\frac{2}{3}$, $27\frac{2}{3}$
 (3) $1\frac{2}{3}$, $1\frac{2}{3}$, $1\frac{2}{3}$, $1\frac{2}{3}$, $287\frac{1}{2}$, $7\frac{1}{2}$, $367\frac{1}{2}$, $756\frac{1}{2}$

3 Find the square roots of —

- (1) 1, 027, 187, 3361, 284, 00027, 4738027
 (2) 00134, 0711, 537, 004, 54, 017, 049382716

II EXTRACTION OF THE CUBE ROOT.

422 A perfect cube is a number whose cube root can be expressed exactly either by an integer or by a fraction

The cubes of the digits 1, 2, 3, 4, 5, 6, 7, 8, 9, are respectively 1, 8, 27, 64, 125, 216, 343, 512, 729, and it is important that these last numbers and the corresponding roots should be committed to memory

423 Given the number of figures in a number, to find the number of figures in its cube root

Since, the cube root of 1 is 1,

the cube root of 1000 is 10,

the cube root of 1000000 is 100, &c,

it follows that the cube root of any number between 1 and 1000 must lie between 1 and 10, i.e., will have one figure in the integral part, of any number between 1000 and 1000000 must lie between 10 and 100, i.e., will have 2 figures in the integral part, of any number between 1000000 and 1000000000 must lie between 100 and 1000, i.e., will have 3 figures in the integral part, and so on. Hence, if a point be placed over the *units'* figure of the number, and thence over every third figure to the left of that place the number of points will shew the number of figures in the integral part of the root. This rule may manifestly be extended to Decimals

424 To extract the cube root of a given number

RULE Place a point over the *units'* figure of the given number and thence over every *third* figure to its left, and also to its right if the number be a decimal, adding ciphers, if necessary, to get periods of three, and remember that each period consists of the figure over

which the dot is placed and the two figures to its left, if there are so many (for the first period may contain 1, 2, 3 figures)

Find the number whose cube is either equal to, or next less than the *first* period on the left hand and place it as the *first* figure of the root. Subtract its cube from the first period, and to the remainder bring down the *next* period.

Multiply the square of the root already obtained by 300 for a *trial* or *partial* divisor, and then find how often this divisor is obtained in the dividend, this quotient gives the *next* figure of the root. Then, multiply this quotient figure by the product of the previous figure of the root by 30, and place the result below the partial divisor. Below these place the square of this last quotient figure and add the three together for a *Complete* divisor. Multiply this complete divisor by the last figure of the root and subtract. To the remainder bring down the next period to form the next dividend.

Multiply the square of the root already obtained by 300, and find how often this trial divisor is contained in the dividend. Put this quotient as the third figure of the root. Then, multiply the figures of the root already obtained by 30 and the product by the last quotient figure, and place the product below the partial divisor. Then place the square of the last quotient figure, and add the three together, for a complete divisor. Multiply this divisor by the last figure and subtract, and bring down the next period to form the next dividend. Proceed in this way till all the periods have been brought down.

Note If at any step, the dividend is less than the divisor, put a cipher to the root, two ciphers to the trial divisor, and bring down the next period.

Ex 1 Find the cube root of 21952

$$\begin{array}{r}
 21952(\underline{28} \\
 8 \\
 2^3 \times 300 = 1200 \quad | \quad 13952 \\
 2 \times 30 \times 8 = 480 \\
 8^2 = 64 \\
 \hline
 1744 \quad | \quad 13952
 \end{array}$$

Here, first divide into periods beginning with 2, the first period on the left contains only 2 figures. The trial divisor 1200 goes into the dividend 13952, 8 times

Ex 2 Extract the cube root of 12812 904

$$\begin{array}{r}
 12812\ 904(\underline{23\ 4} \\
 8 \\
 2^3 \times 300 = 1200 \quad | \quad 4812 \\
 2 \times 30 \times 3 = 180 \\
 3^2 = 9 \\
 \hline
 1389 \quad | \quad 4167 \\
 23^3 \times 300 = 158700 \quad | \quad 645904 \\
 23 \times 30 \times 4 = 2760 \\
 4^2 = 16 \\
 \hline
 161476 \quad | \quad 645904
 \end{array}$$

First divide into periods of three beginning with 2, both left and right. The first period is 12 and the greatest cube root in 12 is 2.

The trial divisor 1200 goes into the dividend 4812, 3 times

The trial divisor 158700 goes into 645904, 4 times

We may shorten the process a little as below —

$$\begin{array}{r}
 63 \\
 \underline{6} \\
 694
 \end{array}
 \quad
 \begin{array}{r}
 12 \\
 \underline{189} \\
 1389 \\
 \underline{9} \\
 1587 \\
 \underline{2776} \\
 161476
 \end{array}
 \left.
 \begin{array}{l}
 12812904(234 \\
 8 \\
 \underline{4812} \\
 4167 \\
 \underline{645904} \\
 645904
 \end{array}
 \right\}$$

In column II instead of writing ciphers, put 9 the units' figure of 189 two places further to the right, in the same manner, write 2776 in the second step

To find the trial divisor in the second step, take the sum of $189 + 1389 + 3$, which is equal to the product of 69×23

425 If the number is not a perfect cube, we may obtain its cube root to any required number of decimal places by annexing ciphers and bringing down periods of three ciphers each

Ex Extract the cube root of 3 to 3 decimal places

$$\begin{array}{r}
 186 \\
 \underline{12} \\
 1989
 \end{array}
 \quad
 \begin{array}{r}
 108 \\
 \underline{1116} \\
 11916 \\
 \underline{36} \\
 13068 \\
 \underline{17901} \\
 1324701
 \end{array}
 \left.
 \begin{array}{l}
 303000000(669 \\
 216 \\
 \underline{84000} \\
 71496 \\
 \underline{12504000} \\
 11922309 \\
 \underline{581691}
 \end{array}
 \right\}$$

Since the root is to be extracted to 3 places of decimals, there must be 3 periods of 3 figures in the decimal part, therefore we must annex 8 ciphers to 3

426 When one more than a half of the figures required in the root have been obtained by the ordinary method, the rest can be found by Contracted Division, as in Art 389

427 In extracting the cube roots of vulgar fractions, if the denominator of the fraction be a *perfect cube*, find the cube roots of both the numerator and the denominator separately, but if the denominator of the fraction be *not a perfect cube*, either reduce the fraction to an equivalent fraction whose denominator is a perfect cube and then extract the cube root of numerator and denominator, or reduce the fraction to a decimal and proceed in the ordinary way

$$\begin{aligned}
 \text{Thus, } \sqrt[3]{\frac{27}{64}} &= \frac{\sqrt[3]{27}}{\sqrt[3]{64}} = \frac{3}{4}, \quad \sqrt[3]{\frac{29}{64}} = \frac{\sqrt[3]{29}}{\sqrt[3]{64}} = \frac{3.072317}{4} = 768079... \\
 \sqrt[3]{\frac{85}{7}} &= \sqrt[3]{\frac{61}{7}} = \sqrt[3]{\frac{2989}{343}} = \frac{\sqrt[3]{2989}}{\sqrt[3]{343}} = \frac{14.4048}{7} = 2.0578 \\
 &\text{or} = \sqrt[3]{871.4285} = 2.0578
 \end{aligned}$$

Examples CXXVII

1 Find the cube roots of —

(1) 1331, 15625, 46656, 2197, 185193, 117649

- (2) 704969, 912673, 33076161, 15069223, 105823817
 (3) 873722816, 198767717056, 702121283072
 (4) 17 576, 132 651, 493 039, 64481 201, 18 609625
 (5) 007645373, 876467493, 001030301, 000026730899
 (6) $\sqrt[4]{343}$, $\sqrt[5]{140608}$, $49\sqrt[3]{2}$, $7558\sqrt[3]{1\frac{9}{16}}$, $465\sqrt[3]{\frac{3}{4}}$, $57\sqrt[3]{\frac{2}{3}}$
 (7) 1034, 5 912, 5, 078759, 3 467 (each to 4 decimal places)
 (8) $\frac{1}{2}$, $\frac{9}{13}$, $\frac{5}{7}$, $7\sqrt[3]{6}$, $18\sqrt[3]{\frac{1}{2}}$, $18\sqrt[3]{\frac{8}{27}}$ (each to 4 decimal places)
 (9) 002, 003, 013, 024, 2 187 (each to 8 places of decimals)

2 Find the cube roots of —

- (1) $\frac{512}{3375}$, $\frac{5030912}{65536}$, $\frac{512}{03375}$, $\frac{1257728}{16384}$
 (2) 3845 296, $\sqrt[3]{37}$, 1587 962, 8, 27, 325142, 81 812703 (the last four to 4 decimal places)

3 A cubical block of stone contains 50653 solid feet, find the length of its side

4 Extract the cube root of 233 744896, and derive the cube root of this number multiplied by 008

III EXTRACTION OF SOME OTHER ROOTS

428 The directions already employed may by a little management be rendered available for the discovery of some other roots as will be evinced in the following notes

- (1) The **Fourth** root of a number is found by extracting its square root, and then the square root of its square root
 (2) The **Sixth** root of a number is found by extracting its cube root, and then the square root of its cube root, or by extracting its square root, and then the cube root of its square root
 (3) The **Eighth** root of a number is found by extracting its square root, then the square root of its square root, and lastly the square root of that square root
 (4) The **Ninth** root of a number is found by extracting its cube root, and then the cube root of its cube root

Ex Find the *fourth* root of 1679616 and the *sixth* root of 308 915776

- (1) Here the *square* root is found to be 1296, and the *square* root of 1296 is 36 Therefore the *fourth* root of 1679616 is 36
 (2) Here the *square* root is found to be 17 576, and the *cube* root of 17 576 is 26 Therefore the *sixth* root required is 26

Examples CXXVIII

- 1 Find the fourth roots of 104976 , 1500625 , 4323738 0096
- 2 Find the sixth roots of 2985984 , 24 137569 , 17596 287801
- 3 Find the eighth roots of 214358881 , 21035 8 , 003532 , 57 $\frac{5}{8}$
(the last three to 5 decimal places)
- 4 Find the ninth roots of 262144 , 134217728 , 5159780352
- 5 Find the fourth root of 21 $\frac{201}{101}$, and the sixth roots of 85 766121 and 260184053769595201

Miscellaneous Examples V

1 What are the prime factors in 45090045, and what is the smallest whole number by which it must be multiplied in order to make it a perfect square ?

2 What is the difference between the values of

$$\frac{2\frac{1}{2} \text{ of } 5\frac{1}{2}}{27\frac{1}{2}} \text{ of Rs } 11 \text{ } 4a \text{ and } \frac{51\frac{1}{2}}{2\frac{1}{2} + 3\frac{1}{2}} \text{ of Rs } 6 \text{ } 4a \text{ ?}$$

3 A chain, 11 yds long, is divided into 50 equal parts, called links , find how many square links there are in an acre

4 Bring $\left\{ \left(\frac{5\frac{1}{2} - \frac{1}{2} \text{ of } 2\frac{5}{7} + \frac{2\frac{1}{2}}{47} \right) - 21\frac{28}{9} \times 3\frac{18}{100} \right\}$ cwt to the fraction of 4 $\frac{1}{2}$ ton

5 A merchant bought 264 gallons of spirit at Rs 12 8a 4 $\frac{3}{4}$ p per gal , 378 gallons at Rs 9 10a 7p per gal , and 420 gallons at Rs 12 15a 6 $\frac{1}{4}$ p per gal . If he sell the whole quantity at Rs 12 4a per gal , what profit will he make by the transaction ?

6 Two numbers have for their G C M 179 and for their L C M 56385 . What must the greater number be, if the less

$$= 105 \text{ times } \frac{2\frac{1}{2}}{4\frac{1}{2}} \text{ of } \frac{363}{84} \text{ ?}$$

7 Which is the greater

$$\frac{3}{5} \text{ of } \frac{13}{16} - \frac{1\frac{1}{2}}{6\frac{1}{2}} \text{ of } \frac{19}{20} + \frac{3}{7} \text{ of } \frac{6\frac{5}{8}}{3\frac{1}{2}} \text{ or } \frac{5}{3} \text{ of } \frac{13}{16} + \frac{6\frac{2}{3}}{1\frac{1}{4}} \text{ of } \frac{19}{20} - \frac{7}{3} \text{ of } \frac{6\frac{1}{2}}{3\frac{1}{2}} \text{ ?}$$

and express the difference as a decimal

8 Express Rs 6 5a 10 $\frac{3}{4}$ p as the fraction of Rs 9 8a 10p

9 Find by Practice the value of 29764 articles at Rs 1 11a 9 $\frac{1}{4}$ p each

$$10 \text{ Simplify } \frac{2}{3}(6\frac{1}{2} + 3\frac{1}{2})\text{L} + \frac{2\frac{1}{2} - \frac{3}{2} \text{ of } 1\frac{1}{2}}{\frac{1}{2} \text{ of } 3\frac{1}{2} + 1\frac{1}{2}} \times 95 \text{ of } 5s + \frac{168}{024} d$$

$$11 \text{ Simplify } (254)^2 + (246)^2 - 127 \times 984 + (6)^2$$

12 Prove that

$$0.76923 = \frac{07}{1-09} = \frac{076}{1-012} = \frac{0769}{1-0003} = \frac{07692}{1-00004}$$

13 A man bequeathes his property amounting to Rs 49166 in such a way that $\frac{1}{3}$ of his wife's share, $\frac{2}{3}$ of his eldest son's, $\frac{1}{3}$ of his younger son's and $\frac{1}{2}$ of his daughter's shares, are all equal Find the shares of each

14 In a subscription list one-half of the subscriptions are a guinea each, one third a half-guinea each, and the 5 shilling subscriptions which complete the list amount to £12, find the whole sum subscribed

15 A can mow 2 ro 32 po of grass in 1 day, B, 2 ac 2 ro in $2\frac{1}{2}$ days, C, 2 ac 2 ro 32 po in 3 days, what is the size of the smallest field which would employ each of them, working alone, an exact number of days? In what time would each mow the field?

16 Express as decimals —

$$(1) \frac{35}{57} + \left(\frac{009}{7} - \frac{0031}{21} \right) \quad (2) \frac{0759}{2\frac{1}{2} - \frac{1}{2}} - \frac{31\frac{4}{5} - \frac{1}{2}}{14\frac{7}{5}}$$

17 If 1 rupee = 1s 10d, 1 sovereign = 4 8d dollars, and 1 dollar = 5 2 francs, find the value in francs of 10 lacs of rupees

18 A certain number of men agree to subscribe as many pies each as there are subscribers, the whole subscriptions being Rs 5797 0a 1p How many subscribers are there?

19 A person after paying an income tax of 1 anna in the rupee, devotes $\frac{1}{10}$ of the remainder of his income to purposes of charity, and finds that he has left Rs 5175, what is his income?

20 Find how much rice a family requires monthly, when a reduction in the price from 7 to 10 measures for the rupee reduces the total monthly expenses from Rs 31 $\frac{1}{2}$ to Rs 30

21 Simplify —

$$(1) \left(\frac{2375}{63} \text{ of } \frac{88}{0625} \right) - \left(\frac{177}{1135} \text{ of } \frac{4}{7} \right) \quad (2) \frac{10\frac{4}{5}}{100} - 10\frac{4}{5} \quad (3) \frac{7}{\frac{1}{5}} - \frac{1}{9}$$

22 Two pieces of cloth of the same length cost Rs 111 12a and Rs 144 respectively, the price of the first was Rs 3 1a 8p per yard, what was the price of the other?

23 Find the value of 03125 of Rs 2 + 729 of Rs 3 1a 4p + 729 of Rs 4 10a

24 Find the square roots of 39 $\frac{1}{8}$ and 00255025

25 Find the least and the greatest number of six digits such that if they be divided by 240, 275, 320 and 400, the remainders will be 210, 245, 290 and 370 respectively

26 If one cubic foot of water weighs 62 37 lbs Avoir, reduce the weight of $\frac{344 \times 344 - 154 \times 154}{41 \times 405}$ cub ft to the decimal of a ton

27 A contractor bought 2250 pharas of unslaked lime at Calcutta at the rate of Rs 45 for 100 pharas. On slaking it every phara gave 3 cub ft of lime, but of this $\frac{1}{8}$ th was unserviceable, the carriage of the remainder to the place where it was required (distant 18 miles, cost 4a per 100 cub ft per mile. At what rate per cub ft must he sell it there, in order to gain Rs 90 on his outlay,

28 How many sheep must a person buy at £7 each, that after paying one shilling a score for folding them at night he may gain £79 16s by selling them at £8 each?

29 Divide 1028 5 by 000017, and $\frac{27}{31}$ by 0006, and multiply the difference of the quotients by 00025

30 What decimal multiplied by the sum of 3 5, 6 $\frac{5}{8}$, $\frac{7}{8}$ and 83 will produce 29?

31 If 625 of an article cost £13 16s 1d, what will 125 of it cost at the same rate?

32 Which is the greatest, and which the least of the expressions

$$(1) \frac{2}{3} + \frac{3}{4}, \quad (2) 141421, \quad (3) \frac{1}{2} + \frac{1}{2} + \frac{1}{4}?$$

33 The year consists of 365 24224 days. In what time would the error arising from taking it as 365 $\frac{1}{4}$ days amount to 1 day?

34 A person received on the death of his aunt $\frac{1}{10}$ of her property, and spent $\frac{1}{4}$ of it in paying off his debts, what fraction of his aunt's property did he then possess?

35 What least numbers must be severally subtracted from 9321, 7381 and 5628 that the three remainders shall have 723 for their common measure?

36 Simplify

$$\frac{525}{42 \text{ of } 516} - \frac{725}{31} \text{ of } \frac{31}{29} (3\frac{1}{2} + 6\frac{1}{2}) \times \frac{Rs 3 \frac{6a}{10} - 2a}{Rs 10 \frac{6a}{2a}}$$

37 What weight must be added to or subtracted from 8 cwt. 3 qrs 12 lbs so that 5 cwt 2 qrs 18 lbs shall be the same fraction of the sum or difference that 3 cwt 2 qrs 17 $\frac{1}{2}$ lbs is of 5 cwt 1 qr 26 $\frac{1}{2}$ lbs?

38 Find the two greatest numbers of 4 digits whose difference is 50 and G C M 25, and the two least numbers of 5 digits whose difference is 15 and G C M 5

39 Four concentric circles have the same centre O . A straight line through O cuts the circles in A, B, C and D on the same side of O . Four points start simultaneously from A, B, C, D and moving

in the same direction describe the circles in 9 hrs, 13 hrs, 21 hrs, and 37 hrs respectively Find after how many hours they will be again in a straight line through O and on the same side of it

40 A gave 5 of an orange to B , 3 of what remained to C , how much of the orange had A left for himself?

41 What is the cost of 12 mds 30 sr 8 ch 2 tolas at Rs 3 5a 9p per maund?

42 Reduce 5 bi 9 kat 8 ch to the fraction of an acre, and $7\frac{7}{8}$ of 3 acres 2 ro 5 po to the fraction of $11\frac{1}{3}$ bighas

43 7 ru-dollars are worth 2 ducats, and 9 ducats worth 4 moldores, and 20 moldores worth £27, how many ru-dollars will there be in £72?

44 Find the value of $3\frac{2}{5} + 4\frac{1}{8} + 1\frac{1}{10} + 3\frac{1}{2}\frac{3}{8}$ both by vulgar fractions and by decimals, and shew that the two results coincide

45 Find the values of $35 + 283 + 6 + 1175$, $1173 - 10916$, $3375 \times 16 \times 48$, $3375 - 45$ and find the product of the results

46 A man travelled $\frac{3}{4}$ of a journey by sea, $\frac{1}{5}$ of the remainder by rail, he then drove $\frac{1}{2}$ of the rest, and finished it by walking 10 miles How long was the journey?

47 Multiply Rs 3 0a 6p by 853125 and divide Rs 38340 3a 6p by 44175

48 If a pound of silver cost £3 6s, what is the price of a cup which weighs 10 lbs 6 oz 10 dwts, subject to a duty of 1s 6d per oz, and also to a charge of 1s 9d per oz for workmanship?

49 Reduce $\frac{1}{4}$ of $\frac{1}{2\frac{1}{2}}$ of $\frac{4}{5}$ of 5 cwt 2 qrs 14 lbs 7 oz to the decimal of 428571 of 15 tons 8 cwt 1 qr 14 lbs

50 Reduce $7\frac{7}{8}$ of $(3\frac{3}{4} - 3\frac{1}{10})$ of 5 cwt 2 qrs $3\frac{1}{2}$ lbs to the fraction of $(5\frac{3}{10} - 3\frac{1}{4})$ of 3 tons 16 cwt 3 qrs $22\frac{2}{5}$ lbs

51 What fraction of 2 lbs 10 oz Avoir must be added to 1 lb 8 oz Troy to give 3 lbs 7 oz 10 dwts?

52 Gold is sold at £3 17s $10\frac{1}{2}$ d per oz and bought at £3 17s 9d per oz, what is the largest unit of value in which both prices can be expressed as integers? and what is the smallest integral number of ounces, the value of which can be exactly expressed in £ at both prices?

53 Extract the square roots of $\frac{100010001}{1000}$ and $\frac{17 \times 29\frac{4}{5}}{000729}$

54 What length of wire will go round the edges of a cube, the surface of which contains 187 sq yds 54 sq in? What is the least number of such cubes which will contain an exact number of cubes whose edges are 1 ft 3 in?

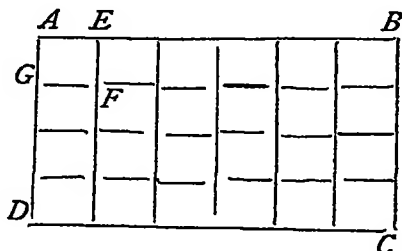
CHAPTER IX

Measurements and Duodecimals

I SQUARE MEASURE

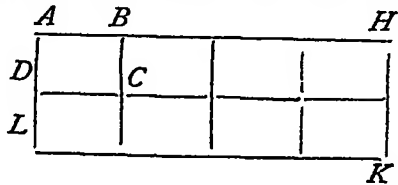
- 429** In Arithmetic, we deal with the areas of rectangles only
- (a) A rectangle is a four-sided figure whose opposite sides are equal and parallel, and whose angles are right angles
- (b) A square is a rectangle which has all its sides equal
- (c) The *length* and *breadth* of a rectangle are called its *dimensions*
- (d) The *area* of a figure is the quantity of surface contained in it, and is numerically measured by the multiple it is of a certain fixed *area*, which is assumed for its measuring unit
- 430** To find the area of a rectangle

Suppose $ABCD$ to represent a rectangular surface as that of a table, of which the length AB is 6 feet, and the breadth AD 4 feet. Divide AB into 6 equal parts, each equal to 1 foot and AD into 4 equal parts, each equal to 1 foot, as in the figure, and through the points of division draw straight lines parallel to AB and AD . Then by this means we shall have divided the whole surface into 6×4 small



figures, such as $AEFG$, all equal to one another, and since each of these figures measures a foot every way—1 foot in length and 1 foot in breadth—it is a *square foot*. Therefore the area of 6×4 small figures, is 6×4 or 24 square feet.

If the side AB , instead of containing an exact number of feet, contain some feet and a fraction, as $3\frac{1}{2}$ feet, and AD in like manner contain $4\frac{1}{2}$ feet produce AB to H , making AH equal to four times AB , also produce AD to L making AL equal to twice AD . Complete the rectangle $AHKL$. Then AH is four times AB , or $4 \times 3\frac{1}{2}$ feet = 13 feet and AL is twice AD or $2 \times 4\frac{1}{2}$ feet = 9 feet. Therefore the rectangle



$AHKL$ contains 9×13 or 117 square feet. But the rectangle $AHKL$ contains 8 such rectangles as $ABCD$, and therefore $ABCD$ contains $\frac{1}{8}$ of 117 square feet, or $14\frac{5}{8}$ sq ft, a result obtained by multiplying $3\frac{1}{2}$ by $4\frac{1}{2}$ for $1\frac{1}{2} \times 3 = 4\frac{1}{2}$. Hence, the number of square feet in the surface is found by multiplying the *number* of feet in its length by the *number* of feet in its breadth.

431 As the same method of proof would apply in every case (whether integral or fractional) we have the following Rule —

RULE Express the length and breadth in units of the same denomination, and then multiply the number of the units denoting the length by the number of the units denoting the breadth. The product will give the area in square units of that denomination

The above Rule is frequently expressed briefly thus —

$$\text{Area} = \text{Length} \times \text{Breadth}$$

We must remember here that it is the *number* of the units in length and the *number* of the units in breadth that we should multiply together, for length being a concrete quantity cannot be multiplied by another concrete quantity such as breadth. Also, if the numbers giving the length and breadth be given in inches, or in feet, or in yards, or in miles, their product will give the area in square inches, or in square feet, or in square yards, or in square miles

432 Since $\text{area} = \text{length} \times \text{breadth}$,

$\therefore \text{Length} = \text{area} \div \text{breadth}$, and $\text{Breadth} = \text{area} \div \text{length}$

Hence, if we divide the number of square units in the area by the number of units *either* the length or breadth, we shall find the number of units in the *other*

433 The length and breadth of a square being equal, its side will be found by extracting the square root of its area

To find the breadth of a rectangle, whose length is twice or thrice its breadth, divide the given area by 2 or 3, and then extract the square root of the result

Note Students should notice here the difference between 3 square feet and 3 feet square. By *3 square feet* is meant an area which is equal to 3 squares, each of which is a foot square. By *3 feet square* is meant the area of a square whose side is 3 feet, and therefore whose area is 3×3 or 9 square feet

Ex 1 Find the area of a floor 8 ft 9 in long by 3 ft 8 in broad

Length = 8 ft 9 in = $8\frac{3}{4}$ or $8\frac{6}{8}$ ft, breadth = 3 ft 8 in = $3\frac{2}{3}$ or $3\frac{1}{3}$ ft.

$$\therefore \text{area of the floor} = (8\frac{6}{8} \times 3\frac{1}{3}) \text{ sq ft} = 12\frac{5}{2} \text{ sq ft} = 32\frac{1}{2} \text{ sq ft} \\ = 3 \text{ sq yds } 5 \text{ sq ft } 12 \text{ sq in}$$

Ex 2 Find the width of a room 50 ft 8 in long having an area of 128 sq yds 2 sq ft 4 sq in

$$\text{Area} = (128 \times 9 + 2 \times \frac{1}{4}) \text{ sq ft} = 1154\frac{1}{2} \text{ sq ft} = 41\frac{5}{8} \text{ sq ft}$$

$$\text{Length} = 50 \text{ ft } 8 \text{ in} = 50\frac{2}{3} \text{ ft} = 15\frac{2}{3} \text{ ft}$$

$$\therefore \text{width} = (41\frac{5}{8} \div 15\frac{2}{3}) \text{ ft} = (41\frac{5}{8} \times \frac{3}{16}) \text{ ft} = 15\frac{1}{4} \text{ ft} \\ = 22 \text{ ft } 9\frac{3}{4} \text{ in}$$

Note Since linear feet multiplied by linear feet gives sq ft, it follows that sq ft divided by linear feet gives linear feet, and so on

Ex 3. Find the area of a square court whose side is 17 ft 11 in
Here, length=breadth=17 ft 11 in = $17\frac{11}{12}$ ft = $21\frac{5}{12}$ ft

$$\therefore \text{area} = (21\frac{5}{12} \times 21\frac{5}{12}) \text{ sq ft} = 452\frac{25}{144} \text{ sq ft} = 321\frac{1}{4} \text{ sq ft} \\ = 35 \text{ sq yds } 6 \text{ sq ft } 1 \text{ sq in}$$

Ex 4 How many bricks $11\frac{1}{2}$ in long by $7\frac{1}{2}$ in wide will be required to pave the floor of a room 18 ft 9 in long by 15 ft 8 in wide?

$$\text{Area of a brick} = (11\frac{1}{2} \times 7\frac{1}{2}) \text{ sq in} = 47 \times \frac{1}{2} \text{ sq in}$$

$$\text{Area of the floor} = (225 \times 188) \text{ sq in}$$

$$\therefore \text{no of bricks reqd} = (225 \times 188) \div (47 \times \frac{1}{2}) \\ = 225 \times 188 \times \frac{4}{47} \times \frac{2}{1} = 480 \text{ Ans}$$

Ex 5 How many yards of fencing are required to enclose a square garden containing 1 ac 1 po 29 yds 6 $\frac{1}{2}$ ft?

$$\text{Area} = 1 \text{ ac } 1 \text{ po } 29 \text{ yds } 6\frac{1}{2} \text{ ft} = 44100 \text{ sq ft}$$

$$\therefore \text{a side} = \sqrt{(44100)} \text{ ft} = 210 \text{ ft}$$

$$\therefore \text{length of fencing reqd} = 4 \times 210 \text{ ft} = 840 \text{ ft} = 280 \text{ yds Ans}$$

Ex 6 The area of the floor of a room is 450 sq ft, its length is twice its breadth, find its length

$$\text{Breadth} = \sqrt{(450)} \text{ ft} = \sqrt{(225)} \text{ ft} = 15 \text{ ft},$$

$$\therefore \text{length} = 2 \times 15 \text{ ft} = 30 \text{ ft Ans}$$

Ex 7 A garden roller is 3 ft 3 in wide, and its circumference is 6 ft 9 in, how many sq ft of ground does it pass over in 8 complete revolutions?

In each revolution, the roller passes over a space of ground whose area is equal to the product of the width and the circumference

$$\therefore \text{area rolled over in one revolution} = (3\frac{1}{2} \times 6\frac{3}{4}) \text{ sq ft} = 23\frac{1}{4} \text{ sq ft}$$

$$\therefore \text{area passed over in 8 revolutions} = 23\frac{1}{4} \times 8 \text{ sq ft} \\ = 187 \text{ sq ft} = 175\frac{1}{2} \text{ sq ft}$$

431 Carpeting and matting always refer to the *floor* only

The amount of carpet or mat required to cover a floor will be equal to the **area of the floor**. Carpet or mats are sold in strips of different widths and when we know the width of a strip, we can find its length by dividing the *area of the floor* by the *width* of the carpet or mat

Ex 1 Find the cost of carpeting a room 22 ft 8 in long and 17 ft 4 in broad at Rs 1 2a per square yard

$$\text{Area of carpet} = (22\frac{2}{3} \times 17\frac{1}{3}) \text{ sq ft} = 68 \times \frac{5}{3} \times \frac{1}{3} \text{ sq yds.}$$

$$\therefore \text{required cost} = 68 \times \frac{5}{3} \times \frac{1}{3} \times Rs 1\frac{2}{3} = Rs 49 \text{ 1a } 9\frac{1}{2}p \text{ Ans}$$

Ex 2 What length of mats, $\frac{3}{4}$ of a yard wide, will be required

to cover a floor 15 ft 8 in long by 11 ft 3 in broad? also find the cost of the mats at 2a 3p per yard

$$\begin{aligned}\text{Area of matting} &= (15\frac{2}{3} \times 11\frac{1}{4}) \text{ sq ft} = \frac{4}{7} \times \frac{4}{5} \times \frac{1}{8} \text{ sq yds} = \frac{2}{15} \text{ sq yds} \\ \text{length of matting} &= (\frac{2}{15} - \frac{1}{4}) \text{ yds} = \frac{2}{9} \text{ yds} = 26\frac{1}{3} \text{ yds} \\ &= \underline{26 \text{ yds } 0 \text{ ft } 4 \text{ in}} \quad \text{Ans}\end{aligned}$$

$$\text{Also, cost of mats} = 2a \ 3p \times \frac{2}{9} = \frac{2}{3} \times \frac{2}{9} = \frac{2}{15} = \underline{Rs \ 3 \ 10a \ 9p}$$

Ex 3 The cost of painting a surface 8 ft 9 in wide at Rs 3 13a 4p per sq yd is Rs 50 5a, find the length of the surface

$$\begin{aligned}Rs \ 3 \ 13a \ 4p &= Rs \ 3\frac{1}{8} = Rs \ 3\frac{3}{8}, \quad Rs \ 50 \ 5a = Rs \ 50\frac{5}{8} = Rs \ 62\frac{5}{8} \\ \text{area of surface} &= (\frac{62\frac{5}{8}}{3\frac{3}{8}}) \text{ sq yds} = \frac{108}{8} \text{ sq yds} = \frac{27}{2} \text{ sq ft} \\ \text{length of surface} &= \frac{27}{2} \text{ sq ft} \div 8\frac{3}{4} \text{ ft} = (\frac{27}{2} \times \frac{4}{7}) \text{ ft} \\ &= \frac{27}{7} \text{ ft} = \underline{13 \text{ ft } 6 \text{ in}} \quad \text{Ans}\end{aligned}$$

Ex 4 The expense of carpeting a room thrice as long as it is broad at Rs 3 12a per sq yd is Rs 551 4a, find the length and breadth of the room

$$\begin{aligned}Rs \ 3 \ 12a &= Rs \ 3\frac{1}{2} = Rs \ 1\frac{1}{2}, \quad Rs \ 551 \ 4a = Rs \ 551\frac{1}{2} = Rs \ 1102\frac{1}{2} \\ \therefore \text{area of the room} &= (\frac{1102\frac{1}{2}}{1\frac{1}{2}}) \text{ sq yds} = 147 \text{ sq yds} \\ \therefore \text{breadth} &= \sqrt{147} \text{ yds} = \sqrt{49} \text{ yds} = 7 \text{ yds} = 21 \text{ ft} \\ \text{and length} &= 21 \text{ ft} \times 3 = 63 \text{ ft} \quad \text{Ans}\end{aligned}$$

Examples CXXIX

1 Find the area of each of the following rectangles —

- | | |
|-----------------------------------|--------------------------------|
| (1) 25 ft by 17 ft | (2) 36 ft by 13 ft |
| (3) 17 ft 6 in by 13 ft 4 in | (4) 19 ft 4 in by 16 ft 8 in |
| (5) 12 yds 1 ft 5 in by 2 ft 9 in | (6) 5 yds 11 in by 23 ft 10 in |
| (7) 15 ft 4½ in by 14 ft 4 in | (8) 19 ft 1 in by 17 ft 1½ in |

2 Find the quantity of matting that will be required for each of the following rooms —

- | | |
|----------------------------------|------------------------------------|
| (1) 52 yds 10 in by 13 yds 2½ in | (2) 2 yds 1 ft 6 in by 4 yds 9 in |
| (3) 22 ft 8 in by 17 ft 4 in | (4) 6 yds 2½ ft by 2 yds 1 ft 5 in |

3 Find the length or breadth of each of the following rooms —

- | |
|---|
| (1) Area = 150 sq ft 27 sq in and length = 6 yds 2½ ft |
| (2) = 20 sq yds, 5 sq ft 101 sq in and length = 15 ft 7 in |
| (3) = 11 sq yds 3 sq ft 30 sq in and = 37 ft 2 in |
| (4) = 17 sq yds 2 sq ft 131 sq in and breadth = 11 ft 11 in |
| (5) = 402 sq ft 72 sq in and breadth = 5 yds 2½ ft |

4 Find the area of a square whose side is 7 yds 1 ft 5 in

5 How many sq ft and inches remain out of 313 sq ft of matting, after covering a floor 16 ft 9 in long by 12 ft 11 in broad?

6 Find the sides of the following squares —
 6sq yds 7sq ft 52sq in , 241sq yds 8sq ft 112sq in , 34225sq in

7 A piece of canvas of uniform width is 7 ft $3\frac{3}{4}$ in long and it covers 2 sq yards 103 $\frac{1}{2}$ sq in , what is its width ?

8 Find the number of acres in a square field whose side is 4 chains 50 links

9 How many sods each 10 $\frac{1}{2}$ in by 7 $\frac{1}{2}$ in will be required to cover a piece of ground 25 yards by 14 yards ?

10 How many planks, each 10 ft 6 in by 10 in , must be used to cover a floor 42 ft by 17 ft 6 in ?

11 How many postage stamps, each 1 $\frac{1}{2}$ in by $\frac{3}{4}$ in would be required to cover a wall 14 ft by 10 ft 6 in ?

12 How many boards 18 ft 6 in long and 7 in wide will be required to floor a room 10 yds 1 ft 9 in long and 8 yds 6 in wide ?

13 A postage stamp is one inch long and $\frac{3}{4}$ in broad , how many will an album of 32 leaves, 8 in long and 5 in broad, contain ?

14 A square space containing 140 sq yds 36 sq in is to be lengthened by 4 ft 3 in in one of its dimensions and shortened by 3 ft 4 in in the other , what will then be its area ?

15 How many tiles 7 in square will be required for the floor of a room 19 ft 3 in long by 13 ft 5 in wide ?

16 A garden roller is 3 ft 7 $\frac{1}{2}$ in wide and 5 ft 10 $\frac{3}{4}$ in in circumference , how many sq feet and inches of ground does it pass over in making 4 revolutions ?

17 A rectangular field is 7 chains 35 links long and 5 chains broad , 1 ac. 3 ro is to be cut off from it by a line parallel to its breadth , where must this line be drawn ?

18 The length of a room is double its width, and the area of the floor is 136 sq yds 1 sq ft 18 sq in , find its length

19 The length of a room is three times its breadth, and its area is 635 sq yds 5 sq ft 48 sq in , find its length in feet

20 How many acres are there in a rectangular tract of country 3 $\frac{1}{2}$ miles long and 2 $\frac{1}{2}$ miles broad ?

21 Find the cost of matting each of the following rooms —

(1) 119 ft 6 in long and 83 ft 4 in broad at 12s per sq ft

(2) 146 ft 9 in long and 88 ft 9 in broad at 7s 6d per sq yd

(3) 25 ft 8 in long and 16 ft 9 in wide at 17s 7d per sq yd

(4) 98 ft 4 in long and 24 ft 6 in broad at 3s 9d per sq foot

22 What length of carpet will be required to cover each of the following floors —

(1) 22 ft 6 in long and 15 ft 9 in wide Carpet 2 ft 3 in wide ?

(2) 35 ft 4 in long and 27 ft 3 in broad Carpet 27 in wide ?

(3) 15 ft 8 in long and 11 ft 3 in wide Carpet $\frac{1}{2}$ yd wide ?

23 Find the expense of carpeting each of the following rooms —

(1) 34 ft 8 in long and 13 ft 3 in wide with carpet $\frac{3}{4}$ yd wide at *Rs* 1 11*a* per yd

(2) 31 ft 6 in by 23 ft 9 in with carpet 18 in wide at *Rs* 1 6*a* per yd

(3) 20 ft 3 in long and 17 ft 4 in broad with carpet $\frac{1}{2}$ yd wide at *Rs* 2 1*a* 4*p* per yd.

(4) 26 ft 8 in by 20 ft 3 in with carpet $\frac{2}{3}$ yd wide at 4*s* 8*d* per yd

24 The cost of carpeting the floor of a room 12 ft 6 in long at 5*a* per sq foot is *Rs* 187 8*a*, find its width

25 The breadth of a room is 11 ft 11 in and the cost of matting the floor at 10*a* 8*p* per sq ft is *Rs* 113 3*a* 4*p*, find the length of the room

26 The length of a room is 23 ft 6 in and the cost of carpeting the floor at 4*s* 6*d* per sq yd is £11 15*s*, find the breadth of the room

27 It costs *Rs* 226 14*a* to cover the floor of a room 8 yds 9 in long and 6 yds 24 in wide, with carpet 2 ft wide Find the price of the carpet per yard

28 If the cost of carpeting a room 11 yds long and 8 yds wide with carpet at 3*s* a yard, be £19 16*s*, find the width of the carpet

29 Find the expense of turfing a plot of ground, which is 40 yds long and 100 ft wide, with turfs each a yard in length and 1 ft in breadth, the turfs, when laid, costing *Rs* 3 6*a* per hundred

30 The expense of carpeting a room, whose breadth is 12 ft 9 in with carpet 24 in wide at *Rs* 3 13*a* 4*p* per yard is *Rs* 126 4*a* 2*p*, find the length of the room

31 On laying down a plot of ground with sods 2 ft 6 in long and 9 in wide, it is found that it requires 75 sods to form one strip extending its whole length, and that a man can lay down $1\frac{1}{4}$ strips each day, find the surface covered in 8 days

32 The roller used for a bowling green being 6 ft 6 in in circumference and 2 ft 3 in in width, is observed to make 12 revolutions from one extremity of the green to the other, find the area rolled, when the roller has passed 10 times the length of it

33 Find the side of a square court-yard the expense of paving which at *Rs* 1 14*a* per sq yard is *Rs* 385 3*a* 4*p*

34 If 1572 tiles each 10 in square would pave a court, how many would be required if the tiles were each 8 in wide and 15 in long ?

35 A lawn, which is twice as long as it is broad, costs £38 10*s* 5½*d* to turf at 11*d* per sq yard Find its length and breadth

36 A lawn is half as long again as it is wide, the cost of levelling it at 6s per sq yd is Rs 1764, find the cost of enclosing it with a fence at Rs 3 12s per yard

435 Papering and painting, always refer to the *walls* only, plastering and whitewashing both to the *walls* and the *ceiling*, and glazing to the *windows* only

In finding the area of a *wall* we multiply the length of the wall by its height. In a room there are 4 walls, and the two that are opposite are equal

The area of a wall running length wise = length (of the room) \times height and that of a wall running breadth wise = breadth (of the room) \times height

$$\therefore \text{area of the 4 walls} = 2 \times \text{length} \times \text{height} + 2 \times \text{breadth} \times \text{height} \\ = 2(\text{length} + \text{breadth}) \times \text{height}$$

But $2 \times (\text{length} + \text{breadth})$ is the perimeter or circuit of the room, i.e., the sum of the lengths of the 4 walls

$$\therefore \text{area of the walls} = \text{perimeter (or circuit)} \times \text{height}$$

This expression gives the area of the walls, including that occupied by windows, doors, fire places, &c. Deductions must be made for these in practice in all questions relating to papering, painting, plastering, &c unless otherwise mentioned. Paper for covering walls is done up in long rolls, and is of a certain width, and to find the length of paper required to cover the walls of a room, we must first find the *area of the walls* and then divide this area by the *width* of the paper, as in Art 434

Note In finding the quantity of paper required to cover the walls of a room, the student must not multiply together the length, the breadth and the height of the room, for this would give the volume necessary to fill the room instead of the quantity of paper required merely to cover the walls. See Art 441

436 The area of the ceiling of a room is the *same* as that of the floor. Hence to find the area of a ceiling multiply the length of the room by its breadth

Ex 1 Find the area of the 4 walls of a room 30 ft 8 in long, 26 ft 5 in broad and 10 ft 6 in high

$$\begin{aligned} \text{Area of the 4 walls} &= (30 \text{ ft } 8 \text{ in} + 26 \text{ ft } 5 \text{ in}) \times 10 \text{ ft } 6 \text{ in} \\ &= 2 \times 57 \text{ ft } 1 \text{ in} \times 10 \text{ ft } 6 \text{ in} = 2 \times 57 \frac{1}{2} \times 10 \frac{1}{2} \text{ sq ft} \\ &= 2 \times 598 \frac{1}{2} \times \frac{1}{2} \text{ sq ft} = 4 \frac{1}{2} \times 598 \text{ sq ft} = 1198 \frac{1}{2} \text{ sq ft} \end{aligned}$$

Ex 2 Find the length of paper 2 ft wide required to cover the walls of the above room

$$\text{Area of the 4 walls} = 4 \frac{1}{2} \times 598 \text{ sq ft, width of paper} = 2 \text{ ft}$$

$$\therefore \text{length of paper reqd} = (4 \frac{1}{2} \times 598 - 2) \text{ ft} = 4 \frac{1}{2} \times 598 \text{ yds} = 1993 \frac{1}{2} \text{ yds}$$

Ex 3 Find the cost of papering the above room at 5a 4p per yard

$$\text{Length of paper} = 47\frac{9}{16} \text{ yds} \quad 5a \ 4p = 5\frac{1}{2}a = 1\frac{1}{2}a = Re \ \frac{1}{2}$$

$$\therefore \text{cost required} = Re \ \frac{1}{2} \times 47\frac{9}{16} = Rs \ 27\frac{9}{32} = \underline{Rs \ 66 \ 9a \ 6\frac{3}{4}p}$$

Ex 4 A room 25 ft 7 in long, 18 ft 11 in broad and 10 ft 6 in high has two doors, each 7 ft by 3 ft 4 in and 3 windows, each 6 ft by 4 ft 3 in, determine the expense of plastering the walls and ceiling at 12a per sq yard

$$\text{Area of the 4 walls} = 2(25 \text{ ft } 7 \text{ in} + 18 \text{ ft } 11 \text{ in}) \times 10 \text{ ft } 6 \text{ in}$$

$$= 2 \times 44 \text{ ft } 6 \text{ in} \times 10 \text{ ft } 6 \text{ in} = 2 \times 44\frac{1}{2} \times 10\frac{1}{2} \text{ sq ft}$$

$$= 2 \times 89 \times 21 \text{ sq ft} = 1899 \text{ sq ft}$$

$$\text{Area of the ceiling} = 25\frac{7}{8} \times 18\frac{11}{16} \text{ sq ft} = 470\frac{7}{16} \times 22\frac{7}{8} \text{ sq ft} = 6099\frac{9}{16} \text{ sq ft,}$$

$$\text{Area of 2 doors} = 2 \times 7 \times 3\frac{1}{2} \text{ sq ft} = 14\frac{1}{2} \text{ sq ft, and that of the 3 windows} = 3 \times 6 \times 4\frac{1}{2} \text{ sq ft} = 15\frac{3}{2} \text{ sq ft}$$

$$\text{whole area to be plastered} = (1899 + 6099\frac{9}{16} - 14\frac{1}{2} - 15\frac{3}{2}) \text{ sq ft}$$

$$= 1899\frac{1}{16} \text{ sq ft} = 1899\frac{1}{16} \text{ sq yds}$$

$$\therefore \text{cost of plastering} = (1899\frac{1}{16} \times 12)a = 1727\frac{15}{8}a = \underline{Rs \ 107 \ 15a \ 0\frac{3}{8}p}$$

437 Having given the length and breadth of a room, and the area of the four walls, to find the height

$$\text{RULE} \quad \text{Height} = (\text{area of the 4 walls}) \div 2 (\text{length} + \text{breadth})$$

Ex 1 The cost of papering a room 15 ft long at Re 1 4a per sq foot is Rs 710, the cost of carpeting the floor of the same room at Re 1 14a per sq foot is Rs 243 12a Find the height and the breadth of the room

$$\text{Area of the floor in sq ft} = Rs \ 243 \ 12a \div Re \ 1 \ 14a = 130$$

$$\text{breadth of the room} = 130 \text{ sq ft} \div 15 \text{ ft} = 8\frac{2}{3} \text{ ft}$$

$$\text{Area of the 4 walls in sq ft} = Rs \ 710 \div Re \ 1 \ 4a = 568$$

$$\text{Also } 2(\text{length} + \text{breadth}) = 2(15 \text{ ft} + 8\frac{2}{3} \text{ ft}) = 2 \times 23\frac{2}{3} \text{ ft} = 47\frac{1}{3} \text{ ft}$$

$$\text{height of the room} = (568 \div 47\frac{1}{3}) \text{ ft} = 568 \times \frac{3}{142} \text{ ft} = \underline{12 \text{ ft}}$$

Ex 2 If the cost of papering a room 8½ yds long and 4 yds high with paper 2 ft wide at 2a 8p per yard, be Rs 29 13a 4p, find the breadth of the room

$$Rs \ 29 \ 13a \ 4p = Rs \ 29\frac{5}{8} = Rs \ 17\frac{5}{8}, \text{ and } 2a \ 8p = Re \ \frac{1}{8}$$

$$\text{length of paper required} = (17\frac{5}{8} \div \frac{1}{8}) \text{ yds} = 179 \text{ yds}$$

$$\therefore \text{area of the four walls} = (179 \times \frac{2}{3}) \text{ sq yds} = 358 \text{ sq yds}$$

$$2 (\text{length} + \text{breadth}) = (358 \div 4) \text{ yds} = 179 \text{ yds}$$

$$\text{the reqd breadth} = (179 \div 8\frac{1}{2}) \text{ yds} = 20 \text{ yds} = \underline{20 \text{ ft}}$$

Ex 3 The expense of carpeting a room 20 ft long was Rs 75, but if the breadth had been 3 ft less than it was, the expense would have been Rs 60 What was the breadth of the room?

The difference between the two costs = Rs $(75 - 60 = \text{Rs } 15)$

But this arises from the fact of the breadth being 3 ft less, and is therefore the cost of 20×3 or 60 sq ft of carpet

Now, since Rs 15 is the cost of 60 sq ft of carpet

Rs 75 is the cost of 60×5 or 300 sq ft of carpet

Hence the area of the room is 300 sq ft and breadth reqd of the room = $(300 - 20 \text{ ft} = \underline{15 \text{ ft}})$

Examples CXXX

1 Find the area of the 4 walls of each of the following rooms —

- (1) 25 ft 7 in long, 19 ft 4 in wide, and 9 ft 9 in high
- (2) 15 ft 6 in long, 13 ft 4 in wide, and 10 ft 6 in high
- (3) 23 ft 5 in long, 18 ft 7 in wide, and 9 ft 6 in high
- (4) 20 ft 10 in long, 16 ft broad, and 10 ft 9 in high

2 Find the length of wall paper required for each of the following rooms —

- (1) 18 ft 9 in square and 13 ft 4 in high, with paper 1 ft 4 in wide
- (2) 21 ft 4 in long, 17 ft 5 in wide and $13\frac{1}{2}$ ft high, with paper $16\frac{1}{2}$ in wide
- (3) 28 ft 6 in long, 18 ft 9 in wide and 12 ft high, with paper 1 ft 9 in wide
- (4) 22 ft 5 in long, 12 ft 1 in broad and 11 ft 3 in high, with paper $\frac{1}{2}$ vd wide

3 Find the expense of papering each of the following rooms —

- (1) 20 ft 6 in long, 17 ft 4 in broad and 9 ft high, with paper 20 in wide at 3a 4p per yard
- (2) 29 ft 7 in long, 21 ft 4 in wide and 12 ft 11 in high, with paper $11\frac{1}{2}$ in wide at $7\frac{1}{2}d$ per yard
- (3) 12 ft 6 in long, 8 ft 4 in wide and 9 ft high, with paper 3 ft wide, in pieces 20 yds long, at 12a per piece
- (4) 5 yds 1 ft $2\frac{1}{2}$ in long, 5 yds $3\frac{1}{2}$ in broad, and 4 yds high with paper 9 in wide at $2\frac{1}{2}d$ per yard

4 Find the cost of painting the walls and ceiling of each of the following rooms —

- (1) 12 ft 6 in long, 27 ft 4 in wide and 20 ft high at 5a 8p per sq yd
- (2) 15 ft long, 10 ft broad and 9 ft 9 in high at 14a per sq yd
- (3) 35 ft 4 in long, 17 ft 6 in wide and 20 ft high at $7\frac{1}{4}d$ per sq yd

5 A room is 20 ft 6 in long, 15 ft 6 in wide and 16 ft high ; it has two doors, each 8 ft high and 3 ft 9 in wide and 3 windows, one 5 ft by 7 ft, the other two 5 ft by 4 ft each. What will it cost to paper the room with paper one yard wide at 6s 8d per yard ?

6 A room whose height is 18 ft 5 in, breadth 20 ft 10 in and length 22 ft 9 in, has a door 7 ft 6 in by 3 ft 4 in and two equal windows each 3 ft 6 in by 5 ft 2 in, find the cost of papering it at Rs 1 11s 6d per square yard

7 A room is to be papered whose length is 23 ft 8 in, breadth 15 ft 10 in and height 11 ft 9 in in it there are two windows each 9 ft 6 in high by 5 ft wide, a fire-place 4 ft 6 in high by 6 ft wide, and a door 7 ft 6 in high by 3 ft 6 in wide. Find the cost of the paper required at 17s 6d per piece of 12 yards, the width of the paper being 26 inches

8 What will be the expense of papering a room 24 ft 4 in long, 19 ft 8 in wide and $13\frac{1}{2}$ ft high, with a paper which is $2\frac{1}{2}$ ft wide and costs 11s per piece of 12 yds the windows and doors not requiring to be papered, making up a sixth part of the whole surface ?

9 How many postage stamps, 1 in long and $\frac{1}{2}$ in broad, will be required to cover the walls of a room 18 ft long, 12 ft broad and 10 ft 6 in high ?

10 Find the expense of whitewashing the ceiling and walls of a square room at 1s 8d per sq yd, whose floor measures 32 sq yds 1 sq ft and height 11 ft 6 in

11 A room whose height is 11 feet and length twice its breadth, takes 143 yds of paper 2 ft wide for its four walls. How much carpet will it require ?

12 A room is 20 ft long and 16 ft wide, what must be its height in order that the area of the floor and ceiling together may be equal to the area of the walls ?

13 A room is 19 ft 5 in long and 16 ft 7 in broad, and the cost of painting the walls at 7s 6d per sq yard is Rs 43 3s. Find the height of the room

14 The cost of carpeting a room twice as long as it was broad at Rs 2 8s per sq yd amounted to Rs 61 4s, and the painting of the walls at 6s a sq yd amounted to Rs 26 4s. Find the height of the room

15 The expense of carpeting a room 18 ft long was £7 4s, but if the breadth had been 4 ft less than it was, the expense would have been £5 8s, what was the breadth of the room ?

16 The length of a room is 14 ft, the cost of painting the walls at Rs 1 5s a sq yd is Rs 75 13s 4d, and the cost of carpeting the room at Rs 2 a sq yd is Rs 37 5s 4d. Find the height and breadth of the room

17 The cost of painting the walls of a room 18 ft long and 9 ft high at $Rs\ 1\ 8a$ a sq foot is $Rs\ 810$. Find the cost of carpeting the room at $Rs\ 1\ 9a$ a sq yard.

18 A room is 14 ft 9 in long, 9 ft 3 in wide and 10 ft 6 in high, it contains two windows each 5 ft 6 in by 4 ft, three doors each 6 ft by 3 ft and a fire place $6\frac{1}{2}$ ft by 4 ft, find how many postage stamps it would take to cover the walls, a stamp being $\frac{1}{4}$ in by $\frac{1}{4}$ in, supposing that the stamps may be cut if necessary.

19 Two rectangular rooms of the same height were papered, the first was 16 ft long and 14 ft wide and the second 14 ft long and 12 ft wide, the paper was 1 ft 9 in wide and $Rs\ 1\ 6a$ per piece of 12 yds, and the hanging cost $6a$ per piece, the whole expense was $Rs\ 42$, find the height of the rooms.

20 The length of a room is 20 ft, the cost of papering the walls with paper $2\frac{1}{2}$ ft wide at $4a$ per yard is $Rs\ 30\ 6a\ 8p$, and that of carpeting the room at $Rs\ 3\ 5a\ 4p$ per sq yd is $Rs\ 122\ 3a\ 6p$. Find the height of the room.

21 A room 10 ft high and 20 ft long costs £190 to paint its walls at 5s per sq foot. What is the cost of the carpet which will cover the floor at $Rs\ 3\ 2a$ per sq yard?

22 It takes 5904 tiles, each measuring 6 in by 10 in, to cover the floor of a room 120 ft long, what is the width of the room? What will be the cost of laying down the tiles at $2a\ 6p$ per sq yard?

23 The cost of carpeting a room is $Rs\ 72$, and of papering the same room with paper at $1a\ 8p$ per sq foot is $Rs\ 106\ 4a$. The length of the room is 18 ft, and if the width had been 4 ft less the cost of the carpet would have been $Rs\ 18$ less. Find the height of the room.

438 **Paths and moats** (1) When a path or a moat of a *uniform* width goes all round *outside* a field, its area may be obtained either (i) by *increasing* each dimension by *twice* the *uniform* width of the path or moat, and then subtracting the area of the given field from the increased area, or (ii) by *increasing* each dimension by the *uniform* width of the path, &c and then using the following RULE —

$\text{Area} = 2(\text{increased length} + \text{increased breadth}) \times \text{uniform width of the path}$

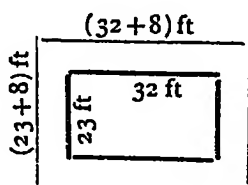
(2) But if the path or the moat go round *inside* the field, then either (i) *diminish* each dimension of the field by *twice* the *uniform* width and subtract the diminished area from the area of the field, or (ii) *diminish* each dimension by the *uniform* width of the path, and use the following RULE —

$\text{Area} = 2(\text{diminished length} + \text{diminished breadth}) \times \text{uniform width of the path}$

Ex 1 A rectangular piece of ground is 32 ft long and 23 ft broad. Find the cost of enclosing it with a path 4 ft broad, at the price of Rs 3 per sq yard, (1) when the path is outside the piece of ground, (2) when the path is part of the ground.

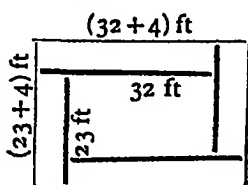
(1) *By the first Method*

$$\begin{aligned} 32 + 2 \times 4 &= 40, \quad 23 + 2 \times 4 = 31 \\ \therefore \text{area of the path} &= (40 \times 31) \text{ sq ft} - (32 \times 23) \text{ sq ft} \\ &= 504 \text{ sq ft} = 56 \text{ sq yds} \\ \therefore \text{cost} &= \text{Rs } 56 \times 3 = \text{Rs } 168 \end{aligned}$$



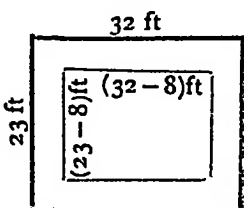
By the second Method

$$\begin{aligned} 32 + 4 &= 36, \quad 23 + 4 = 27 \\ \therefore \text{the area of the path} &= 2(36 + 27) \times 4 \text{ sq ft} \\ &= 504 \text{ sq ft} = 56 \text{ sq yds} \\ \therefore \text{cost} &= \text{Rs } 3 \times 56 = \text{Rs } 168 \end{aligned}$$



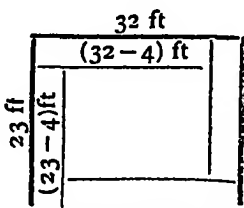
(2) *By the first Method*

$$\begin{aligned} 32 - 2 \times 4 &= 24, \quad 23 - 2 \times 4 = 15 \\ \therefore \text{area of the path} &= (32 \times 23) \text{ sq ft} - (24 \times 15) \text{ sq ft} \\ &= 376 \text{ sq ft} = 41\frac{2}{3} \text{ sq yds} \\ \therefore \text{cost} &= \text{Rs } 3 \times 41\frac{2}{3} = \text{Rs } 125 \text{ } 5\text{a } 4\text{p} \end{aligned}$$



By the second Method

$$\begin{aligned} 32 - 4 &= 28, \quad 23 - 4 = 19 \\ \therefore \text{the area of the path} &= 2(28 + 19) \times 4 \text{ sq ft} \\ &= 376 \text{ sq ft} = 41\frac{2}{3} \text{ sq yds} \\ \therefore \text{cost} &= \text{Rs } 3 \times 41\frac{2}{3} = \text{Rs } 125 \text{ } 5\text{a } 4\text{p} \end{aligned}$$



Ex 2 A room 35 ft long by 18 ft broad is enclosed by walls 18 in thick and all round the outside there is a verandah 9 ft deep. Find the cost of paving the verandah at 8a per sq yard.

Since the walls are 18 in or $1\frac{1}{2}$ ft thick, the external length of the room is $(35 + 2 \times 1\frac{1}{2})$ or 38 ft, and external breadth is $(18 + 2 \times 1\frac{1}{2})$ or 21 ft.

$$\text{Now } 38 + 9 = 47, \quad 21 + 9 = 30$$

$$\begin{aligned} \therefore \text{area of the verandah} &= 2(47 + 30) \times 9 \text{ sq ft} = 2 \times 77 \times 9 \text{ sq ft} \\ &= 154 \text{ sq yds} \end{aligned}$$

$$\therefore \text{cost} = \text{Rs } 1 \times 154 = \text{Rs } 77$$

Ex 3 A rectangular court-yard 40 yds 2 ft 7 in long and 75 ft 5 in broad, has 2 foot-paths, each $7\frac{1}{2}$ ft wide, the one running the whole length of it and the other the whole breadth of it, crossing each other at right angles. Find the total expense of paving the court-yard with pebbles at Rs 2 4a per sq yd, and the foot-paths with flag stones at Rs 1 8a per sq yd

Area of court-yard,

(including the paths) = $122\frac{1}{2} \times 75\frac{5}{8}$ or $122\frac{1}{2} \times 75\frac{5}{8}$ sq ft

(excluding the paths) = $115\frac{1}{2} \times 67\frac{1}{2}$ or $115\frac{1}{2} \times 67\frac{1}{2}$ sq ft

∴ area of the paths = $(122\frac{1}{2} \times 75\frac{5}{8} - 115\frac{1}{2} \times 67\frac{1}{2})$ sq ft

= $105\frac{1}{2} \times 7\frac{5}{8}$ sq ft = $0\frac{3}{4}$ sq yds

∴ cost of paving the court-yard = $115\frac{1}{2} \times 67\frac{1}{2} \times Rs\ 2\frac{4}{10} = Rs\ 11\frac{2}{3} \times 11\frac{1}{2}$
 = Rs 1954 or 3²/₁₀

and cost of paving the paths = $Rs\ \frac{3}{4} \times 0\frac{3}{4} = Rs\ 2\frac{3}{8}$ or 2a

∴ the total cost reqd = Rs 2192 2a 3²/₁₀

Examples CXXXI

1 A field is 300 yds long and 200 yds broad, if a belt of tree 30 yds wide be planted round it inside, find the area of the interior space

2 A rectangular piece of ground is 60 yds long and contains half an acre. It consists of a walk 8 ft wide surrounding a grass plot. Find the area of the plot

3 A grass plot is 23 ft 8 in long and 16 ft 7 in broad, round it a walk 10 ft wide is made and paved at 15a per sq yd. What is the cost of the paving?

4. How many paving stones, each of them one foot long and $\frac{1}{2}$ of a foot wide, will be required for paving a street 45 ft wide, surrounding the outside of a square, the side of which is 225 ft?

5 A hall 70 ft long and 36 ft broad is enclosed by walls 18 in thick and all round the outside there is a verandah $13\frac{1}{2}$ ft deep. What will be the cost of paving this verandah at the rate of 12a per sq yard?

6 A rectangular court is 80 yds long and 50 yds broad. It has paths, joining the middle points of the opposite sides 6 ft wide, and it has also paths of the same breadth running all round it on the inside. The remainder is covered with grass. If the paths cost 13a 4p per sq ft and the grass 1a 8a per sq yd, find the whole cost of laying out the court

7 How many flag stones each 5 ft 6 in long and 4 ft 15 in wide are required for paving a cloister which encloses a rectangular court 45 ft 77 yds long and 41 ft 93 yds wide, the cloister being 12 ft 45 in wide?

8 A rectangular court has a path of the uniform width of 3 yds. 1 ft running round it, the length of the court (including the path)

is 40 yds and the breadth 30 yds, find the cost of paving the path at 4s 6d per sq yd and of covering the remainder of the court with turf at 13s per 200 sq ft

9 The area of a square cricket field is 9 ac 3 10 8 16 po, a path of the uniform width of 39 yds is made close to the boundary (on the inner side) of the field at a cost of 4d per sq yd, and the remainder of the field is laid down in turf at a cost of 5s 6d per 100 sq yds, find the total cost of preparing the field

10 A room is 60 ft long by 29 ft wide, how many people can be seated in it on chairs $1\frac{1}{2}$ ft wide, and placed 2 ft apart from back to back, allowing a clear passage 3 ft wide down the middle of the room, and a space 15 ft deep at one end?

11 A box with a lid is to be made of $1\frac{1}{2}$ in plank, the external dimensions to be 3 ft 6 in, 2 ft 6 in and 1 ft 9 in. How many square feet of plank will be used in the construction?

12 It costs Rs 6435 6a to level and turf a square cricket ground at 6a per sq yd, find the cost of enclosing it with an iron railing at Rs 3 12a per yard

13 A cistern, without a lid, whose floor and sides are $1\frac{1}{2}$ in thick, is 5 ft 3 in long, 3 ft 7 in wide and 2 ft $5\frac{1}{2}$ in high in its external dimensions. Find the area of the inside surface, and the cost of painting the inside at the rate of 2a 8p per sq foot

14 A rectangular grass plot measures 320 yds by 160 yds, all round it (on the outer side of the boundary) is a gravel path 6 ft broad. The price for making the grass plot is 4a per sq yd, what must be the price of the gravel path per sq yard, that the path may cost Rs 11832 less than the grass plot?

15 A court-yard 15 yds by 12 yds is to be paved with pebbles at Re 1 8a per sq yd except two foot paths at right angles to the sides, each 4 ft wide which meet in the centre, forming a cross, these are to be laid in paving at Re 1 10a per sq yd, find the cost of the whole

16 A cistern $12\frac{3}{4}$ cubits long, $8\frac{1}{2}$ cubits broad and $6\frac{1}{2}$ cubits deep is to be lined with lead. How many sq cubits of lead will be required? What will be the weight of the lead, if each sq cubit weigh 8 seers? What will it cost at 2a per seer?

II CUBIC MEASURE

439 In Arithmetic, we deal with the volumes of rectangular solids only

(a) A solid is that which has length, breadth and thickness

(b) A rectangular parallelepiped or solid is a solid figure bounded by six rectangular surfaces of which every opposite two are equal and parallel, as a brick

- (c) A **cube** is a rectangular solid bounded by six squares, as a *die*
 (d) The length, breadth and thickness (or height or depth) of a rectangular solid are called its **dimensions**
 (e) The **capacity** or **volume** of a solid, is the quantity of space, comprehending length, breadth and thickness, which it contains
 (f) The word **content** is also frequently used to denote length, area and capacity or volume

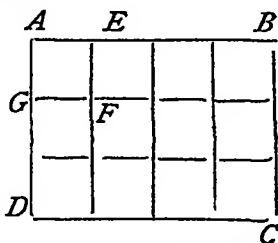
440 A rectangular solid measuring a yard each way is called a **cubic yard**, a rectangular solid measuring a foot each way is called a **cubic foot**, and so on

441 *To find the volume of a rectangular solid*

Let $ABCD$ represent a rectangle whose length AB is 4 inches and breadth AD , 3 inches

Then $ABCD$ will contain 12 square inches

If on the figure $ABCD$ there be placed a solid in the form of a dice, each edge being one inch in length, on every square as $AEFG$, we shall have a layer of solids containing as many cubic inches as there are square inches in $ABCD$, or 12 cubic inches, if on this layer we place a second layer of exactly the same form and size, we shall have the whole solid two inches high and containing twice 12 cubic inches, and so on. Hence the whole number of cubic inches in any such solid will be expressed by the product of the number of square inches in the base or rectangle $ABCD$, multiplied by the number of inches in the height. Therefore the process for measuring the **volume** or **solidity** or **capacity** of any rectangular solid is expressed by the following **RULE** —



RULE *Express the length, breadth and height or thickness in units of the same denomination, their product will give the volume in cubic units of that denomination*

442 It should be noticed here, as in Art 431, that if the number of inches in the length, and breadth, and height be mixed numbers or fractions, still their product will give the number of cubic inches in the volume. Also, if the numbers giving the three dimensions be given in feet, or in yards, or in miles, their product will give the volume in cubic feet, or in cubic yards, or in cubic miles

443 The above *Rule* is more briefly stated thus —

Volume = Length \times Breadth \times Height

\therefore Length = volume \div (breadth \times height), Breadth = volume \div (length \times height), and Height = volume \div (length \times breadth)

Ex 1 Find the cubic content of a rectangular solid whose dimensions are $10\frac{1}{2}$ ft long, $8\frac{1}{2}$ ft broad and $6\frac{1}{2}$ ft high

The cubic content = $10\frac{1}{2} \times 8\frac{1}{2} \times 6\frac{1}{2}$ cub ft = $\frac{2^2}{2} \times \frac{2^2}{2} \times \frac{2^2}{2}$ cub ft
 = $\frac{4 \times 4 \times 4}{8}$ cub ft = $548\frac{5}{8}$ cub ft = 548 cub ft 1080 cub in

Ex 2 A square block of stone 2 ft in thickness, is in cubic content 5 cub ft 24 cub in, what is the length of its edge?

The area of square stone = $5\frac{2}{3}$ cub ft $\div 2$ ft = $2\frac{2}{3}$ sq ft

\therefore each edge reqd = $\sqrt{(2\frac{2}{3})}$ ft = $1\frac{1}{2}$ ft = 1 ft 7 in

Ex 3 How many bricks will be required to build a wall 75 ft long, 6 ft high and 18 in thick, each brick being 9 in long, $4\frac{1}{2}$ in wide and 3 in deep?

The volume of a brick = $(\frac{9}{12} \times \frac{9}{12} \times \frac{3}{12})$ cub ft = $\frac{1}{128}$ cub ft

The volume of wall = $75 \times 6 \times \frac{1}{2}$ cub ft = 75×9 cub ft

\therefore the number of bricks = 75×9 cub ft $\div \frac{1}{128}$ cub ft = $75 \times 9 \times \frac{128}{1}$
 = $75 \times 128 =$ 9600

Ex 4 What is the length of a room, whose width is 10 ft 4 in and height 10 ft 6 in, and which contains 1519 cub ft of air?

The length = $\frac{1519 \text{ cub ft}}{10\frac{1}{2} \times 10\frac{1}{2} \text{ sq ft}} = 1519 \times \frac{2}{31} \times \frac{2}{31} \text{ ft} = 14 \text{ ft}$

Ex 5 A reservoir is 24 ft 8 in long by 12 ft 9 in wide, how many cubic feet of water must be drawn off to make the surface sink $3\frac{1}{4}$ ft?

As the surface sinks $3\frac{1}{4}$ ft, the depth of the quantity of water drawn off is also $3\frac{1}{4}$ ft

\therefore quantity of water drawn off = $24\frac{2}{3} \times 12\frac{3}{4} \times 3\frac{1}{4}$ cub ft
 = $\frac{2^2}{3} \times \frac{3^2}{4} \times \frac{3^2}{4}$ cub ft = 1100 $\frac{3}{4}$ cub ft.

Ex 6 Water flowing at the rate of 60 ft every 4 min through a cylindrical pipe, whose sectional area is 16 sq in fills a square tank 32 ft deep in 8 hours Find the length and breadth of the tank

The quantity of water flowing through the pipe in 4 min = $16 \text{ sq in} \times 60 \text{ ft} = \frac{1}{2} \times 60$ cub ft = $\frac{30}{1}$ cub ft

\therefore quantity of water flowing in 8 hours = $(\frac{30}{1} \times 15 \times 8)$ cub ft
 = 800 cub ft

\therefore the area of the tank = $800 \text{ cub ft} \div 32 \text{ ft} = 25 \text{ sq ft}$

As the tank is a square, each side = $\sqrt{(25)} \text{ ft} =$ 5 ft

Ex 7 A rectangular pond 60 ft long, 49 ft 6 in broad and 6 ft 8 in deep is full of water, 100 water carts are employed to take away water from it, the water box in each cart being 5 ft 6 in long, 4 ft 6 in broad and 16 in high How much will the water sink down, when each of the carts has been used 3 times?

The cubic content of each cart = $(5\frac{1}{2} \times 4\frac{1}{2} \times \frac{16}{12})$ cub ft = 33 cub ft

\therefore quantity of water drawn off = $33 \text{ cub ft} \times 100 \times 3 = 9900 \text{ cub ft}$

The area of the surface of water in the tank = $(60 \times 49\frac{1}{2})$ sq ft
 $= 2970$ sq ft

∴ the water will sink down 9900 cub ft $\div 2970$ sq ft $= 1\frac{0}{3}$ or $1\frac{1}{3}$ ft

Ex 8 The exterior dimensions of a box made of wood of half an inch in thickness are 5 ft 4 in long, 4 ft 3 in broad and 3 ft 9 in high respectively, find the expense of painting it inside and outside at 6s 9d per square yard

The wood being $\frac{1}{2}$ in thick, the inner dimensions are 5 ft 3 in, 4 ft 2 in and 3 ft 8 in respectively

The exterior surface to be painted = $2(5 \text{ ft } 4 \text{ in} + 4 \text{ ft } 3 \text{ in}) \times 3 \text{ ft } 9 \text{ in}$
 $+ 5 \text{ ft } 4 \text{ in} \times 4 \text{ ft } 3 \text{ in} \times 2 = 2 \times 9 \text{ ft } 7 \text{ in} \times 3 \text{ ft } 9 \text{ in} + 5 \text{ ft } 4 \text{ in} \times 4 \text{ ft } 3 \text{ in} \times 2$
 $= 2 \times 9\frac{7}{12} \times 3\frac{3}{4}$ sq ft $+ 5\frac{1}{2} \times 4\frac{3}{4} \times 2$ sq ft
 $= (1\frac{2}{3} + 1\frac{1}{6})$ sq ft $= 2\frac{5}{6}$ sq ft

The interior surface to be painted = $2(5 \text{ ft } 3 \text{ in} + 4 \text{ ft } 2 \text{ in}) \times 3 \text{ ft } 8 \text{ in}$
 $+ 5 \text{ ft } 3 \text{ in} \times 4 \text{ ft } 2 \text{ in} \times 2 = 2 \times 9 \text{ ft } 5 \text{ in} \times 3\frac{2}{3} \text{ ft} + 5\frac{1}{2} \times 4\frac{1}{2} \times 2$ sq ft
 $= (2 \times 9\frac{5}{6} \times 3\frac{2}{3})$ sq ft $+ (5\frac{1}{2} \times 4\frac{1}{2} \times 2)$ sq ft
 $= (1\frac{2}{3} + 1\frac{1}{4})$ sq ft $= 2\frac{7}{8}$ sq ft

∴ the whole area to be painted = $(2\frac{5}{6} + 2\frac{7}{8})$ sq ft $= 4\frac{59}{24}$ sq ft
 $= 1\frac{27}{8}$ sq yds

∴ cost reqd = $1\frac{27}{8} \times 6\frac{3}{4} \text{ s} = 17\frac{1}{8} \text{ s} = \underline{\underline{Rs 10 \text{ } 12\text{s } 6\frac{1}{2}\text{d}}}$

Examples CXXXII

1 Find the cubic content of each of the rectangular solids whose dimensions are the following —

- | | | | |
|-----|-------------------|-------------------|----------------------|
| (1) | Length 6 ft 4 in, | breadth 5 ft 3 in | and height 3 ft 6 in |
| (2) | 12 ft 8 in | 9 ft 10 in | 8 ft 5 in |
| (3) | 15 ft 7 in | 12 ft 5 in | 8 ft 4 in |
| (4) | 10 yds 7 in | 8 yds 2 ft 3 in | 2 yds 2 ft 9 in |
| (5) | 6 yds 2 ft 3 in | 4 yds 1 ft 7 in | 6 ft 10 in |

2 Find the volume of a cube whose edge is 13 ft 8 in

3 Find the cost of a piece of timber, whose length, breadth and thickness are respectively $54\frac{1}{2}$ ft, 5 ft and 2 ft 5 in at 6s per solid foot

4 What must be the length of a trench 6 ft 8 in deep and 9 ft 2 in wide, that it may contain 13 cub yds 15 cub ft 1152 cub in?

5 If 56 cub ft 1044 cub in of timber are required to floor a room 29 ft 3 in broad by 35 ft 4 in long, find the thickness of the boards

6 Find the height of a room 26 ft 4 in long, 13 ft 9 in broad, the content of which is 44898 cub ft 4 cub in

7 If 473088 bricks, each 9 in long, $4\frac{1}{2}$ in broad and $2\frac{1}{2}$ in thick be required for a wall $\frac{1}{2}$ a mile long, 7 ft high and of a certain thickness, find the thickness

8 A wall is to be built 15 yds long, 7 ft high and 13 in thick, with a door way 6 ft high and 4 ft wide, how many bricks will it require, if each brick including mortar occupy 108 cub in ?

9 Find the edge of a cube which contains 15 cub ft 1080 cub in

10 The bottom of a cistern contains 16 sq ft 128 sq in, how deep must it be to contain 1216 gallons ? 1 gallon contains 277½ cub in nearly

11 A cubic foot of water weighs 31½ seers Find the length of the side of a cubic vessel whose contents (water) weigh 101 mds 39 sr 3½ ch

12 A space 8 ft square and 10 ft high is enclosed all round with earth of the uniform thickness of 2 feet, what is the quantity of earth ?

13 A cubic foot of water weighs 62½ lbs, and a room 18 ft 9 in by 13 ft 4 in is flooded to the depth of 2 inches, what is the weight of water in the room ?

14 How many bricks of which the length, breadth and thickness are 12, 9 and 6 in respectively will be required to build a wall, whereof the length, height and thickness are 64, 9 and 1½ ft respectively ?

15 If 56 cub ft 912 cub in be the content of an open cistern, 6 ft 2 in long and 3 ft 4 in wide, what will be the cost of lining the inside of it with lead at 10s 1½d per sq yd ?

16 If a cubic foot of iron weigh 78 times as much as a cub foot of water, find the weight of a block of iron 2028 ft long, 258 ft broad and 25 ft thick, supposing a cub foot of water to weigh 1000 oz Avoir

17 The depth of water in a cistern whose base contains 1344 sq in is 2 ft 10 in Find the depth of the same quantity of water in another cistern whose base contains 1088 sq in

18 The weight of a cubic foot of water being 31½ seers, find the weight of a rectangular block of gold 8 in in length, 2 in in thickness and 3 in in breadth, the weight of a mass of gold being 1925 times the weight of an equal bulk of water

19 A cistern whose length, depth and breadth are 6 ft 3 in, 5 ft and 4 ft 2 in respectively, is filled with water and leaks till the water sinks 7 in, find the volume of water left

20 How many bricks, each 12 in long, 4 in wide and 3 in thick will be required to build a wall 18 ft 8 in long, 12 ft 6 in high and 9 in thick, leaving in it a door way 6 ft 3 in high and 2 ft 8 in wide ?

21 A cubic foot of water weighs 1000 oz Avoir Find in tons the weight of a rain-fall of one inch over an acre of ground

22 The breadth of a room is twice its height, and half its length, and the volume of air in the room is 4096 cub ft Find its length

23 If in a box 3 ft 6 in long, 1 ft 7 in deep and 1 ft 10 in wide, I pack 160 books each $9\frac{1}{2}$ in long, $5\frac{1}{2}$ in wide and $1\frac{1}{2}$ in thick, find how many more, of a size 7 in long, $4\frac{1}{2}$ in wide and 1 in thick, are required to fill it

24 A gentleman wishes to raise his lawn (which is 634 yds long and 340 yds broad) 2 ft and for that purpose digs a moat round it 17 yds broad in every part, supposing the depth of the moat to be uniform, how deep must it be in order that he may have soil sufficient for his purpose?

25 A room 21 ft long by $13\frac{1}{2}$ ft wide is surrounded by walls $1\frac{1}{2}$ ft thick and 14 ft high. There are two doors each $4\frac{1}{2}$ ft by 6 ft, and one window 3 ft by $4\frac{1}{2}$ ft. Find (1) the cost of building the walls at the rate of Rs 5 1a per cub yard and (2) the number of bricks each measuring 9 in \times 4 in \times $2\frac{1}{4}$ in required for the work

26 In the middle of a rectangular field, 350 yds long by 250 yds broad, a tank 50 yds square is dug and the earth is thrown evenly on the field to a height of one yard. Find the depth of the tank and the cost of digging it at 9p per cub yard

27 The content of a box, whose length is twice its breadth, and whose breadth is twice its depth, is 1 cub yd, find its dimensions, and the cost of gilding it at 14s per sq ft

28 Find the cost of painting the surface of a cube, the edge of which is 2 ft 5 in at 10a per sq yd

29 A closed vessel of metal 1 in thick, the external dimensions of which are 8 ft 3 in, 7 ft 5 in and 4 ft 3 in weighs 3 cwt 1 qr 8 lbs, what would be the weight of a solid mass of the metal of the same dimensions?

30 Find the cost of making a road 110 yds in length and 18 ft wide, the soil being first excavated to the depth of 1 foot at a cost of 8a per cub yd, rubble being then laid 8 inches deep at 8a per cub yd and gravel placed on top, 9 in thick at Re 1 4a per cub yd

31 The content of a cistern is the sum of two cubes whose edges are 10 in and 2 in and the area of its base is the difference between two squares whose sides are $1\frac{1}{2}$ ft and $1\frac{3}{4}$ ft. Find its depth

32 A cubic foot of gold is extended by hammering so as to cover an area of 6 acres. Find the thickness of the gold as a decimal of an inch to 7 places of decimals

33 A cistern 32 ft long, 25 ft wide and 11 ft deep is emptied in 20 minutes by a pipe whose sectional area is 14 sq in. How fast, does the water flow in the pipe?

34 A cubic foot of water weighs 1000 oz Avoir, a pipe whose bore is 5 sq in, discharges $312\frac{1}{2}$ lbs of water per minute, find the rate per hour at which the water issues

35 The contents of a reservoir of square bottom are let out through a rectangular hole 3 in by 4 in. If an edge of the bottom be 6 ft and the height 4 ft 8 in, and water flow at the rate of $1\frac{1}{2}$ ft per second, what time will be spent in discharging the contents?

36 The cubic content of a room 20 ft long and $12\frac{1}{2}$ ft high is 4875 cubic feet, find the cost of painting its walls at 6s per sq yd

37 An iron chest 5 ft long, 3 ft broad and 2 ft 5 in high is made with sheet of iron 1 in thick. Find the inside content of the chest and the weight of the chest, if 1 cub foot of iron weighs 6 mds 8 sr

38 A tank is 100 ft long, 60 ft broad and 18 ft deep, 20 water carts, each carrying 30 mds of water at a time are employed to empty the tank. If a cub foot of water weighs 30 seers, in how many times will the tank be emptied?

39 A hollow rectangular iron pillar 10 ft in length is made with iron sheets 1 in thick and the breadth of the hollow part is 6 in. Find the weight of the pillar, if 1 cub foot of iron weighs 6 mds, and its price at Rs 5 8s per maund

40 A cistern, measuring 13 ft in length, 5 ft in breadth and 4 ft in depth has a tap which, not being properly opened, discharges 54 gals an hour less than it would otherwise do and empties the cistern in $7\frac{1}{2}$ instead of 6 hours. How many cub inches are there in the gallon?

41 Water flows into a rectangular cistern whose dimensions are 12 ft 1 in long, 11 ft 8 in wide and 5 ft 4 in deep through a pipe of 10 sq in aperture at the rate of $2\frac{1}{2}$ ft per second, and flows out through an orifice at the rate of 2 ft 5 8 in per second, if the cistern is filled in 2 hours, find the size of the orifice

42 A stream of water 8 yds broad at the surface and 6 yds at the bottom and 2 yds deep flows at the rate of $1\frac{1}{3}$ miles an hour into a tank 220 yds long and 56 yds broad, which holds 74250 tons of water. Find the depth of the tank and the time in which it will be filled, a cubic foot of water weighing 1000 oz Avoir

III DUODECIMALS

444 Artificers take the dimensions of their work in *yards, feet, inches, parts, &c*, and it is usual to reduce the yards to feet so that the different denominations are *all* connected by the same number 12, or decrease in a *twelve-fold* proportion, from the place of feet towards the right hand. For the sake of uniformity, the denominations after feet are termed *primes, seconds, thirds, &c*, distinguished respectively by accents, ' , " , ' , &c, placed a little to the right, contiguous to the figures to which they belong

Thus, 20 feet, 8 inches, 5 parts, 7 thirds, &c is written 20 ft 8' 5" 7''' &c and is read 20 *ft*, 8 *primes*, 5 *seconds*, 7 *thirds*, &c

From the circumstance above mentioned, the process is frequently called *Duodecimal Multiplication* or *Duodecimals*, and it is also sometimes termed *Cross Multiplication*, but the former of these names are evidently misapplied, because the *different* digits of the various denominations are not connected with each other by the number 12 though the *denominations* themselves are

445 In Duodecimals, the sub multiples of the foot whether linear, square or cubic—follow the scale of 12, so that,

LINEAR MEASURE

1 foot = 12 primes

1 prime = 12 seconds

SQUARE MEASURE

1 sq ft = 12 (superficial) primes

1 supt prime = 12 seconds

1 second = 12 thirds

1 third = 12 fourths, &c

1 second = 12 thirds

1 third = 12 fourths, &c

CUBIC MEASURE

1 cub ft = 12 (solid) primes

1 solid prime = 12 seconds

1 second = 12 thirds

1 third = 12 fourths, &c

446 The whole of the above statements can be briefly stated thus —

$$\left. \begin{array}{l} 1 \text{ linear foot} \\ 1 \text{ square foot} \\ 1 \text{ cubic foot} \end{array} \right\} = 12' = 144'' = 1728''' = 20736^{iv} = \&c ,$$

therefore in linear measure the *inch* is the same as the *prime*, in square measure as the *second*, and in a cubic measure as the *third*. Hence, we can easily pass from quantities expressed in duodecimals to those expressed in foot and inches, and *conversely*

Ex 1 Convert 37 ft 5' 3" into *ft* and *in*, and 47 ft 5½ in. into *Duodecimals*

$$(a) \ 37 \text{ ft } 5' 3'' = 37 \text{ ft } 5\frac{1}{2} \text{ in} = \underline{37 \text{ ft } 5\frac{1}{2} \text{ in}}$$

$$(b) \ 47 \text{ ft } 5\frac{1}{2} \text{ in} = 47 \text{ ft } 5\frac{9}{2} \text{ in} = \underline{47 \text{ ft } 5' 9''}$$

Ex 2 Express 27 sq ft 118½ sq in in *Duodecimals*, and 46 sq ft 10' 7" 8" in *sq ft* and *sq in*

$$(a) \ 27 \text{ sq ft } 118\frac{1}{2} \text{ sq in} = 27 \text{ sq ft } 118\frac{1}{2} \text{ ' } = \underline{27 \text{ sq. ft } 9' 10'' 8'''}$$

$$(b) \ 46 \text{ sq ft } 10' 7' 8'' = 46 \text{ sq ft } 127\frac{1}{2}'' = \underline{46 \text{ sq ft } 127\frac{1}{2} \text{ sq in}}$$

Ex 3 Convert 25 cub ft 1267½ cub in into *Duodecimals*, and 48 cub ft 6' 7" 8" 9" 4" into *cub ft* and *cub in*

$$(a) \ 25 \text{ cub ft } 1267\frac{1}{2} \text{ cub in} = 25 \text{ cub ft } 105'' 7''' 6\frac{1}{2}^{iv} \\ = \underline{25 \text{ cub ft } 8' 9'' 7''' 6^{iv} 8^{v}}$$

$$(b) \ 48 \text{ cub ft } 6' 7' 8' 9'' 4''' = 48 \text{ cub ft } 79'' 8''' 9\frac{1}{2}^{iv} \\ = 48 \text{ cub ft } 956\frac{1}{2} \text{ cub in.} \\ = \underline{48 \text{ cub ft } 956\frac{1}{2} \text{ cub in}}$$

Examples CXXXIII .

1 Express in yards, feet and inches —

- (1) 7 ft 3' 6", 25 ft 11' 10" 8", 146 ft 6' 9" 9"
 (2) 18 sq ft 11' 6" 8", 216 sq ft 6' 0" 10" 6", 274 sq ft 4' 10" 0" 9"
 (3) 20 cub ft 9' 6" 9", 11 cub ft 4' 8' 3", 83 cub ft 6' 5" 10" 8" 8"
 (4) 25 ft 8' 3" 9", 24 sq ft 11' 3", 2341 cub ft. 5' 6" 8" 8"

2 Express in *Duodecimals* —

- (1) 13 ft 5½ in , 19 yds 2 ft 6½ in , 7 yds 1 ft 6½ in , 9 yds 2 ft 4¼ in
 (2) 50 sq ft 68 sq in , 47sq ft 63sq in , 33sq yds 3sq ft 28¼sq in
 (3) 4 cub ft 1088 cub in , 42 cub ft 334 cub in , 39 cub ft
 1120½ cub in , 18 cub ft 1664 cub in

447 Before reckoning areas and volumes by *Duodecimals*, we must notice the relation which exists between the following products

SQUARE MEASURE	CUBIC MEASURE
feet × primes = (superficial, primes	supl ft × primes = (solid) primes
feet × seconds = „ seconds	„ ft × seconds = „ seconds
feet × thirds = „ thirds	„ ft × thirds = „ thirds
and so on	and so on
primes × primes = „ seconds	„ primes × primes = „ seconds
primes × seconds = „ thirds	„ primes × seconds = „ thirds
and so on	and so on
seconds × seconds = „ fourths	„ seconds × seconds = „ fourths
seconds × thirds = „ fifths, &c	„ seconds × thirds = „ fifths, &c
for 1 ft × 1' = 1 × 1½ sq ft = 1½ sq ft = 1' , 1 ft × 1" = 1 × 1¼ sq ft = 1¼ sq ft = 1" , 1 ft = 1" = 1 × 17½ cub ft = 17½ cub ft = 1" , &c	

448 The operation employed to compute superficial and solid contents is that of Multiplication, conducted by means of a mixed *Decimal* and *Duodecimal* scale of Notation, the figures of the feet being expressed and multiplied in the ordinary way, whilst in the other places the number 12 is always made use of instead of 10. The denomination on the left hand of the multiplier is used first, those of the multiplicand being taken as in other cases, then the next in order, and so on, and for the reason that we put the first figure of a *partial* product one place to the *left* of that of the preceding one when we begin with the least denomination of the multiplier, the terms of the product here must each be put one place to the *right* of those of the preceding, in order to possess their proper relative values, and the addition is effected by beginning with the lowest denomination, as in compound quantities. The practical applications of the Rule will be best understood by the following Examples

Ex 1 Find the area of a rectangle whose adjacent sides are 5 ft 3 in and 4 ft 9 in

$$\begin{array}{r} 5 \text{ ft} \quad 3' \\ 4 \quad 9 \\ \hline 21 \text{ sq ft. } 0' \\ 3 \quad 11 \quad 3'' \\ \hline 24 \text{ sq ft } 11' \quad 3'' \end{array}$$

$$\begin{aligned} 4 \text{ ft } \times 3' &= 12' = 1 \text{ ft } 0', \text{ carry } 1 \text{ ft} \\ 4 \text{ ft } \times 5 \text{ ft} + 1 \text{ ft} &= 21 \text{ ft} \\ 9 \times 3 &= 27'' = 2' 3'', 9' \times 5 \text{ ft} + 2' = 47' = 3 \text{ ft } 11' \\ \text{Thus the area} &= \underline{24 \text{ sq ft } 11' 3''} \end{aligned}$$

Ex 2 Required the area of a square whose side is 7 ft 8' 9"

$$\begin{array}{r} 7 \text{ ft} \quad 8' \quad 9'' \\ 7 \quad 8 \quad 9 \\ \hline 52 \text{ sq ft } 1' \quad 3'' \\ 5 \quad 1 \quad 10 \quad 0'' \\ \hline 59 \text{ sq ft } 8' \quad 10' \quad 6'' \quad 9''' \end{array}$$

$$\begin{aligned} 7 \text{ ft } \times 9'' &= 63' = 5' 3'', \text{ carry } 5' \\ 7 \text{ ft } \times 8' + 5' &= 61' = 5 \text{ ft } 1', \text{ \&c} \\ 8 \times 9'' &= 72' = 6' 0'', \text{ \&c} \\ 8' \times 8' + 6' &= 70' = 5' 10', \text{ \&c} \\ 9' \times 9'' &= 81''' = 6''' 9''', \text{ \&c} \\ \text{Thus the area} &= \underline{59 \text{ sq ft } 10' 6'' 9'''} \end{aligned}$$

Ex 3 Find the content of a rectangular solid whose lineal dimensions are 5 ft 6 in, 4 ft 5 in and 3 ft 4 in

$$\begin{array}{r} 5 \text{ ft} \quad 6' \\ 4 \quad 5 \\ \hline 22 \text{ sq ft. } 0' \\ 2 \quad 3 \quad 6'' \\ \hline 24 \quad 3 \quad 6 \\ 3 \quad 4 \\ \hline 72 \text{ c. ft } 10 \quad 6 \\ 8 \quad 1 \quad 2 \quad 0''' \\ \hline 60 \text{ c. ft. } 11 \quad 8 \quad 0'' \end{array}$$

$$\begin{aligned} \text{Thus the required volume} &= 80 \text{ cub ft } 11' 8'' \\ &= \underline{80 \text{ cub ft } 1680 \text{ cub in}} \end{aligned}$$

Ex 4 Required the capacity of a cube the length of whose edge is 2 ft 9 in

$$\begin{aligned} \text{The capacity} &= 2 \text{ ft } 9' \times 2 \text{ ft } 9' \times 2 \text{ ft } 9' = 7 \text{ sq ft } 6' 9'' \times 2 \text{ ft } 9' \\ &= 20 \text{ cub ft. } 9' 6'' 9''' = \underline{20 \text{ cub ft } 1377 \text{ cub in}} \end{aligned}$$

Examples CXXXIV

1. Find by *Duodecimals* the areas of the following rectangles —

- | | |
|---|--|
| (1) 14 ft 6 in by 12 ft 7 in | (2) 25 ft 7 in by 7 ft 10 in |
| (3) 16 ft 5' by 12 ft. 11' | (4) 11 ft 11' by 2 ft 3' 4". |
| (5) 9 ft 4' 7" by 5 ft 6' 4" | (6) 17 ft. 3' 4" by 19 ft 5' 11" |
| (7) 15 yds 2 ft $4\frac{1}{2}$ in by 9 yds 2 ft $4\frac{1}{2}$ in | |
| (8) 207 ft. $4\frac{1}{2}$ in by 95 ft $7\frac{1}{2}$ in | (9) 17 ft $3\frac{1}{2}$ in by 12 ft $6\frac{1}{2}$ in |
| (10) 19 yds. 2 ft $6\frac{1}{2}$ in by 7 yds 1 ft $3\frac{1}{2}$ in | |
| (11) 10' 3" 4''' by 5' 0" 6''' | (12) 13 ft 2' 6' by 1' 9' 10''' |

2 Find by *Cross Multiplication* the volumes of each of the following solids —

(1) 8 ft 6 in by 3 ft 9 in by 5 ft 4 in

(2) 3 ft 7 in \times 5 ft 8 in \times 2 ft 11 in

(3) 4 ft 6' by 5 ft 7' by 6 ft 8' (4) 8 ft 9' \times 5 ft 10' \times 3 ft 6' 4"

(5) 18 ft 7' 4" by 17 ft 3' 9" by 11 ft 11"

3 Find by *Duodecimals* the area of a square whose side is—

(1) 2 yds 1 ft $3\frac{1}{2}$ in

(2) 123 ft $6\frac{1}{2}$ in

4 Find by *Duodecimals* the volume of a cube whose edge is

(1) 12 ft 3 in

(2) 11 ft 6' 5"

(3) 3 yds 1 ft $7\frac{1}{2}$ in

5 Divide 1532 ft 9' 9" superficial measure by 18 ft 9' lineal measure

IV SQUARE & CUBIC MEASURES OF BENGAL

449 In Bengal, the areas of rectangular fields, &c., are found by a method similar to the above, and is called *Suvankar's Method*

Suvankar gives the following RULE for finding areas —

Bigha \times bigha = bigha

Bigha \times katha = katha

Katha \times katha = dhûl

Bigha \times chatak = chatak

Katha \times chatak = ganda

Chatak \times chatak = kâg

20 dhûls = 1 katha, 16 gandas = 1 dhûl, 16 kâgs = 1 ganda

Ex Find the area of a rectangular field 5 bi 14 kat long and 4 bi 13 kat broad

5 bi 14 kat

4 bi 13

22 16

3 14 2 dhûl

26 bi 10 2 dhûl

4 bi \times 14 kat = 56 kat = 2 bi 16 kat,

carry 2 bi

5 bi \times 4 bi + 2 bi = 22 bi, 13 kat \times 14 kat

= 182 dhûl = 9 kat 2 dhûl, carry 9 kat,

5 bi \times 13 kat + 9 kat = 74 kat = 3 bi 14 kat

2 dhûl = 2 \times 16 or 32 ga = 1 ch 12 ga

area = 26 bi 10 kat 2 dhûl = 26 bi 10 kat 1 ch 12 ga

450 The volume of a rectangular solid is found in the same way as in Art 441

Ex Find the volume of a wall 48 cubits long, 12 cubits high, and $1\frac{1}{2}$ cubits thick

Volume of the wall = $48 \times 12 \times 1\frac{1}{2}$ cub cubits = 864 cub cubits

Examples CXXXV

1 Find by *Suvankar's* method the area of —

(1) 3 bi by 2 bi

(2) 5 bi by 16 kat

(3) 1 bi 12 kat by 16 kat

(4) 8 bi 13 kat by 3 bi 16 kat

(5) 6 bi 17 kat by 5 bi 6 kat

(6) 12 bi 18 kat by 10 bi 12 kat.

- (7) 10 bi 18 kat by 6 bi 12 kat (8) 8 bi 12½ kat by 6 bi 8 kat
 (9) 12 bi 12½ kat by 10 bi 16 kat 5 ch
 (10) 12 bi 16 kat 10 ch by 8 bi 10 kat 6 ch
 (11) 380 cubits by 260 cubits (12) 724 cubits by 248 cubits
- 2 Find the volume from the following dimensions —
- (1) 72, 14, 8 cubits (2) 312, 16, 1½ cubits
 (3) 480, 62, 5½ cubits (4) 248, 15, 7¼ cubits
 (5) 24 yds 18 yds, 6 yds (6) 58 yds, 25 yds, 6 ft

$\therefore \text{Rs } 410 \dots$ in 12 days by $\frac{29 \times 16 \times 54 \times 410}{1169 \times 12}$ men
or 720 men *Ans*

Ex 3 If 6 men can do a piece of work in 30 days of 9 hours each, how many men will take to do 10 times the amount, if they work 25 days of 8 hours each?

The work can be done in 30 days of 9 hrs each by 6 men,
 1 day of 9 hrs each by (6×30) men,
 1 day of 1 hr each by $(6 \times 30 \times 9)$ men,
 25 days of 1 hr each by $\frac{6 \times 30 \times 9}{25}$ men,
 25 days of 8 hrs by $\frac{6 \times 30 \times 9}{25 \times 8}$ men
 10 times the work 25 days of 8 hrs by $\frac{6 \times 30 \times 9 \times 10}{25 \times 8}$ men
 or 81 men Ans

Ex 4 If 252 men in 5 days of 11 hours each, can dig a trench 210 yds long, 3 yds wide and 2 vds deep, in how many days of 10 hours each, can 24 men dig a trench 420 yds long, 5 vds wide and 3 yds deep?

The solid content of the first trench = $210 \times 3 \times 2$ or 1260 cub yds
 second = $420 \times 5 \times 3$ or 6300 cub yds
 Now, 252 men can dig 1260 cub yds in 5 days of 11 hrs each
 \therefore 1 man 1260 cub yds in (5×252) days of 11 hrs
 \therefore 1 man 1 cub yd in $\frac{5 \times 252}{1260}$ days of 11 hrs
 \therefore 24 men 1 cub yd in $\frac{5 \times 252}{1260 \times 24}$ days of 11 hrs
 \therefore 24 men 6300 cub yds in $\frac{5 \times 252 \times 6300}{1260 \times 24}$ days of 11 hrs
 \therefore 24 men 6300 cub yds in $\frac{5 \times 252 \times 6300 \times 11}{1260 \times 24}$ days of 11 hrs
 \therefore 24 men 6300 cub yds in $\frac{5 \times 252 \times 6300 \times 11}{1260 \times 24 \times 10}$ days of 10 hrs
 or 288 $\frac{1}{2}$ days Ans

Ex 5 If I get 8 chataks of bread for 4a when wheat is Rs 7 8a a maund, what ought a maund of wheat to be when I get 12 ch of bread for 2a 8p?

The 4a bread weighs 8 ch when wheat is at Rs 7 $\frac{1}{2}$ per md
 1a 8 ch Rs $(7\frac{1}{2} \times \frac{1}{4})$ per md
 1a 1 ch Rs $(7\frac{1}{2} \times \frac{1}{4} \times 8)$ per md
 2 $\frac{2}{5}$ a 1 ch Rs $(15 \times \frac{1}{4})$ per md
 2 $\frac{5}{8}$ a 12 ch Rs $(\frac{15}{4} \times 8)$ per md
 or Rs 3 5a 4p Ans

Ex 6 If 10 cannon, which fire 3 rounds in 5 minutes, kill 270 men in $1\frac{1}{2}$ hours, how many cannon, which fire 5 rounds in 6 minutes will kill 500 men in one hour?

The first set in $1\frac{1}{2}$ hours firing 3 rounds in 5 min make $(3 \times 1\frac{1}{2} \times 60 - 5)$ or 54 rounds and the second set in one hour firing 5 rounds in 6 min make $(5 \times 60 - 6)$ or 50 rounds

Now, in 54 rounds 270 men are killed by 10 cannon,

\therefore 1 round 270 men 10×54

\therefore 1 round 1 man is $\frac{10 \times 54}{270}$

\therefore 50 rounds 1 man $\frac{10 \times 54}{270 \times 50}$

\therefore 50 rounds 500 men are $\frac{10 \times 54 \times 500}{270 \times 50}$

or 20 cannon *Ans*

Examples CXXXVI

1. If the wages of 4 men for 12 days be Rs 6, what would be the wages of 6 men for 10 days?

2. If 9 men earn Rs 40 8a in 24 days, how many men must work 16 days to earn Rs 450?

3. If a regiment of 930 soldiers consume 351 mds of wheat in 168 days, how many soldiers will consume 1404 mds in 56 days?

4. If I pay Rs 5 for the carriage of 2 tons for 6 miles, what must I pay for the carriage of 12 tons 17 cwt for 34 miles?

5. If the wages of 29 men for 54 days amount to £80 9s 6d how many men must work 12 days to receive £407?

6. If the gas for 5 burners, lighted 5 hours every day, for 10 days, cost Rs 2 2a, how many burners may be lighted 4 hours every evening for 15 days at a cost of Rs 38 4a?

7. If the carriage of 60 cwt for 20 miles cost Rs 145, what weight can I have carried 30 miles for Rs 54 6a?

8. If 5 men can reap a field of $12\frac{1}{2}$ bighas in $3\frac{1}{2}$ days, working 16 hrs a day, in what time can 7 men reap a field of 15 bighas working 12 hrs a day?

9. If 200 men in 12 days of 8 hours each can dig a trench 160 yds long, 6 wide and 4 deep, in how many days of 10 hours each will 90 men dig a trench 450 yds long, 4 wide and 3 deep?

10. If the carriage of goods weighing 5 cwt 2 qrs 12 lbs for 150 miles come to Rs 32 11a 4p, what will be the charge for carrying 4 wagon loads of the same, each weighing 7 cwt 2lbs a distance of 450 miles?

11 If 15 pumps, working 8 hours a day, can raise 1260 tons of water in 7 days, how many pumps, working 12 hours a day, will be required to raise 7560 tons of water in 14 days?

12 If with a capital of Rs 10000 a person gains by trade Rs 500 in 16 months, in how many months will he gain Rs 1250 with a capital of Rs 4000?

13 If when wheat is at Rs 3 per maund, the 4a loaf weighs 8 chataks, what should be the price of wheat per maund, when 3 sr 2 ch of bread cost 12a 6p?

14 If the 4d loaf weigh 1 lb 11 oz 12 drs, when wheat is at 7s 6d per bushel, what ought the 6d loaf to weigh when wheat is at 5s 3d per bushel?

15 If 15 horses and 148 sheep can be kept for 9 days for Rs 757 8a, what sum will keep 10 horses and 132 sheep for 8 days, supposing 5 horses eat as much as 84 sheep?

16 If Rs 240 be paid for bread for 49 persons for 20 months, when wheat is at Rs 3 per maund, how long will Rs 234 find bread for 91 persons, when wheat is at Rs 3 8a per maund?

17 If 5 men and 7 boys can reap a field of 125 acres in 15 days, in how many days will 10 men and 3 boys reap a field of 75 acres, each boy's work being one-third of a man's?

18 If 44 cannon, firing 30 rounds an hour for 3 hours a day, consume 300 barrels of powder in 5 days, how long will 40 barrels last 66 cannon, firing 40 rounds an hour for 5 hours a day?

452 Problems in Simultaneous Equations

Proceed as in the following Examples

Ex 1 If 9 horses and 7 cows cost Rs 850, and 5 horses and 8 cows cost Rs 575, find the cost of a horse and of a cow

The cost of 9 horses and 7 cows = Rs 850,

∴ the cost of 45 horses and 35 cows = Rs 850 × 5 or Rs 4250

Again, the cost of 5 horses and 8 cows = Rs 575,

the cost of 45 horses and 72 cows = Rs 575 × 9 or Rs 5175

Hence, by subtraction, we get

the cost of 37 cows = Rs 925,

∴ the cost of a cow = Rs 925 ÷ 37 = Rs 25 Ans

Again, the cost of 9 horses and Rs 175 = Rs 850, from (1)

∴ the cost of 9 horses = Rs 675,

∴ the cost of a horse = Rs 675 ÷ 9 = Rs 75 Ans

Ex 2 If 2 men and 3 boys can do $\frac{1}{4}$ of a piece of work in 2 days, and 3 men and 5 boys can do $\frac{1}{8}$ of it in 6 days, in what time can a man do the work?

In 2 days, 2 men and 3 boys can do $\frac{1}{4}$ of the work,
 \therefore 1 day, 2 men and 3 boys $\frac{1}{8}$
 \therefore 1 day, 10 men and 15 boys $\frac{5}{8}$
 Again, in 6 days, 3 men and 5 boys $\frac{3}{8}$
 \therefore 1 day, 3 men and 5 boys $\frac{1}{8}$
 \therefore 1 day, 9 men and 15 boys $\frac{4}{8} \times 3$ or $\frac{3}{2}$ of the work.

Hence, by subtraction we get
 in 1 day, 1 man can do $(\frac{3}{2} - \frac{1}{8})$ or $\frac{1}{8}$ of the work.

\therefore a man can do the whole in 32 days Ans

Ex 3 If 3 men with 4 boys earn Rs 58 in 8 days, and 2 men with 3 boys earn Rs 40 in the same time, in what time will 6 men and 7 boys earn Rs 210?

Since 3 men and 4 boys earn in 8 days Rs 58,
 \therefore 3 men and 4 boys earn in 1 day $Rs\ 58 - 8 = Rs\ 7\ 4a$
 Also 2 men and 3 boys earn in 1 day $Rs\ 40 - 8 = Rs\ 5$

Hence, by subtraction we get

1 man and 1 boy earn in 1 day Rs 2 4a

\therefore 2 men and 2 boys $Rs\ 2\ 4a \times 2 = Rs\ 4\ 8a$

But 2 men and 3 boys $Rs\ 5$

\therefore by subtraction, 1 boy earns in 1 day 8a

Again since 1 man and 1 boy earn in 1 day Rs 2 4a

\therefore 6 men and 6 boys earn in 1 day $Rs\ 2\ 4a \times 6 = Rs\ 13\ 8a$.

and since 1 boy earns in 1 day 8a

\therefore 6 men and 7 boys earn in 1 day Rs 14

\therefore the no of days required = $\frac{Rs\ 210}{Rs\ 14} = 15$ Ans

Examples CXXXVII

1 6 horses and 7 cows can be bought for Rs 2500, and 13 cows and 11 horses can be bought for Rs 4610 What is the value of each animal?

2 If 15 lbs of tea and 17 lbs of coffee together cost Rs 32 12a, and 25 lbs of tea and 13 lbs of coffee together cost Rs 43 1a 4a, find the price of each per pound

3 The price of 2 turkeys and 9 fowls is £3, and the price of 5 turkeys and 3 fowls is £4 5s, find the price of a turkey and of a fowl

4 If 3 men and 5 women do a piece of work in 8 days, which 2 men and 7 children can do in 12 days find how long 13 men, 14 children and 15 women working together will take to do it

5 If 5 men with 7 boys can earn Rs 76 8a in 6 days, and 2 men with 3 boys can earn Rs 21 in 4 days, in what time will 6 men with 12 boys earn Rs 600?

6 If 8 men and 5 boys can reap 29 acres in 3 days, and 6 men and 7 boys can reap 50 acres in 6 days, how long will it take 3 men and 6 boys to reap 15 acres ?

7 If 5 men and 3 boys can reap 23 acres in 4 days, and if 3 men and 2 boys can reap 7 acres in 2 days, how many boys must assist 7 men, in order that they may reap 45 acres in 6 days ?

8 If 2 boys and 1 man can do a piece of work in 4 hours and 2 men and 1 boy can do the same in 3 hours, find in what time a man, a boy, and a man and a boy together, respectively, can do the same.

9 If 2 men and 5 boys can do $\frac{1}{2}$ of a work in 3 days, and 3 men and 7 boys can do $\frac{1}{3}$ of it in 2 days, in what time will a boy be able to do the whole work ?

10 A farmer parting with his stock sells to one person 9 horses and 7 cows for Rs 300, and to another at the same prices, 6 horses and 13 cows for the same sum. What is the price of each ?

11 In a certain employment, 9 men and 7 women received together Rs 4 7a 2p for their wages, and it is found that 7 men receive Re 1 3a 8p more than 5 women, required the wages of each

12 If 17 ducks and 20 chickens are worth Rs 29 14a and at the same average prices 15 chickens and 31 ducks are worth Rs 38 6a, how many ducks are worth 14 chickens ?

453 Least Common Multiple

Ex 1 Find the *least* number which, being divided by 2, 3, 4, 5, 6, 7, shall give the remainders 1, 2, 3, 4, 5, 6, respectively

Since $2-1=1$, $3-2=1$, $4-3=1$, and so on,

the remainder in each case is less than the divisor by 1

Now, the L C M of 2, 3, 4, 5, 6 and 7 is 420

the reqd least number $= 420 - 1 = 419$ Ans

Ex 2 Find the *least* number which, being divided by 2, 4, 6, 8, 10 and 12, leaves in each case a remainder 1, but when divided by 13, leaves no remainder

The L C M of 2, 4, 6, 8, 10 and 12 is 120

the reqd number $= 120k + 1$, where k is a positive integer

Making $k=1, 2, 3, 4, 5$, &c, the reqd number is one of the numbers 121, 241, 361, 481, 601, &c

Again, since the number is divisible by 13, therefore of the above numbers that which is divisible by 13 is the reqd number

Now, the first of the above numbers which is divisible by 13 (by trial) is 481

Hence the reqd least number $= 481$ Ans

Examples CXXXVIII

1 Find the *least* number which, when separately divided by 6, 9, 12, 15 and 21, leaves in each case a remainder which is the G C M of the above divisors

2 Find the *least* number which, being divided by 2, 3, 4, 5, 6, 7, and 8, shall give the remainders 1, 2, 3, 4, 5, 6 and 7 respectively

3 Find the *least* number which, when divided by 6, 9, 12, 15 and 18, leaves in each case a remainder 3, but when divided by 21 leaves no remainder

4 Find the *least* number which, when divided by 8, 10, 12, 14 and 16, leaves the remainders 6, 8, 10, 12 and 14 respectively, but when divided by 17 leaves no remainder

5 Find the *least* number that is divisible by 22, but being divided by 6, 9, 12, 15 and 18 gives 4, 7, 10, 13 and 16 as remainders respectively

6 Find the *least* number which, being divided by 2, 3, 4, 5, 6 and 7, gives the remainders 1, 2, 3, 4, 5 and 0 respectively

7 What *greatest* number and what *least* number can be subtracted from 902510 that the remainders may be divisible by 28, 32, 40 and 45?

8 What *least* number must be added to 17346 that the sum may be divisible by 48, 64, 72, 96 and 108?

9 What *least* number must be added to 17287 that the sum being divided by 26, 35, 38, 91 and 95 respectively shall leave in each case a remainder 2?

10 What *greatest* number and what *least* number must be subtracted from 1100 that the remainders being severally divided by 4, 12, 20, 24, 36 and 45 shall give a remainder 3 in each case?

11 What *greatest* number of 7 digits is that, which, being divided by 7, 9, 11, 13 and 15, leaves the remainders 5, 7, 9, 11 and 13 respectively?

12 What *least* number of 8 digits is that, which, being divided by 24, 30, 36, 42 and 50, leaves the remainders 14, 20, 26, 32 and 40 respectively?

454 **Races and Games of Skill** If *A* and *B* start from the same place at the same time to run a distance of 100 yds, and if while *A* arrives at the goal, *B* has run only 80 yds, then *A* gives *B* (100-80) or 20 yds in 100, and *B* gets or takes 20 yds in 100. Also, if *A* and *B* start at the same time, but *B* 20 yds in advance of *A* and arrive at the goal at the same instant, then *A* is

said to give B 20 yds start, and that they run a dead heat. So in a game of skill, if B can make only 90 points while A makes 100, A gives B $(100-90)$ or 10 points out of 100, and B gets 10 out of 100.

Ex 1 In a race of 500 yds, A can beat B by 50 yds, and B can beat C by 50 yds, by how much will A beat C in a race of 200 yds?

A can run 500 yds while B runs $(500-50)$ or 450 yds

A . $\frac{500}{450}$ yds 1 yd

$\therefore A$ $\frac{500}{450} \times 500$ or $\frac{50000}{9}$ yds, while B runs 500 yds

But when B runs 500 yds, C can run $(500-50)$ or 450 yds

$\therefore A$ can run $\frac{50000}{9}$ yds, while C runs 450 yds

A 5000 yds 450×9 yds

A 100 yds $\frac{450 \times 9}{50}$ yds

$\therefore A$ 200 yds $\frac{450 \times 9 \times 2}{50}$ or 162 yds

Hence A can beat C by $(200-162)$ or 38 yds in 200 yds Ans

Ex 2 At a game of billiards, A can give B 15 points in 50 and he can give C 20 in 50, how many can B give C in a game of 70?

While A makes 50 points, B makes $(50-15)$ or 35,

and C makes $50-20$ or 30,

while B makes 35 points, C makes 30,

while B makes 70 points, C makes 30×2 or 60

Hence B can give C $(70-60)$ or 10 points in a game of 70 Ans

Examples CXXXIX

1 A can beat B by 5 yds in a 100 yds race, and B can beat C by 10 yds in a 200 yds race. By how much can A beat C in a 400 yds race?

2 In a race of 200 yds, P can beat Q by 31 yds and R by 18 yds, by how many yds could R beat Q in 350 yds?

3 In a mile race, A can beat B by 20 yds, and B can beat C by 20 yds. How many yds start can A give C that there may be a dead heat?

4 In a game at rackets A can give C 10 points out of 15, and B can give C 8. How many points can A give B ?

5 In a mile race, A beats B by 60 yds, and B beats C by 80 yds. By how much will A beat C in a race of 400 yds?

6 In a mile race A wins, B being 11 yds behind, and C 64 yds behind A . How much would C be behind B in a 2 miles race?

7 In a game of skill, A can give B 8 points out of 40 and B can give C 10 points out of 50, how many can A give C out of a game of 25?

8 In a mile race A gives B 100 yds start, and beats him by 20 yds. If B can run the mile in 5 min 8 sec, how long did A take?

9 In a 440 yds race, A beats B by 20 yds, and C by 41 yds, B can also give C a start of 12 sec in a mile race. In what time can each run a mile?

10 In a race P gained 25 yds upon Q in every 125 yds, and finally won by 70 yds, find the length of the course.

11 At billiards A can give B 15 points and to C 20 points out of 50, how many can B give C in 70 for an even match?

12 A runs 20 yds while B runs 21 yds, B runs 31 yds while C runs 30 yds, if A can run a mile in 5 min 15 sec, what time will C take to do it?

13 In a half-mile race A gives B 10 yds start and beats him by 20 yds, B gives C 30 yds start in half a mile, and is beaten by 60 yds. Which runs the faster, A or C ?

14 At a game at fives, out of 15 points A can give B 3, also A can give C 7 points, how many points can B give C so as to make an even match?

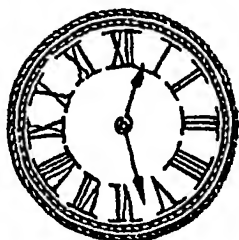
15 A , B , C and D run a race over 1 mile. First A and B race, when A wins by 20 yds, then C and D race, when C wins by 60 yds, next B and D race, when B wins by 40 yds. If A and C race, which will win and by how much?

16 In a mile race A beats B by 80 yds and C by 20 sec, also B beats C by 5 seconds in the same race. How long will A take to run the whole distance?

17 At a game of skill A can give B 8 points out of 38, and to C 10 points out of 95. Of B and C , which is the better player, and how many points can he give the other in 340?

18 A can give B 400 yds and C 500 yds in a mile race, if C can run the mile in 10 minutes, in what times can A and B run the same?

455 Hands of a Clock On the right is the diagram of a clock with its two hands that move about a common centre. The longer of these hands is called the *minute hand* and indicates the minutes, and the shorter the *hour hand* and indicates the hours. The minute-hand takes one hour or 60 minutes to travel round the dial plate once, while the hour-hand moves only 5 minutes round. Therefore the minute-hand travels 12 minutes for every minute of the hour-hand or in other words the minute-hand *gains* (12-1) or 11 minutes for every 12 minutes it advances, or 1 minute for every $1\frac{1}{11}$ minutes it advances. Hence, to find the time in which the minute-hand is to gain a certain number of minutes over the hour-hand, multiply the given number of minutes by $1\frac{1}{11}$.



It should also be noticed here that if the minute-hand is 15 min either *before* or *behind* the hour-hand, the hands are at *right angles*, and if the minute hand is 30 min either *before* or *behind* the hour-hand, the hands are *opposite to each other*.

Ex 1 At what times between 7 and 8 o'clock will the hands of a clock be (i) together, (ii) at right angles, and (iii) opposite to each other?

(i) At 7 o'clock the minute-hand points to 12 and the hour-hand to 7, so that the minute hand is 5×7 or 35 min behind the hour-hand. Now, in order that the hands may be together, the minute hand shall have to gain these 35 min over the hour hand. But the minute hand takes $1\frac{1}{11}$ min to gain 1 min, therefore the required time = $(\frac{1}{11} \times 35)$ min or $3\frac{2}{11}$ min past 7.

(ii) At 7 o'clock the minute hand is 35 min behind the hour-hand. Now to be at right angles, the minute hand shall be 15 min either *behind* or *before* the hour-hand, therefore the minute hand shall have to gain either $(35-15)$ min or $(35+15)$ min, i.e. either 20 min or 50 min. Hence the required time is either $(\frac{1}{11} \times 20)$ min or $(\frac{1}{11} \times 50)$ min past 7, i.e. either $2\frac{2}{11}$ min or $5\frac{4}{11}$ min past 7.

(iii) At 7 o'clock the minute hand is 35 min behind the hour-hand. Now to be opposite to each other, the minute hand shall be 30 min either *behind* or *before* the hour hand, and this can happen only once *behind*, when the minute-hand has gained $(35-30)$ min or 5 min over the hour-hand. Therefore the required time is $(\frac{1}{11} \times 5)$ min or $5\frac{5}{11}$ min past 7.

Ex 2 When will the hands of a clock be (i) at right angles, (ii) 10 min apart, between 10 and 11 o'clock?

(i) At 10 o'clock the minute-hand is 10 min before the other so that the two hands will be at right angles when the minute-hand

has gained either $(15 - 10)$ or 5 min more, or $(45 - 10)$ or 35 min more. Hence the required times are either $(\frac{1}{2} \times 5)$ min or $(\frac{1}{2} \times 35)$ min, i.e., either $5\frac{1}{2}$ min or $38\frac{1}{2}$ min past 10.

(ii) At 10 o'clock the minute-hand is 10 min before the other. Again, the hands will be 10 min apart, when the minute-hand has gained $(10 + 30)$ or 40 min over the hour-hand. Hence the required time is $(\frac{1}{2} \times 40)$ min or $43\frac{1}{2}$ min past 10.

Ex 3 I left home between 12 and 1 o'clock P.M., and on my return between 5 and 6 o'clock P.M., the hands have exactly changed places. Find the time of my departure.

When the *H* hand and the *M* hand change places, the *H* hand moves through a space equal to the original interval between *H* and *M* and the *M* hand moves through a space equal to a certain number of rounds of the dial, minus the original interval between *H* and *M*.

Now the *M* hand moves 12 times as fast as the *H* hand and in this case the number of rounds of the dial is evidently 5. ∴ the *M* hand moves through 5 rounds of the dial minus the original interval between *H* and *M* and this space must be equal to 12 times the original interval between *H* and *M*. Hence 5 rounds of the dial = 13 times the original interval between *H* and *M*, ∴ the original interval between *H* and *M* = $\frac{1}{13} \times 60$ min spaces = $23\frac{1}{13}$ min spaces (See Fig Art 455).

Again, at 12 o'clock the *H* and *M* hands were coincident, ∴ the *M* hand must have to gain $23\frac{1}{13}$ min spaces over the *H* hand, and it gains 11 min spaces in every 12 min spaces, ∴ the time in which it gains $23\frac{1}{13}$ min spaces = $\frac{1}{11} \times \frac{13}{13} \times 60$ min = $25\frac{2}{11}$ min. Hence the time of my departure is 12 $25\frac{2}{11}$ min P.M.

Examples CXL

1 At what times between the hours stated below are the hour and minute hands of a clock (i) together? (ii) at right angles? and (iii) directly opposite to each other?

- | | | | |
|--------------|----------------|----------------|---------------|
| (1) 1 and 2 | (2) 2 and 3 | (3) 3 and 4 | (4) 4 and 5 |
| (5) 5 and 6 | (6) 6 and 7 | (7) 7 and 8 | (8) 8 and 9 |
| (9) 9 and 10 | (10) 10 and 11 | (11) 11 and 12 | (12) 12 and 1 |

2 When will the hour and minute-hands of a clock be (i) 10 min apart? (ii) 25 min apart? between the hours of —

- | | | | |
|-------------|-------------|-------------|-------------|
| (1) 3 and 4 | (2) 4 and 5 | (3) 6 and 7 | (4) 7 and 8 |
|-------------|-------------|-------------|-------------|

3 At 3 o'clock the hour and minute-hands of a clock are at right angles. How often will they be at right angles to each other before striking 5?

4 At 12 o'clock the two hands of a clock are coincident. How often will they be coincident before striking 4?

5 At 6 o'clock the two hands are in the same straight line. How often will they be in the same straight line before striking 9?

6 In the astronomical clock, the hours are marked upon the dial from 1 up to 24, find the time between 8 and 9 o'clock when the hands are together.

7 I left home between 4 and 5, and on my return between 7 and 8, the hands have exactly changed places. Calculate the hour of departure.

8 It is between 6 and 7, and the number of minute spaces between the two hands of a clock is $\frac{1}{2}$ of what it was $8\frac{1}{4}$ min ago. What is the time?

9 I left home between 3 and 4, and on my return between 8 and 9, I found that the hands of the watch had exactly changed places. When did I return?

10 If the hands of a clock coincide every 65 minutes, how much does the clock gain or lose in a day?

11 A clock in which the hour hand has been displaced shows the time to be 16 min past 3, and the two hands are together, the time is between 3 and 4 o'clock. Find by how many minute divisions the hour hand has been displaced.

12 A clock is 10 min too fast at noon, it loses 2 min in an hour, find the true time when the hands are (i) at right angles, (ii) directly opposite, and (iii) coincident, between 4 and 5 o'clock?

13 A clock is 10 min too slow at noon, and gains $2\frac{1}{2}$ min in an hour, what will be the true time when the hands are (i) coincident, (ii) directly opposite and (iii) at right angles, for the fourth time afternoon?

14 A clock which loses 10 sec per hour is set to the correct time at 9 15 A.M. on Monday. What will be the correct time by the clock when its hour and minute hands point in exactly opposite directions between 9 and 10 o'clock P.M. in the evening? How must the hands be altered, so that the clock may show correct time at noon on Tuesday?

456 Clocks When a clock indicates 10 minutes *before* the true time, it is said to be 10 min *too fast*, and when it indicates 10 min *behind* the true time, it is said to be 10 min *too slow*.

Thus, if a clock indicate 9 10 when the correct time is 9, it is said to be 10 min *too fast*, but if, on the other hand, it indicates 8 50 at the same hour, it is called 10 min *too slow*.

Ex 1 A clock was 10 min too slow at noon on Monday, on Friday at the same hour it is 10 min too fast. When will it again shew correct time?

From Monday noon to Friday noon, there are 4 days, and in these 4 days the clock has gained $(10+10)$ or 20 min. Therefore in one day the clock gains $(20-4)$ or 5 min. Now at noon on Friday the clock indicates 12 hrs 10 min, therefore it shall have to gain $(12 \text{ hrs} - 10 \text{ min})$ or 710 min in order to shew correct time. But it gains 5 min per day. \therefore it will gain these 710 min in $(710 \div 5)$ or 142 days.

Ex 2 Two clocks are set right at noon on Monday, one gains 4 min and the other loses 2 min in 24 hrs. (i) When will there be a difference of one hour between the times indicated by them? (ii) What time will the first indicate when the second indicates noon, 4 days after? (iii) What time will the second indicate, when the first indicates 6 P.M. on the following Friday? (iv) What will be the true time when the first indicates 3 P.M. on the following Wednesday? (v) What will be the true time when the second indicates 8 A.M. on the following Saturday?

(i) Since the first gains 4 min and the second loses 2 min in 24 hours, \therefore they will differ by $(4+2)$ or 6 min in 24 hrs or 1 day. Therefore they will differ by 1 hour or 60 min in 10 days. Hence the required time is Thursday noon, 10 days later.

(ii) When the second indicates 23 hr 58 min or $2\frac{10}{15}$ hrs, the first indicates 24 hrs 4 min or 6' more.

\therefore when the second indicates 1 hr, the first indicates $6 \times \frac{10}{15}$ min more.

\therefore (4×24) hrs $2\frac{10}{15} \times 4 \times 24$ min, or $24\frac{24}{15}$ min.

Hence the first will indicate 12 hrs $24\frac{24}{15}$ min P.M., when the second indicates noon, 4 days after.

(iii) From Monday noon to Friday 6 P.M. there are 102 hours. When the first indicates $24\frac{1}{2}$ hrs, the second indicates $23\frac{20}{60}$ or 6' less;

\therefore 1 hr $6 \times \frac{10}{60}$ min "

\therefore 102 hrs $2\frac{10}{60} \times 102$ min " or $25\frac{15}{60}$ min "

Hence the second will indicate $25\frac{15}{60}$ min less, or 5 hrs $34\frac{20}{60}$ min P.M. when the first indicates 6 P.M. on the following Friday.

(iv) From Monday noon to Wednesday 3 P.M. there are 51 hours. When the first indicates $24\frac{1}{2}$ hrs, the true time is 4' less,

\therefore 1 hr $4 \times \frac{10}{60}$ min less,

\therefore 51 hrs $2\frac{10}{60} \times 51$ min " or $8\frac{15}{60}$ min "

Hence the true time is $8\frac{15}{60}$ min behind or 2 hrs $51\frac{15}{60}$ min P.M. when the first indicates 3 P.M. on the following Wednesday.

(v) From Monday noon to Saturday 8 A M there are 116 hours
 When the second indicates $23\frac{3}{8}$ hrs, the true time is 2 min more
 \therefore $\qquad\qquad\qquad$ 1 hr $\qquad\qquad\qquad$ \therefore $2 \times \frac{3}{8}$ min more
 \therefore $\qquad\qquad\qquad$ 116 hrs $\qquad\qquad\qquad$ $\frac{3}{8} \times 116$ min
 $\qquad\qquad\qquad$ $\qquad\qquad\qquad$ or $9\frac{3}{4}$ min

Hence the true time is $9\frac{3}{4}$ min before, or 8 hrs $9\frac{3}{4}$ min A M when the second indicates 8 $\frac{1}{4}$ on the following Saturday

Ex 3 Two clocks commence to strike 10 together The 5th stroke of the second is coincident with the third stroke of the first If the first strike all its strokes in 18 seconds, find the interval between the 8th strokes of the two clocks

Since the clocks begin striking together, and the 5th stroke of the second is coincident with the 3rd stroke of the first, it is clear that the time taken by the second for 4 strokes is the same as that taken by the first for 2 strokes, or time for 2 strokes of the second = time for 1 stroke of the first Again, since the first strikes all its strokes, i.e., 9 strokes after it begins striking, in 18 seconds, we see that each of the strokes of the first comes after an interval of 2 seconds Therefore each stroke of the second clock comes after an interval of 1 second The 8th stroke of the first would thus come 14 seconds, and the 8th stroke of the second would come 7 seconds after the striking begins Therefore the interval between the 8th strokes would be (14 - 7) or 7 seconds *Ans*

Examples CXLI

1 A clock was 10 min slow 25 days ago, and to day at the same hour is 10 min fast, when will it again shew true time?

2 A watch which is 10 minutes too fast at 12 o'clock on Monday, gains 3 min 10 sec per day, what will be the time by the watch at a quarter-past 10 A M on the following Saturday?

3 Of two clocks, one gains 10 min and the other loses $7\frac{1}{2}$ min in 24 hrs, what will be the difference of the times indicated by them at 6 o'clock A M on Friday, if they are together at noon on the preceding Tuesday?

4 Two clocks point out 12 at the same instant; one of them gains 7 sec and the other loses 8 sec in 12 hours, after what interval will one have gained half an hour on the other and what o'clock will each then shew?

5 A church clock is set at 12 o'clock on Saturday night, at noon on Tuesday it is 3 min too fast, supposing the rate regular, find the true time when the clock strikes four on Thursday afternoon

6 A watch set accurately at 12 o'clock indicates 10 min to 5 at 5 o'clock P M What is the time when the watch indicates 5 o'clock?

7 If one watch loses and another gains at the rate of 1 min a day, and they are both set right at noon on Monday, what time will be indicated by the latter, when the former points to 10 hrs 49 $\frac{1}{2}$ min 1 M on the following Saturday?

8 A clock, which is 4 min 8 $\frac{3}{4}$ sec too fast at half past 9 A M on Tuesday loses 2 min 45 sec daily, what will be the time indicated by the clock at a quarter past 5 P M on the following Friday?

9 Two clocks, one gaining 3 min and the other losing 2 min. per day, are set right at noon. What is the time by the first clock when the second indicates noon a week afterwards?

10 A clock which loses 4 min in 12 hours is 10 min too fast at midnight on Sunday. What o'clock will it indicate at 6 o'clock on Wednesday evening?

11 A watch, which was 5 min 40 sec fast on Monday at noon, is 2 min 51 sec fast at midnight on the following Sunday, what did it lose in a day?

12 A clock which gains 7 $\frac{1}{2}$ min in 24 hours is 12 min fast at midnight on Sunday. What o'clock will it indicate at 4 o'clock on Wednesday afternoon?

13 A clock gains 3 $\frac{1}{2}$ min a day, how must the hands be placed at noon so as to point to the true time at 7 hrs 30 min P M?

14 One clock gains 1 min in 12 hours, and another loses 4 min in 24 hours. They are set right at noon on Monday. Determine the time indicated by each clock, when the one appears to have gained 16 $\frac{1}{2}$ min on the other.

15 Two clocks begin to strike twelve together, one strikes in 35 seconds, the other in 25, what fraction of a minute is there between their seventh strokes?

16 Two clocks strike 9 together on Tuesday morning. On Wednesday morning one wants 10 min to 11 when the other strikes 11. How much must the faster be put back, that they may strike 9 together on Wednesday evening?

17 One clock strikes four times while another strikes three. It is observed that they both begin striking at a certain hour together, and that the last stroke of one is simultaneous with the last stroke but two of the other. What o'clock is it?

18 A clock, which was 12 min fast at a quarter to 11 P M on Nov 28, was exactly right at 11-30 P M on the following day. How many minutes was it slow at a quarter to 2 P M on Dec 7?

19 A clock, which was 14 min fast at a quarter to 11 P M on Dec 2, was 8 min slow at 9 A M. on Dec 7. When was it exactly right?

20 A watch gains 3 min every day How should it be set at 9 P M on Friday, so that it should denote right time at 12 noon the next day ?

21 Two clocks, of which one gains 3 min and the other 2 min a day, are set right at 10 A M What time will the first clock denote when the second denotes 40 min past 8 P M ?

22 On Monday it was 8 A M by a watch, and 55 min past 7 A M by a clock, 3 days afterwards, it was 8 A M by the watch and 58 min past 7 A M by the clock If the clock gains 30 seconds per day, how much does the watch gain or lose daily ?

23 Two clocks begin to strike 12 together The second stroke of one is coincident with the third stroke of the other If the first strike all the strokes in 18 seconds, find the time between their 11th strokes

24 Two clocks commence striking a certain hour at the same instant The third stroke of one is simultaneous with the fourth stroke of the other, and the first strikes thrice after the second has stopped What is the hour ?

25 At 10 minutes to 2 in the afternoon a clock is 55 seconds slow, and at 6 in the evening it is 30 seconds slow, at what hour will it shew true time ?

26 A clock loses 5 min a day It shews correct time at noon on a Monday, after how many days will it again shew correct time on a Monday ?

27 A clock which was 16 min too slow 24 days ago, is 16 min too fast to day at the same hour When did it last shew correct time, and when will it shew correct time again ?

28 I set my watch at noon on Saturday, at 10 30 A M on Monday it had gained $3\frac{1}{2}$ min, what will be the real time on Tuesday when my watch is at 3 30 P M ?

29 Two clocks are together at noon on March 1st, if one gains uniformly 3 min a day, and the other 36 sec a day, when will the difference between them be $3\frac{1}{2}$ hours ?

30 One clock gains 3 min in 24 hours and a second gains 3 min in 15 hours, the first is put right on January 1st, the second on January 2nd, when will they indicate the same time ?

457 **Temperature** *Temperature* in bodies is measured by an instrument called a **Thermometer**, which consists of an air-tight glass tube having a reservoir of mercury or alcohol at one end of it. The tube is *graduated* or marked off into small equal lengths, and the divisions are numbered in regular order It has been ascertained by experiment that the atmospheric pressure remaining the same,

water always boils and freezes at the *same* temperatures. The two marks on the stem of the thermometer which correspond to these are called the *boiling* and *freezing* points respectively.

458 The following **Thermometers** are in common use —

- (1) **Fahrenheit's Thermometer** is commonly used in England. In it the *freezing point* is denoted by 32° and the *boiling point* by 212° . The space between these two points is divided into 180 equal parts, each called a *degree* ($1^{\circ} F$).
- (2) The **Centigrade Thermometer** is in use on the Continent. The *freezing point* is denoted by 0° , and the *boiling point* by 100° , the intervening space being divided into 100 equal parts, each called a *degree* ($1^{\circ} C$).
- (3) **Reaumur's Thermometer** is used in Russia. The *freezing point* is marked $0^{\circ} R$, and the *boiling point* $80^{\circ} R$.

459 In converting a temperature expressed in the Fahrenheit scale into the Centigrade or the Reaumur scale, and *vice versa*, we have the following facts to remember —

$$(i) 32^{\circ} F = 0^{\circ} C = 0^{\circ} R, \quad (ii) 180^{\circ} F = 100^{\circ} C = 80^{\circ} R$$

$$1^{\circ} F = \frac{5}{9}^{\circ} C = \frac{4}{9}^{\circ} R, \quad 1^{\circ} C = \frac{9}{5}^{\circ} F = \frac{8}{5}^{\circ} R, \quad 1^{\circ} R = \frac{5}{4}^{\circ} F = \frac{9}{8}^{\circ} C$$

$$\text{Ex 1} \quad 85^{\circ} F = 32^{\circ} F + 53^{\circ} F = 0^{\circ} C + \frac{5}{9} \times 53^{\circ} C = \frac{265}{9}^{\circ} C = 29\frac{4}{9}^{\circ} C$$

$$= 0^{\circ} R + \frac{4}{9} \times 53^{\circ} R = \frac{212}{9}^{\circ} R = 23\frac{5}{9}^{\circ} R$$

$$\text{Ex 2} \quad -36^{\circ} C = 0^{\circ} C - 36^{\circ} C = 32^{\circ} F - \frac{9}{5} \times 36^{\circ} F$$

$$= 32^{\circ} F - 64\frac{2}{5}^{\circ} F = -12\frac{4}{5}^{\circ} F = -32\frac{4}{5}^{\circ} F$$

$$= -\frac{4}{9} \times 36^{\circ} R = -16^{\circ} R = -28\frac{4}{5}^{\circ} R$$

The following relation which connects the three scales will be very useful in these reductions —

$$\frac{F - 32}{180} = \frac{C}{100} = \frac{R}{80}, \quad \text{or} \quad \frac{F - 32}{9} = \frac{C}{5} = \frac{R}{4}$$

Examples CXLII

1 What degrees (i) C , (ii) R , correspond to—

- (1) $40^{\circ} F$ (2) $68^{\circ} F$ (3) $118^{\circ} F$ (4) $160^{\circ} F$ (5) $-28^{\circ} F$?

2 What temperatures (i) F , (ii) R , are represented by—

- (1) $20^{\circ} C$ (2) $45^{\circ} C$ (3) $58^{\circ} C$ (4) $92^{\circ} C$ (5) $-10^{\circ} C$?

3 What temperatures (i) F , (ii) C , are the same as—

- (1) $35^{\circ} R$ (2) $60^{\circ} R$ (3) $76^{\circ} R$ (4) $-10^{\circ} R$ (5) $-25^{\circ} R$?

460 **Time and Distance** If a man walks at the rate of 4 miles an hour, he will walk in 5 hours 4×5 or 20 miles; *conversely*,

if a man walks at the rate of $3\frac{1}{2}$ miles an hour, he takes $(15 \div 3\frac{1}{2})$ or $4\frac{2}{7}$ hours to walk 15 miles, and if he takes $4\frac{1}{2}$ hours to walk 12 miles, he walks at the rate of $(12 \div 4\frac{1}{2})$ or $2\frac{2}{3}$ miles per hour

461 The following remarks will be found useful in solving problems of this class

(i) When two bodies are moving in a straight line in *opposite* directions, the speed of their approach or their *relative* speed is equal to the *sum* of their *absolute* speeds

(ii) When two bodies are moving in a straight line in the *same* direction, the speed of their approach or their *relative* speed is equal to the *difference* of their *absolute* speeds

(iii) When two bodies are approaching each other from *opposite* directions, they *meet* when they have travelled the whole distance between them

(iv) When a boat goes down-stream, the rower is *helped* by the current, but the current *opposes* his progress when the journey is up-stream. Hence in the first case the total work done on the boat is the *sum* of the works done by the rower and current separately, in the second case it is their *difference*

(v) A railway train in motion passes a particular point, when the train has gained its own length over the point, and of two trains running on parallel rails either in opposite directions or in the same direction, the one passes the other only when it has gained over the other a distance equal to the *sum* of the lengths of the trains

Ex 1 If two persons *A* and *B* start at the same time from two towns *C* and *D* distant 300 miles from each other, when and where will they meet, if they travel respectively at the rate of $6\frac{1}{2}$ and $8\frac{1}{4}$ miles an hour?

Here, the relative speed of *A* and *B* is $(6\frac{1}{2} + 8\frac{1}{4})$ or 15 miles per hour, and since they meet when they have together travelled 300 miles,

therefore the required time is $(300 \div 15)$ or 20 hours
 Also the distance travelled by *A* = $(6\frac{1}{2} \times 20)$ or 135 miles,
 and *B* = $(8\frac{1}{4} \times 20)$ or 165 miles } *Ans*

Ex 2 *A* travelling at the rate of 12 miles an hour starts 15 miles behind *B* who travels only 10 miles an hour, find when *A* will overtake *B* and the distance travelled by each

Here the relative speed of *A* and *B* is $(12 - 10)$ or 2 miles per hour, and since *A* shall have to gain 15 miles on *B*, in order to overtake him,

therefore the required time = $(15 \div 2)$ or $7\frac{1}{2}$ hours
 Also the distance *A* has travelled = $(12 \times 7\frac{1}{2})$ or 90 miles,
 and *B* .. $(10 \times 7\frac{1}{2})$ or 75 miles } *Ans*

Ex 3 A starts from P to walk to Q a distance of 84 miles at 3 miles an hour, two hours after, B starts from Q for P at $3\frac{1}{2}$ miles an hour. When will A and B meet?

When B starts, A has already travelled (3×2) or 6 miles, therefore the distance between them is then only $(84 - 6)$ or 78 miles.

Now, the relative speed of A and B $(3 + 3\frac{1}{2})$ or $6\frac{1}{2}$ miles per hour, therefore to travel 78 miles, they will take $(78 \div 6\frac{1}{2})$ or 12 hours.

Hence, A and B will meet $(12 + 2)$ or 14 hours after A started. *Ans*

Ex 4 A starts from P for Q at the rate of 4 miles an hour. Three hours later B also starts from P for Q and reaches the destination at the same time as A . If B 's rate exceeds A 's by $\frac{1}{2}$ mile an hour, find the distance between P and Q .

In 3 hours, A goes (4×3) or 12 miles ahead of B . Now, since B goes $\frac{1}{2}$ mile faster per hour than A , therefore he overtakes A at the end of $(12 \div \frac{1}{2})$ or 24 hours. But B 's rate of walking is $4\frac{1}{2}$ miles an hour, therefore in 24 hours he goes $(24 \times 4\frac{1}{2})$ or 108 miles.

Hence the distance between P and Q is 108 miles. *Ans*

Ex 5 A train leaves Calcutta at 6:45 A.M. and travels 30 miles an hour, another train leaves Calcutta at 3 P.M. and travels 40 miles an hour, when and where will the second train overtake the first?

From 6:45 A.M. to 3 P.M. there are 8 hrs 15 min or $8\frac{1}{4}$ hours.

The first train starts 8 hrs 15 min before the other, and is therefore $(30 \times 8\frac{1}{4})$ or $247\frac{1}{2}$ miles ahead of it. Now, the second train shall have to gain these $247\frac{1}{2}$ miles to overtake the first. But the relative speed of the two trains is $(40 - 30)$ or 10 miles per hour, therefore the required time is $(247\frac{1}{2} \div 10)$ or $24\frac{3}{4}$ hours.

Hence the second overtakes the first at 3:45 P.M. next day }
and the distance travelled is $(24\frac{3}{4} \times 40)$ or 990 miles from Calcutta. } *Ans*

Ex 6 Two trains starting from the same station and travelling in opposite directions, are 315 miles apart in 5 hrs 15 min, had they been travelling in the same direction, they would have been 74 miles apart in 7 hrs 24 min. Find the speed of each train.

Since 5 hrs 15 min = $5\frac{1}{4}$ hours, and 7 hrs 24 min = $7\frac{2}{3}$ hrs
the sum of their rates per hour = $(315 \div 5\frac{1}{4})$ or 60 miles,
and the difference = $(74 \div 7\frac{2}{3})$ or 10 miles.

Hence, the rate of the faster = $\frac{1}{2}(60 + 10)$ or 35 miles per hour, }
and slower = $\frac{1}{2}(60 - 10)$ or 25 miles per hour. } *Ans*

Ex 7 A starts from Calcutta for Mankar, a distance of 91 miles, at 6 A.M., walking $3\frac{1}{2}$ miles an hour, B starts from Mankar 12 hrs later and reaches Calcutta at the same time as A . What was B 's speed per hour?

Here B takes 12 hours less than A to travel 91 miles

Now, A takes $(91 - 3\frac{1}{2})$ or 26 hrs to travel ,

$\therefore B$ takes $(26 - 12)$ or 14 hrs

$\therefore B$'s speed per hour is $(91 - 14)$ or $6\frac{1}{2}$ miles *Ans*

Ex 8 The distance from A to B is 16 miles, 4 miles of which is up-hill and 8 down-hill , find the difference between the times in which a person would walk from A to B and back again respectively, supposing his pace up-hill to be 4 miles, down-hill 8 miles and on level 6 miles per hour

From A to B is 4 miles up-hill, 4 mi level and 8 mi down-hill ,

\therefore the whole time taken $= (\frac{1}{4} + \frac{1}{6} + \frac{1}{8})$ hrs $= 2\frac{3}{4}$ hrs $= 2$ hrs 45 min

From B to A is 8 miles up-hill, 4 mi level and 4 mi down-hill ,

\therefore the whole time taken $= (\frac{1}{2} + \frac{1}{6} + \frac{1}{8})$ hrs $= 3\frac{1}{4}$ hrs $= 3$ hrs 15 min

Hence the diff reqd $= 3$ hrs 15 min $- 2$ hrs 45 min $= 30$ min *Ans*

Ex 9 If a man rows 40 miles in 10 hrs against a stream, the rate of which is 3 miles an hour, how long will he be in rowing 40 miles with the stream ?

Since he rows in 10 hrs a distance of 40 miles against the stream,

\therefore 1 hr $\frac{40}{10}$ or 4 miles

And since the rate of the stream is 3 miles per hour, \therefore he can row $(4+3)$ or 7 miles per hour Therefore, with the aid of the stream, he can row $(7+3)$ or 10 miles per hour Hence to row 40 miles with the stream, he will take $\frac{40}{10}$ or 4 hours *Ans*

Ex 10 A hare is 210 of her own leaps before a greyhound , she takes 7 leaps for every 6 that the greyhound takes, but 3 of the greyhound's leaps are equal to 4 of the hare's , how many leaps will the greyhound take before she is caught ?

3 leaps of the greyhound $= 4$ leaps of the hare ,

\therefore 1 leap $= \frac{4}{3}$

\therefore 6 leaps $= \frac{4}{3} \times 6$ or 8

But when the greyhound takes 6 leaps, the hare takes 7 leaps , \therefore the greyhound gains $(8-7)$ or 1 leap of the hare on 6 of his own leaps Hence, he will gain 210 of the hare's leaps on (210×6) or 1260 leaps of his own *Ans*

Ex 11 I have to be at a certain place in a certain time, and I find that, if I walk at the rate of 4 miles per hour, I shall be 5 min too late , if at the rate of 5 miles per hour, I shall be 10 min too soon How far have I to go ?

Since 4 miles per hour $= 1$ mi in 15 min , 5 miles per hour $= 1$ mi in 12 min Therefore, in each mile $(15-12)$ or 3 min is

gained, i.e., 1 min in each $\frac{1}{2}$ mile by increasing the speed from 4 to 5 miles per hour

Now the whole time gained in the reqd distance is 15 min

∴ the required distance = $(15 \times \frac{1}{2})$ or 5 miles *Ans*

Ex 12 If a snail, on the average, creep 1 ft $3\frac{1}{2}$ in up a pole during 12 hours in the night, and slip down 8 in during the 12 hours in the day, how many hours will he be in getting to the top of a pole 70 ft high?

Here, 1 ft $3\frac{1}{2}$ in = $15\frac{1}{2}$ in, and 70 ft = 840 in

In 24 hours, the snail gains $(15\frac{1}{2} - 8)$ in or $7\frac{1}{2}$ in

Now, by alternate creeping and slipping when the snail shall arrive for the first time at such a position of the post whose distance from the top is either $15\frac{1}{2}$ in, or less than $15\frac{1}{2}$ in, it will climb up within the next 12 hours to the top. Hence we should have first to find what *least* number, not less than $(840 - 15\frac{1}{2})$ or $824\frac{1}{2}$ is a multiple of $7\frac{1}{2}$. Now this number is 825, for $110 \times 7\frac{1}{2} = 825$, and then $(840 - 825)$ or 15 in remains

Since the snail, on the average, creeps $7\frac{1}{2}$ in in 24 hours,

∴ the snail, the first 825 in in (24×110) hrs

Again, since the snail creeps $15\frac{1}{2}$ in in 12 hours,

∴ 1 in in $(12 - 15\frac{1}{2})$ or $\frac{3}{4}$ hour

15 in in $\frac{3}{4} \times 15$ or $11\frac{1}{4}$ hours

Hence the whole time reqd = $(24 \times 110 + 11\frac{1}{4})$ or $2651\frac{1}{4}$ hours *Ans*

Ex 13 Two guns are fired from the same place after an interval of 23 min, but a person approaching the place hears the reports after an interval of 22 min 30 sec. Find his rate of progress, sound travelling at the rate of 1142 ft per second

The distance which the man travels in 22 min 30 sec or $22\frac{1}{2}$ min is the same as that travelled by the report in $(23 \text{ min} - 22 \text{ min } 30 \text{ sec})$ or 30 sec

But in 30 sec, the sound travels 1142×30 ft,

∴ the man travels in $22\frac{1}{2}$ min, a distance of 1142×30 ft

∴ in 1 min, $(1142 \times 30 - 22\frac{1}{2})$ or 4598 ft

∴ in 1 hour 4598×60 ft or $4598 \times 60 \times \frac{1}{5280}$ mi or $17\frac{1}{4}$ miles *Ans*

Ex 14 Two trains 110 yds and 88 yds long respectively, run at the rates of 30 and 15 miles an hour on parallel rails, find how long a person sitting in the first train would take to pass the other train, and how long the two trains would take to pass each other, supposing the trains were running (i) in opposite directions, (ii) in the same direction

Hence $PQ = (4000 + 19200 + 74400)$ or 97600 ft
 $= 18 \text{ mi } 853 \text{ yds } 1 \text{ ft}$ Ans

Ex 16 A, B, C and D are four Railway stations. The roads from A to B and from C to D are inclined planes of different inclinations, and that from B to C is level. A train starts from A and stopping at B and C for 5 min and 10 min respectively arrives at D . At D it stops for 15 min and then returns directly to A without stopping at B and C . The down and up speed of the train along AB are 12.5 miles and 10.5 miles per hour respectively, the down and up speed along CD are 15 miles and 9.375 miles per hour respectively, and along the level BC 11.75 miles per hour. If the whole time occupied by the train in going and returning be 4.83 hours, find the length of $AB + BC + CD$ in miles.

Along AB , to run down 1 mile, the train takes $(1 - 12.5)$ or $\frac{2}{5}$ hr
 up $(1 - 10.5)$ or $\frac{2}{5}$ hr
 down and up 2 miles $(\frac{2}{5} + \frac{2}{5})$ or $\frac{4}{5}$ hr
 Along BC , to run 1 mile, the train takes $(1 - 11.75)$ or $\frac{1}{8}$ hr
 2 miles, $2 \times \frac{1}{8}$ or $\frac{1}{4}$ hr
 Along CD , to run down 1 mile, the train takes $(1 - 15)$ or $\frac{1}{3}$ hr
 up 1 mile $(1 - 9.375)$ or $\frac{2}{3}$ hr
 down and up 2 miles $(\frac{1}{3} + \frac{2}{3})$ or 1 hr

Here, we see that the average speed in the three planes is the same and the time of going and returning is $(4.83 \text{ hrs} - 30 \text{ min})$ or $4\frac{1}{2}$ hrs

Since in $\frac{1}{4}$ hr the train runs 2 miles,

\therefore 1 hr $2 \times \frac{1}{4}$ miles

\therefore $4\frac{1}{2}$ hrs $(2 \times \frac{1}{4} \times 4\frac{1}{2})$ or 50 miles

Hence the required distance is (1×50) or 25 miles Ans

Examples CXLIII

1 A and B are 6 miles apart, and walk at the rate of $4\frac{1}{2}$ and $3\frac{1}{2}$ miles an hour respectively. How long will elapse before they meet, (i) if they walk towards each other, (ii) if they walk in the same direction?

2 A walking $5\frac{1}{2}$ miles an hour gives B walking $3\frac{1}{2}$ miles an hour an hour's start. How long will A take to catch B and how far will he have to walk?

3 Two men start to meet each other at 9 P.M. from places 31 miles apart. If one of them walks $4\frac{1}{2}$ miles an hour and the other $3\frac{1}{2}$ miles an hour, when will they meet, and how far will each have travelled?

4 A and B walk respectively $5\frac{1}{2}$ and $3\frac{1}{2}$ miles an hour. They

are 25 miles apart and walk to meet one another, but B starts 2 hours before A . How far will A have to walk?

5 A mail train leaves Calcutta for Buxar (400 miles) at 8 20 A M and travels at the rate of 45 miles an hour, another train leaves Buxar for Calcutta at 9 A M, and travels at the rate of $47\frac{1}{2}$ miles an hour. Find when they will meet, and at what distance from Calcutta.

6 A policeman sets off after a thief at 7 A M, the thief having had $2\frac{1}{2}$ hours start. If the thief goes 4 miles an hour, and the policeman $5\frac{1}{2}$ miles an hour, when will the policeman catch the thief?

7 A train starts from a terminus at 9 A M, travelling 25 miles an hour. An express starts at 10 30 A M and travels 43 miles an hour. At what time and how far from the terminus will the express overtake the slow train?

8 A starts from Calcutta to walk to Burdwan, a distance of 68 miles, at 3 miles an hour, two hours later B starts from Burdwan for Calcutta at $3\frac{1}{2}$ miles an hour. When will A and B meet?

9 A starts from a place X , for a place Y , a distance of 80 miles at 6 A M, walking $3\frac{1}{2}$ miles an hour, B starts from X 2 hours later and reaches Y at the same time as A . What was B 's speed?

10 A Hackney coach starts from Calcutta at 6 o'clock A M and runs towards Burranagur which is about $4\frac{1}{2}$ miles distant, at the rate of 9 miles an hour, 10 min. after, A starts for Burranagur and rides the same road at the rate of $13\frac{1}{2}$ miles an hour. When will the coach be overtaken by A and at what distance from Burranagur?

11 Two men A and B leave Howrah for Karmatar, a distance of 168 miles, just at the same time that another man C leaves Karmatar for Howrah. C , walking at the rate of 5 miles an hour, meets A 14 hours and B 15 hours after starting on the road. What are A 's and B 's speed per hour?

12 Two couriers pass through a place at an interval of 4 hours, travelling at the rates of $11\frac{1}{2}$ and $17\frac{1}{2}$ miles an hour, how long and how far must the first travel before he is overtaken by the second?

13 A and B walk in opposite directions from the same place, and are at the end of 5 hours, 38 miles distant. If A walk at the rate of 4 miles per hour, find the rate at which B walks.

14 The distance from P to Q is 143 miles. How many hours before A , who walks at $3\frac{1}{2}$ miles an hour, must B start to reach Q in the same time as A , B 's rate of walking being $2\frac{1}{2}$ miles an hour?

15 Two men A and B leave Mogul Serai for Jaunpur (46 miles) 15 min before C who leaves Jaunpur for Mogul Serai. After A has walked for 5 hrs 15 min he meets C , who again overtakes B 45 min later on. If A walks at the rate of 4 miles per hour, find at what rate was B and C walking per hour.

16 A sets out to walk from P to Q 5 min after B left Q for P . After walking for 45 min at 30 yds per minute, he overtakes B , and on arriving at Q stays there for a time. On his return he again overtakes B just at P after an interval of 2 hrs 30 min from the first meeting. Find the distance between P and Q , B 's rate per minute and the time A stays at Q .

17 The train which leaves Calcutta at 3-10 P.M. arrives at Ranaghat at 5 o'clock, and the train which leaves Ranaghat at 3-30 P.M. arrives in Calcutta at 5-42 P.M., when do they pass each other?

18 M starts from C and travels towards D at the rate of 6 miles per hour, two hours afterwards N starts from C and going 10 miles per hour reaches D 4 hours before M . Find the distance from C to D .

19 A train starts to go from A to B , a distance of 72 miles. Its proper rate of travelling is 20 miles an hour, but after having gone 24 miles it meets with an accident which delays it 10 min and diminishes its speed to 15 miles an hour, how much will the train be behind time?

20 Two trains starting from the same station and travelling in opposite directions, are 405 miles apart in 6 hrs 45 min, had they been travelling in the same direction, they would have been 144 miles apart in 14 hrs 24 min. Find the speed of each per hour.

21 A man walks a certain distance, and rides back in 3 hrs 45 min, he could ride both ways in 2½ hours. How long would it take him to walk both ways?

22 A hare starts 40 yds before a greyhound, and is not perceived by him till she has been up 40 seconds, she runs away at the rate of 10 miles an hour, and the dog makes after him at the rate of 18 miles an hour. How long will the course hold, and what ground will the greyhound run over?

23 The distance from A to B is 12 miles, 2 miles of which are up hill and 3 down-hill, find the difference between the times in which a person would ride from A to B and back again respectively, supposing his pace up hill to be 4 miles, down hill 5 miles, and on level ground 10 miles per hour.

24 A person sets out to walk from A to B at the rate of 4 miles an hour. After he has walked 10 miles he is overtaken by the coach which started a quarter of an hour after him. At a distance of 13 miles from A he meets the coach returning from B where it has stayed for half an hour. Find the distance from A to B .

25 Two places A and B , are distant from each other 324 miles by railway. A train leaves A for B at the same time that a train leaves B for A , the trains meet at the end of 6 hours, the train from A to B having travelled 16 miles an hour more than the other. How many miles did each travel an hour?

26 The road between two towns A and B , distant 15 miles, goes over a hill whose summit is 3 miles from A . Two pedestrians set out at the same time from A and B , the former going 1 mile an hour up-hill and $5\frac{1}{4}$ down, the latter $3\frac{1}{2}$ up hill, and $4\frac{1}{2}$ down, how far will the slower one have to walk when the first arrives at his journey's end?

27 Two trains start at the same time from Calcutta and Buxar, and proceed towards each other at the rate of 30 miles and 50 miles per hour respectively. When they meet, it is found that one train has run 100 miles further than the other. Find the distance between Calcutta and Buxar.

28 Sound travels at the rate of 1140 ft per second. If a shot be fired from a ship moving at the rate of 10 miles an hour, how far will the ship have moved before the report is heard 14 $\frac{1}{2}$ miles off?

29 A train, having to perform a journey of 250 miles, is obliged after 103 miles to reduce its speed by $\frac{1}{8}$. The result is that the train arrives at its destination 1 hr 10 min behind time, what is its ordinary rate?

30 A stream flows at the rate of $2\frac{1}{2}$ miles an hour, a man rows 18 miles against the stream in 6 hours. How long will he be in returning?

31 If a man rows 10 miles in $2\frac{1}{2}$ hours against a stream, the rate of which is 3 miles an hour, how long would he be in rowing 25 miles with the stream?

32 A man rows down a river 18 miles in 4 hours with the stream and returns in 12 hours, find the rate at which he rows and the rate at which the stream flows.

33 P can row from A to B (a distance of 24 miles) and back in still water in 12 hours, how long will it take him to do the same when there is a stream flowing from A to B at the rate of 2 miles an hour?

34 A hare is 50 leaps before a greyhound, and takes 4 leaps to the greyhound's 3 leaps, but 2 of the greyhound's = 3 of the hare's, how many leaps must the greyhound take to catch the hare?

35 A hare is 10 of her own leaps before a greyhound and takes 5 leaps for the greyhound's 4. 3 of the greyhound's leaps are equal to 4 of the hare's, how many leaps must the hare take before she is caught?

36 The whole time occupied by a train 120 yds long, travelling at the rate of 20 miles an hour, in crossing a bridge is 18 seconds, find the length of the bridge.

37 Two guns are fired at the same place after an interval of 21 minutes, but a person approaching the place observes that 20 min 15 sec elapse between the reports, what was his rate of progress, sound travelling 1125 ft per second?

38 A person saw the flash of a gun fired from a frigate at sea, distant 1 mile 480 yds, and 2 seconds afterwards saw the flash of another gun fired from a vessel in a line between the frigate and himself, and 4 seconds later still heard the two reports simultaneously, what was the distance between the vessels?

39 A man near the sea shore sees the flash of a gun fired from a vessel steaming directly towards him and hears the report in 15 sec. He then walks towards the ship at the rate of 3 miles an hour, and sees a second flash 5 min after the first, and immediately stops, the report follows in 10.5 sec. Find the rate of the ship, the velocity of sound being 1200 feet per second.

40 How many seconds will a train 184 ft in length, travelling at the rate of 21 miles an hour, take in passing another train 223 ft long, proceeding in (i) the same direction, (ii) opposite directions, at the rate of 16 miles an hour?

41 A man rides at the rate of 12 miles an hour, but stops 5 minutes to change horses at the end of every 8th mile, how long will it take him to perform a journey of 167 miles?

42 A man rides at the rate of 15 miles an hour, but stops 7 minutes to change horses at the end of every 10th mile, how long will he take to go a distance of 126 miles?

43 A snail creeps 17 in up a pole during 12 hrs in the night, and slips down 10 in during the 12 hrs in the day. If the pole is 12 ft high, in how many hours will it get to the top?

44 A monkey, climbing up a greased pole, ascends 13 ft and slips down 6 ft in alternate minutes. If the pole is 63 ft high, how long will it take him to reach the top?

45 A train is 88 yds long, and is running at 20 miles an hour, in what time will it pass a particular point on a telegraph post standing by the side?

46 A train 66 yds long passes a particular point on a telegraph post in 6 seconds. Find the speed of the train.

47 Two trains, 88 yds and 44 yds long respectively, are running on parallel rails at the rates of 15 and 20 miles an hour. In what time will they pass one another if they are running in opposite directions?

48 Two trains, 66 yds and 99 yds long respectively, are running with uniform velocities on parallel rails at the rates of 20 and 25 miles per hour. In what time will they pass one another, if they are running in the same direction?

49 Two trains, 92 ft and 84 ft long respectively, are moving with uniform velocities on parallel rails in opposite directions, and are observed to pass each other in 1.5 sec, but when they are moving in the same direction, their velocities being the same as before,

the faster train is observed to pass the other in 6 seconds Find the rates at which the trains are moving

50 Two trains running at the rates of 25 and 20 miles an hour respectively on parallel rails in opposite directions are observed to pass each other in 8 seconds, and when they are running in the same direction at the same rates as before, a person sitting on the faster train observes that he passes the other in $31\frac{1}{2}$ seconds Find the lengths of the trains

51 A train, 88 yds in length, overtook a person walking along the line at the rate of 4 miles an hour, and passed him in 10 seconds Twenty minutes after, the train overtook another person and passed him in 9 seconds When will the first person overtake the last?

52 *A* and *B* are two Railway stations A train 195 yds long starts from *A* for *B* 26 min after a man started from *A* to travel in the direction of *B* The train overtakes the man and passes him in 15 seconds After an interval of one hour the man is again overtaken by the train returning from *B*, where it has stayed for 5 min and it passes him in 13 seconds Find the distance of *B* from *A* and the rates of the man and of the train per hour

53 A passenger train 130 ft long leaves station *A* for *B* 10 min before a mail train 110 ft long, which also leaves *A* for *B* The mail overtakes and passes the other in 6 sec After an interval of 45 min the mail on its return from *B*, where it has stayed for 5 min again overtakes the passenger and passes it in 3 sec Find the distance between *A* and *B*, and the rate of each

54 *A*, *B*, *C* and *D* are four Railway stations The roads from *A* to *B*, *B* to *C*, and *C* to *D* are inclined planes of different inclinations A train leaves *A* and passing through *B* and *C*, arrives at *D*, and staying there 30 min returns to *A* The down and up speed of the train along *AB* are 17.5 miles and 8.75 miles per hour respectively, along *BC* are 14 miles and 10 miles per hour respectively, and along *CD* are 12.5 miles and 10.9375 miles per hour respectively If the whole time occupied by the train be $3\frac{1}{2}$ hours, find the length of *AB+BC+CD* in miles

55 A ship 40 miles from the shore springs a leak which admits $3\frac{3}{4}$ tons of water in 12 minutes 60 tons would suffice to sink her, but the ship's pumps can throw out 12 tons of water in an hour Find the average rate of sailing so that she may reach the shore just as she begins to sink?

56 I have to attend a meeting at a certain place in a certain time, and I find that, if I walk at the rate of 4 miles per hour, I shall be 40 minutes too late, if at the rate of 6 miles per hour, I shall be 35 minutes too soon How far have I to go?

57 A train 88 yards long overtook a person walking along the line at the rate of 4 miles an hour and passed him completely in

10 seconds, it afterwards overtook another person and passed him in 9 seconds. At what rate per hour was this second person walking?

58 Two trains start from A to arrive at a fixed time at B travelling at the rates of 25 and 30 miles an hour respectively. The first reaches B 40 min behind time and the second 26 min before. Find the distance between A and B .

59 Two trains 88 yds and 96 yds long run with uniform velocities at the rates of $67\frac{1}{2}$ and $112\frac{1}{2}$ miles per hour respectively. How long will a person sitting in the faster train take to pass the other, when going in (i) opposite directions, (ii) the same direction?

60 A and B start from P and Q , a distance of 60 miles, for Q and P respectively, at 4 and 5 miles an hour. They meet at R , reach Q and P , return immediately, and meet again at S . Find the distance between R and S .

61 Two boats row a race over a straight course 1 mile 995 yds long, their rates of speed being 12 miles and $11\frac{1}{5}$ miles an hour respectively. Assuming that sound travels at the rate of 1140 feet in a second, find how much the faster boat will be ahead of the other when the sound of the gun fired at starting is heard at the winning post.

62 Two boats start to row in a race at 3 o'clock. The winning boat comes in at $6\frac{1}{2}$ min past 3, 40 yds ahead of the other. At 4 min past 3 the losing boat was 1140 yds from the winning post. Find the length of the course, and the speed of the winning boat in miles per hour.

63 A railway train having left a terminus at noon is overtaken at 6 P.M. by another train, which left the same terminus at 1 P.M. If the former train had been 10 miles further on the road when the latter started, it would not have been overtaken till 8 P.M. Find the rates of the trains.

64 A , B and C are three stations. A and B are connected by a railway along an inclined plane, but B and C by a Ferry. A man started from A for B (by train) at 4.45 P.M. and 30 min after his arrival at B took a seat in steamer for C . Having stayed at C for 2 hours, he returned to A (through B) next day at 8.30 A.M. If the down and up-speed of the train along AB be 15 miles and 10 miles per hour respectively, and the rate of the steamer with and against the stream be 18 miles and 9 miles per hour respectively, find the length of $AB+BC$ in miles.

Examples worked out

Ex. 1 The products of the sum of two numbers multiplied by each separately are 3825 and 3400, find the numbers.

The sum of the products of the sum of two or more numbers each separately = the square of their sum.

Now $3825 + 3400 = 7225$, and $\sqrt{7225} = 85$,

one number $= 3825 - 85 = \underline{45}$, and the other $= 3400 - 85 = \underline{40}$ *Ans*

Ex 2 The product of two numbers is 1215, and the quotient when one is divided by the other is $\frac{5}{4}$. Find the numbers

The product \times the quotient $=$ the square of the dividend

Now $1215 \times \frac{5}{4} = 2025$, and $\sqrt{2025} = 45$,

one number $= \underline{45}$, and the other $= 1215 - 45 = \underline{27}$ *Ans*

Ex 3 A horse is sold at a gain for Rs 450. Had it been sold for Rs 390, the loss would have been three times the gain. What did it cost?

The difference between Rs 450 and Rs 390 is $(3 + 1)$ or 4 times the gain, therefore the gain $= \frac{1}{4}$ of Rs $(450 - 390) = \frac{1}{4}$ of Rs 60 = Rs 15

the cost price $=$ Rs 450 - Rs 15 = Rs 435 *Ans*

Ex 4 A man, walking a distance of 18 miles, finds at the end of 1 hr 48 min that the distance which he has walked is $\frac{5}{8}$ ths of the remaining distance. Find the average pace

Here, $(1 + \frac{4}{5})$ or $\frac{9}{5}$ of the remaining distance $=$ whole distance $=$ 18 miles

the remaining distance $= (18 \times \frac{5}{9})$ or $10\frac{1}{2}$ miles,

and $\frac{5}{8}$ of the remaining distance $= (\frac{5}{8} \times 10\frac{1}{2})$ or $7\frac{1}{2}$ miles

Now, in 1 hr 48 min or $1\frac{4}{5}$ hr he has walked $7\frac{1}{2}$ miles,

in 1 hr he can walk $(7\frac{1}{2} - 1\frac{4}{5})$ or $\frac{1}{2} \times \frac{5}{4}$ miles $= 4\frac{1}{4}$ miles

Hence the man's rate per hour $= \underline{4\frac{1}{4}}$ miles *Ans*

Ex 5 The majority was a fifth of the number on the winning side, if 10 voters change to the losing side the majority would be 1. How many voted on each side?

Since a transfer of 10 votes would reduce the majority to 1, therefore the majority $= 2 \times 10 + 1 = 21$

Thus, $\frac{1}{5}$ of the votes on the winning side $= 21$, hence the number of votes on the winning side $= 21 \times 5 = 105$. Therefore the number of votes on the losing side $= 105 - 21 = \underline{84}$ *Ans*

Ex 6 Rs 49 was divided amongst 150 children, each girl had 8a and each boy 4a, how many boys were there? (*E E* 1879)

Each girl may be paid 4a along with the boys, and 4a afterwards separately, so that she may get 8a altogether

The 150 children would in this way first get $4a \times 150$ or Rs 37 8a

The remaining Rs $49 - Rs 37 8a = Rs 11 8a$ would have to be distributed amongst the girls at the rate of 4a each. Therefore the no of girls $= Rs 11 8a \div 4a = 46$, and the no of boys $= 150 - 46 = \underline{104}$ *Ans*

Ex 7 A workman was engaged for 28 days at $Rs\ 1\ 4a$ a day, but instead of receiving anything, was to pay $8a$ a day on all days on which he was idle, he received altogether $Rs\ 26\ 4a$. How many days was he idle?

Had he worked for 28 days, he would have received $Rs\ 1\ 4a \times 28$ or $Rs\ 35$, but as he received only $Rs\ 26\ 4a$, the loss for his idleness is $Rs\ 35 - Rs\ 26\ 4a = Rs\ 8\ 12a$.

But his loss per day for being idle $= Rs\ 1\ 4a + 8a = Rs\ 1\ 12a$

\therefore the no of days he was idle $= \frac{Rs\ 8\ 12a}{Rs\ 1\ 12a} = \frac{140a}{28a} = 5$ Ans

Ex 8 A gentleman wishing to relieve a number of beggars, finds that if he give them 6 pice a head, he will have $5a$ left, and that he has not enough by $3\frac{1}{2}a$ to give them $2a$ a head. Find the number of beggars, and the money he possesses.

Raising the rate from 6 pice to 8 pice or by 2 pice, increases the money required, by $(5 + 3\frac{1}{2})$ or $8\frac{1}{2}a$ i.e. $3a$ pice.

Therefore the no of beggars $= 34 - 2 = 17$,
and the man has $(17 \times 6\text{ pice} + 5a = 122\text{ pice} = Rs\ 1\ 14a\ 2\text{ pice})$ Ans

Ex 9 In rifle shooting, a bull's eye counts 4, a centre 3, an outer 2. Eleven men fire five shots each at a target, and score 113, 8 misses are made and 3 bull's eyes. Find the number of centres and outers.

The 11 men fire 5×11 or 55 shots, 8 being misses, 47 shots score 113. The bull's eyes score 3×4 or 12. Therefore the remaining $(47 - 3)$ or 44 shots, which are either centres or outers, score $(113 - 12)$ or 101. If the 44 shots had been all centres, the score would have been 44×3 or 132. Therefore $(132 - 101)$ or 31 points are due to outers only.

Hence the no of outers is 31, and the no of centres $(44 - 31)$ or 13 Ans

Ex 10 A has three times as much money as B, and $Rs\ 10$ more than C, the sum of their money is $Rs\ 165$. Find A's money.

If C had $Rs\ 10$ more, then A's money would be equal to C's, and the sum of A, B and C's money would be $Rs\ (165 + 10)$ or $Rs\ 175$. Now representing B's money by 1, A's money would be 3, and C's money 3.

And $3 + 1 + 3 = 7$, A's money $= \frac{3}{7}$ of $Rs\ 175 = Rs\ 75$ Ans

Ex 11 I have a certain sum of money, wherewith to buy a certain number of nuts, and I find that if I buy at the rate of 40 a penny, I shall spend $5d$ too much, if at the rate of 50 a penny, $10d$ too little. How much money have I spent?

A nut in the first case costs $\frac{1}{40}$ pence and in the second $\frac{1}{50}$ pence. Therefore the difference in the price of a nut in the two cases is $(\frac{1}{40} - \frac{1}{50})$ or $\frac{1}{200}$ pence. Also the diff in money $= (10 + 5)$ or $15d$.

Now $200d$ is the diff. in price for 1 nut

$1d$..	200 nuts
$\therefore 15d$		15×200 nuts

Then the price of 15×200 nuts at 40 a penny = $(15 \times 200 - 40)$ or $75d$ Hence the money expended = $(75 - 5)$ or $70d$

Ex 12 A viaduct is made of 3 series of arches built over one another, the spans of the arches being $12\frac{2}{3}$ yds, $8\frac{1}{2}$ yds and $4\frac{1}{2}$ yds respectively. The piers on which they stand are in each series 4 yds wide. Find the least length of the viaduct.

Leaving one of the extreme piers, i.e., 4 yds the remaining length of the viaduct must be a multiple of $(12\frac{2}{3} + 4)$, $(8\frac{1}{2} + 4)$ and $(4\frac{1}{2} + 4)$. Hence the least length of the viaduct must be the L.C.M. of $16\frac{2}{3}$, $12\frac{1}{2}$ and $8\frac{1}{2}$ increased by 4 yds. Now the L.C.M. of $16\frac{2}{3}$, $12\frac{1}{2}$ and $8\frac{1}{2}$ = 350. Hence the reqd length = 354 yds.

Miscellaneous Examples VI

1 Given that the sum of the divisor and quotient = 33600, also that the quotient = divisor $\times 15$, and that the remainder = divisor - 15, find the dividend.

2 What will it cost to make a gravel walk, 7 ft wide, along the sides of a square field, containing $2\frac{1}{2}$ ac at 10s per sq yd, the walk being part of the field?

3 A piece of cloth, when measured with a void measure which is 6 of an inch too short, appears to be $88\frac{1}{2}$ yards long, what is its true length?

4 A person having paid an income tax of 4d in the £ during the first half of the year, and $3\frac{1}{2}d$ in the £ during the second half of the year, has £359 5s $11\frac{1}{4}d$ left. What was his gross income?

5 A square room 6 yards long or broad and 5 yards high has the ceiling painted at 6s 4p a sq yd, its 4 walls papered with paper $\frac{3}{4}$ of a yard wide at 6s per yard, and its floor covered with carpet $\frac{3}{4}$ of a yard wide at Rs 2 4s per yard. Find the cost.

6 Of three pipes A, B and C, A fills a cubic inch in a second, B a cubic foot in a minute, C a cubic yard in an hour, if all were running together, in what time would they fill 1069 cubic inches?

7 A rectangular pile is 12 yds high and stands on a base 10 ft square, find the number of oblong pieces 2 ft long, 8 in broad, 4 in deep, contained in it. Also the cost of covering the pile with matting $\frac{3}{4}$ of a yard wide at 3s 2p per yard.

8 A plate of gold 3 in square and $\frac{1}{4}$ of an inch thick is extended by hammering so as to cover surface of 7 sq yds, find its present thickness.

9 A room is 20 ft long, 16 broad and 12 high. If pure gold be worth £4 5s per oz Troy, and a cubic foot of gold weigh 19260 oz Avoirdupois, what is the value of the gold which will exactly fill the room?

10 A creditor receives upon a debt of Rs 3270 a dividend of 9a 2p in the rupee, and afterwards he receives a further dividend upon the deficiency of 3a 4p in the rupee, how much does he receive on the whole?

11 If £1 be worth 25 2 francs, 9½ thalers worth 35 francs, and 60 thalers worth 107 Austrian paper florins, find how many Austrian paper florins should be received for £10

12 A gentleman has a bowling green 300 ft long and 200 ft broad, which he would raise one foot higher, by means of the earth to be dug out of a ditch that goes round it, to what depth must the ditch be dug, supposing the breadth to be everywhere 8 ft?

13 A room is 57 feet long by 32 ft wide. How many people can be seated in it on chairs, which are 1½ feet wide, and placed 2 feet apart from back to back, allowing a clear passage 4 feet wide down the middle of the room, and a clear space 13 feet deep at the end?

14 A cistern 12 ft long, 2 ft wide, and 6 in deep, contains pulp for making paper. If ½ the volume of the pulp is lost in the process of drying, how many sheets of paper, 8 in by 6, will be obtained, if 300 sheets in thickness go to the inch?

15 A train consisting of 3 first class, 4 second class and 5 third class carriages, travelled from Calcutta to Jagadispur, a distance of 191 miles. The rates charged per mile were, 1st class 1a 6p, 2nd class 1a 2p, 3rd class 8p, the amount paid by the passengers was Rs 4659 9a 8p, each 2nd class and each 3rd class carriage contained 32 and 48 passengers respectively. Find the number of passengers in each first class carriage.

16 A heap of cocoanuts can be made up exactly into groups of 25, but when made up into groups of 18, 27 and 32, there is always a remainder 11, find the least number of cocoanuts such a heap can contain.

17 A person who can walk down a hill at the rate of 3¼ and up at the rate of 2½ miles an hour, ascends and comes down to his starting point after walking for 4 hrs 36 min. How far did he walk?

18 A man rides at the rate of 11 miles an hour, but stops 5 min to change horses at the end of every 7th mile. how long will he take to go a distance of 94 miles?

19 What least number is that which leaves a remainder 5 when divided either by 9, 99, 999 and 9999?

20 Find the least number of seven digits such that if it be divided by 27, 33, 45, 60 and 75, the remainder in each case will be 5

21 Find the value of

$$\frac{2\frac{1}{2} + \frac{3}{4} \text{ of } 2\frac{2}{3} - 1\frac{1}{2}}{36 + 15 \times 4 - 24 - 21} \times \frac{12\,345 - 9\,48}{3\,472 + 668} \times \frac{\frac{1}{3} \text{ of } 81 \text{ po } 4\frac{1}{2} \text{ yds}}{\frac{1}{4} \text{ of } 101 \text{ po } 4\frac{1}{2} \text{ yds}} \times$$

$$\frac{201 \text{ of } 1 \text{ md } 30\frac{1}{2} \text{ sr}}{17\,5625 \text{ of } 2 \text{ mds } 20\frac{1}{2} \text{ sr}} \times Rs\,2\,8a$$

22 A man travels 60 miles in 3 hours, partly by rail and partly by coach. If he had gone all the way by rail he would have arrived at his destination an hour earlier, and would have saved $\frac{2}{5}$ ths of the time he was on the coach. How far did he travel by coach?

23 It costs *Rs* 1 10a 8p less to feed 1 horse a month when gram is 36 seers for the rupee than when it is 24, find the cost when gram is 40 seers for the rupee.

24 The quotient in a division sum equals six times the divisor, and the divisor equals six times the remainder, the three amount together to 516, find the dividend.

25 Three lines of palings run side by side for a distance of 90 feet. The rails are respectively 2, 3 and 5 feet apart. How often will a person walking outside the palings, on looking across them, see three rails in a line?

26 If a piece of work can be finished in 45 days by 35 men working continuously, and if the men drop off by 7 at the end of every 15 days, find in what time the work will be finished.

27 A garrison of 100 men had provisions for 27 days, at the end of 10 days a reinforcement arrives, and there are now only provisions for 5 days. What was the reinforcement?

28 By what factor less than 1000 must 7983 be multiplied so that the last 3 figures of the product may be 986?

29 The G.C.M. of two numbers of 4 digits is 187, and their L.C.M. is 21879. Find the numbers.

30 A riband is 47 3824 yards long, how many pieces, each 7871 yd, can be cut off? and how many inches will be left?

31 A gravel walk 6 ft wide runs round a grass plot 60 ft long and 40 ft wide. If gravel be *Rs* 1 8a per cubic yard, find the cost of a coat of gravel on the walk 3 in deep.

32 A man sells a horse for *Rs* 120 more than he gave for it, and realises a profit equal to $\frac{2}{5}$ ths of its cost price. What was the cost price?

33 When 52 lbs of coffee are worth as much as 12 lbs of tea, 22 lbs of tea are worth as much as 572 lbs of sugar, a cask of sugar, costs 2 guineas, and 1 cwt of coffee costs 8 guineas, what is the weight of a cask of sugar?

34 A square field is bordered by a path 3 yds wide, the field and path together occupying $2\frac{1}{2}$ acres. Find the cost of covering the path with gravel at 12s per sq yd.

35 On a stream, B is intermediate to and equidistant from A and C . A boat can go from A to B and back in 5 hrs 15 min, from A to C in 7 hrs. How long would it take to go from C to A ?

36 One man walks 3 miles in 32 min, and another walks $2\frac{1}{2}$ miles in 28 min, how much start must the slower walker have that in a 7 mile race they may walk a dead heat?

37 If a cubic foot of gold be made to cover uniformly and perfectly 43200000 square inches find the thickness of the gold.

38 A barter some sugar with B for flour, which is worth 25 3d per stone, but uses a false stone-weight of $13\frac{1}{2}$ lbs, what value should B set upon his flour, that the exchange may be fair?

39 A and B are the termini of a Railway 144 miles long. A fast train starts from B at 9 h 0 m, another fast train travelling at the same rate, starts from A at 10 h 0 m. A slow train starts from B at 10 h 20 m, the fast train from A meets the other fast train at 11 h 30 m, and the slow train at 12 h 32 m, find the rates at which the trains travelled.

40 A crow wishing to quench its thirst came to a vessel which contained 28 cub in of water. The crow being unable to reach the water picked up several small stones, each three quarters of a cubic inch in size, and let them drop into the vessel until the water came to the top of the vessel. If the size of the vessel was such that it would exactly hold 73 cub in of water, find the number of stones dropped in by the crow.

41 A down train usually travels at the rate of 30 miles an hour and meets an up train 50 miles from the terminus. On one occasion, on account of an accident it only went at the rate of 20 miles an hour and met the up-train $41\frac{1}{2}$ miles from the terminus. Find the speed of the up train.

42 A works for 6 days at the rate of 8 hours per day, B works for 5 hours on the first day and on each of the five subsequent days one hour longer than on the preceding day, A does as much in 4 hours as B does in 5 hours. If the total sum paid to A and B as wages for the week be Rs 21, how much should each receive?

43 If one watch gain, and another lose, at the rate of a minute a day, and they are both set right at noon on Monday, what time will be indicated by the latter when the former points 6 hours $3\frac{3}{4}$ min on the following Friday morning? Also what is the correct time?

44 In paving a court-yard 1295 bricks are employed, the exposed surface of each brick measuring $9\frac{1}{2}$ in by $4\frac{1}{2}$ in, how many tiles 6 in square would be required for paving a yard one ninth of the size of the former?

45 How many degrees, &c., must be added to or subtracted from $15^{\circ} 12' 20''$ that the sum or difference shall be the same fraction of $22^{\circ} 10' 20''$ that $8^{\circ} 10' 15''$ is of $12^{\circ} 15' 22' 5''$?

46 A supply of water would fall short of the calculated time by 12 days if 4 seers leak off every day, but it would fall short by 20 days if 8 seers leak off daily. Find the total quantity of water in the supply.

47 If 5 men working 6 hrs a day and 8 boys working $7\frac{1}{2}$ hrs a day can complete a piece of work in 10 days or 6 men working $6\frac{1}{2}$ hrs a day and 5 boys working 7 2 hrs a day can complete the same piece of work in 11 days, how many days will it take 8 men working 7 hrs a day and 17 boys working 8 hrs a day to finish a piece of work twice as great?

48 A watch which gains 24 min in 24 hours is set right at 9 P M on Monday. What will be the true time when the watch indicates 2 P M on Thursday next?

49 The external dimensions of a box without a lid are, length 4 ft, breadth 3 ft, depth 2 ft and the thickness of the sides and bottom is the same, namely 1 inch, if the cost of a cubic yard of the material is Rs 4 8a, and the cost of making the box = $\frac{1}{11}$ of the cost of the material, what will the box cost?

50 If gold can be beaten out so thin that a grain will form a leaf of 56 square inches, how many of these leaves will be required to make up the thickness of a sheet of paper, the weight of a cubic foot of gold being 1215 lbs Troy and 400 sheets of paper making a book 1 inch thick?

51 A viaduct consists of 3 series of arches built upon each other, the breadths of the arches in each being respectively 8 vds 2 ft, 6 vds and 5 yds, whenever the piers in all the series are vertically above each other, there occurs a mass of masonry 4 yds wide, of such there are 3, find the length of the viaduct.

52 To complete a piece of work, B would take twice as long as A and C together, and C thrice as long as A and B together, A, B and C by their united exertions can do it in 5 days. In what time could each do it by himself?

53 A can run at the rate of 8 miles an hour, B at the rate of $7\frac{1}{2}$ miles an hour, what is the greatest number of yards start that A can give B so as to beat him in a race of 440 yards?

54 The distance between two stations A and B is 65 miles. A train starts from A to go to B at the rate of 15 miles an hour, and is delayed 10 min on the way, another train starts from A two hours after the former at the rate of 25 miles an hour, find the interval between their times of arrival at B.

55 Five men do 6005 of a piece of work in 2 12 hours, how

long will 6 boys take to finish it, it being known that 3 men and 7 boys have done the whole piece of work in 3 hours ?

56 Two clocks, one of which gains and the other loses one minute in an hour, strike one o'clock together, shew that the interval between their respective striking 2 will be $2\frac{2}{35}$ minutes by a correct clock

57 An express train owing to a defect in the engine goes at $\frac{5}{8}$ ths of its proper speed, and arrives at 6 49 P M instead of 5-55 P M At what hour did it start ?

58 A peon walks from A to B at the rate of 3 miles an hour, and after transacting some business which occupies him an hour, returns to A by the tram-way at the rate of 5 miles an hour He then finds he has been absent 2 hrs 20 min Find the distance from A to B

59 The products of the sum of two numbers multiplied by each separately are 12400 and 11625 Find the numbers

60 The products of the sum of three numbers multiplied by each separately are 4674, 4920 and 5535 Find the numbers

61 I sold an article for Rs 450 at a loss, had I sold it for Rs 498, the gain would have been 5 times the loss Find the cost

62 A number of rupees is divided amongst four people A receives $\frac{2}{3}$ ths of the whole, B $\frac{1}{4}$ ths of the remainder, C $\frac{1}{5}$ ths of what then remains, and the number of rupees given to D is the square root of the whole number to be distributed What sum does each receive ?

63 The product of two numbers is 1575 and the quotient of the one divided by the other is $\frac{7}{2}$ Find the numbers

64 A is twice as old as B and 10 years older than C , the sum of their ages is 105 years, find B 's age

65 A person meeting a company of beggars gave $4a$ to each and had $Re\ 1$ left, he found that he should have required $12a$ more to enable him to give the beggars $6a$ each How many beggars were there ?

66 A carpenter agreed to work for 60 days on condition that he should receive for each day that he worked $Re\ 1\ 4a$ and his board, and pay $4a\ 8p$ for his board each day he was idle At the end of the term he received Rs 50 $5a\ 4p$ How many days did he work ?

67 A person distributes Rs 45 amongst 50 men and women, giving each woman $9a$ and each man $15a$ Find the number of men

68 Bought 12 yards of broad cloth and silk for Rs 107 For the silk I paid Rs 9 $8a$ per yard and for the broad cloth Rs 8 $8a$ per yard How many yards of silk were bought ?

69. An undergraduate rowed down the river a distance of

11 miles in $1\frac{1}{2}$ hours with the stream, and on his return met the same stream and with a uniform stroke throughout he rowed back again in $3\frac{1}{2}$ hours Find the rate of the current per hour

70 A train is to arrive in Calcutta from Allahabad at 9 A M Had it travelled at 40 miles an hour it would have been 48 minutes late, but if at 45 miles an hour 46 minutes earlier Find the distance between Allahabad and Calcutta

71 Which will be the more advantageous to employ to do a piece of work, 6 men who work 10 hrs a day for 15s or 9 boys who work 8 hrs a day for 9s, it being given that a man can do half as much work again as a boy in an hour?

72 I want to buy a certain number of mangoes for a certain sum, if I buy at the rate of 4 for an anna, I shall spend 5a too much, if at the rate of 5 for an anna, 10a too little, what is the sum?

73 A man walking $13\frac{1}{2}$ miles finds that in 1 hr 15 min he has walked $\frac{1}{3}$ of the remaining distance, find his rate of walking

74 If the hands of a clock coincide every $65\frac{1}{2}$ minutes, how much does the clock gain or lose in a day?

75 A supply of water would fall short by 10 days if 12 gallons leak off daily, but it would fall short by 14 days if 20 gallons leak off daily Find how long the supply would last if 5 gallons leak off daily

76 36 men can do a piece of work in 24 days After working for a certain number of days they take 12 men to their help, and then finish the work 4 days sooner When do these 12 men join?

77 A person leaves A for B at 4 miles an hour between 4 and 5 P M, and returns after staying at B for 10 minutes, between 8 and 9 P M on the same day and finds the hands of his watch have exactly changed places When does he return and what is the distance from A to B?

78 A squad of 11 boys fired 10 shots each at a target, and scored 286, 20 bull's eyes were made and 11 misses How many centres and outers were there? (A bull's eye scores 4, a centre 3 and an outer 2)

79 If a snail, on the average, creep 2 ft 3 in up a pole during 12 hours in the night and slip down 1 ft 4 in during the 12 hours in the day, how many hours will he be in getting to the top of a pole 25 feet high?

80 The cost of papering the walls of a room 30 ft long at 4s a sq foot is £200, and the cost of matting the floor at Rs 3 per 5 sq ft is Rs 360 Find the height If this room has a verandah 9 ft deep all round the outside, what will it cost to pave it at Rs 2 a sq yd, the walls being 18 in thick?

81 Find the cost of painting the walls of a square room 14 ft high and 18 ft long with two doors 8 ft by 4 ft, and three win-

jows 10 ft by 5 ft, the amount saved by each window being Rs 28 2a
What additional height would increase the cost by Rs 4 8a?

82 A viaduct consists of two series of arches built over one another, the spans of the arches are 12 yds and 11 yds respectively. The piers on which they stand are 5 and 4 yds respectively. Find the least length of the viaduct

CHAPTER XI

Ratio and Proportion

I RATIO

462 Ratio is the relation which one number has to another, or, which one quantity numerically considered bears to another of the *same kind*, the comparison being made by observing what *multiple*, *part* or *parts*, the former is of the latter

Thus, the ratio of the *abstract numbers* 4 and 2 is written 4 : 2, and read *four is to two*, and it will be expressed by $\frac{4}{2}=2$, the same, being used to denote the ratio of the *concrete quantities* 4 ft and 2 ft, provided they be of the *same kind* and of the *same denomination*

463 Of the numbers or quantities *compared* and called the *terms* of the ratio, the former is styled the *antecedent* and the latter the *consequent*, also, the ratio is said to be a *ratio of greater or less inequality* according as the antecedent is *greater or less* than the consequent, and it is a *ratio of equality* when these terms are equal

Thus, in the ratio £4 to £5, £4 is called the *antecedent* and £5 the *consequent*, also £4 and £5 are its *terms*. Again, 6 : 5 is a ratio of *greater inequality*, 4 : 9 is one of *less inequality*, and a ratio of *equality* may be denoted by 1 : 1 or 2 : 2 or 3 : 3, &c., at pleasure

464 Hence, the magnitude of a ratio is expressed by the *vulgar fraction* whereof the antecedent is the *numerator* and the consequent the *denominator*

Thus, the ratio of £9 and £12, written 9 : 12, will have its magnitude expressed by the fraction $\frac{9}{12}$, or, reduced to its lowest terms, by the fraction $\frac{3}{4}$, whereas, the ratio of 9d to 6s will be that of 9d

to 72d, which = $\frac{9d}{72d} = \frac{1}{8} = \frac{1}{8}$, and this is therefore the same as that of 9 lbs to 72 lbs

Also, if the terms of the ratio be vulgar fractions or decimals the fraction expressing its magnitude may be simplified by the rules already given

465 It appears from the last Article, that the value of a ratio does not depend upon the nature of the quantities involved. Thus, the ratios 4 yds 5 yds, 4s 5s, 4 lbs 5 lbs are all equal, each of these being determined by the fraction $\frac{4}{5}$. Hence, in treating of ratios we usually consider the terms to be numbers, for at any time we can pass from quantities of the same kind to the numbers which measure them, and *vice-versa*, whenever we find it necessary to do so.

466 The magnitudes of two or more ratios may therefore be compared, by comparing the values of the vulgar fractions which represent them, according to the principle of the last Article.

Thus, if the ratios be 3 4 and 5 7, then their magnitudes will be represented by $\frac{3}{4}$ and $\frac{5}{7}$,

$$\text{but } \frac{3}{4} = \frac{3 \frac{1}{2}}{5 \frac{1}{2}} \text{ and } \frac{5}{7} = \frac{2 \frac{1}{2}}{3 \frac{1}{2}},$$

and $\frac{3 \frac{1}{2}}{5 \frac{1}{2}}$ being greater than $\frac{2 \frac{1}{2}}{3 \frac{1}{2}}$ it follows that the ratio 3 4 is greater than the ratio 5 7 in other words, 3 has to 4 a greater ratio than 5 has to 7.

467 One ratio is said to be the *inverse* or *reciprocal* of another, when the antecedent and consequent of the one are respectively the consequent and antecedent of the other.

Thus, the *inverse* ratio of 5 7 is the ratio of 7 5.

468 *A ratio of greater inequality is diminished, and a ratio of less inequality is increased, by adding the same quantity to both its terms.*

First, let us take the ratio of *greater inequality* 7 5, and add 1 to both its terms, so that it becomes 8 6,

then the original ratio = $\frac{7}{5} = \frac{1 \frac{2}{5}}{1}$, and the new ratio = $\frac{8}{6} = \frac{4}{3}$, therefore the new ratio is *less* than the original one.

Secondly, taking the ratio of *less inequality* 8 11, and adding 2 to each term, so as to make it 10 13, we have

the original ratio = $\frac{8}{11} = \frac{1 \frac{2}{11}}{1}$, and the new ratio = $\frac{10}{13} = \frac{1 \frac{10}{13}}{1}$,

the latter of which fractions being greater than the former, the new ratio is the *greater* of the two.

Exactly in the same manner, it may be shewn that a ratio of *greater inequality* is *increased*, and a ratio of *less inequality* is *diminished*, by subtracting the same quantity from each of its terms.

469 *If the terms of a ratio be multiplied or divided by the same quantity, the magnitude of the ratio will not be altered.*

Let the ratio be 3 8, then its magnitude is $\frac{3}{8}$ which is equivalent to

$$\frac{6}{16} \text{ or } \frac{9}{24}, \text{ or } \frac{12}{32}, \text{ or } \frac{15}{40}, \&c,$$

that is, the ratio 3 8 is equal to each of the ratios 6 16, 9 24, 12 32, 15 40, &c which arise from the equal *multiplication* of its terms.

and *conversely*, each of the latter ratios is reducible to the original one by the equal *division* of its terms

470 If the antecedents of two or more ratios be multiplied together for a new antecedent, and their consequents be multiplied together for a new consequent, the resulting ratio is said to be **Compound** of the others, and it is called their **compound ratio**

Thus, if the ratios be $2 : 3$, $4 : 7$ and $8 : 13$, the ratio which arises from their composition will be $2 \times 4 \times 8 : 3 \times 7 \times 13$, or $64 : 273$

Examples CXLIV

1 What are the simplest expressions of the magnitudes of the following ratios?—

(1) $3 : 5$, $4 : 12$, $9 : 21$, $64 : 48$, $48 : 64$, $20 : 32$

(2) $7\frac{1}{2} : 37\frac{1}{2}$, $37\frac{1}{2} : 7\frac{1}{2}$, $6\frac{1}{2} : 75$, $16\frac{1}{8} : 60\frac{1}{2}$, $6\frac{1}{2} : 3\frac{1}{2}$, $2\frac{1}{2} : 6$

(3) $225 : 75$, $65 : 775$, $\frac{1}{6} : 571\frac{1}{2}$, $35 : 375$, $625 : 025$

(4) $R : 57$ $8a$, $R : 2$ $8a$, $R : 2$ $8a$, $R : 57$ $8a$, $2 : 8\frac{1}{2}$, $8\frac{1}{2} : 2$

(5) $R : 10$, $R : 2$ $15a$, 14cwt 3qrs 7lbs , 1ton , 3mds , 5mds , 10sl

(6) 3yds , 2ft , 16hrs , 40min , 40sec , 1day , 3cwt , 2qrs , 2tons

2 Which of the following ratios is the greater?

(1) $5 : 9$ or $7 : 11$, $10 : 17$ or $17 : 23$, $34 : 27$ or $37 : 31$

(2) $17 : 18$ or $11 : 12$, $7\frac{1}{2} : 10$ or $3 : 4$, $\frac{1}{8} : 3\frac{1}{2}$ or $1\frac{1}{2} : \frac{1}{2}$

(3) $11 : 12$ or $17 : 18$, $\frac{1}{8}$ or $\frac{1}{7}$, $\frac{1}{2} : \frac{4}{5}$, $2\frac{1}{2} : 3$ or $3 : 1\frac{1}{2}$

3 Of the following ratios which is the greatest?

(1) $6 : 7$, $8 : 9$ or $21 : 24$, $2\frac{1}{2} : 6\frac{1}{2}$, $2\frac{1}{2} : 3\frac{1}{2}$ or $4\frac{1}{6} : 11\frac{1}{6}$

(2) $7 : 15$, $1\frac{1}{2} : 2\frac{1}{5}$ or $75 : 96$, $4 : 7$, $8 : 15$ or $13 : 24$

(3) 14sl , 1gu , 16lbs , 1qr , 2ft , 1in , 1vd , 9in or 3gals , 3qts , 42pts

4 Find whether the ratios $7 : 9$, $11 : 17$ and $10 : 7$ are increased or diminished by adding 1, 2, 3, respectively to their terms

5 Are the ratios $4 : 3$, $9 : 13$ and $15 : 22$ increased or diminished by subtracting 2, 3, 4, respectively from their terms?

6 What are the ratios arising from the composition of the following ratios?

(1) $5 : 12$, $8 : 7$ and $6 : 25$, $5 : 7$, $7 : 18$ and $18 : 35$

(2) $5 : 7$, $13 : 15$, $21 : 91$ and $45 : 52$, $1\frac{1}{2} : 2\frac{1}{5}$ and $2\frac{1}{3} : 7\frac{1}{3}$

(3) $7 : 15$, $9 : 16$ and $24 : 35$, $4 : 9$, $3 : 4$, $5 : 6$ and $12 : 7$

7 If the consequent be 32, and the value of the ratio $\frac{1}{3}$, what is the antecedent?

8 If the antecedent be 156 and the value of the ratio 3 , what is the consequent?

9 If the consequent be 3 acres, and the value of the ratio $\frac{11}{100}$; what is the antecedent?

10 If $A = 3\frac{1}{2}$ of B , and $C = 5\frac{1}{2}$ of B , find the ratio of A to C

11 Compare the rates of two trains, one of which travels $397\frac{5}{8}$ miles in $11\frac{2}{3}$ hours, and another which travels $262\frac{1}{3}$ miles in $8\frac{1}{3}$ hours

12 If, when A makes a profit of Rs 2, B makes Rs 3, and when B makes a profit of Rs 4, C makes Rs 5, and when C makes a profit of Rs 6, D makes Rs 7, compare the profits of A , B , C and D

II PROPORTION

471 Proportion is the equality of two ratios

Thus, the ratios 2 3 and 6 9, being expressible by the equal fractions $\frac{2}{3}$ and $\frac{6}{9}$, are equal, and the four numbers 2, 3, 6, 9 form a proportion which is written 2 3 6 9, and is read 2 is to 3 as 6 is to 9, or $2 : 3 = 6 : 9$ and is then read 2 to 3 equals 6 to 9, the numbers 2, 3, 6, 9 being its terms which taken in order are called proportionals

Hence, in every proportion, the first term is greater than, equal to, or less than the second, according as the third term is greater than, equal to, or less than the fourth

472 In the proportion above expressed, the numbers 2 and 9 are called the extremes, and the numbers 3 and 6 the means, 9 is called a fourth proportional to 2, 3, and 6, and it follows immediately from the equality of the ratios denoted by

$\frac{2}{3} = \frac{6}{9}$, and the multiplication of them both by 27, that

$\frac{2}{3} \times 27 = \frac{6}{9} \times 27$, that is, $2 \times 9 = 6 \times 3$,

in words, if four numbers constitute a proportion, the product of the extremes is equal to the product of the means (the first and last being the extremes and the second and third the means)

473 This property of a proportion proves immediately that either of the extremes may be obtained by dividing the product of the means by the other, and that either of the means may be had by the division of the product of the extremes by the other, also, these qualities constitute the general practical application of Proportion

474. The terms of a proportion may be made to undergo changes and modifications in the same way as the corresponding terms of the vulgar fractions

Thus, 3 4 9 12, gives $3 \times 12 = 4 \times 9$, whence

$\frac{3}{4} = \frac{9}{12}$, or 3 9 4 12, and $\frac{4}{9} = \frac{12}{9}$, or 4 3 12 9,

and we observe that in each of these the product of the extremes equals that of the means

Also, if four numbers form a proportion and any equi-multiples whatever of the first and second be taken, and any equi-multiples whatever of the third and fourth, the resulting numbers taken in order will still form a proportion

For, since $5 : 3 = 15 : 9$, or $\frac{5}{3} = \frac{15}{9}$, and also $\frac{2}{3} = \frac{4}{6}$,

$$\text{we have } 5 \times 2 = 15 \times \frac{2}{3}, \text{ or } \frac{5 \times 2}{3 \times 2} = \frac{15 \times 2}{9 \times 3},$$

$$\text{Whence, } 5 \times 2 : 3 \times 2 = 15 \times 2 : 9 \times 3$$

Again, if any equi-multiples whatever of the first and third numbers be taken, and also any equi-multiples whatever of the second and fourth, the numbers thence arising will form a proportion

Thus, if we take the proportion above, we have

$$\frac{5}{3} \times \frac{4}{7} = \frac{15}{9} \times \frac{4}{7}, \text{ or } \frac{5 \times 4}{3 \times 7} = \frac{15 \times 4}{9 \times 7},$$

$$\text{whence, } 5 \times 4 : 3 \times 7 = 15 \times 4 : 9 \times 7$$

The *new* ratios constituting these proportions being *equal* to the *original*, the division of the terms of a proportion, in accordance with this Article, will often facilitate practical computations by diminishing the number of figures employed

475 If four quantities of the *same* kind taken in order be proportionals, it will be *useful* to recollect that,

- (i) The first the third the second the fourth
- (ii) The second the first the fourth the third
- (iii) The sum of the first and second the first the sum of the third and fourth the third
- (iv) The sum of the first and second the second the sum of the third and fourth the fourth
- (v) The difference of the first and second the first the difference of the third and fourth the third
- (vi) The difference of the first and second the second the difference of the third and fourth the fourth
- (vii) The sum of the first and second the difference of the first and second the sum of the third and fourth the difference of the third and fourth

These may easily be shewn to be correct by any of the proportions hitherto given

476 Of two or more proportions if the corresponding terms be multiplied together, the numbers thence arising will also form a proportion

Thus, if the proportions be

$$3 : 7 = 6 : 14 \text{ and } 4 : 9 = 12 : 27 \text{ then } \frac{3}{4} = \frac{6}{12} \text{ and } \frac{7}{9} = \frac{14}{27},$$

$$\text{whence } 3 \times \frac{4}{9} = \frac{6}{12} \times \frac{14}{27}, \text{ or } \frac{3 \times 4}{7 \times 9} = \frac{6 \times 12}{14 \times 27},$$

$$\text{and } 3 \times 4 : 7 \times 9 = 6 \times 12 : 14 \times 27$$

This operation is called the **compounding** of proportions and the resulting proportion is said to be **compounded** of the others

477 In the above Articles, *abstract* numbers have been considered, but when the quantities are *concrete*, we must take care to exclude such propositions as *express* ratios between things of *different kinds* thus, the ratio of 10 lbs to 15 lbs being the *same* as that of 2s to 3s, we have the proportion

$$10 \text{ lbs} \quad 15 \text{ lbs} = 2s \quad 3s,$$

but we cannot have the proportion

$$10 \text{ lbs} \quad 2s = 15 \text{ lbs} \quad 3s$$

as no ratio subsists between 10 lbs and 2s or between 15 lbs and 3s

Nor indeed can we even in the first of these forms multiply together the *concrete* quantities so that the product of the extremes equals the product of the means, but what we do in finding any term in such cases, is to consider merely their *numerical* values, because the ratios being *abstract magnitudes* will remain the same whatever be the *nature* of the quantities they are used to compare

478 Three quantities of the same kind are said to be in **continued proportion** when the ratio of the first to the second is equal to the ratio of the second to the third. The second quantity is called a **mean proportional** between the first and third, and the third quantity is called a **third proportional** to the first and second

Thus, 16, 8 and 4 are in *continued proportion*, for $16 : 8 = 8 : 4$, 8 is a *mean proportional* between 16 and 4, and 4 is a *third proportional* to 16 and 8

479 In a continued proportion expressed as above, it is obvious that the *square of the mean proportional* between two numbers is equal to their *product*, and consequently the mean proportional between two numbers is equal to the *square root* of their product

Ex 1 Find a fourth proportional to 5, 7 and 15

$$5 : 7 = 15 : \text{number required},$$

$$5 \times \text{number required} = 15 \times 7$$

$$\text{number required} = (15 \times 7 \div 5) = 21 \quad \text{Ans}$$

Ex 2 What number has the same ratio to 9 that 3 has to 5?

$$3 : 5 = \text{number required} : 9,$$

$$5 \times \text{number required} = 3 \times 9,$$

$$\therefore \text{number required} = (3 \times 9 \div 5) = \frac{27}{5} = 5\frac{2}{5} \quad \text{Ans}$$

Ex 3 Find a mean proportional between 14 and 56

$$\text{Square of the required number} = 14 \times 56 = 784$$

$$\therefore \text{the required number} = \sqrt{784} = 28 \quad \text{Ans}$$

Ex 4 One gowala to 19 sr of milk adds 5 sr of water, and another to 27 sr of milk adds 7 sr of water, compare the amount of milk in the two mixtures

The first mixture consists of $(19+5)$ or 24 sr, of which $\frac{19}{24}$ is milk,
 second $(27+7)$ or 34 sr, of which $\frac{27}{34}$ is milk,
 the ratio of milk in the two mixtures is $\frac{19}{24} : \frac{27}{34}$ or $\frac{19}{24} \times \frac{34}{27}$,
 $i.e., 19 \times 17 : 27 \times 12$ or $323 : 324$ Ans

Ex 5 A mixture is composed of 9 parts brandy and 1 water, 4 gallons of water are added, and the mixture contains 6 times as much brandy as water, how many gallons of brandy does it contain?

In the original mixture, brandy water = 9 1 or 18 2,
 new brandy water = 6 1 or 18 3

Now, brandy remaining the same 18 parts, the water is increased by $(3-2)$ or 1 part

for 1 gal of water added there are 18 gals of brandy,
 for 4 gals (18×4) or 72 gals

Hence the required quantity of brandy = 72 gals Ans

Examples CXLV

1 Find the fourth proportional to —

(1) 15, 27 and 20 (2) 11, 80 and 70 (3) 1590, 53 and 1710

(4) $9\frac{1}{2}$, $7\frac{1}{2}$ and $28\frac{1}{2}$ (5) $18\frac{1}{2}$, $5\frac{1}{2}$ and 75 (6) 102, 51 and 103

(7) 11 1, 16 38 and 17 76 (8) Rs 15, Rs 17 $\frac{1}{2}$ and Rs 100

(9) 11 $\frac{1}{10}$ in, $55\frac{1}{2}$ yds and Rs 2 4a

(10) 57 m, 38 m and 17 tons 13 cwt 1 qr

(11) $19\frac{3}{4}$ ac, 11 ac 2 ro 20 po and 79 tons.

(12) 13s 7 $\frac{1}{2}$ d, £2 os 10 $\frac{1}{2}$ d and 5s 8 $\frac{1}{4}$ d

2 Find a number which shall have the same ratio to 7 as 27 has to 3, also, a magnitude to which 39 has the same relation as 3 $\frac{1}{2}$ has to 2 $\frac{3}{4}$

3 Required the number which has to 40, the ratio of 375 to 3, and find a fourth proportional to $\frac{2}{7}$, 17 and 125

4 Complete the proportion of which the first, second and fourth terms are $\frac{1}{10}$, 35 and 3 $\frac{3}{4}$, also, that whose first, third and fourth terms are 35, 125 and 0145

5 Find a mean proportional between —

(1) 16 and 4 (2) 5 and 125 (3) 057 and 513

(4) $\frac{5}{8}$ and $\frac{15}{8}$ (5) $3\frac{3}{8}$ and 10 $\frac{3}{8}$ (6) 1 and 001

6 Find the term not given in each of the following proportions —

(1) 144 740 370 (2) 1 $\frac{1}{2}$ 3 $\frac{1}{2}$ 2 $\frac{1}{2}$ (3) $\frac{5}{7}$ $\frac{19}{7}$ $\frac{16}{7}$

(4) 12 36 39 (5) 01 7 5775 (6) 8 79 1264

7 Find a third proportional to (1) 25 and 400 (2) $\frac{7}{5}$ and $\frac{5}{7}$

8 The ratio of A to B is $2 : 3$, of B to C is $5 : 6$, and of C to D is $7 : 8$, find the continued ratio of A , B , C and D

9 If 3 men and 11 boys working together, can do 5 times as much work per hour as a man and a boy together, compare the work of a boy with that of a man

10 A buys 15 cows and 130 sheep for a certain sum, and B buys 9 cows and 175 sheep, at the same rates as A , for the same sum. Compare the values of a sheep and of a cow

11 A cask of 72 gallons consists of 11 parts brandy and 1 part water, how much water must be added that it may consist of 9 parts brandy and 1 water?

12 270 sheep and 14 horses eat 101 acres of grass in 30 days and 155 sheep and 21 horses eat 185 acres of grass in 75 days. Compare the amount eaten by a sheep and by a horse in the same time

13 A greyhound pursues a hare and takes 3 leaps for every 4 leaps of the hare, but 2 leaps of the hound are equal to 3 of the hare, compare the rates of hound and hare

14 Six coins of equal weight, made of gold and silver mixed, are melted together and re cast. In one of them the gold and silver were in the ratio of $2 : 1$, in two others of $3 : 5$, and in the rest $7 : 5$. In what ratio will the gold and silver be mixed in the new coins?

15 The values of gold and silver are as 1428 to 1, in what proportions must these metals be combined, in order that the compound may be twice as valuable as an alloy of two parts of silver and one of gold?

III RULE OF THREE

480 The object of the **Rule of Three** is, by means of *three* quantities given, to determine a *fourth*, which shall be the same multiple, part or parts of one of them, that one of the remaining quantities is of the other, and it therefore follows that the operation, by which this may be accomplished, depends on the rule already laid down in Art 473, that in a proportion if three of the numbers or quantities are given, the fourth number or quantity can easily be found

RULE For the Statement Of the three quantities proposed, put down as the last on the right hand, that which is of the *same kind* or under the *same circumstances* as the one required, and the *greater* or *less* of the two others in the second place, according as the required one ought, from the nature of the case, to be *greater* or *less* than the last, and the remaining one in the first place

For the Operation Reduce, if necessary, the first and second terms to the *same* denomination and the third to the *lowest* denomination contained in it multiply together the second and third

terms thus reduced, and the quotient arising from division of the product by the first, will be the quantity required, expressed in the *denomination* to which the *last* term was reduced, and it may be had in other denominations by the proper divisions or multiplications

481 It is sometimes necessary to consider what *preparation* may be required before the rule is applied, and when the statement is made, the first, and the second or third terms may be divided by any factor common to them, either *before* or *after* the reductions, without affecting the result inasmuch as no alteration is produced from multiplication and division by the same number

Ex 1 If 11 mds of ice cost Rs 41 7a 8p, what sum must be paid for 45 mds?

Here, what is required being *money*, the *last* term of the statement will be Rs 41 7a 8p, since it is of the *same kind*, and because the price of 45 mds must manifestly be *greater* than that of 11 mds, the *second* term must be 45 mds and the *first* will be 11 mds, that is, the statement and operation will be as follows —

11 mds	45 mds	Rs 41 7a 8p	the required price,
or 11 mds	45 mds	7964p	the required price

$$\therefore \text{the required price} = \frac{7964 \times 45}{11} p = (724 \times 45)p \\ = 32580p = \underline{\text{Rs } 169 \text{ } 11a} \quad \text{Ans}$$

Ex 2 If a person can walk a certain distance in 8 days of 7 hrs 30 min each, in how many days can he do the same, when 10 hrs of each are available for the purpose?

Since *days* are *required* here, and the number of days is necessarily *less* as the number of hours employed in each is *greater*, we shall have,

10 hrs	7 hrs 30 min	8 days	reqd time
or 10 hrs	7½ hrs (= 7½ hrs)	8 days	reqd time
reqd time = $\frac{15 \times 8}{10 \times 2}$ days = 6 days Ans			

482. The RULE OF THREE is applicable in two different kinds of cases, according to which it is called the **Rule of Three Direct** or the **Rule of Three Inverse**, but the rules for the *Statement* and *Operation* above given are applicable to both kinds of cases

The RULE OF THREE DIRECT is that in which *more* requires *more* and *less* requires *less*, or, in other words, in which a *greater* number requires a *greater* answer, and a *less* number a *less* answer. The first of the above two examples is an instance of the **Rule of Three Direct**, for 45 mds being greater than 11 mds, the price of 45 mds is necessarily greater than that of 11 mds

The RULE OF THREE INVERSE is that in which *more* requires *less* and *less* requires *more*, or, in other words, in which a *greater*

number requires a *less* answer, and a *less* number a *greater* answer. The second of the above Examples is an instance of the Rule of Three Inverse, for 10 hrs being greater than 7 hrs 30 min, it is plain that walking 10 hrs a day will take a *less* time than walking 7 hrs 30 min daily.

483 The following Examples are instances of the RULE OF THREE DIRECT

Ex 1 What will 5 cwt 2 qrs 24 lbs cost, if 18 lbs cost Rs 6 15a ?

18 lbs 5 cwt 2 qrs 24 lbs Rs 6 15a reqd cost,
or 18 lbs 640 lbs Rs $1\frac{11}{16}$ reqd cost,

$$\therefore \text{the reqd cost} = \text{Rs } \frac{111 \times 640}{18 \times 16} = \text{Rs } 246\frac{10}{16} = \text{Rs } 246\ 10a\ 8p \text{ Ans}$$

Ex 2 If 17 mds 6 sr can be bought for Rs 142 14a 8p how much can be bought for Rs 373 5a 4p ?

Rs 142 14a 8p Rs 373 5a 4p 17 mds 6 sr the reqd weight
or Rs $1\frac{17}{16}$ Rs $11\frac{20}{16}$ $3\frac{4}{16}$ mds the reqd weight

$$\therefore \text{the reqd weight} = \frac{343 \times 1120 \times 12}{20 \times 3 \times 1715} \text{ mds} = 2\frac{2}{5} \text{ mds} = 44 \text{ mds } 32 \text{ sr}$$

Ex 3 A bankrupt's debts are Rs 5255 4a and his assets Rs 3753 12a, how much can he pay in the rupee ?

Rs 5255 4a Re 1 Rs 3753 12a what he can pay per Re,
or Rs $219\frac{1}{4}$ Re 1 Rs $159\frac{1}{5}$ what he can pay per Re

$$\text{what he can pay per Re} = \frac{15015 \times 4}{21021 \times 4} = \text{Re } 1 = 11a\ 51p \text{ Ans}$$

Ex 4 A man's income is Rs 2755, and the income-tax is 3a 4p in the rupee, what tax does he pay ?

Re 1 Rs 2755 3a 4p reqd tax

$$\therefore \text{the reqd tax} = 3a\ 4p \times 2755 = \text{Rs } 573\ 15a\ 4p \text{ Ans}$$

Ex 5 If after paying an income tax of 7d in the pound, a person has £776 13s 4d remaining, what is his actual gross income ?

After paying 7d in the £, he has left (240 - 7) or 233d

233d £776 13s 4d £1 reqd income,
or £ $2\frac{2}{3}$ £ $1\frac{1}{3}$ £1 reqd income

$$\text{the reqd income} = \text{£ } \frac{2330 \times 240}{3 \times 233} = \text{£ } 800 \text{ Ans}$$

Ex 6 If I can travel 198 miles by railway for Rs 24 12a, how far at the same rate of charge ought I to be carried for Rs 80 7a ?

Rs 24 12a Rs 80 7a 198 miles reqd distance,
or Rs $2\frac{3}{4}$ Rs $1\frac{28}{16}$ 198 miles reqd distance,

$$\text{the reqd distance} = \frac{1287 \times 198 \times 4}{16 \times 99} \text{ miles} = 643\frac{1}{2} \text{ miles Ans}$$

Ex 7 If 5 cows eat as much as 3 horses, and that the charge per annum for pasturing 5 horses is Rs 200, what will be the cost of the annual pasture of 15 cows?

Here, 5 cows = 3 horses, ∴ 15 cows = 9 horses
 5 horses 9 horses Rs 200 cost required

$$\therefore \text{the reqd cost} = \text{Rs } \frac{200 \times 9}{5} = \text{Rs } \underline{360} \text{ Ans}$$

484. The following are Examples of the RULE OF THREE INVERSE

Ex 1 In what time will 45 men do a piece of work, which 36 men can do in 35 days?

45 men 36 men 35 days reqd no of days

$$\therefore \text{the reqd time} = \frac{35 \times 36}{45} \text{ days} = \underline{28} \text{ days Ans}$$

Ex 2 What sum of money must be advanced on loan for 63 days, as an equivalent for the loan of Rs 1107 12a for 125 days?

63 days 125 days Rs 1107 12a the reqd sum,
 or 63 days 125 days Rs 1107 12a reqd sum,

$$\therefore \text{the reqd sum} = \text{Rs } \frac{125 \times 1107 \frac{1}{2}}{63 \times 4} = \text{Rs } \underline{2197 \ 14a \ 8p} \text{ Ans}$$

Ex 3 What length of carpet 2 ft 3 in wide, will be required to cover a room which is 27 ft 6 in long, and 22 ft 6 in wide?

2 ft 3 in 22 ft 6 in 27 ft 6 in the reqd length,
 or 27 in 270 in 330 in the reqd length,

$$\therefore \text{the reqd length} = \frac{330 \times 270}{27} \text{ in} = 3300 \text{ in} = \underline{91 \text{ yds } 2 \text{ ft}} \text{ Ans}$$

Ex 4 How many yards of cloth at Rs 1 13a per yard must be given in exchange for 942½ yards of silk at Rs 9 10a per yard?

Rs 1 13a Rs 9 10a 942½ yds reqd cloth,
 or Rs 1½ Rs 7½ 2885 yds reqd cloth,

$$\text{the reqd cloth} = \frac{1885 \times 77 \times 16}{2 \times 8 \times 20} \text{ yds} = \underline{5005} \text{ yds Ans}$$

Ex 5 If 2000 men have provisions for 95 days, and if after 15 days 400 men go away, find how long the remaining provisions will serve the number left

After 15 days there will be provisions left of 2000 men for (95 - 15) or 80 days, while the number of men will be reduced to (2000 - 400) or 1600 men

1600 men 2000 men 80 days no of days reqd,

$$\therefore \text{no of days reqd} = \frac{2000 \times 80}{1600} = \underline{100} \text{ Ans}$$

485 There are certain Examples, in which, at first sight, more than three terms appear to be given, but they, in certain cases, come under this Rule, as in the following instances

Ex 1 If the 4d loaf weighs 2 lbs 3 oz when wheat is at 7s 1½d a bushel, what should it weigh when wheat is at 7s 11d a bushel?

Here 4d may be left out of consideration, as being the same in both cases

7s 11d 7s 1½d 2 lbs 3 oz reqd weight,
or 95d 85½d 35 oz reqd weight

$$\text{the reqd weight} = \frac{35 \times 171}{2 \times 95} \text{ oz} = 31\frac{1}{2} \text{ oz} = \underline{1 \text{ lb } 15\frac{1}{2} \text{ oz}} \text{ Ans.}$$

Ex 2 If the carriage of 5 cwt 7 lbs for 84 miles cost Rs 39, what will it cost to have 21 cwt 1 qr 14 lbs carried the same distance?

Here 84 miles may be neglected, as it is the same in both cases

5 cwt 7 lbs 21 cwt 1 qr 14 lbs Rs 39 reqd cost,
or 567 lbs 2394 lbs Rs 39 reqd cost,

$$\text{the reqd cost} = \text{Rs } \frac{2394 \times 39}{567} = \text{Rs } \frac{494}{3} = \text{Rs } \underline{164 \text{ } 10\text{s } 8\text{p}} \text{ Ans}$$

486 Examples such as the following are easily worked by the RULE OF THREE

Ex 1 A person gives away annually Rs 200 in charity, and his weekly bills amount to Rs 75, what additional daily expenditure may he incur with an income of Rs 5925? (A year = 52 weeks)

For the annual amount of his weekly bills, we have

1 wk 52 wks Rs 75 annual weekly bills,
∴ annual weekly bills = Rs (75 × 52) = Rs 3900

Therefore his charity and weekly bills amount to Rs (3900 + 200) or Rs 4100, and he has Rs (5925 - 4100) or Rs 1825 left to be expended in 365 days, whence,

365 days 1 day Rs 1825 additional daily expenses,

$$\text{addl daily expenses} = \text{Rs } \frac{1825}{365} = \text{Rs } \underline{5} \text{ Ans}$$

Ex 2 Two trains 210 ft and 180 ft in length respectively are going in opposite directions, the first at the rate of 23 miles per hour and the other at the rate of 27 miles per hour, find how long they will take to pass each other

As the trains are going in opposite directions, they are approaching each other by (24 + 27) or 51 miles per hour, and one shall pass the other, when (210 + 180) or 390 ft or ⅓ mile is passed over

51 miles ⅓ mile 1 hr the reqd time,

$$\text{the reqd time} = \frac{13}{51 \times 176} \text{ hr} = \underline{5\frac{10}{176}} \text{ sec} \text{ Ans}$$

Examples CXLVI

- 1 Required the price of 450 lbs, at Rs 2 5a 8p per lb
- 2 A persons salary is £191 12s 6d for 365 days, in how many days will he have a claim for £31 10s?
- 3 Required the price of 4 cwt 1 qr 4 lbs 8 oz of sugar, when 1 lb costs 7s 10½d
- 4 If an artificer earn Rs 190 8a in 20 days, in what time will he earn Rs 238 2a?
- 5 If 17 ells 3 qrs, each ell containing 5 qrs, be bought for Rs 68 12a, how much must be paid for 18 yards?
- 6 If 1000 sovereigns weigh 21 lbs 5 oz 16 dwts 6 grs, what weight of gold will be contained in 384 sovereigns?
- 7 How much wheat can be purchased for Rs 550 2a, at the rate of Rs 3 6a 4p per maund?
- 8 If a farm of 375 bighas be let for Rs 4015 10a a year, what is that for each bigha?
- 9 If lodgings be let at Rs 6 12a per week, what will the demand amount to for 273 days?
- 10 Required the price of 36 cwt 1 qr of rice, when 2 cwt 2 qrs 10 lbs cost £4 7s 9½d
- 11 If a servant's wages be £30 or 8½d a year, what will be his demand for a service of 338 days?
- 12 If a person can walk 3 mi 6 fur 25 po in an hour, in what time will he complete a journey of 99 mi 4 fur 10 po?
- 13 If the carriage of 3 cwt 2 qrs 14 lbs for 51 miles come to Rs 9 3a 6p, what will be the charge for carrying 10 tons 3 cwt, the same distance?
- 14 At the rate of 11s 7½d in the pound, what is the sum paid by a bankrupt for a debt of £2735 10s?
- 15 If 67 mds 8 sr cost Rs 746 10a 8p, how much can be bought for Rs 285 13a 4p?
- 16 If 11 mds 8 sr cost Rs 2240, what will 4 mds 11 sr 8 ch cost at the same rate?
- 17 If 3 qrs 7 lbs of tobacco cost £17 13s 6d, what is the value of 5 cwt 1 qr 23 lbs?
- 18 If 17 cwt 2 qrs 14 lbs can be obtained for £8 13s 3½d what weight can be obtained for £21 10s 2½d?
- 19 The clothing of a regiment of 735 men costs Rs 13987 12a, what will the clothing of a regiment of 903 men cost at the same rate?

20 The interest on Rs 2719 2a 8p for 77 days is Rs 31 13a 8p find the interest on the same sum for 245 days

21 A person in 87 days spends Rs 389 11a, in how many days will he spend Rs 1634 14a 4p at the same rate?

22 If 15 workmen can do a piece of work in 25 days, in what time can 25 men do the same?

23 How much in length, that is 3 ft 9 in broad, will be equivalent to 37 ft 9 in in length, which is 7 ft 6 in broad?

24 If 69 yds of carpet 3 qrs wide, cover a room 8 yds 2 qrs 2 nls long, find the width of the room

25 If a person's annual income be 650 guineas, how much will he have saved at the end of the year, after spending £10 13s 9½d a week?

26 What may a person, having an income of Rs 10000 a year spend daily, so as to lay by Rs 4342 8a yearly?

27 If I lend a friend Rs 2500 for 6 months, how long ought he to lend me Rs 1875, to requite the kindness?

28 What is the tax upon Rs 3021 12a 8p, when Rs 4294 2a is rated at Rs 6 12a?

29 If the rate levied upon a rental of Rs 7637 8a amount to Rs 1336 9a, how much is it in the rupee?

30 A person buys 136 yds of cloth for Rs 1500, and retails it at Rs 19 per yard, what does he gain by the transaction?

31 A person's daily income is Rs 17 8a, and his quarterly expenditure is Rs 1355, how much will he have saved at the end of 9 years?

32 If a gentleman spend £152 10s every week, what must be his daily income that in 15 years he may lay by £7522 10s? (a year = 52 weeks)

33 How many ducats of 4s 11½d each are equal in value to 55926 rls dollars of 4s 10½d each?

34 After payment of an income-tax of 7d in the £ a person has left £249 19s 9½d, find his full income

35 A bankrupt's debts amount to Rs 5355 3a 4p and his assets to Rs 3213 2a How much can he pay in the rupee?

36 A certain number of reapers can reap 84 ac 3 ro 14 po in 13½ hours, in how many hours can they reap 40½ ac 8 po?

37 If a person having an income of Rs 1855 has to pay an income-tax of Rs 54 1a 8p, what is the income of a person who pays Rs 306 4a?

38 A merchant exchanged 1134 yds of velvet for 5313 yds of silk at Re 1 11a per yard, find the value of the velvet per yard

39 A bankrupt pays $9\alpha\ 3\frac{2}{5}\text{p}$ in the rupee, what will be lost on a debt of Rs 27350?

40 A person after paying for the first half of a year an income-tax of $4\frac{1}{2}\text{p}$ in the rupee, and for the second half one of $7\frac{1}{2}\text{p}$ in the rupee on his income, has Rs 3535 15α left, what was the income on which he paid?

41 A merchant paid Rs 187 8α for a year's income-tax, but after the tax has been increased to 72 pies in the rupee, he paid Rs 525, what was his income, and at what rate in the rupee was the tax levied at first?

42 A person bought 180 gallons of wine for Rs 1250, find the quantity of water to be added that he may retail the mixture at Rs 6 4α a gallon.

43 If an estate produce £1680 a year, and the land tax be payable upon this sum at 3s 6d in the pound, what is its clear annual value?

44 When a bankrupt's effects pay three dividends of 4s $2\frac{1}{2}\text{d}$, 3s $2\frac{1}{2}\text{d}$ and 2s $4\frac{1}{2}\text{d}$ in the £, what do his creditors lose upon his entire debt which is £4265?

45 A person bought 125 yds of cloth, at the rate of 2 yds for Rs 2 8α and 125 yds at the rate of 3 yds for Rs 2 8α , what will he gain or lose by selling the 250 yards at the rate of 5 yds for Rs 5?

46 A garrison of 638 men has provisions for 124 days, how long will the provisions last if the garrison be reinforced by 418 men?

47 If $1\frac{1}{2}$ yds of silk cost Rs 5 $7\alpha\ 6\text{p}$, what will be the cost of 75 yards? and how many yards can be got for Rs 50?

48 If $2\frac{1}{2}$ lbs of tea cost Rs 4 $5\alpha\ 8\text{p}$, how much will $27\frac{1}{2}$ lbs of the same quality cost?

49 The price of 0625 lb of coffee is 4583s, what is the value of 075 of a ton?

50 If when the price of wheat is 55 5s a quarter, the 6d loaf weighs 34375 lbs, what is the price of wheat when the loaf weighs 28125 lbs?

51 If 5 lbs of sugar cost 0703125 of Rs 8, what will 0625 cwt of the same quality of sugar cost?

52 A has 3 lbs $1\frac{3}{4}\text{oz}$ of ginger worth $10\alpha\ 2\text{p}$ a lb which he exchanges with B for 3 lbs 8oz of pepper, what is the value of the pepper per lb?

53 If 7 oxen or 11 horses can eat the grass of a field in 37 days, in how many days will 5 oxen and 8 horses eat it?

54 A clock, set on Friday at 8 P.M. loses $2\frac{1}{2}$ min daily, what is the correct time when the clock strikes 8 next Tuesday morning?

55 If 1000 men have provisions for 50 days, and if after

10 days, 150 men go away, find how long the remaining provisions will serve the number left

56 Find the height of a tower which casts a shadow of 75 ft 6 in long, when the length of the shadow of a walking stick 3 ft 3 in long is 2 ft 9 in

57 If 7 boys earn as much as 4 men, and 48 men assisted by 14 boys earn Rs 423 8a, what number of boys assisting 20 men will earn Rs 272 4a in the same time?

58 A besieged town containing 22,400 inhabitants has provisions for 3 weeks, how many must be sent away that they may be able to hold out 7 weeks?

59 Two watches one of which gains 3 min while the other loses 5 min a day are set right at 10 o'clock A M on Wednesday the 26th February, 1868, when will there be a difference of 90 minutes?

60 If 428571 of a barrel of beer be worth 72 of £2 10s, what is the value of 625 of the remainder?

61 If 4 men working 15 hours, 3 men working 12 hours, and 8 men working 3 hours earn Rs 32 8a, what will a man's wages for 6 days come to, if he works 11 hours a day?

62 If $\frac{1}{12}$ of 3, of $\frac{1}{2}$ of $5\frac{1}{2}$ of 22 lbs of sugar cost 5a 6d, how much will 1 ton 11 cwt 3 qrs cost?

63 If 4 men or 6 women or 9 boys can perform a piece of work in $27\frac{1}{2}$ days, in what time can (i) 5 men and 9 women perform it? and (ii) 5 men and 8 boys perform it?

64 A clock which was 12 min fast at a quarter to 11 P M on Nov 28, was exactly right at 11 30 P M the following day. How many minutes was it slow at a quarter to 2 P M on Dec 7?

65 If 17 men can do a piece of work in 89 days, and if after working for 33 days, 3 men leave, in how many days in all will the work be done?

66 If 13 men, 10 women and 17 boys can complete a piece of work in 50 days, how long would 7 men, 13 women and 14 boys be in doing twice as much work, the parts done by each in the same time being as the numbers 3, 2 and 1?

67 If a besieged garrison have 1 months provisions at the rate of 18 chataks per man per day, how long would they be able to hold out, if each man were allowed only 12 chataks per day?

68 If the 6d loaf weighs 3 chataks when wheat is at Rs 3 per maund, what ought it to weigh when wheat is at Rs 3 6a per md?

69 A hare starts with 25 of her leaps in advance of a hound, and takes 4 leaps to the hound's 3, but 2 of the hound's leaps are equal to 3 of the hare's, how many leaps must the hound take to overtake the hare?

70 A besieged fortress has provisions for 3 weeks, at the

rate of 14 ch a day for each man , at what rate per day must the provisions be distributed, so that the place may hold out 5 weeks ?

71 A person contracts to make a road 189 miles long in 15 months . He employs 256 men, but after 3 months finds that he has only finished 28 miles . How many men must he now employ to finish it within the time required ?

72 If 7 gallons of brandy cost as much as 9 gallons of rum , and 9 gallons of rum as much as 12 gallons of gin, and the cost of 3 gallons of these, taken one of each kind, be £2 2s 6d, what is the value of each per gallon ?

73 A servant's wages are fixed at Rs 231 a year , he enters his situation on the 12th of July, and leaves on Christmas day in the same year , his master had advanced him Rs 36 12a, what sum ought he to receive on leaving ?

74 A man takes 55 min to walk to Barrackpore by the road, and 58 min 40 sec to return by the fields, walking at the same pace . The distance by the road is $3\frac{1}{2}$ miles , what is it by the fields ?

75 If 4 men and 5 boys earn Rs 46 1a 4p in 7 days, and 3 men and 8 boys earn Rs 60 6a in 9 days , in what time will 12 men and 12 boys earn Rs 388 8a ?

76 A piece of gold at Rs 38 15a per oz is worth Rs 1500 , what will be the value of a piece of silver of equal weight at Rs 27 4a per lb ?

77 A contractor undertook to build a house in 21 days, and engaged 15 men to do the work . But after 10 days he found it necessary to engage 10 men more, and then he accomplished the work one day too soon . How many days behindhand would he have been if he had not engaged the 10 additional men ?

78 A ship leaves port with sufficient provisions to last 14 weeks , 6 of the crew absconded upon setting sail, and the voyage lasted 16 weeks, at the end of which time the provisions were just exhausted , find the number of the full crew

79 A fixed rent of £1170 per annum is converted into a corn rent of one-half wheat at the average price of 48s per quarter , and the other half barley at the average price of 30s per quarter , what will be the rent when wheat has advanced to 56s and barley to 32s per quarter ?

80 A railway train travels 27 miles an hour, including stoppages, and 30 miles an hour when it does not stop , in what distance will it lose 45 minutes by stopping ?

IV. DOUBLE RULE OF THREE

487 Double Rule of Three or Compound Rule of Three is a process in which five quantities are given to find a sixth,

and four of the five given quantities form two parts of different kinds, and the fifth and the answer required form a third pair of another kind also the quantities of the first and second kind are directly or inversely proportional to the given quantity of the third kind

488 This definition of Double Rule of Three may be extended to cases where seven quantities are given to find an eighth, nine to find a tenth, and so on

489 For the sake of convenience, we may divide each question in Double Rule of Three into two parts, the *supposition* and the *demand*, the *supposition* being the part which expresses the conditions of the question, and the *demand* the part which mentions the thing demanded or sought

Thus, in the question—If 16 horses eat 54 mds of corn in 6 days, how many horses will eat 135 mds in 8 days?—we might arrange the terms thus

Supposition—16 horses, 54 mds 6 days

Demand—horses reqd 135 mds 8 days

Here, we see that four of the given quantities form two pairs of different kinds,—*maunds* and *days*, and the fifth and the answer form a third pair of another kind —*horses*

Also the number of horses is inversely proportional to the number of days and directly proportional to the number of maunds

490 We will now explain the ordinary method of Solution, which is by two or more Rule of Three statements, and by compounding them into one final statement, hence the name Double Rule of Three and Compound Rule of Three

Taking the preceding Example, we can divide it into two parts —

First—If 16 horses eat 54 mds of corn in 6 days, how many horses will eat 135 mds in the same time?

Here, since the time is the same in the two hypotheses, the number of horses will depend on, and is *directly* proportional to the number of maunds Hence we have the following statement —

(i) 54 mds	135 mds	16 horses	no of horses reqd
54	135	16	$(16 \times 135) \div 54$ or 40 horses.

Secondly—If 40 horses eat 135 mds in 6 days, how many horses will eat the same number of maunds in 8 days?

Here, the number of maunds being the same in the two hypotheses, the number of horses depends on, and is *inversely* proportional to, the number of days, hence the statement, —

(ii) 8 days	6 days	40 horses	no of horses reqd
or 8	6	40	no of horses reqd

Compounding the statements (i) and (ii), we have

54 × 8	135 × 6	16 × 40	40 × no of horses reqd
		16	no of horses reqd

This last statement is generally written thus —

$$\frac{54}{8} \quad \frac{135}{6} \quad \left. \vphantom{\frac{54}{8} \quad \frac{135}{6}} \right\} = 16 \text{ no of horses reqd}$$

$$\therefore \text{no of horses reqd} = \frac{16 \times 135 \times 6}{54 \times 8} = \underline{30} \text{ Ans}$$

491 From the above considerations, we deduce the following Rule —

RULE *For the Statement* Write down as the third term that quantity which corresponds to the quantity required. Then take the first pair of given quantities of the same kind, and with them make the Rule of Three statements with reference to the third term *only*, do the same with the second pair, and with each succeeding pair writing these ratios under one another.

For the Operation Reduce the first and second terms in each of these statements to the same denomination. Also, reduce the common third term to a single denomination if it be not already in that state, and then treat the terms as abstract numbers. Multiply all the first terms together for a final first term, and all the second terms together for a final second term, and retain the former third term. In this final statement multiply the second and third terms together and divide the product by the first. The quotient will be the answer to the question in the denomination to which the third term was reduced.

Ex 1 If 5 men earn Rs 187 8a in 12 weeks, how much will 16 men earn in 20 weeks?

$$\begin{array}{l} 5 \text{ men} \quad 16 \text{ men} \\ 12 \text{ wks} \quad 20 \text{ wks} \end{array} \left. \vphantom{\begin{array}{l} 5 \text{ men} \quad 16 \text{ men} \\ 12 \text{ wks} \quad 20 \text{ wks} \end{array}} \right\} = \text{Rs } 187 \text{ 8a} \quad \text{Rs reqd} \left\{ \begin{array}{l} \text{More men, more earning,} \\ \text{more wks, more earning,} \end{array} \right.$$

$$\frac{5 \times 12}{16 \times 20} \quad \text{Rs } 32^5 \quad \text{Rs reqd}$$

$$\therefore \text{no of Rs reqd} = \frac{16 \times 20 \times 375}{5 \times 12 \times 2} = \text{Rs } \underline{1000} \text{ Ans}$$

492 Every question in Double Rule of Three may be solved by the *Unitary Method* —

Taking the preceding Example we proceed thus —

Since 5 men in 12 weeks earn Rs 187¹,

\therefore 5 men in 1 week earn Rs $187\frac{1}{12} \times \frac{1}{12}$,

\therefore 1 man in 1 week earns Rs $187\frac{1}{12} \times \frac{1}{5} \times \frac{1}{12}$,

\therefore 1 man in 20 weeks earns Rs $187\frac{1}{12} \times \frac{1}{5} \times \frac{20}{12}$,

\therefore 16 men in 20 weeks earn Rs $187\frac{1}{12} \times \frac{16}{1} \times \frac{20}{12} = \text{Rs } \underline{1000} \text{ Ans}$

Ex 2 If 3 tons 16 cwt can be carried 25 miles for Rs 118 12a, what weight can be carried 52 miles for Rs 59 9a 4p?

Rs 118 12a = Rs 118³, Rs 59 9a 4p = Rs 59¹, 3 tons 16 cwt = 76cwt

$$\frac{52 \text{ mi} \quad 25 \text{ mi}}{\text{Rs } 118^3 \quad \text{Rs } 59^1} \left. \vphantom{\frac{52 \text{ mi} \quad 25 \text{ mi}}{\text{Rs } 118^3 \quad \text{Rs } 59^1}} \right\} = 76 \text{ cwt wt reqd} \left\{ \begin{array}{l} \text{More miles, less weight,} \\ \text{more money, more weight} \end{array} \right.$$

$$\therefore \text{weight reqd} = \frac{76 \times 25 \times 715 \times 4}{52 \times 475 \times 12} = \underline{18\frac{3}{4}} \text{ cwt Ans}$$

Ex 3 If a penny loaf weigh 6 oz when wheat is 5s 6d a bushel, what should be the weight of a shilling loaf when wheat is 8s 3d a bushel?

$$\left. \begin{array}{ll} 1d & 12d \\ 8\frac{3}{4}s & 5\frac{1}{2}s \end{array} \right\} = 6 \text{ oz} \quad \text{weight reqd} \quad \left\{ \begin{array}{l} \text{More price of loaf, more weight,} \\ \text{more price of wheat, less weight.} \end{array} \right.$$

$$\text{weight reqd} = \frac{6 \times 12 \times 11 \times 4}{2 \times 33} \text{ oz} = 48 \text{ oz} = \underline{3 \text{ lbs}} \quad \text{Ans}$$

Ex 4 If 144 men in 5 days of 11 hours each can dig a trench 132 yds long, 5 ft wide and 2 ft deep, in how many days of 9 hours each can 56 men dig a trench 210 yds long, 8 ft wide and 3 ft deep?

$$\left. \begin{array}{ll} 56 \text{ men} & 144 \text{ men} \\ 9 \text{ hrs} & 11 \text{ hrs} \\ 132 \text{ yds} & 210 \text{ yds} \\ 5 \text{ ft} & 8 \text{ ft} \\ 2 \text{ ft} & 3 \text{ ft} \end{array} \right\} = 5 \text{ days} \quad \text{days reqd} \quad \left\{ \begin{array}{l} \text{More men, less days,} \\ \text{more hours, less days,} \\ \text{more length, more days;} \\ \text{more breadth, more days,} \\ \text{more depth, more days} \end{array} \right.$$

$$\text{days reqd} = \frac{5 \times 144 \times 11 \times 210 \times 8 \times 3}{56 \times 9 \times 132 \times 5 \times 2} = \underline{60} \quad \text{Ans}$$

Ex 5 A garrison of 4500 men is supplied with provisions for 15 months at the rate of 13 chataks per day per man, how many must leave, that the same provisions may supply those that remain 27 months at 10 chataks per day per man?

$$\left. \begin{array}{ll} 27 \text{ mo} & 15 \text{ mo} \\ 10 \text{ ch} & 13 \text{ ch} \end{array} \right\} = 4500 \text{ men} \quad \text{men reqd} \quad \left\{ \begin{array}{l} \text{More months, less men,} \\ \text{more chataks, less men} \end{array} \right.$$

$$\therefore \text{men reqd} = \frac{4500 \times 15 \times 13}{27 \times 10} = 3250$$

$$\text{the no of men that must leave} = 4500 - 3250 = \underline{1250} \quad \text{Ans}$$

Ex 6 If 48 cannon firing 4 rounds in 7 minutes kill 540 men in 1½ hours, how many cannon firing 8 rounds in 9 minutes will kill 840 men in 40 minutes?

$$\left. \begin{array}{ll} 8 \text{ rounds} & 4 \text{ rounds} \\ 7 \text{ min} & 9 \text{ min} \\ 540 \text{ men} & 840 \text{ men} \\ \frac{3}{4} \text{ hr} & 1\frac{1}{2} \text{ hrs} \end{array} \right\} = 48 \text{ cannon} \quad \text{cann reqd} \quad \left\{ \begin{array}{l} \text{More rounds, less} \\ \text{more interval, more} \\ \text{more men, more} \\ \text{more hours, less} \end{array} \right\} \text{cannon}$$

$$\therefore \text{cannon reqd} = \frac{48 \times 4 \times 9 \times 840 \times 3 \times 3}{8 \times 7 \times 540 \times 2 \times 2} = \underline{108} \quad \text{Ans}$$

Examples CXLVII

1 If 6 men can mow 48 bighas in 4 days, how long will it take 10 men to mow 120 bighas, at the same rate?

2 If 67 maunds carried 87 miles cost Rs 242 14a, what will 73 maunds carried 93 miles cost?

3 If 939 men consume 364 maunds of wheat in 7 months, how many will consume 1404 maunds in 13½ months?

4 If a tradesman with a capital of *Rs* 10000 gains *Rs* 900 in 7 months, how long will he be in gaining *Rs* 202 8*a* with a capital of *Rs* 3150 ?

5 If 37 tons carried 57 miles cost *Rs* 87 14*a* , what weight can be carried 83 miles for *Rs* 217 14*a* ?

6 If 29 mds 16 sr be carried 40 miles for *Rs* 5 13*a* 4*p*, how far ought 9 mds 32 sr to be carried for *Rs* 8 7*a* 4*p* ?

7 If a regiment of 1878 soldiers consume 245 mds 28 sr of wheat in 336 days, how many maunds will an army of 22356 men consume in 112 days ?

8 If the penny loaf weigh 6 oz when wheat is at *Rs* 2 12*a* per maund, what should be the price of a loaf weighing 4½ lbs when wheat is at *Rs* 4 2*a* per maund ?

9 If the carriage of 5 cwt 1 qr 12 lbs for 39 miles be £2 8*s* 6*d*, what must be paid for the carriage of 7 cwt 16 lbs for 48¼ miles ?

10 If the wages of 3 men be *Rs* 150 for 20 days, how many men will earn *Rs* 157 8*a* in 9 days at the same rate ?

11 If a quantity of provisions will serve a besieged garrison of 1500 men for 12 weeks at the rate of 20 oz a day for each man, how many men would the same provisions maintain for 20 weeks at the rate of 8 oz a day for each man ?

12 If 5 men can reap a rectangular field whose length is 800 feet and breadth 700 feet in 3½ days of 14 hours each, in how many days of 12 hours each can 7 men reap a field 1800 feet long and 960 feet broad ?

13 If 21 horses and 217 sheep can be kept 10 days for *Rs* 564 2*a* 8*p*, what sum will keep 9 horses and 60 sheep for 27 days, supposing that 3 horses eat as much as 50 sheep ?

14 If the 4*d* loaf weigh 1 lb 9½ oz when wheat is at 9*s* 3*d* per bushel, how much bread can be got for 5*s* 7½*d* when wheat is at 76*s* per quarter ?

15 If 30 cannon firing 4 rounds in 5 min kill 640 men in an hour how many men would be killed in an hour and a half by 10 cannon firing at the rate of 3 rounds in 4 min ?

16 If 60 guns firing 5 rounds in 6 min kill 350 men in 1¼ hrs , how many guns firing 7 rounds in 9 min will kill 980 men in 25 min at the same rate ?

17 If 5 horses require as much corn as 8 ponies, and 15 quarters of corn last 12 ponies for 64 days, how long may 25 horses be kept for £41 5*s*, when corn is 22*s* a quarter ?

18 If 20 men could perform a piece of work in 12 days, find the number of men who would perform another work three times as great in one fifth of the time

19 If 9 men can reap 15 ac 1 ro 28 po in 5 days of 10½ hours each, how many men will reap 401 ac 8 po in 7 days of 11½ hrs each?

20 If when copper is at £7 14s 4½d per cwt I can get 3 cwt 2 qrs 14 lbs of brass for £27 0s 3½d, how much brass shall I get for £153 17s 6d when copper is at £9½ per cwt ?

21 A person is able to perform a journey of 142 2 miles in 4½ days when the day is 10 16½ hours long, how many days will he be in travelling 505 6 miles when the days are 8 4 hours long ?

22 If the 6d loaf weigh 4 35 lbs when wheat is at 5 75s per bushel, what ought to be paid for 49 3 lbs of bread when wheat is at 9 2s per bushel ?

23 If 240 men working 10 hours a day can dig a trench 300 yds long, 3½ ft deep and 2½ ft wide in 6 days, in how many days would 80 men, working 9 hours a day, dig a trench 500 yds long, 3 ft wide and 2½ ft deep ?

24 If 30 men, 6 women and 5 boys can dig a trench 120 yds long, 5 wide and 2 deep in 16 days of 10 hours each, in how many days of 8 hours each will 24 men, 8 women and 4 boys dig a trench 144 yds long, 6 wide and 4 deep, supposing that 5 women can do the work of 3 men, and 2 boys that of 1 man ?

25 Two cogged wheels, of which one has 15 cogs and the other 28, work in each other. If the first turn 16 times in 7½ seconds, how often will the other turn in 21 seconds ?

26 Two gangs of 6 and 9 men are set to reap two fields of 35 and 45 bighas respectively. The first gang works 7 hours in the day, and the latter 8 hours. If the first gang complete their work in 12 days, in how many days will the second complete theirs ?

27 If 10 compositors, in 16 days of 14 hrs each, can compose 20 sheets of 24 pages in each sheet, 50 lines in a page, and 40 letters on the average in a line, in how many days of 7 hours each can 20 compositors compose a volume, to be printed in the same letter, containing 40 sheets, 16 pages in a sheet 60 lines in a page and 50 letters in a line ?

28 A piece of work is to be done in 36 days, 15 men work at it 15 hours a day, but after 24 days only ¾ of it is done, if 3 more men are put on, how many hours a day must all work to finish it in the given time ?

29 If 1000 men, besieged in a fort and supplied with provisions for 5 weeks at the rate of 10 oz a day for each man, be reinforced by 500 men, to what daily amount must the provisions be reduced for each man if it be desired to make the original supply last for 8 weeks ?

30 If 7 women earn as much as 4 men, and 48 men assisted by 14 women earn 121 guineas in 17 days, what number of women assisting 20 men will earn £21 3s 6d in one third of the time ?

31 Two sets of men perform the same amount of work. Each man in the first set is stronger than each one in the second in the

ratio of 7 to 6, the first set works 6 days a week for 10 weeks, and the second set, 5 days a week for 7 weeks. If there are 9 men in the first set, how many are there in the second?

32 If 48 pioneers in 5 days of 12 $\frac{1}{2}$ hours each can dig a trench 13075 yards long, 45 yds wide and 25 yds deep, how many hours per day must 90 pioneers work during 42 days in order to dig a trench 16366875 yds long, 4875 yds wide and 32 yds deep?

33 If the rent of 77 bu 10 kat 4 ch of land be Rs.1572 8a for 1 year 20 days, of how much land will the rent be Rs 1258 for 44 days?

34 If the rent of a farm of 53 ac 1 ro 6 po, be Rs 1008 12a what would be the rent of another farm containing 17 ac 3 ro 2 po if 6 acres of the latter be worth 7 acres of the former?

35 If the wages of 25 men amount to Rs 766 10a 8p in 16 days how many men must work 24 days to receive Rs 1035, the daily wages of the latter being one half of those of the former?

36 The cost of paving a yard with tiles 9 in square is Rs 181 8a what would have been the cost had the tiles been 9 in long and 6 in broad, their price being to that of the former as 5 : 8, and the cost of labour being $\frac{1}{8}$ and $\frac{1}{4}$ of the price of each tile in the former and the latter case respectively.

37 If 17 men by working 8 hours a day made an excavation 120 ft 6 in long, 25 ft 6 in broad and 24 ft deep in 54 days, how many hours daily must 18 men work during 51 days in order that they may make an excavation whose length and breadth are 1 ft 6 in less and depth 1 ft 6 in greater than the preceding one, supposing that 9 men of the latter do as much as 10 men of the former?

38 A contract is to be finished in 200 days and 50 men are put on to work at once, at the end of $\frac{1}{3}$ ths of this time, it is found that only $\frac{1}{4}$ th of the work is done. What extra number of men will be required to complete the contract in the given time, the last employed men to work 12 hours per day, whilst the first 50 men work, until the contract is finished, only 10 hours per day?

39 If 38 men working 6 hours a day can do a piece of work in 12 days, find in what time 57 men working 8 hrs a day can do a piece of work twice as great, supposing 2 men of the first set to do as much work in 1 hour, as 3 men of the second set can do in 1 $\frac{1}{2}$ hours.

40 A contractor agrees to execute a certain piece of work in a certain time. He employs 55 men who work 9 hrs daily. When $\frac{2}{3}$ ths of the time expired, he finds that only $\frac{1}{3}$ ths of the work is done. How many men must he employ during the remaining part of the time, working 11 hrs daily, in order that he may fulfil his contract?

41 If 5 pumps, each having a length of stroke of 3 feet, working 15 hours a day for 5 days, empty the water out of a mine, what must be the length of stroke of each of 15 pumps which, working 10

hours a day for 12 days, would empty the same mine, the strokes of the former set of pumps being performed 4 times as fast as those of the latter ?

41 If 10 men reap 4 acres of corn in 4 days, working 10 hours a day, how many men would be required to reap 21 acres in 12 days, working 12 hours a day, supposing that a reaper who works 10 hours a day does $\frac{3}{5}$ th part more work in an hour than one who works 12 hours a day ?

43 If 20 men can perform a piece of work in 12 days, how many men will perform a piece of work half as large again in a fifth part of the time, if they work the same number of hours per day, supposing that 2 of the second set can do as much work in an hour as 3 of the first set ?

44 If it require 4199 bricks, each 9 in long, 4 in wide and 3 in thick to build a certain wall, how many will be wanted to build a wall of double the length, half the height, and half the thickness, each brick being $9\frac{1}{2}$ in long, $4\frac{1}{2}$ in wide and $3\frac{1}{2}$ in thick ?

45 A town is garrisoned with 10000 troops and has provisions sufficient for that number for 6 months, allowing a ration of $3\frac{1}{2}$ lbs daily to each man. How many must be sent away so that by giving $\frac{1}{2}$ lb less to each man, the provisions may last for 8 months ?

46 A boat is propelled by 8 oars which take 10 strokes per minute, and it goes at the rate of 9 miles an hour. Find the rate of a boat propelled by 6 oars which take 8 strokes per minute, when 5 strokes of each man in the latter boat are equivalent to 6 strokes of each man in the former.

47 If a person drink 5 dozen bottles of wine in 3 months when the wine is Rs 2 a bottle, how many bottles of wine may he drink in 3 months, without increasing the expense, when the price is Rs 2 8a a bottle ?

48 If a wall 660 yds long, 10 ft high and $22\frac{1}{2}$ in thick, be built by 27 men in $31\frac{1}{4}$ days of 11 hours each, in how many days of 12 hours each could 60 men build a wall 12 ft high and $2\frac{1}{2}$ ft thick round a park $4\frac{1}{2}$ miles in circumference ?

49 If it cost Rs 118 2a to supply a family of 12 persons for 5 weeks, when rice is at Rs 7 8a per maund, how much will it cost to supply a family of 18 persons for 7 weeks, when rice is at Rs 10 per maund ?

50 If a family of 9 people in Calcutta spend Rs 16380 in a year, what must be the expenses of a family of 8 people to live in Jessore in the same style for 7 months, the prices there being $\frac{7}{8}$ of what they are in Calcutta ?

51 A farmer engages 30 men and 45 women to cut down his crop in 20 days of 12 hours each, but after 12 days' reaping, he wishes the remainder of the crop to be cut down in 4 days of 10

hours each, how many additional men must he employ, supposing 2 men to do as much work as 3 women?

52 If 8s 2d worth of bread be consumed in 7 days, by a family consisting of 6 adults and 5 children, when the price of the quartern loaf is $7\frac{1}{2}d$, how many adults, along with 3 children, will consume 12s 9d worth of bread in 15 days, when the price of the quartern loaf is $8\frac{1}{2}d$, supposing that 3 children consume as much per day as 2 adults?

53 If Rs 50 is sufficient to maintain 8 coolies for a fortnight, when rice is $2\frac{1}{2}$ seers for 6s, how much will be required to maintain 6 coolies for 25 days, when rice is 2 maunds for Rs 7?

54 If 5 steam engines of 9 horse-power in 3 weeks (when employed 3 days in the week and 10 hours a day) raise through a certain altitude 25 three bushel sacks of wheat, weighing 60 lbs a bushel, in what time will 9 engines of 8 horse power (when employed 5 days in the week and 9 hours a day) raise through 15 times the former altitude, 75 two bushel sacks of wheat, weighing 63 lbs a bushel?

55 A canal, which was $14\frac{1}{2}$ miles long, 21 yds wide and $19\frac{1}{2}$ ft deep, was dug by 1200 men in 8 months. What was the depth of another canal, which was dug by 1500 men in a year, it being $17\frac{1}{2}$ miles long and 72 ft wide, remembering that the time employed for completing a mile of the second canal, with a given number of men, was (owing to greater difficulties in cutting the rock) to the time employed over a mile of the first as 13 8?

V THE RULE OF PROPORTION

493 As has been observed in The Rule of Three of which this is only another name, we have here *three* quantities either simple or compound given, to find a *fourth* which shall complete the proportion, and this is a *fourth proportional* to the three quantities proposed.

494 Assuming as an *Axiom*, that effects have the same relation or ratio to each other as the causes which produce them under the same circumstances, it is evident that in any two causes of the same kind we shall have the following proportion —

First Cause Second Cause First Effect Second Effect,
and then, what was said in Articles 472 and 473 will enable us to find any *one* term if the *three* others be supposed to be given.

495 To avoid the trouble of writing the name of the required term or quantity at length, we shall always denote it by the simple symbol x which must be treated in the same way as any other number, and it may occupy any place in the proportion either by itself or as a *factor*, either *integral* or *fractional* with given numbers, as in the following Examples

Ex 1 If 5 men can mow 12 acres of grass in a certain time,

now many acres will 16 men be able to mow in the same or an equal time?

Here, 5 men } are the first and second { *Causes*,
 16 men }
 12 acres } are the first and second { *Effects*
 7 acres }

whence, we have the following proportion —

$$5 \text{ men } 16 \text{ men } . 12 \text{ ac } x \text{ ac } ,$$

and therefore by the Articles just referred to, we find

$$5 \times x = 16 \times 12 = 192 ,$$

$$\text{whence } x = 1\frac{2}{5}^2 \text{ ac} = \underline{38 \text{ ac } 1 \text{ ro } 24 \text{ po}} \quad \text{Ans}$$

Ex 2 If 8 oz of bread be sold for 6d when wheat is at £15 a load, what should be the price of wheat when 12 oz are sold for 4d?

If the price of a load of wheat be *regulated* by, so as to be *proportional* to, the price of an ounce of bread, since,

in the former case the price of 1 oz = $\frac{3}{4}d = \frac{3}{4}d$,

and in the latter the price of 1 oz = $\frac{1}{2}d = \frac{1}{2}d$,

we must have the following proportion,

$$\frac{3}{4}d \quad \frac{1}{2}d \quad £15 \quad £x ,$$

whence $x = (\frac{1}{2} \times 15) - \frac{3}{4} = £6\frac{13}{4} = £6 \underline{13s \ 4d}$, which is the reqd price

These Examples, the causes in which are simple terms being dependent upon only *one* magnitude, are instances of what is called **Direct Proportion**, because the effect is *greater* or *less* in the same proportion as the cause is *greater* or *less*

Ex 3 If 10 men can perform a piece of work in 12 days, how many days will it take 8 men to do the same?

Here, the causes will evidently be to each other as 10×12 to $8 \times x$, and the effects are the *same*, and may therefore be represented by 1, or any other symbol

$$\text{whence, } 10 \times 12 \quad 8 \times x \quad 1 \quad 1 ,$$

$$\text{therefore } 8 \times x = 10 \times 12 = 120, \text{ and } x = 120 \div 8 = \underline{15 \text{ days}} \quad \text{Ans}$$

Ex 4 How much in length, that is 3 ft 9 in broad, will be equal to what is 37 ft 9 in long and 7 ft 6 in broad?

Here, the first cause = 45 in $\times x$ in, the second cause = 90 in \times 453 in, and the effects are to be equal

$$\text{therefore } 45 \times x \quad 90 \times 453 \quad 1 \quad 1 , \text{ whence } 45 \times x = 90 \times 453,$$

$$\text{and } x = (90 \times 453) \div 45 = 906 \text{ in} = \underline{75 \text{ ft } 6 \text{ in}} \quad \text{Ans}$$

In these two Examples, the *entire* causes are *compound* quantities depending upon two *subordinate* causes, and because the effect is the *same*, each subordinate cause is *less* or *greater* according as

the other is *greater* or *less*, constituting what is called **Inverse Proportion**

Ex 5 If a person can perform a journey of 100 miles in 12 days of 8 hours each how far will he be able to travel in 15 days of 9 hours each?

Here, 12×8 and 15×9 are the causes, and the distances travelled 100 and x are the effects, whence

$$12 \times 8 \quad 15 \times 9 \quad 100 \quad x,$$

$$\text{and } x = \frac{15 \times 9 \times 100}{12 \times 8} = 140\frac{1}{2} \text{ miles } \text{Ans}$$

Ex 6 If 60 maunds of corn feed 6 horses for 50 days, in how many days will 15 horses consume 75 maunds?

The causes are 6×50 and $15 \times x$, and the effects are 60 and 75 maunds, therefore

$$6 \times 50 \quad 15 \times x \quad 60 \quad 75, \text{ or } 2 \times 10 \quad x \quad 4 \quad 5,$$

$$\text{whence, } x = \frac{2 \times 10 \times 5}{4} = 25 \text{ days } \text{Ans}$$

In the former of these Examples, the distance travelled are in the *compound* ratio of the numbers of days and their lengths, and in the latter, the numbers of maunds have the same ratio as that which is *compounded* of the numbers of horses and days

Ex 7 If 25 labourers can dig a trench 220 yards long, 3 ft 4 in wide and 2 ft 6 in deep, in 32 days of 9 hours each, how many would it require to dig a trench half a mile long, 2 ft 4 in deep and 3 ft 6 in wide, in 36 days of 8 hours each?

First cause = $25 \times 32 \times 9$ } being the products of the *subordinate*
 Second cause = $x \times 36 \times 8$ } causes,
 First effect = $220 \times \frac{16}{3} \times \frac{1}{2}$ } the *mixed* quantities being reduced to
 Second effect = $880 \times \frac{1}{2} \times \frac{1}{2}$ } fractions of 1 yard

Hence, we have the following proportion —

$$25 \times 32 \times 9 \quad x \times 36 \times 8 \quad 220 \times \frac{16}{3} \times \frac{1}{2} \quad 880 \times \frac{1}{2} \times \frac{1}{2},$$

$$\text{or, } 25 \quad x \quad 1 \times 10 \times 5 \quad 4 \times 7 \times 7,$$

$$\text{whence, } x = \frac{25 \times 4 \times 7 \times 7}{1 \times 10 \times 5} = 98 \text{ labourers } \text{Ans}$$

These Examples, the causes and effects being simple and compound quantities consisting of their respective subordinate *partial* causes and effects, are instances of **Compound Proportion** in its fullest meaning

Ex 8 If 10 excavators can dig 12 loads of earth in 16 hours, whilst 12 others can dig 12 loads in 15 hours, find the time in which they will jointly dig 108 loads

Since, the ratio $10 \times 16 \quad 12 \times 15$ is *not* equal to the ratio 12

it follows that the individuals of the two sets do not work at the same rate, but the rate of one of the *first* set being represented by

$$\frac{12}{10 \times 16} = \frac{3}{40}, \text{ that of one of the second set will be equal to } \frac{9}{12 \times 15} = \frac{1}{20},$$

whence, $\{10 \times \frac{1}{40} + 12 \times \frac{1}{20}\} \times \text{the required time} = 108$,

or $(\frac{1}{2} + \frac{3}{10})$ of the required time = 108 hours,

that is, the required time = $\frac{10}{7}$ of 108 hrs = 80 hrs Ans

496 In practice, when the *partial* causes and effects consist of compound quantities, it is most convenient to express them by vulgar fractions or decimals, and when the *entire* causes and effects are compound quantities, to proceed as in *Sections III and IV* (all the Examples of which are instances of this Rule) shortening the operation as much as possible by means of Article 474

Examples CXLVIII

1 If when malt costs 63s a quarter the price of a quart of ale be $4\frac{1}{2}d$, what should its price be when malt is at 66s 6d per quarter?

2 If a person can perform a journey in 24 days of $10\frac{1}{2}$ hours each, what time will it take him to do the same when the days are $12\frac{1}{2}$ hours long?

3 If the expenses of 7 persons for 3 months amount to Rs 735, what will be the expenditure of 10 persons for 12 months at the same rate?

4 If 10 horses consume 7 mds 20 sr of oats in 7 days, in what time will 28 horses consume 30 maunds at the same rate?

5 If 10 men reap 20 acres of corn in 4 days, how many men can reap 70 acres in 10 days, at the same rate of labour?

6 If 7 lbs of sugar be sold for 4s 8d when the cost of a cwt is £3 7s 8d, what should be the cost of a cwt when 11 lbs is sold for 7s $1\frac{1}{2}d$?

7 If the 4a loaf weighs 1 lb 11 oz 12 drs when wheat is at Rs 3 12a per maund, what ought the 6a loaf to weigh when wheat is at Rs 2 10a per maund?

8 If 21 maunds be carried 40 miles for Rs 5 13a 4p, how far ought 7 maunds to be carried for Rs 8 7a 4p?

9 If 6664 men consume 357 maunds of wheat in 57 days, how many maunds of wheat will 1596 men consume in 119 days?

10 If the carriage of 13 cwt 2 qrs 19 lbs for 35 miles come to Rs 48 12a, what must be paid for the carriage of 41 cwt 1 lb for 49 miles?

11 If 12 men can perform a piece of work in 20 days, required the number of men who could perform another piece of work four times as great in a fifth part of the time

12 If with a capital of Rs 10000, a tradesman gain Rs 1000, in 7 months, in what time will he gain Rs 605, with a capital of Rs 3850 ?

13 If the 4^d loaf weighs 3 35 lbs when wheat is 4 75^s a bushel, what ought to be the price of wheat per bushel, when 47 5 lbs of bread cost 13 3^s ?

14 If 7 men can build a wall 245 yds long, 8 ft high and 18 in thick, in 35 days of 12 hours each, what length of wall, 10 ft high and 27 in thick, could 12 men build in 43 days of 10 hours each ?

15 If 27 men can do 1 piece of work in 14 days, working 10 hrs a day, how many hours a day must 24 boys work, in order to complete the same in 45 days, the work of a boy being half that of a man ?

16 If 1 artillery men can fire a gun 48 times and 5 men 52 times in an hour, how much more time will be required for firing 21216 shots from 26 guns, when there are 4 men to 1 gun than when there are 5 men ?

17 If 10 cannon which fire 3 rounds in 5 minutes, kill 270 men in 1½ hours, how many cannon which fire 5 rounds in 6 minutes, will kill 500 men in 1 hour, at the same rate ?

18 If 120 men in 3 days of 12 hours each, can dig a trench 30 yds long, 2 ft broad and 4 ft deep, how many men would be required to dig a trench 50 yds long, 6 ft deep and 1½ yds broad in 9 days of 15 hours each ?

19 If 6 men can reap 15 acres in 3 days of 14 hrs each and 10 boys can reap 10½ acres in 5 days of 9 hours each, find the ratio of the work of a man to that of a boy, and determine what number of acres 4 men and 7 boys together reap in a day

20 If beer which is brewed with 3 bushels of malt to the barrel cost 1^s 3^d per gallon, when malt is at 62^s 8^d the quarter, how much will beer cost per gallon, which is brewed with 5 bushels of malt to the barrel, when a quarter of malt costs 50^s ?

21 A town is garrisoned with 50000 troops and has provisions sufficient for that number for 3½ months, allowing 1 ration of 2 lbs daily to each man, how many must be sent away so that by giving an additional ½ lb to each man the provisions may last 14 months ?

22 A wall 700 yds long was to be built in 29 days. At the end of 11 days 18 men had built 220 yds of it, how many additional men was it then necessary to engage to work at the same rate, in order that the wall might be completed in the given time ?

23 If 6 horses eat 3 tons of hay in 15 days, how long will it take 18 sheep to eat 5 acres of grass, the voracity of 2 horses being equal to that of 7 sheep, and one acre producing 3 tons of grass, each ton being as satisfying as 1½ tons of hay ?

24 If 5 men, 4 women, and 7 boys can complete a piece of work in 60 days, how long will 9 men, 15 women and 18 boys take to complete a piece of work 5 times as great, the parts done by each man, woman, and boy respectively, in the same time, being as the numbers 3, 2, 1?

25 If 4 men and 2 boys can paper a room 23 ft long, 19 broad and 16 high, with paper 2 ft 8 in wide, in 7 days, working 12 hours a day, in how many days of 9 hours each can 4 men and 4 boys paper 3 rooms, each 32 ft long, 27 broad and 15 high, with paper 3 ft broad, a boy's working being 25 per cent of a man's?

CHAPTER XII

Miscellaneous Propositions

497. Percentage The term *per centum* or *per cent* means *for a hundred*

If a man has 75 sheep and if 3 of them are lost, we say that 4 per cent of the man's sheep are lost, meaning thereby that if the man had possessed 100 sheep, 4 would have been lost according to this rate of loss. The 4 is called the *rate per cent*

The symbol % or *p c* is used as an abbreviation for the words *per cent*

498 It is not usual to calculate gain, loss, increase, decrease, &c, at so much per unit, but at so much per cent or per 100 units, whatever the *unit* may be. When, however, the rate per unit or, more briefly, the rate is known, it is obvious that the *rate per cent* is found by multiplying the *rate per unit* by 100. It is worth noting that 5 per cent = $\frac{5}{100}$ of the whole, $7\frac{1}{2}$ per cent = $\frac{7\frac{1}{2}}{100}$ of the whole, 8 per cent = $\frac{8}{100}$ of the whole, whatever may be the whole or whatever may be the equal units constituting the whole, and so on with other percentages

Ex 1 Find how much per cent 15 is of 96, *ie*, find what number bears the same ratio to 100, that 15 bears to 96

96 100 15 rate per cent ,

$$\text{rate per cent} = \frac{100 \times 15}{96} = 1\frac{25}{8} = 15\frac{625}{100} \text{ Ans}$$

Ex 2 What rate per cent does the fraction $\frac{7}{80}$ denote?

$$\text{Rate per cent} = \frac{7}{80} \times 100 = 7\frac{1}{8} = 23\frac{1}{4} \text{ Ans}$$

Ex 3 The number of boys in a school increases in a certain period from 125 to 180, what is the increase per cent?

On 125 the increase is $(180 - 125)$ or 55 ,

. 125 100 55 increase per cent ,

$$\therefore \text{increase per cent} = \frac{55 \times 100}{125} = 44 \quad \text{Ans}$$

Ex 4 The population of a town increased 8 per cent from 1875 to 1885, and its population in the latter year was 9720, find its population in 1875

Every 100 men in 1875 increased to 108 men in 1885 ,

108 9720 100 pop in 1875 ,

$$\therefore \text{pop in 1875} = \frac{100 \times 9720}{108} = 9000 \quad \text{Ans}$$

Ex 5 Of the gross profits of a Railway Company in a certain year 41 per cent are spent to pay the working expenses, 56 per cent are paid to the shareholders, and the remainder Rs 150000 is reserved and the total profits

$100 - (41 + 56)$ or 3 per cent is reserved ,

. Rs 3 Rs 150000 Rs 100 total profits ,

$$\text{total profits} = \frac{\text{Rs } 150000 \times 100}{3} = \text{Rs } 5000000 \quad \text{Ans}$$

Ex 6 A house depreciates in value each year at the rate of 10 per cent of its value at the beginning of the year, and its value at the end of 3 years is Rs 14580. What was its original value ?

At the end of each year the value of the house is $\frac{90}{100}$ or $\frac{9}{10}$ of what it was at the beginning of the year ,

\therefore its value at the end of 1st year = $\frac{9}{10}$ of its original value,

its value at the end of 2nd year = $\frac{9}{10}$ of $\frac{9}{10}$ of its original value, and

its value at the end of 3rd year = $\frac{9}{10}$ of $\frac{9}{10}$ of $\frac{9}{10}$ of its original value
= $\frac{9^3}{10^3}$ of its original value

. $\frac{9^3}{10^3}$ of its original value = Rs 14580 ,

$$\therefore \text{its original value} = \text{Rs } \frac{14580 \times 1000}{729} = \text{Rs } 20000 \quad \text{Ans}$$

Examples CXLIX.

1 What fractions are denoted by the following rates per cent ?

(1) $12\frac{1}{2}$ (2) 15 (3) $\frac{1}{3}$ (4) $76\frac{1}{2}$ (5) 130 (6) $16\frac{2}{3}$

2 Find the values of —

(1) 6 per cent of 360

(2) $7\frac{1}{2}$ p c of Rs 240

(3) 18% of 325 mds

(4) $\frac{1}{4}\%$ of Rs 13675

(5) 75% of 250 bu 10 kat

(6) $9\frac{1}{2}$ p c of 1800 yds

(7) $62\frac{1}{2}\%$ of Rs 720 10 s

(8) 50 per cent of 4 hrs 25 min 12 sec

3 What percentage is —

- (1) 7 of $24\frac{1}{2}$? (2) 37 of 75? (3) 23 of 256?
 (4) $43\frac{3}{4}$ of 165? (5) $185\frac{1}{4}$ of 7321 75? (6) 57 men of 600 men?
 (7) Rs 782 8 of Rs 760? (8) 90 8.48 of 801 6?
 (9) £34 17s 9 $\frac{3}{4}$ d of £607 1s 11 $\frac{1}{4}$ d?

4 What rates per cent do the following fractions denote?

- (1) $\frac{1}{8}$ (2) $\frac{1}{3}$ (3) $4\frac{1}{2}$ (4) $1\frac{1}{2}$ (5) 35 (6) $5\frac{1}{2}$

5 The population of a city increased from 15635 to 18762 in five years, what was the rate per cent of the increase?

6 The population of a village decreased in a certain time from 1916 to 1437, find the rate per cent of decrease.

7 The population of York in 1861 was 40433, and it increased 832 per cent between 1861 and 1871, find the population in 1871.

8 The population of a city decreased 3311 per cent between 1861 and 1871, in 1861 it was 113357, find what it was in 1871.

9 The population of a certain country increased 5 per cent from 1875 to 1885 and its population in the latter year was 40841010, find its population in 1875.

10 The population of a town increased 35 per cent between 1851 and 1861, and 19 per cent between 1861 and 1871, the population in 1871 was 93177, find the population in 1851.

11 A farmer gave 100 bighas of land to each of his sons. After three years the eldest lost 50 per cent, and the second increased as much, what per cent was the eldest son's land now of that of the second?

12 A man took from a bank Rs 393, which was $131\frac{1}{6}$ per cent of what he had deposited, how much then remained?

13 A man spends $\frac{1}{2}$ of his money and then 5 per cent of what remains, what percentage of his money has he left?

14 Of what number is 57 men, $9\frac{1}{2}$ per cent?

15 A regiment which lost in an engagement $31\frac{1}{2}$ per cent of its men, had 440 men left, how many had it at first?

16 The population of a certain village is 4059, which is $12\frac{1}{2}$ per cent more than it was 5 years ago, what was it then?

17 A High School has 4 classes. There are 20 per cent of the whole number in the first class, 26 per cent in the second, 29 per cent in the third, and the remaining 150 are in the fourth class. Find the number of boys in each class.

18 Archimedes discovered that the crown made for King Hiero consisted of gold and silver in the ratio of 2 : 1, how much per cent was gold and how much per cent silver?

19 A bought goods to the value of Rs 3457 8a and sold them to B at a gain of 15 per cent on his outlay, and B sold them to C at a loss of 15 per cent on his outlay, how much did C give for them?

20 A man's capital increased 20 per cent every year, at the end of 4 years it was Rs 5184, what was his capital at first?

21 A house depreciates in value each year at the rate of 10 per cent of its value at the beginning of the year, and its value at the end of 3 years is Rs 10735, find its original value

22 After spending 50 per cent of his income and then 20 per cent of the remainder, a man has Rs 600 left, find his income

23 A person lays out Rs 4800 and Rs 3600 respectively in two different speculations, in the first he loses 10 per cent and in the second he gains 15 per cent. Find his gain or loss per cent on the sum invested

24 A person loses in his first year 10 per cent of his capital, but in the next year he gains 20 per cent of what he had at the end of the first year and his capital is now Rs 1110 more than it was at first, find his original capital

25 90 per cent of the boys of a school pass in spelling, and 85 per cent in arithmetic, 150 pass in both subjects and no boy fails in both. How many boys are there in the school?

499 Average The Average or mean value of two or more numbers is found by adding the numbers together and dividing the sum by the number of those given numbers

Hence, the sum of a number of given numbers is their average value multiplied by the number of them

Ex 1 Find the average of 14, 26, 9, 18, 13, 24, 27 and 39

Here, the sum of the numbers = 170, and as the number of the given numbers is 8, we get the average as $170 \div 8$ or 21 25 Ans

Ex 2 The average age of a school of 750 boys is 15 4 years, 50 boys leave, thereby diminishing the average age of the school to 15 3, find the average age of those who leave

The sum of the ages of the 750 boys = $750 \times 15 4$ or 11550 years

The sum of the ages of the remaining (750-50) or 700 boys = $700 \times 15 3$ or 10710 years

∴ the sum of the ages of the 50 boys who leave is (11550 - 10710) or 840 years

∴ the average age reqd = $(840 \div 50)$ or 16 8 years Ans

Examples CL

1 Find the average value of the following numbers —

- (1) 13, 15, 74, 23, 6 and 31 (2) 1600, 276, 974, 0, 236, 845 and 1239
 (3) $9\frac{3}{4}$, $12\frac{1}{3}$, 14, 0, $28\frac{1}{2}$ and 19 (4) 5 063, 7 00285, 12, 8 7396 and 5 69352
 (5) $15\frac{1}{2}$, $36\frac{3}{4}$, $17\frac{2}{3}$, 0, $10\frac{5}{8}$, $74\frac{1}{8}$, $28\frac{1}{2}$ and 33
 (6) 12 48, 21, 7 75, 034, 3 125, 0, 24 5 and 12 35

2 At a competitive examination there were 7 candidates of the age of 19, 12 of 18, 4 of 22, 9 of 20, and 15 of 17 Find the average age

3 On Sunday I spent no money, on Monday Rs 43 14a, on Tuesday Rs 51 12a 8p, on Wednesday Rs 46 14a 6p, on Thursday Rs 52 8a, on Friday Rs 32 15a 6p, on Saturday Rs 26 4a, find my average daily expenditure during the week

4 The weights of a boat's crew are respectively 9 st 5 lbs, 10 st, 10 st 5 lbs, 11 st 1 lb, 12 st, 11 st, 6 lbs, 11 st 4 lbs, and 10 st, 7 lbs, what is the average weight of the crew? If the average weight of the crew be diminished by 2 lbs, when the weight of the coxswain is included, find the weight of the coxswain

5 The population of five parishes being 1236, 452, 364, 516 and 3430 respectively, find what the population of a sixth parish must be, in order that the average population of the six may be 1256 5

6 The populations of three towns in the year 1881 were 21326, 42324, and 6700, and in the year 1891 it was found that the first two had increased 12 and 10 per cent respectively, and the last had decreased 18 per cent, find the average population of the three towns in the year 1891

7 In a school, 17 children average 6 years, 26, $7\frac{1}{2}$ years, 35, $9\frac{1}{4}$ years, 20, 10 years, and 8, $12\frac{1}{2}$ years Find the average age of all the children

8 The average age of 27 men is 57 years, that of the first eleven is 53 years, and that of the last eight $59\frac{1}{2}$ years Find the average age of the rest

9 The average of 21 results is 61, that of the first eight being 64, and of the next eleven 59 Required the average of the last two

10 The average age of a school of 200 boys is 14 75 years, what will be the average age, if 10 new boys come whose average age is 12 3 years?

11 The average weight of 8 men is increased by 2 lbs, when one of them who weighs 12 stones is replaced by a fresh man, what is the weight of the new man?

12 The average temperature for Monday, Tuesday and Wednesday was 53° , the average for Monday, Wednesday and

Thursday was 56° , that for Thursday being 60° , what was the temperature on Tuesday?

13 The average salary of A , B and C is Rs 40 per month. That of B , C and D is Rs 50 per month, and D 's salary is Rs 60. What is the salary of A ?

14 The average of 25 results is 18, that of the first 12 being 14, and that of the last 12 being 17. Find the 13th result.

15 In a class there are 40 boys, and their average age is 15 years. One boy, however, aged 18, leaves the school, and another joins, and then the average age of the class is 14.875. Find the age of the new boy.

16 The average of ten results was $17\frac{1}{2}$, that of the first three was $16\frac{1}{2}$, and of the next four $16\frac{1}{2}$, the eighth was 3 less than the ninth, and 4 less than the tenth. What was the last result?

500 Profit and Loss All questions which relate to gain or loss in mercantile transactions fall under the head of Profit and Loss.

We have already given the student some idea of *Gain* or *Loss* (See Art 181). Under the head of *Profit and Loss*, we estimate a profit or a loss not absolutely, but in relation to the cost price. Men of business adopt 100 as a standard cost price, and reduce the gain or loss on a particular cost price to the corresponding gain or loss on 100, that is, to a gain or loss of so much per cent.

Again, when the cost price is represented by 100, the selling price is represented by $100 + \text{gain per cent}$, or $100 - \text{loss per cent}$, according as a gain or loss has been made.

501 But although questions in Profit and Loss can always be solved by the Rule of Three, yet it is often useful to remember that since a gain of 18 per cent means a gain of 18 or 100, where 100 represents the cost price, it is a gain of $\frac{18}{100}$ of the cost price. And in like manner a loss of 12 per cent means a loss of $\frac{12}{100}$ of the cost price.

Ex 1 A house, bought for £4250, is sold at a profit of 12 per cent, how much was gained, and for what was the house sold?

Here, the selling price of £100 goods is £(100 + 12) or £112

\therefore £100 £4250 £112 selling price,

\therefore selling price = $\frac{£112 \times 4250}{100} = £4760$

Hence gain = $£(4760 - 4250) = £510$ } *Ans.*

Ex 2 Goods were sold for Rs 216, at a loss of 10 per cent, what did they cost?

Here, the selling price of Rs 100 goods is Rs (100 - 10) or Rs 90

Rs 90 Rs 216 Rs 100 cost price,

\therefore cost price = $Rs\ 100 \times \frac{216}{90} = Rs\ 240$ } *Ans.*

Ex 3 A watch which costs Rs 40 was sold for Rs 49, find the gain per cent

Here, gain on Rs 40 is Rs (49 - 40) or Rs 9

Rs 40 Rs 100 Rs 9 gain per cent ,

$$\therefore \text{gain per cent} = \frac{100}{40} \times \text{Rs } 9 = \underline{\text{Rs } 22\frac{1}{2}} \quad \text{Ans}$$

Ex 4 I buy a horse for Rs 120, what do I sell it at, to lose 10%?

Here, the selling price of Rs 100 goods is Rs (100 - 10) or Rs 90

\therefore Rs 100 Rs 120 Rs 90 selling price,

$$\text{selling price} = \frac{90}{100} \times \text{Rs } 120 = \underline{\text{Rs } 108} \quad \text{Ans}$$

Ex 5 If I gain 5 per cent by selling an article for Rs 6 9 $\frac{1}{2}$, how much shall I gain or lose per cent by selling it for Rs 6?

Here, the selling price of Rs 100 goods is Rs (100 + 5) or Rs 105

. Rs 105 Rs 6 $\frac{1}{2}$ Rs 100 cost price ,

$$\therefore \text{cost price} = \text{Rs } \frac{105 \times 100}{105 \times \frac{16}{4}} = \text{Rs } \frac{25}{4} = \text{Rs } 6\frac{1}{4}$$

Hence, loss on every Rs 6 $\frac{1}{4}$ is Rs (6 $\frac{1}{2}$ - 6) = Rs $\frac{1}{4}$

\therefore Rs 6 $\frac{1}{4}$ Rs 100 Rs $\frac{1}{4}$ loss per cent ,

$$\therefore \text{the loss per cent} = \text{Rs } \frac{100 \times 4}{4 \times 25} = \underline{\text{Rs } 4} \quad \text{Ans}$$

Ex 6 A merchant buys 4000 mds of rice, one-fifth of which he sells at a gain of 5 p c, one-fourth at a gain of 10 p c one half at a gain of 12 p c, and the remainder at a gain of 16 p c. If he had sold the whole at a gain of 11 p c, he would have made Rs 728 more. What was the cost of the rice per maund?

$\frac{1}{5}$ of 4000 mds = 800 mds , $\frac{1}{4}$ of 4000 mds = 1000 mds ,

$\frac{1}{2}$ of 4000 mds = 2000 mds , remainder = 200 mds

Gain on 800 mds at 5% = $\frac{1}{20}$ of 800 mds at cost price, and so on

. total actual gain = $\frac{1}{20}$ of 800 mds + $\frac{1}{10}$ of 1000 mds + $\frac{12}{100}$ of 2000 mds + $\frac{16}{100}$ of 200 mds

$$= 40 \text{ mds} + 100 \text{ mds} + 240 \text{ mds} + 32 \text{ mds}$$

$$= 412 \text{ mds at cost price}$$

If the whole be sold at a gain of 11%, the gain would be $\frac{11}{100}$ of 4000 mds at cost price = 440 mds at cost price

. difference of the two gains = (440 - 412) or 28 mds at cost price

. cost price of 28 mds = Rs 728 ,

. cost price of 1 md = $\text{Rs } 728 \div 28 = \underline{\text{Rs } 26} \quad \text{Ans}$

Examples CLI

1 If goods are bought at Rs 22 14s 8d per cwt and sold at Rs 25 10s 8d per cwt, what is the gain per cent ?

2 A piano is bought for $\text{Rs } 631\ 4a$, what must it be sold for that a gain of 4 per cent may be made?

3 If I pay $\text{Rs } 122\ 8a$ for 84 lbs of tea, what price per lb must I sell it at to gain 15 per cent?

4 A house was bought for $\text{Rs } 312\ 8a$ and sold at a loss of 12 per cent, at what price was he sold?

5 Cloth is sold for $6s\ 1\frac{1}{4}d$ per yard at a loss of $12\frac{1}{2}$ per cent, find the prime cost

6 By selling an article for $\text{Rs } 1\ 14a$ a person gains 16 per cent, find the cost of the article

7 By selling a horse for $\text{Rs } 722\ 8a$, I lose 15 per cent, find the cost price of the horse

8 By selling an article for $3s\ 9d$ a person loses 5 per cent, at what price must he sell it to gain $4\frac{1}{2}$ per cent?

9 If by selling wine at $\text{Rs } 7\ 8a$ per gallon I lose 10 per cent at what price must I sell it to gain 15 per cent?

10 The cost of a 38 gallon cask of wine was $\text{Rs } 250$, and 8 gallons are lost by leakage, at what price per gallon must the remainder be sold to realize 10 per cent on the outlay?

11 If $5\frac{1}{2}$ per cent be gained by selling butter at $\text{Rs } 52\ 12a$, per cwt, how much per cent will be gained by selling it at $10a$ per lb,

12 By selling a horse for $\text{Rs } 1168\ 8a$ a person lost 5 per cent? what would have been his gain or loss per cent had he sold him for $\text{Rs } 1322\ 4a$?

13 If a tradesman gains $4s\ 10\frac{1}{4}d$ on an article which he sells for $16s\ 3d$, what is his gain per cent?

14 A contractor bought 250 sheep and sold them for $\text{Rs } 5322\ 14a\ 8p$ at a gain of $16\frac{2}{3}$ per cent, what was the cost price of each sheep?

15 If by selling an article for $\text{Rs } 2\ 12a$, I gain $\frac{3}{8}$ of my outlay, what should I gain per cent by selling it for $\text{Rs } 3\ 4a$?

16 A stationer sold quills at $11s$ a thousand, clearing $\frac{3}{8}$ of the money, what would he clear per cent by selling them at $13s\ 6d$ a thousand?

17 By selling goods for $\text{Rs } 800$, I lost $\frac{1}{5}$ of their cost, for what should I have sold them to gain $\frac{1}{10}$ of their cost?

18 A person, having bought goods for $\text{Rs } 400$, sells half of them at a gain of 5 per cent, for how much must he sell the remainder so as to gain 20 per cent on the whole?

19 I bought 100 articles for $\text{Rs } 4\ 9a\ 4p$, and sold them so as to gain $\frac{1}{8}$ of the selling price, find the selling price and the gain per cent

20 A draper bought 3672 yards of linen at $Rs\ 1\ 9a\ 4p$ per yard. He sells $\frac{1}{3}$ of it at $Rs\ 1\ 12a$ per yard, and $\frac{1}{3}$ of the remainder at $Rs\ 1\ 15a\ 4p$, at what price per yard must he sell the rest to gain on the whole $12\frac{1}{2}$ per cent?

21 A tradesman's prices are 20 per cent above cost price, if he allows a customer $12\frac{1}{2}$ per cent on his bill, what profit per cent does he make?

22 A merchant sells to a retailer at 40 per cent profit, but the latter failing, the former receives only $4a$ in the rupee, find his gain or loss per cent.

23 By selling tea at $Rs\ 2\ 10a\ 8p$ per lb a grocer clears $\frac{1}{3}$ of his outlay, if he raises the price to $Rs\ 3\ 1a\ 4p$, what does he clear per cent?

24 Bought eggs at $10a$ per score, how many may be sold for $Rs\ 1\ 2a$ so as to gain $12\frac{1}{2}$ per cent?

25 If a trader uses a light weight of $13\frac{1}{2}$ lbs instead of a stone, how much per cent does he gain fraudulently?

26 A person sold 55 yards of silk for $Rs\ 257\ 13a$, thereby gaining the cost of $13\frac{3}{4}$ yards, find the prime cost and the gain per cent.

27 A person sold 20 horses for $Rs\ 1750$ thereby losing the cost of 6 horses, find the prime cost of the horses and the loss per cent.

28 If oranges are bought at 20 for a half rupee, how many should be sold for $Rs\ 14$ to gain 40 per cent?

29 A grocer buys 1 cwt of tea at $4s\ 2d$ per lb and mixes it with tea at $2s\ 11d$ per lb, how much of the latter must he add to the former that by selling the mixture at $3s\ 8d$ per lb he may gain 20 per cent on his outlay?

30 A person sold 72 yards of cloth for $Rs\ 87$, his profit being the cost of 11 $\frac{1}{2}$ yards, how much did he gain per cent?

31 A grocer buys coffee at the rate of $Rs\ 85$ per cwt and chicory at $Rs\ 25$ per cwt and mixes them in the proportion of 5 parts chicory to 7 parts coffee, at what rate per lb must he sell the mixture so as to gain $16\frac{2}{3}$ per cent on his outlay?

32 A watch is bought for 25 guineas, at what price must it be sold to secure a clear profit of 30 per cent after allowing a discount of $2\frac{1}{2}$ per cent to the purchaser?

33 How much tea at $Rs\ 1\ 5a\ 4p$ per lb must be mixed with 4 cwt 47 lbs at $Rs\ 1\ 6a$ per lb, so that by selling the mixture at $Rs\ 1\ 8a$ per lb, 10 per cent may be gained?

34 $Rs\ 61\ 4a$ was spent in buying apples at $Rs\ 1\ 7a\ 4p$ a score. When they came to be sold part of them were worthless,

but the rest, on being sold at a profit of 30 per cent, realized Rs 68 4a, how many scores were there of worthless ones?

35 A merchant buys 1260 mds of rice, one fifth of which he sells at a gain of 5 per cent, one-third at a gain of 8 per cent and the remainder at a gain of 12 per cent if he had sold the whole at a gain of 10 per cent he would have obtained Rs 28 14a more, what was the prime cost per maund?

36 A merchant buys 3150 yards of cloth. He sells $\frac{1}{4}$ of it at a gain of 6 per cent, $\frac{1}{4}$ at a gain of 8 per cent, $\frac{1}{4}$ at a gain of 12 per cent and the remainder at a loss of 3 per cent. Had he sold the whole at a gain of 5 per cent he would have received Rs 120 12a more than he did, what was the prime cost of 1 yard?

37 A grocer had 150 lbs of tea, of which he sold 50 lbs at Rs 4 8a per lb, and found that he was thereby gaining $7\frac{1}{2}$ per cent. At what rate must he sell the remaining 100 lbs so as to clear 10 per cent upon the whole?

38 An article when sold at a profit of $7\frac{1}{2}$ per cent yields 2s 1d more than when it is sold at a loss of $12\frac{1}{2}$ per cent, find its prime cost.

39 If 3 per cent more be gained by selling a horse for Rs 832 8a than by selling him for Rs 810, what was the original cost?

40 An article when sold at a profit of 5 per cent yields 12a more than when sold at a loss of 5 per cent, find its prime cost.

41 A person by selling an article, which costs Rs 140 per cwt at Rs 1 6a 6s per lb, makes 5 per cent more profit than he would do if he sold the whole for Rs 557 10a 6s, what was the quantity sold?

42 A person sells an article at 5 per cent profit, if he had bought it at 5 per cent less, and sold it for 8a less, he would have gained 10 per cent, what was the cost price of the article?

43 An article passes successively through the hands of three dealers, each of whom in selling adds as his profit 10 per cent of the price at which he bought it, if the third dealer sells the article for Rs 332 12a, what did the first dealer pay for it?

44 The cost of freight and insurance on a certain quantity of goods was 15 per cent, and that of duty 10 per cent on the original outlay the goods were sold at a loss of 5 per cent, but if they had brought Rs 30 more there would have been a gain of 1 per cent, find how much they cost.

45 A man sells a horse, at a loss, for 40 guineas, had he sold it for 50 guineas his gain would have been $\frac{2}{3}$ of his former loss, find the cost price.

46 A man having bought a quantity of goods for Rs 1500, sells $\frac{1}{3}$ at a loss of 4 per cent, by what increase per cent must he

raise that selling price that by selling the remainder at the increased rate he may gain 4 per cent on the whole transaction ?

502 Division into Proportional Parts A given quantity is said to be divided into Proportional Parts, when it is divided into parts which have the same *ratio* to each other that certain given numbers have

503 *To divide a given quantity into parts which shall be proportional to certain given numbers*

RULE (i) Divide the given quantity by the sum of the given numbers expressing the ratios of the parts, multiply the quotient by each of these numbers, and the products will give the parts required (Rule of Unitary Method)

(ii) The sum of the given parts any one of them the given quantity to be divided the corresponding part of it (Rule of Proportion)

This proportion must be repeated for each of the parts, or at all events for all but the last part, which may be found either by this proportion or by subtracting the sum of the values of the other parts from the given quantity to be divided

Ex 1 Divide Rs 837 among A, B and C, so that their shares may be as 5, 9 and 13 respectively

(i) Here $5+9+13=27$, $\text{Rs } 837 \div 27 = \text{Rs } 31$

$$\left. \begin{aligned} \therefore A's \text{ share} &= \text{Rs } 31 \times 5 = \text{Rs } 155, \\ B's \text{ share} &= \text{Rs } 31 \times 9 = \text{Rs } 279, \\ \text{and } C's \text{ share} &= \text{Rs } 31 \times 13 = \text{Rs } 403 \end{aligned} \right\} \text{Ans}$$

(ii) Here $5+9+13=27$

$$\left. \begin{aligned} \therefore \frac{27}{5} & \quad \text{Rs } 837 & A's \text{ share}, & \therefore A's \text{ share} = \text{Rs } 155 \\ \frac{27}{9} & \quad \text{Rs } 837 & B's \text{ share}, & \therefore B's \text{ share} = \text{Rs } 279 \\ \frac{27}{13} & \quad \text{Rs } 837 & C's \text{ share}, & \therefore C's \text{ share} = \text{Rs } 403 \end{aligned} \right\} \text{Ans}$$

$$\text{or, } C's \text{ share} = \text{Rs } 837 - \text{Rs } (155 + 279) = \text{Rs } 403$$

504 If the given numbers are fractions we may follow the same method but it will be more convenient to find integral numbers proportional to the given fractions, by multiplying each fraction by the LCM of their denominators

Ex 2 Divide 3925 nuts among three persons A, B and C, so that their shares may be to each other in the ratio of $\frac{2}{3}$, $\frac{3}{8}$ and $\frac{1}{15}$

(i) Here, $\frac{2}{3} + \frac{3}{8} + \frac{1}{15} = \frac{157}{120}$, $3925 \div \frac{157}{120} = 3000$

$$\left. \begin{aligned} \therefore A's \text{ share} &= 3000 \times \frac{2}{3} = 2000, \\ B's \text{ share} &= 3000 \times \frac{3}{8} = 1125, \\ \text{and } C's \text{ share} &= 3000 \times \frac{1}{15} = 200 \end{aligned} \right\} \text{Ans}$$

(ii) The L C M of 3, 8 and 15 is 120, multiply each fraction by this, then the shares are in the ratios 80 45 32

Now, $80+45+32=157$ and $3925-157=25$

. A 's share $= 25 \times 80 = 2000$, B 's share $= 25 \times 45 = 1125$,
and C 's share $= 25 \times 32 = 800$

Ex 3 Divide Rs 1050 among A , B , C and D so that A 's share
 B 's share 2 3, B 's C 's 4 5, and C 's D 's 6 7

A 's share B 's $= 2 \quad 3 = 16 \quad 24$,

B 's share C 's $= 4 \quad 5 = 24 \quad 30$,

C 's share D 's $= 6 \quad 7 = 30 \quad 35$,

A 's B 's C 's D 's $= 16 \quad 24 \quad 30 \quad 35$

But $16+24+30+35=105$, and Rs 1050 \div 105 = Rs 10,
 $\therefore A$'s share $=$ Rs 10 \times 16 = Rs 160, B 's share $=$ Rs 10 \times 24 = Rs 240, }
 C 's share $=$ Rs 10 \times 30 = Rs 300, D 's share $=$ Rs 10 \times 35 = Rs 350 }

Ex 4 Divide Rs 288 12a among A , B and C , so that B 's share may be half as much again as A 's, and C 's share one third as much again as both A 's and B 's

B 's share $= \frac{3}{2}$ of A 's share, and A 's share $+ B$'s share $= \frac{5}{2}$ of A 's share and C 's share $= \frac{4}{3}$ of $\frac{5}{2}$ of A 's share $= \frac{10}{3}$ of A 's share,

. A 's share B 's C 's $= 1 \quad \frac{3}{2} \quad \frac{10}{3} = 6 \quad 9 \quad 20$

But $6+9+20=35$, and Rs 288 12a \div 35 = Rs 8 4a

. A 's share $=$ Rs 8 4a \times 6 = Rs 49 8a,

B 's share $=$ Rs 8 4a \times 9 = Rs 74 4a,

and C 's share $=$ Rs 8 4a \times 20 = Rs 165 } Ans

Ex 5 Gunpowder is composed of nitre, charcoal and sulphur in the proportion of 33, 7 and 5. How many lbs of sulphur are there in 180 lbs of powder?

$33+7+5=45$, also 180 lbs \div 45 = 4 lbs

quantity of sulphur $= 4$ lbs \times 5 = 20 lbs Ans

Ex 6 A debt of Rs 28 14a is paid in rupees, 8-anna pieces, and 2-anna pieces. How many coins of each kind were there, if the numbers were proportional respectively to 3, 2, 1?

The values of the three groups of coins (rupees, 8-anna pieces, and 2-anna pieces) with two-anna as unit, are as $3 \times 8 \quad 2 \times 4 \quad 1 \times 1$ or 24 8 1

Now $24+8+1=33$, and also Rs 28 14a \div 33 = 14a

Hence the groups of coins are worth 14a \times 24, 14a \times 8, and 14a \times 1 or 336a, 112a, and 14a respectively

\therefore there are $336-16$ or 21 rupees, $112-8$ or 14 eight anna pieces, and $14-2$ or 7 two anna pieces Ans

Ex 7 600 coins consist of guineas, half sovereigns and half-crowns, the values of the guineas, the half sovereigns and half-crowns are as 14 8 3, find the number of guineas

Suppose the values to be all expressed in shillings

The numbers of guineas, half-sovereigns and half-crowns are as

$$14-21 \quad 8-10 \quad 3-2\frac{1}{2}, \text{ or as } \frac{2}{3}, \frac{4}{5}, \frac{6}{5}$$

Now $\frac{2}{3} + \frac{4}{5} + \frac{6}{5} = \frac{8}{3}$, and $600 - \frac{8}{3} = 225$

the number of guineas = $\frac{2}{3} \times 225 = 150$ *Ans*

Examples CLII

1 Divide —

(1) 1008 into three parts proportional to 2, 3, 4

(2) Rs 25000 into parts proportional to 2, 3, 7, 8

(3) Rs 84700 $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$

(4) £1064 2, 2 $\frac{1}{2}$, 2 $\frac{3}{4}$

(5) 694 bu 13 kat 12 ch 13, 17, 20

(6) Rs 71 10a 8p 10, 2 $\frac{1}{2}$, 1, $\frac{1}{2}$, $\frac{1}{3}$

(7) 360 lbs 3 3, 7, 5

(8) £593 8s 6d 2 3, 3 45, 5 629

2 The proportions used in making English gunpowder are saltpetre 75 parts, sulphur 10 parts, charcoal 15 parts How many seers of each material is required to manufacture 11 mds 8 sr of gunpowder?

3 In the alloy of which brass cannon are made, there are 11 parts of tin to 100 of copper Calculate the weight of tin in a piece weighing 33 mds 12 sr

4 Gunpowder is composed of nitre 33 parts, charcoal 7 parts and sulphur 5 parts How many pounds of powder can be made with 30 lbs of sulphur, and how much nitre is required?

5 In England, gunpowder is made of 75 parts of nitre, 10 of sulphur and 15 of charcoal, in France 77 parts of nitre, 9 of sulphur and 14 of charcoal If 14 maunds of each be mixed, what weight of nitre, sulphur and charcoal will there be in the compound?

6 Divide Rs 26 between 5 men, 7 women and 14 boys, so that each woman may have $\frac{7}{8}$ of each man's share, and each boy $\frac{2}{3}$ of each woman's share

7 Divide Rs 19089 among A, B and C, so that A's share B's = 3 5, and B's C's = 10 11

8 £1630 is divided among A, B and C, A's share B's share is as $\frac{1}{2}$ $\frac{2}{3}$, and B's C's as $\frac{5}{8}$ $\frac{7}{8}$, find the share of each

9 Divide Rs 14787 8a among A, B, C and D, so that A's share B's = 3 8, B's C's = 4 9, and C's D's = 15 4

10 A certain sum is divided among A , B and C , so that A gets three times as much as B , and B three times as much as C , B 's share is Rs 100, what is the sum divided?

11 An examiner wishes to mark three questions in the proportion of the numbers 2, 3, 4. The sum of the marks is to be 150. How must he distribute them?

12 Divide Rs 390 among A , B and C , so that as often as A receives Rs 3, B may receive Rs 4, and as often as B receives Rs 6, C may receive Rs 9.

13 6270 persons meet together, of whom the number of women is 18, boys 29 and girls 10. Find how many were there of each description.

14 A mixed metal consists of 87 parts silver and 13 parts copper, what is the value of 29 lbs 2 oz. Troy of this mixture, when pure silver is at 5s 6d per oz. Troy, and copper at 1s per lb Avoirdupois?

15 Of 2180 mds, A 's share is 2, B 's 3, C 's 4, and D 's 5. Find the share of each.

16 A person has £12 4s in half crowns, florins, and shillings, and the number of coins of each kind are respectively as the numbers 7, 5 and 3. Find the number of coins of each kind.

17 A purse contains £42 8s made up of pence, shillings, half-crowns, and sovereigns, the half crowns, pence, sovereigns and shillings are as the numbers 1, 2, 3 and 8 respectively, find how many of each coin are in the purse.

18 310 coins consist of sovereigns, half sovereigns and florins, whose values are as 5, 3, 2. Find the number of each.

19 Divide Rs 9510 among A , B , C and D , so that $\frac{1}{2}$ of A 's share may be equal to $\frac{2}{3}$ of B 's, and $\frac{3}{4}$ of B 's may be equal to $\frac{1}{2}$ of C 's and $\frac{1}{3}$ of C 's may be equal to $\frac{1}{4}$ of D 's.

20 378 coins consist of rupees, half rupees and quarter-rupees, whose values are as 13, 11, 7. Find the number of each.

505 When two or more men are partners in a common concern, they each contribute a certain sum of money called the Capital, to carry on business.

Fellowship or Partnership is a method by which gains or losses are distributed among partners in any mercantile transaction, in proportion to the capital which each has contributed.

506 Fellowship is either Simple or Compound.

In *Simple Fellowship* the sums of money put in by the several partners continue in the business for the same time, in *Compound Fellowship*, for different periods of time.

507 Simple Fellowship In Simple Fellowship, the gain or loss arising at the end of any given time is divided among the partners in *proportion* to the capital subscribed by each of them

Ex *A*, *B* and *C* form a partnership, *A* subscribes Rs 1750, *B* Rs 2100 and *C* Rs 2650. At the end of 9 months they dissolve, and share the profits amounting to Rs 422 8a, what will be the share of each?

Here, we have to divide Rs 422½ in the proportion of Rs 1750, Rs 2100, and Rs 2650, or in the proportion of the numbers 35, 42, 53

Now, $35 + 42 + 53 = 130$, and $\text{Rs } 422\frac{1}{2} \div 130 = \text{Rs } 1\frac{1}{2}$

$$\left. \begin{array}{l} A's \text{ share} = \text{Rs } 1\frac{1}{2} \times 35 = \text{Rs } 113 \quad 12a, \\ B's \text{ share} = \text{Rs } 1\frac{1}{2} \times 42 = \text{Rs } 136 \quad 8a, \\ \text{and } C's \text{ share} = \text{Rs } 1\frac{1}{2} \times 53 = \text{Rs } 172 \quad 4a \end{array} \right\} \text{Ans}$$

508 Compound Fellowship In Compound Fellowship, the gain or loss is divided among the partners not only in *proportion* to the *capital* subscribed by each, but also to the *time* for which it has been subscribed. Hence we obtain the following Rule —

RULE Reduce all the times into the same denomination, and multiply each man's stock by the time of its continuance, and then state thus —

The sum of all the products each particular product the whole quantity to be divided the corresponding share

Ex 1 *A*, *B* and *C* are partners, *A* puts in Rs 5000 for 7 months, *B* Rs 6000 for 8 months, and *C* Rs 9000 for 9 months. The profit is Rs 4100, what is the share of each?

$$\text{Rs } 5000 \times 7 = \text{Rs } 35000, \text{ Rs } 6000 \times 8 = \text{Rs } 48000,$$

$$\text{Rs } 9000 \times 9 = \text{Rs } 81000$$

$$\text{Also } \text{Rs } 35000 + \text{Rs } 48000 + \text{Rs } 81000 = \text{Rs } 164000$$

$$\left. \begin{array}{lll} \text{Rs } 164000 & \text{Rs } 35000 & \text{Rs } 4100 \\ \text{Rs } 164000 & \text{Rs } 48000 & \text{Rs } 4100 \\ \text{Rs } 164000 & \text{Rs } 81000 & \text{Rs } 4100 \end{array} \right\} \begin{array}{l} A's \text{ share} = \text{Rs } 875, \\ B's \text{ share} = \text{Rs } 1200, \\ C's \text{ share} = \text{Rs } 2025 \end{array} \text{Ans}$$

Ex 2 There were at first 20 men, 30 women, and 15 servants, for every 10a that a man paid, a woman paid 6a and a servant 2a, the bill amounted to Rs 410 how much did each man, woman, and servant pay?

$$10a \times 20 = 200a, 6a \times 30 = 180a, \text{ and } 2a \times 15 = 30a$$

$$\text{Also } 200a + 180a + 30a = 410a$$

$$\begin{array}{lll} 410a & 200a & \text{Rs } 410 \\ 410a & 180a & \text{Rs } 410 \\ 410a & 30a & \text{Rs } 410 \end{array} \begin{array}{l} 20 \text{ men's share} = \text{Rs } 200, \\ 30 \text{ women's share} = \text{Rs } 180, \\ 15 \text{ servants' share} = \text{Rs } 30 \end{array}$$

∴ a man pays Rs 10, a woman Rs 6, and a servant Rs 2 *Ans*

Ex 3 *A* and *B* begin to trade in partnership *A* puts in £400 at first, and £500 at the end of 2 months, *B* puts in £300 at first and £600 at the end of 3 months The profit at the end of the year is £470 How should this be divided?

A's equivalent capital = £400 × 12 + £500 × 10 = £9800,

B's = £300 × 12 + £600 × 9 = £9000

Also £9800 + £9000 = £18800

$\frac{£18800}{£18800} \quad \frac{£9800}{£9000} \quad \frac{£470}{£470}$
 $\left. \begin{array}{l} \text{A's share, } \therefore \text{A's share} = £245, \\ \text{B's share, } \therefore \text{B's share} = £225 \end{array} \right\} \text{Ans}$

Examples CLIII

1 Three partners put into business the sums of Rs 300, Rs 400, and Rs 500, and at the end of a certain time they gained Rs 600, find the share of each

2 Three persons forming a joint stock of Rs 45000, gain by trading Rs 15000, and of this their shares are Rs 7500, Rs 5000 and Rs 2500 find the portion of stock contributed by each

3 Three partners *A*, *B* and *C* invest Rs 2000, Rs 3500, and Rs 4750 for 6, 3 and 2 months respectively, how should a profit of Rs 4000 be divided among them?

4 If *A* contribute Rs 6000 for 5 months, *B* Rs 5000 for 6 months, *C* Rs 4000 for $7\frac{1}{2}$ months and *D* Rs 2500 for 12 months, in the formation of a joint stock, divide a profit of Rs 4760 equitably among them

5 *A* is a working, *B* a sleeping partner in a bookseller's business Their capital amounts to £6400, of which £2400 belongs to *A*, the rest to *B* Their profits, at the end of the first year, amounted to £1600 *A* receives 10 per cent of the profits for managing the business How ought the remaining part of the profits to be divided?

6 Two traders *A* and *B* form a partnership for a year, *A* contributes Rs 5000 and at the end of 4 months he adds Rs 3000, *B* contributes Rs 8000 but at the end of 9 months withdraws Rs 2500 At the end of the year a profit of Rs 4600 is to be divided, how much should each partner get?

7 *A* and *B* enter into partnership for three years *A* contributes Rs 5000, at the end of the first year he adds another Rs 5000, and at the end of the second year Rs 4500, *B* contributes Rs 8000, at the end of the first year and a half he adds Rs 5000, and six months before the partnership concludes he withdraws Rs 3500 Eight months after the commencement of the partnership *C* is admitted with Rs 10000, which he leaves in till the end, how should a profit of Rs 40135 8a be divided among the partners?

8 *A*, *B* and *C* rent a field for Rs 710. *A* puts in 12 oxen for 4 months, *B* 15 oxen for 3 months, and *C* 20 oxen for 6 months. How much of the rent should each pay?

9 *A*, *B* and *C* are partners, *A*'s money has been in the business 3 months, and he claims $\frac{1}{3}$ of the gain, *B*'s money has been in the business 9 months and *C* has had £756 in the business 4 months, and he claims half the gain. How much money did *A* and *B* contribute to the capital?

10 Four merchants *A*, *B*, *C* and *D*, trading with a capital of Rs 238000, find after a certain time their respective shares increased by Rs 265, 13a 4p, Rs 372 2a 8p, Rs 531 10a 8p and Rs 638. How much did they respectively subscribe to the original capital?

11 *A* and *B* enter into partnership with capitals as 4 : 5. At the end of 3 months they withdraw respectively $\frac{1}{3}$ and $\frac{2}{5}$ of their capitals. When the year closes they find their profit to be Rs 4364 12a, how must it be divided between them?

12 *A* and *B* rent a field for 21 guineas. *A* puts into 10 horses for $1\frac{1}{2}$ months, 30 oxen for 2 months and 100 sheep for $3\frac{1}{2}$ months, *B* 40 horses for $2\frac{1}{2}$ months, 50 oxen for $1\frac{1}{2}$ months and 115 sheep for 3 months. If the food consumed in the same time by a horse, an ox and a sheep be in the ratio 3 : 2 : 1, what portion of the rent must each pay?

13 Three persons with a joint stock gain £3650, the first advances $\frac{1}{3}$ of the capital for $\frac{1}{2}$ of the time, the second $\frac{1}{4}$ of the capital for $\frac{1}{3}$ of the time and the third the remainder of the capital for the whole time, find their shares.

14 *A* employs his capital of Rs 2000 in a trade for 8 months. *B*, who joins later on, keeps his capital for 6 months, and receives in the end Rs 150 out of a total profit of Rs 400. How much capital does *B* put in the trade?

15 The capitals contributed by *A* and *B* to a joint stock are in the proportion of 4 : 5, and the profits received by them are in the proportion of 5 : 4. If *B*'s capital is in the joint stock for 15 months, how long is *A*'s?

16 *A* and *B* hired a pasture for Rs 125 for 50 days. *A* put in 25 oxen for a certain number of days, and paid Rs 75, *B* put in 20 oxen for the remaining days, and paid the remaining sum. How long had *B* put in his oxen?

17 The profits received by *A* and *B* by a partnership are in the proportion of 2 : 3, and their capitals are in the proportion of 6 : 7. If *A*'s capital is in the joint stock for 8 months, how long is *B*'s?

18 *A* and *B* rent a pasture for £200 for 80 days. *A* puts in 60 oxen for a certain number of days, *B* puts in 40 for the remaining days, and pays $\frac{1}{3}$ of the total rent. Find the time for which *A* puts in his oxen.

509 Equation of Payments The Equation of Payments is the finding of a proper time, called the *equated time*, at which two or more debts due at *different* times should be discharged at *one* payment, and it is here *assumed* that the interests of *all* the debts for their respective periods are together equal to the interest of their *sum* for the *equated time*

510 To find the equated time of different payments

RULE Multiply each debt into the time which will elapse before it becomes due, and then divide the sum of the products by the sum of the debts, the quotient will be the equated time required

Ex 1 If Rs 100 be due in 3 months, Rs 210 in 2 months and Rs 160 in 5 months, find the equated time

Here, $100 \times 3 + 210 \times 2 + 160 \times 5 = 300 + 420 + 800 = 1520$, and $100 + 210 + 160 = 470$

\therefore the equated time $= \frac{1520}{470}$ months $= 3\frac{11}{7}$ months *Ans*

Ex 2 A owed B £100, to be paid at the end of 9 months, he pays however £20 at the end of 3 months, and £30 at the end of 8 months, when ought the remainder to be paid?

The remainder $= £(100 - 20 - 30) = £50$

Here, $20 \times 3 + 30 \times 8 + 50 \times \text{no of mo reqd} = 100 \times 9$,

or, $60 + 240 + 50 \times \text{no of mo reqd} = 900$,

$\therefore 50 \times \text{no of mo reqd} = 900 - 60 - 240 = 600$,

$\therefore \text{no of mo reqd} = 600 \div 50 = 12$ *Ans*

Examples CLIV

1 If Rs 75 be due in 4 months, Rs 125 in 5 months and Rs 150 in 7 months, what is the equated time?

2 What will be the equated time of payment of £200 due at 3 months, £300 at 8 months and £500 at 12 months?

3 Find the equated time of payment, when $\frac{1}{3}$ of a sum of money is due in 3 months, $\frac{1}{2}$ in 8 months and the remainder in 15 months

4 A finds on the 1st of March that he owes B the following sums Rs 1400 due on the 20th of April, Rs 1200 due on the 14th of May, and Rs 3800 due on the 15th of June On what day may A pay these debts together?

5 A owes B £4600, whereof £1000 is to be paid in 50 days, £1300 in 40 days and the remainder in 140 days, find the equated time

6 A owes B Rs 7300 to be paid in $4\frac{2}{3}$ months, he pays however Rs 1500 at the end of 3 months, and Rs 2100 at the end of 5 months, when was the remainder due?

7 Of a sum of money due 15 months hence, $\frac{1}{2}$ was paid at 4 months, $\frac{1}{4}$ at 6 months, and $\frac{1}{4}$ th at 12 months, how many months may the payment of the residue be deferred?

8 At the beginning of the year *A* finds that he owes *B* four debts, viz, Rs 1200 payable on the 3rd February, Rs 350, payable on the 17th March, Rs 1000, payable on the 5th May, and Rs 850, payable on the 18th August. To simplify accounts he proposes to pay *B* Rs 3400 in one sum, on what day should the payment be made?

511 Alligation is the rule by means of which the rate of quality of a composition or mixture is found from the rates or qualities of the ingredients of which it is made up

512 Alligation is of two kinds, Medial and Alternate. *Alligation Medial* is that in which the price, and quantity, of each of the things composing the mixture are given, to find the price of the mixture. This is equivalent to finding an *average* or *mean* price (See Arts 179 and 184)

RULE Multiply the number of each quantity expressed in the same denomination by its price, and divide the sum of these products by the sum of the numbers. The quotient will be the rate of the mixture

Ex. A wine merchant mixes 60 gals of sherry at Rs 12 per gal, 50 gals at Rs 13 per gal, and 70 gals, at Rs 16 per gal, find the price of a gallon of the mixture

$$\begin{array}{rcl}
 60 \text{ gals at Rs 12 a gal} & = \text{Rs } 720 & \\
 50 & \text{at Rs 13} & = \text{Rs } 650 \\
 70 & \text{at Rs 16} & = \text{Rs } 1120 \\
 \hline
 180 \text{ gals of the mixture} & = & \text{Rs } 2490
 \end{array}
 \quad \left| \begin{array}{l}
 \cdot 1 \text{ gal of the mixture} \\
 = \text{Rs } \frac{2490}{180} \\
 = \text{Rs } 13 \frac{13}{18} \text{ } 13\text{a } 4\text{p} \text{ Ans}
 \end{array} \right.$$

513 *Alligation Alternate* is the method of finding from the price of each of the things which compose a mixture, what quantity must be taken of each, in order that the mixture may be of a given price (Examples of alligation alternate have generally an indefinite number of solutions)

RULE Place the given prices under one another in order, and to the left place the mean price. Link all the prices, so that one *under* and one *above* the mean price shall always be together, and write against each price the difference between the price with which it is linked and the mean price,—these differences, or any equimultiples of them, will give the quantities required

Ex. 1 How must a grocer mix tea at Re 1 2a per lb and Re 1 7a per lb to make a mixture worth Re 1 5a a lb?

$Re\ 1\ 2a = 18a$, $Re\ 1\ 7a = 23a$, $Re\ 1\ 5a = 21a$

21 $\begin{array}{l} 18 \\ 23 \end{array} \left. \begin{array}{l} 2\ \text{lbs at } Re\ 1\ 2a \\ 3\ \text{lbs at } Re\ 1\ 7a \end{array} \right\}$

To make the mixture at $Re\ 1\ 5a$ per lb
1 lb at $Re\ 1\ 2a$ brings a gain of $3a$
and 1 lb at $Re\ 1\ 7a$ loss of $2a$

the mixture must be made
in the ratio of 2 3 Ans

In order therefore, that the gain in
using the former may be equal to the
loss in using the latter, for every 2 lbs
of the former we must take 3 lbs of the

latter, for then the gain would be $2 \times 3a$ and the loss $3 \times 2a$. We must
therefore take the quantities in the ratio of 2 and 3, that is, in the
inverse ratio of the differences of the two prices and the mean price

Ex 2 How must a grocer mix teas at $Re\ 1\ 5a\ 4p$, $Re\ 1\ 7a\ 4p$,
and $Re\ 1\ 10a$ per lb, to make a mixture worth $Re\ 1\ 8a$ a lb?

$Re\ 1\ 5a\ 4p = 21\frac{1}{2}a$, $Re\ 1\ 7a\ 4p = 23\frac{1}{2}a$, $Re\ 1\ 10a = 26a$,
 $Re\ 1\ 8a = 24a$

24 $\begin{array}{l} 21\frac{1}{2} \\ 23\frac{1}{2} \\ 26 \end{array} \left. \begin{array}{l} 2\ \text{lbs at } Re\ 1\ 5a\ 4p \\ 2\ \text{lbs at } Re\ 1\ 7a\ 4p \\ 2\frac{1}{2} + \frac{1}{2}\ \text{lbs at } Re\ 1\ 10a \end{array} \right\}$

Hence, mixing at all the three prices, he must take the kinds in
the following proportion -

2 lbs 2 lbs $(2^2 + 1^2)$ or $3\frac{1}{2}$ lbs, or 1 1 $1\frac{1}{2}$ or 3 3 5 Ans

Ex 3 How may a grocer mix teas at $2s\ 6d$, $2s\ 9d$, $3s\ 1d$
and $3s\ 4d$ per lb, to form a mixture worth $2s\ 10d$ per lb?

$2s\ 6d = 30d$, $2s\ 9d = 33d$, $3s\ 1d = 37d$, $3s\ 4d = 40d$, $2s\ 10d = 34d$

or

34 $\begin{array}{l} 30 \\ 33 \\ 37 \\ 40 \end{array} \left. \begin{array}{l} 3\ \text{lb at } 2s\ 6d \\ 6\ \text{lbs at } 2s\ 9d \\ 4\ \text{lbs at } 3s\ 1d \\ 1\ \text{lb at } 3s\ 4d \end{array} \right\}$

34 $\begin{array}{l} 30 \\ 33 \\ 37 \\ 40 \end{array} \left. \begin{array}{l} 6\ \text{lbs at } 2s\ 6d \\ 3\ \text{lbs at } 2s\ 9d \\ 1\ \text{lb at } 3s\ 1d \\ 4\ \text{lbs at } 3s\ 4d \end{array} \right\}$

Hence a mixture may be made
by mixing them in the proportion
of 3 6 4 1 Ans

Hence a mixture may be made
by mixing them in the proportion
of 6 3 1 4 Ans

Ex 4 How much coffee at $13a$ per lb and chicory at $3a\ 10p$
per lb must a person take to make a mixture of 33 lbs worth $9a\ 8p$
per lb?

$13a = 150p$, $3a\ 10p = 46p$, $9a\ 8p = 116p$

116 $\begin{array}{l} 150 \\ 46 \end{array} \left. \begin{array}{l} 70\ \text{lbs at } 13a\ \text{per lb} \\ 40\ \text{lbs at } 3a\ 10p\ \text{per lb} \end{array} \right\}$

coffee must be to chicory as 70 40 or 7 4, and $7+4=11$;

\therefore coffee in the mixture $= \frac{7}{11}$ of 33 lbs = 21 lbs
and chicory $= \frac{4}{11}$ of 33 lbs = 12 lbs } Ans

Ex 5 I buy one kind of tea at $Rs\ 1\ 2a$ per lb, and another kind at $Rs\ 1\ 12a$ per lb, in what ratio must I mix them in order that by selling the mixture at $Rs\ 2$ per lb I may gain 20 per cent?

$Rs\ 120$ $Rs\ 2$ $Rs\ 100$ cost price of the mixture per lb,
cost price of the mixture per lb = $Rs\ 1\ 10a\ 8p$

$Rs\ 1\ 2a = 18a$, $Rs\ 1\ 12a = 28a$, $Rs\ 1\ 10a\ 8p = 26\frac{2}{3}a$

$26\frac{2}{3}$ 18) 1^1 lbs at $Rs\ 1\ 2a$ per lb
 28 $8\frac{2}{3}$ lbs at $Rs\ 1\ 12a$ per lb

Hence, the proportion of cheaper to dearer is $1\frac{1}{3}$ $8\frac{2}{3}$ or $2\frac{13}{13}$ Ans

514 If there be a lump of two ingredients, and if an equal quantity of another lump with the proportion of the ingredients interchanged be added to it, the quantities of the ingredients become equal in the fresh compound

Ex The price of gold is $Rs\ 38\ 15a$ per oz. A composition of gold and silver weighing 18 lbs is worth $Rs\ 7246\ 8a$, but if the weights of the gold and silver in the composition were interchanged, it would be worth only $Rs\ 1717\ 8a$. Find the proportion of gold and silver in the composition and the price of silver per oz.

If the two lumps were added together, there would be 18 lbs of gold and 18 lbs of silver, and the price of the two lumps together would be $Rs\ 7246\ 8a + Rs\ 1717\ 8a$ or $Rs\ 8964$

Since 18 lbs of gold + 18 lbs of silver is worth $Rs\ 8964$,
and 18 lbs of gold is worth $Rs\ 38\ 15a \times 18 \times 12 = Rs\ 8410\ 8a$
18 lbs of silver is worth $Rs\ 553\ 8a$

1 oz of silver is worth $Rs\ 553\ 8a - (18 \times 12) = Rs\ 2\ 9a$ Ans

Again, cost of 1 oz of the mixture = $Rs\ 7246\ 8a - (18 \times 12) = Rs\ 33\ 8\frac{5}{8}a$
cost of 1 oz of gold = $Rs\ 38\ 15a = 623a$, $= 536\frac{7}{8}a$
and cost of 1 oz of silver = $Rs\ 2\ 9a = 41a$

$536\frac{7}{8}$ 623)	$495\frac{5}{8}$ oz of gold		Hence gold	silver = $495\frac{5}{8}$ $86\frac{2}{3}$
41	$86\frac{2}{3}$ oz of silver			$= 446\frac{2}{3}$ 776
				$= 23\frac{4}{4}$ <u>Ans</u>

Examples GLV

1. A wine merchant mixes 20 gals of wine at $Rs\ 6$ per gal, 25 gals at $Rs\ 7$ per gal, and 36 gals at $Rs\ 8$ per gal, what will be the price of a gallon of the mixture?

2. A grocer mixes 47 lbs of tea at $Rs\ 1\ 1a\ 2p$ per lb, 25 lbs at $Rs\ 1\ 2a\ 8p$ per lb, and 20 lbs at $Rs\ 1\ 6a\ 10p$ per lb, what is the price of a lb of the mixture? If he had also added 8 lbs of sloe leaves at $2a\ 2p$ per lb, what then would be the price?

3. In what ratio must tea worth $2s\ 5d$ per lb be mixed with tea worth $3s\ 4d$ per lb to make a mixture worth $2s\ 9d$ per lb?

4 How must a person mix teas worth $Rs\ 1\ 8a\ 8p$, $Rs\ 1\ 6a$ and $Rs\ 1\ 12a$ per lb respectively, to make a mixture worth $Rs\ 1\ 8a$ per lb?

5 In what proportions must spirits worth $8s\ 3d$, $7s\ 9d$, $6s\ 6d$ and $8s\ 1a$ per gallon respectively be mixed, so that the compound may be worth $8s$ per gallon?

6 I buy some tea at $Rs\ 1\ 2a$ per lb and some at $Rs\ 1\ 12a$ per lb, in what ratio must they be mixed so that by selling the mixture at $Rs\ 2\ 2a\ 8p$ per lb, I may gain 30 per cent?

7 A green grocer sells potatoes at $Rs\ 1\ 5a\ 4p$, $Rs\ 1\ 7a\ 4p$ and $Rs\ 1\ 10a$ per maund, what quantities of each kind must he sell that the average price obtained shall be $Rs\ 1\ 8a$ per maund?

8 A merchant buys wheat at $Rs\ 19\ 8a$ per quarter, and another kind at $Rs\ 3$ per bushel, in what ratio must he mix them to gain 25 per cent by selling the mixture at $Rs\ 28\ 12a$ per quarter?

9 A druggist makes from ingredients worth $Rs\ 2\ 8a$, $Rs\ 3$, $Rs\ 4$ and $Rs\ 4\ 8a$ per lb respectively, $1\frac{1}{2}$ cwt. of a mixture worth $Rs\ 3\ 8a$ per lb how much of each ingredient does he use?

10 A grocer buys teas at $Rs\ 1\ 5a\ 1p$, $Rs\ 1\ 8a$ and $Rs\ 1\ 14a$ per lb respectively how is he to mix them that by selling the mixture at $Rs\ 2\ 0a\ 8p$ per lb, he may gain 16½ per cent?

11 I mix wines at $12s$, $13s$, $14s$ and $14s\ 6d$ per gallon respectively if the mixture is worth $13s\ 6d$ a gallon, how much of each kind of wine is there in 63 gallons of it?

12 How much tea at Rs per lb must I mix with 12 lbs at $Rs\ 1\ 13a\ 1p$ per lb to make a mixture worth $Rs\ 2\ 2a\ 8p$ a lb?

13 It is required to mix teas at $Rs\ 1\ 7a$, $Rs\ 1\ 4a$ and $Rs\ 1\ 2a$ per lb with sloe leaves at $2a$ per lb so that the mixture being sold at $Rs\ 1\ 6a\ 8p$ per lb, one fourth of the receipts may be clear profit.

14 I have 50 lbs of copper worth $10a\ 8p$ per lb and with this I wish to melt pewter worth $7a$ per lb and brass worth $9a\ 1p$ per lb, how much brass and pewter I must use to make a mixture worth $8a$ per lb.

15 How many lbs of tea at $2s\ 8d$ per lb must be mixed with 495 lbs at $2s\ 9d$ per lb so that a profit of 10 per cent may be made by selling the mixture at $3s$ per lb?

16 A silversmith gave $Rs\ 485\ 6a\ 8p$ for 16 lbs 8 oz of silver, giving $Rs\ 2\ 9a\ 8p$ an oz for one part and $Rs\ 2\ 3a$ for the rest, how many oz of each kind did he buy?

17 How much gold at $£4\ 5s$ per o., silver at $5s$ an oz and copper considered as of no value comparatively, may be melted together that the compound may be worth $£2\ 15s$ per oz?

18 The price of gold is £3 17s 10½d per oz, a composition of gold and silver weighing 18 lbs. is worth £657 7s, but if the proportions of gold and silver were interchanged, it would be worth only £259 1s. Find the proportion of gold and silver in the composition, and the price of silver per oz.

19 A person bought apples and pears for Rs 2 3a. If the numbers of apples and pears be interchanged, it would have cost him only Rs 1 9a. If apples cost 9a and pears 3a a dozen, how many of each did he buy?

20 A mass of gold and silver weighing 9 lbs. is worth £318 13s 6d, if the proportions of gold and silver in it were interchanged, it would be worth £129 10s 6d, it is known that 1 oz of gold and 2 oz of silver are worth £4 8s 1½d, what is the price of gold and silver per ounce?

515 **Pasture with Growing Grass** The following Solutions will illustrate the method of operation to be adopted in such cases

Ex 1 A meadow of 15 acres, grass growing uniformly, is consumed by 20 oxen in 96 days, but by 30 oxen in 60 days. How many oxen will eat it in 24 days?

Original grass + 96 days' growth keeps 20 oxen for 96 days,

1 ox for 1920 days

Also, original grass + 60 days growth keeps 30 oxen for 60 days,

∴ 1 ox for 1800 days

Hence, by subtraction, we get

36 days' growth keeps 1 ox for 120 days,

(36 × 16) days' 1 ox for 1920 days,

but original grass + 96 days' growth keeps 1 ox for 1920 days,

original grass + 96 days' growth = (36 × 16) or 576 days' growth,

∴ original grass + 24 days' = (576 - 72) or 504 days'

Now, 36 days' growth keeps 1 ox for 120 days,

∴ 36 days' 5 oxen for 24 days,

∴ 504 days' ∴ (5 × 14) oxen for 24 days

Hence the reqd no of oxen = 5 × 14 or 70 Ans

Ex 2 If 133 oxen consume the grass of a meadow in 13 days, and 112 of the oxen could consume the grass of the same meadow in 16 days—the grass growing uniformly, in what time could 125 of the oxen do it?

Original grass + 13 days' growth maintains 133 oxen for 13 days,

∴ 1729 oxen for 1 day

Also original grass + 16 days' growth maintains 112 oxen for 16 days,
 1792 oxen for 1 day

Hence, by subtraction, we have

3 days growth maintains 63 oxen for 1 day,
 \therefore 1 days 21 oxen for 1 day,
 \therefore 13 days' 21×13 or 273 oxen for 1 day

Hence, original grass of the field is eaten by $(1729 - 273)$ or 1456 oxen in 1 day

Now, in 1 day 21 oxen consume 1 day's growth of grass and there remain $(125 - 21)$ or 104 oxen, to consume the original grass of the field

\therefore 104 oxen 1456 oxen 1 day no of days required,
 \therefore no of days reqd = $1456 - 104 = 14$ Ans

Ex 3 If 25 oxen consume 10 acres of pasture, grass growing uniformly, in 30 days, and 30 oxen consume 8 acres of it in 18 days, how many acres will be consumed by 80 oxen in 60 days?

Let g denote the daily growth of grass per acre

Since 25 oxen in 30 days consume 10 ac of pasture + what grows in it in 30 days, and 30 oxen in 18 days consume 8 ac of pasture + what grows in it in 18 days,

\therefore 1 ox in 1 day consumes $\frac{10 \text{ ac} + 30 \times 10g}{25 \times 30}$, in 1st case,

and 1 ox in 1 day consumes $\frac{8 \text{ ac} + 18 \times 8g}{30 \times 18}$, in 2nd case

Hence, $\frac{10 \text{ ac} + 30 \times 10g}{25 \times 30} = \frac{8 \text{ ac} + 18 \times 8g}{30 \times 18}$, or $\frac{2 \text{ ac} + 60g}{5} = \frac{4 \text{ ac} + 72g}{9}$

$\therefore 18 \text{ ac} + 540g = 20 \text{ ac} + 360g$ $180g = 2 \text{ ac}$, and $g = \frac{2}{180} \text{ ac} = \frac{1}{90} \text{ ac}$

Now, $10 \text{ ac} + 30 \times 10g = 10 \text{ ac} + 30 \times \frac{1}{9} \text{ ac} = \frac{10}{3} \text{ ac}$

$\therefore 25 \times 30$ 80×60 $\frac{10}{3} \text{ ac}$ no of ac reqd + what grows in it in 60 da

\therefore no of acres reqd + what grows in it in 60 days = $\frac{80 \times 60 \times 40}{25 \times 30 \times 3} = 25\frac{2}{3}$

But no of acres reqd + what grows in it in 60 days

= no of acres reqd + $60 \times$ no of acres reqd $\times \frac{1}{90}$,

= no of acres reqd + $\frac{2}{3} \times$ no of acres reqd,

= $\frac{5}{3} \times$ no of acres reqd

$\therefore \frac{5}{3} \times$ no of acres reqd = $25\frac{2}{3}$, \therefore no of acres reqd = $25\frac{2}{3} \times \frac{3}{5} = 51\frac{1}{3}$ Ans.

Examples CLVI

1 If 29 oxen would eat up a field of grass in 7 weeks, or 25 oxen would eat up the same field in 9 weeks, — the grass growing uniformly, how many oxen would eat it up in 6 weeks?

2 Suppose that in a certain meadow the grass is of uniform quality and growth, and that 20 oxen would exhaust the grass in $12\frac{1}{2}$ days, or 21 oxen would do so in 12 days, in what time would 26 oxen do it?

3 A meadow of 50 acres, grass growing uniformly, is consumed by 80 oxen in 60 days, but by 60 oxen in 90 days. How many oxen will eat half of it in 30 days?

4 In a pasture ground wherein grass grows uniformly and which contains several days' growth of grass, 29 oxen eat all the grass in 7 days. If 25 oxen be kept on the same pasture ground, the grass would last for 9 days. How many oxen should be allowed to graze so that all the grass may be eaten up in 6 days?

5 A field of uniformly growing pasture is consumed by 120 oxen in 20 days or by 150 oxen in 15 days. How many oxen will consume it in 12 days, if at the end of 8 days 30 of them are removed?

6 If 23 oxen consume 8 acres of pasture in 26 days, and 25 oxen consume 7 acres of the same in 20 days, — the grass growing uniformly, how many acres of it would 33 oxen consume in $5\frac{1}{2}$ days?

7 Suppose that 17 oxen in 30 days, or 19 oxen in 24 days, could consume a field of uniformly growing pasture, find what number of oxen, diminished by the removal of 4 at the end of 6 days, would eat up the same field in 8 days.

8 Suppose that in a meadow of 20 acres the grass grows at a uniform rate, and that 133 oxen could consume the whole grass in 13 days, or that 28 of the oxen could eat up 5 acres of it in 16 days, how many of the oxen could eat up 4 acres of it in 14 days?

9 Suppose that a tank receives a regular and continual supply of water, and that when it contains a certain quantity, 12 equal taps being set open would empty it in $7\frac{1}{2}$ minutes, or 7 of the same taps would empty it in 16 minutes, how many of the taps would empty it in 50 minutes?

10 In a certain meadow there is a crop of 91 mds 35 sr of grass which grows uniformly. If 11 oxen would consume all the grass in 48 days, but 6 oxen would require 98 days, what weight of grass would each ox eat in a day?

11 A cistern constantly flowing is emptied by a number of waste pipes. If 10 of these be opened the cistern is emptied in 15 minutes, but if 25 be opened it is emptied in 5 minutes. In how many minutes will it be emptied if 15 are opened?

12 In a field in which grass grows uniformly, suppose that 31 oxen can consume $8\frac{1}{2}$ acres in $\frac{1}{2}$ of the time in which 15 oxen would consume $5\frac{1}{2}$ acres, and that 22 oxen would require 3 days longer to consume $7\frac{1}{2}$ acres than 20 oxen would require for $6\frac{1}{2}$ acres. — In what time would the 31 oxen eat up the $8\frac{1}{2}$ acres?

Examples worked out

Ex 1 A fruiterer buys a certain number of mangoes at 20 for 31 and an equal number at 30 for 32. He mixes and sells them at 25 for 32. What is his gain or loss per cent ?

Cost price of one of each of the two kinds is $\frac{31}{20}a$ and $\frac{32}{30}a$ respectively

$$\text{average cost of a mango} = \frac{1}{2} \left(\frac{31}{20} + \frac{32}{30} \right) a = \frac{1}{2} a,$$

$$\text{and selling price of a mango} = \frac{25}{32} a,$$

$$\therefore \text{loss on 1 mango, i.e., on } \frac{1}{2} = \left(\frac{1}{2} - \frac{25}{32} \right) a = \frac{1}{16} a$$

$$\text{Hence } \frac{1}{16} \times 100 = \frac{100}{16} \text{ percentage required,}$$

$$\therefore \text{percentage required} = 800 \div 200 = 4 \text{ Ans}$$

Ex 2 One lb of tea and 3 lbs of sugar cost Rs 3, but, if sugar were to rise 50 per cent. and tea 10 per cent, they would cost Rs 3 8a, find the prices per lb of tea and sugar

If both tea and sugar were to rise 50 per cent,

$$\text{the cost of 1 lb of tea and 3 lbs of sugar would be} = \frac{3 \times 150}{100} = \text{Rs } 4 \text{ 8a}$$

But tea rises only 10 per cent,

$$\therefore 40 \text{ p c of the cost of a lb of tea} = \text{Rs } 4 \text{ 8a} - \text{Rs } 3 \text{ 8a} = \text{Rs } 1$$

$$\therefore \text{the cost of a lb of tea} = \text{Rs } 1 + \frac{10}{100} = \text{Rs } 1 \text{ 2a} \text{ Ans}$$

$$\text{Hence the cost of 3 lbs of sugar} = \text{Rs } 3 - \text{Rs } 1 \text{ 2a} = 8a$$

$$\therefore \text{the cost of a lb of sugar} = \frac{8}{3} a = 2a \frac{8}{3} \text{ Ans}$$

Ex 3 Four parcels of gold, weighing respectively 10, 4, 2 and 4 oz and of 13, 12, 11 and 10 carats fineness, being mixed, what is the fineness of the compound ?

$$\text{Here, } 10 \times 13 + 4 \times 12 + 2 \times 11 + 4 \times 10 = 130 + 48 + 22 + 40 = 240$$

$$\text{The weight of the compound in oz} = 10 + 4 + 2 + 4 = 20$$

$$\therefore \text{the fineness of the compound} = (240 \div 20) \text{ or } 12 \text{ carats Ans.}$$

Ex 4 A man engages a servant on the understanding that he would get Rs 60, and a broad cloth after a year's service. The servant wishing to go away after 8 months' service, gets the broad cloth and Rs 36. Find the value of the broad cloth

Since 8 months is $\frac{2}{3}$ of a year, therefore the servant ought to get $\frac{2}{3}$ of the value of the broad cloth and $\frac{2}{3}$ of Rs 60 or Rs 40

But he is given $\frac{1}{3}$ of the value of the broad cloth more, and for that Rs (40 - 36) or Rs 4 are deducted from cash payment

$$\text{Hence } \frac{1}{3} \text{ of the value of the broad cloth} = \text{Rs } 4$$

$$\therefore \text{the value of the broad cloth} = \text{Rs } 4 \times 3 = \text{Rs } 12 \text{ Ans}$$

Ex 5 The sum of the ages of A, B and C is 150 years. Ten

years ago, their ages were in the proportion of 7 8 9, find their ages

Ten years ago, each was 10 years younger, and therefore the sum of their ages was 30 years less, i.e., was $(150 - 30)$ or 120 years
Also $7 + 8 + 9 = 24$

the age of A was $\frac{7}{24}$ of 120 yrs = 35 yrs, the age of B was $\frac{8}{24}$ of 120 yrs = 40 yrs and the age of C $\frac{9}{24}$ of 120 yrs = 45 yrs

the present age of $A = (35 + 10)$ years = 45 years,
 $B = (40 + 10)$ years = 50 years,
 $C = (45 + 10)$ years = 55 years } *Ans*

Ex 6 Two passengers have together 9 mds of luggage and are charged for the excess above the weight allowed Rs 2 15a 4p, and Rs 4 5a 8p respectively, but if the luggage had all belonged to one of them he would have been charged Rs 7 14a Find the quantity of luggage allowed free, as also the charge per maund

A man is allowed to carry a certain weight of his luggage free of charge, so two men are allowed twice that weight

Also Rs 2 15a 4p + Rs 4 5a 8p = Rs 7 5a

\therefore charge on 9 mds = Rs 7 14a + charge on free allowance,
 and = Rs 7 5a + twice

\therefore charge on free allowance = Rs 7 14a - Rs 7 5a = 9a

charge on 9 mds = Rs 7 14a + 9a = Rs 8 7a

proper charge per maund = Rs 8 7a - 9a = 15a *Ans*

Again, charge on 1 md = 15a, and that on free allowance 9a

. 15a 9a 1 md the free allowance,

the free allowance = $\frac{9}{15}$ md = $\frac{3}{5}$ md = 24 sr *Ans*

Ex 7 A certain number is divided into two parts, such that 5 times one part added to 18 times the other may give 7 times the whole Find the ratio of the parts

Since 5 times the 1st part + 18 times the 2nd part = 7 times the whole,
 and 5 times the 1st part + 5 times the 2nd part = 5 times the whole,

\therefore by subtraction, 13 times the 2nd part = twice the whole
 Hence the 2nd part = $\frac{2}{13}$ of the whole, the 1st part = $\frac{11}{13}$ of the whole

1st part 2nd part $\frac{11}{13}$ $\frac{2}{13}$ or 11 2 *Ans*

Ex 8 A person bought 20 railway tickets for Rs 71 Each first class ticket costs Rs 6, and each second class ticket costs Rs 2.8a What will another lot of 20 tickets, in which the present numbers of first and second-class tickets, are interchanged, cost?

Had all the 20 tickets been second-class, he should have to pay Rs $2\frac{1}{2} \times 20 =$ Rs 50 Thus he shall have to pay Rs $(71 - 50)$ or Rs 21

more than he pays at present. But he has to pay Rs $(6 - 2^1)$ or Rs $3\frac{1}{2}$ more for a first class than for a second class ticket. therefore the no of first class tickets = Rs $21 - Rs \frac{1}{2} = 6$, and the no of second class tickets = $20 - 6 = 14$

∴ on the interchange of tickets, the cost = Rs $6 \times 14 + Rs \frac{1}{2} \times 6 = Rs (84 + 3) = Rs 87$ Ans

Ex 9. A person bought 30 animals, consisting of oxen and cows, for Rs 860. If he had bought as many oxen as he had bought cows, and as many cows as he had bought oxen, the whole lot would have cost him Rs 790. If the cost of an ox and two cows together be Rs 65, find the cost of an ox and of a cow.

Since the original lot of 30 animals cost Rs 860

and the second lot of 30 animals cost Rs 790,

∴ 30 oxen + 30 cows cost Rs 1650 (Art 514)

∴ 1 ox and 1 cow cost Rs 55

But 1 ox and 2 cows cost Rs 65

∴ by subtraction 1 cow costs Rs 10

Hence the cost of 1 ox = Rs $(55 - 10) = Rs 45$

} Ans

Ex 10. In a vessel water and milk are mixed in the proportion of 2 : 7. In another they are mixed in the proportion of 2 : 9. In what proportion should quantities be taken from the vessels to form a mixture in which milk and water will be in the proportion of 4 : 1?

Since $2+7=9$, ∴ in 9 sr of the first vessel, we have 2 sr water and 7 sr milk. If, therefore, 1 sr be taken out of the first vessel, it will contain $\frac{7}{8}$ sr of milk. Similarly, 1 sr from the second vessel contains $\frac{2}{11}$ sr of milk, and 1 sr of the final mixture contains $\frac{1}{10}$ sr of milk.

$\frac{1}{8} \times \frac{7}{8}$ of 1st vessel | Hence the reqd proportion is
 $\frac{1}{11} \times \frac{2}{11}$ of 2nd | $\frac{7}{8} \times \frac{1}{8}$ or 45 : 55 or 9 : 11

Miscellaneous Examples VII

1. A bill of 37 lbs of sugar and 24 lbs of tea amounted to Rs 21 7s 2d, the tea cost 1s 2d more per lb than the sugar. Find the price of the sugar per lb.

2. A merchant's average rate of profit for five years was 5 per cent on his capital and for the first four years his average profit was 4 per cent. What was his rate of profit in the fifth year?

3. If mangoes be bought at the rate of seven for an anna, how must they be sold to gain 33 per cent?

4. Fifty coolies are employed for a day and receive at the end of the day for the whole work they have done Rs 11 2s, the men

being paid at the rate of $4a$, and the women at the rate of $3a$ per day. How many of them were women?

5 A , B and C are in partnership, and $\frac{1}{3}$ of A 's share is equal to $\frac{1}{2}$ of B 's share or $\frac{1}{4}$ of C 's and is Rs 250 less than B 's. What is each man's share?

6 The price of rice being raised 50 per cent, how much per cent must a house-holder reduce his consumption of that article so as not to increase his expenditure?

7 One-third of a population can read, of the remainder 45 per cent can read and write, of what still remains 9 per cent can read, write and count, the rest is 500500 who can neither read, write nor count. Find the total population.

8 A merchant buys some cloth at such a price that by selling it at Rs 4 6a per yard he will gain 5 per cent on his outlay. What percentage will he gain or lose if the cloth be sold at Rs 3 14a per yd?

9 In a company of 100 people, of whom some are rich and some poor, the rich subscribe and give 1a 3p to each poor man, this costs the rich men 7a 1p each, how many rich and how many poor men are there?

10 In sending 1000 cheroots to England I paid freight $\frac{1}{3}$ ds of their prime cost, landing charges $\frac{1}{3}$ rd of their cost, including freight, and duty $2\frac{1}{2}$ times their cost including freight and landing charges. Altogether the cheroots, duty paid, in London cost me £7. What did I give for them at Calcutta?

11 Divide 1800 into two parts such that 8 times the first added to 12 times the second may become 17600.

12 The average weight of students in a class of 30 boys is 100 lbs. If however, the weight of the teacher be included the average weight will increase by 1 lb. Find the weight of the teacher.

13 A 's salary is 40 per cent more than B 's. How much per cent is B 's less than A 's?

14 In an examination, a candidate must get 40 per cent marks to pass. A candidate who gets 210 marks fails by 40 marks. Find the maximum number of marks.

15 An oz of gold is worth £4 and an oz of silver is worth 8s, and a mass of gold and silver weighing 80 oz is worth £101. What is the worth of another mass, in which the number representing gold and silver in oz in the first mass are interchanged?

16 If 10 oz of gold 13 carats fine, 14 oz of gold 12 carats fine, 12 oz of gold 11 carats fine, and 24 oz of gold 10 carats fine be mixed together, find the fineness of the compound.

17 A person bought 100 chairs, some at Rs 4 each and others at Rs 3 each. The total price that he paid was Rs 1350. How many of each sort did he buy?

18 2 lbs of tea and 5 lbs of sugar cost 7s 6d, but, if sugar were to rise $33\frac{1}{3}$ per cent, and tea 20 per cent, they would cost 9s 4d, find the cost of the tea and the sugar per lb

19 12 lbs of tea and 25 lbs of coffee together cost Rs 43 5a 4p, but, if the tea were to rise $2\frac{1}{3}$ per cent, and the coffee to fall $4\frac{1}{2}$ per cent, the same quantities would cost Rs 42 15a 4p, find the price of the tea and the coffee per lb

20 There are two compound metals, the one consisting of a mixture of copper and gold, and the other of a mixture of copper and silver. The value of an ounce of gold, silver and copper are £5, 5s and 5d respectively. Find how much copper must be mixed with the first mixture in order that the value of a given quantity of the first mixture may be 15 times the value of an equal weight of the second mixture containing 80 per cent of pure silver

21 Gold costs £3 17s 10½d per oz and silver 5s 6d per oz, in what proportion must these metals be mixed that a lb of the mixture may be worth £32 5s?

22 The external length, breadth and height of a wooden box are 18, 10 and 6 in respectively, and the thickness of the wood is half an inch, when the box is empty it weighs 15 lbs and when filled with sand 100 lbs. Compare the weights of equal bulks of sand and wood

23 A well is fed by a spring which flows continuously and uniformly in it. When there are 10000 cub ft of water in the well, 7 men can empty it in 20 days, and when there are 15000 cub ft of water in the well, 5 men can empty it in 50 days. How many cub ft of water flow into the well in one day?

24 There are two fields whose lengths are 150 yds and 244 yds and the breadth of the second half as great again as that of the first, these are respectively dug by men and boys who each takes a strip of one given breadth. If the men be paid £49 9s 7d, what will be due to the boys, the workmanship of each these being $\frac{1}{3}$ ths as good as a man's?

25 A merchant buys 5000 mds of rice, one fifth of which he sells at a profit of 5 per cent, one-fourth at a profit of 10 per cent and the remainder at a profit of 16 per cent. If he had sold the whole at a profit of 15 per cent he would have made Rs 438 12a more, what was the cost of the rice per maund?

26 A person buys 5 shares in a company, and sells three of them at a gain of 10 per cent and the remaining two at a gain of $16\frac{2}{3}$ per cent. The gain on the latter sale is £2 19s 7½d more than on the former. Find the price of a single share

27 Divide 320 nuts among three boys A, B and C, in such a way that as often as A receives 8, B shall receive 5, and for every 6 B receives, C shall receive 10

28. A shareholder in a commercial company receives one year a dividend of 5 per cent on his shares. The next year he receives a dividend of $7\frac{1}{2}$ per cent, and finds that he is Rs 412 8a richer. Find the amount of his shares.

29. A person bought 600 animals—horses and oxen—for Rs 40000, each horse cost Rs 80 and each ox Rs 60. Find the number of horses.

30. Divide Rs 50 between A and B , so that $\frac{2}{3}$ of A 's share may exceed $\frac{1}{2}$ of B 's share by Rs 5.

31. A sum of money is divided between A and B so that A gets 10 per cent more than B . If A 's share exceed B 's by Rs 4, find the sum distributed and the share of each.

32. Divide Rs 240 among 8 men, 12 women and 16 boys, so that each man will receive Rs 2 more than each woman, and each woman Rs 2 more than each boy.

33. A certain number is divided into two parts, such that 5 times the first part added to 11 times the second makes 7 times the whole. Find the ratio of the parts.

34. A gentleman engaged a servant on the understanding that he was to receive a coat and Rs 60 after serving for 6 months. He served only for 4 months and received the coat and Rs 30. Find the price of the coat.

35. A boy buys a number of apples at 4 for 3d and a third of the number at 5 for 4d. He sells the whole lot at 6 for 5d. How much does he gain per cent? If his total gain is 7s 1d, how many did he buy?

36. A market woman buys a certain number of mangoes at 20 a rupee, and an equal number at 25 a rupee. She mixes them together and sells them at 21 a rupee. What does she gain or lose per cent?

37. A contributes a certain capital for 4 months, and B contributes Rs 400 for 5 months. If their profits be in the proportion of 3 : 4, what was the capital contributed by A ?

38. The times for which A and B had contributed their capitals are in the proportion of 1 : 2, and the profits received by them are in the proportion of 3 : 4. Compare their capitals.

39. The sum of the ages of three men is 108 years. Twelve years before, their ages were in the proportion of 5 : 4 : 3, find their ages.

40. A lump composed of gold and silver measures 6 cub in and weighs 100 oz. If a cubic inch of gold weighs 20 oz and an equal bulk of silver 12 oz, find the weight of gold in the mixture.

41. In a certain school, 85 per cent of the boys passed in English, 80 per cent passed in Arithmetic, 5 per cent failed in both subjects and 210 passed in both subjects. Find the number of boys in the school.

42 A, B and C rent 1 field for Rs 2878 A puts in 12 horses for 5 months and 45 sheep for 3 months, B puts in 15 oxen for 6 months and 54 sheep for 2 months, C puts in 6 horses and 48 oxen for 3 months. Now, 4 horses and 3 sheep together eat as much as 5 oxen and 1 horse, and 2 oxen eat as much as 7 sheep, how much of the rent should A, B and C pay respectively?

43 A mass of copper and brass weighs 11 seers, and is worth Rs 7 2a. If the proportion of copper and brass be interchanged, the mass would be worth Rs 7 5a. If 1 seer of copper and 2 seers of brass cost Re 1 14a, find the cost of copper and brass per seer and the proportion of the mixture.

44 Divide the number 540 into two parts such that 25 times the greater may exceed 11 times the less by 15 times the whole number.

45 A person buys a certain number of animals for Rs 8000, consisting of horses at Rs 150 each and oxen at Rs 80 each. If he interchanges the numbers of horses and oxen, he requires Rs 1050 less. Find the number of horses and also that of oxen.

46 8 oz of gold 10 carats fine, and 2 oz. of gold 11 carats fine, are mixed with 6 oz of gold of unknown fineness. If the fineness of the compound be 12 carats, find the fineness of the 6 oz gold.

47 1 person bought 60 apples, some at 3 for 2ps and others at 4 for 3ps, and gave 10a 3ps in all. How many were bought at the first rate?

48 Divide 50 into two parts such that 4 times the first part added to 8 times the second may exceed 5 times the whole by 10.

49 A person bought 36 fruits, consisting of mangoes and plantains, for 10a 2ps. Had he bought as many plantains as he had bought mangoes, and as many mangoes as he had bought plantains, they would have cost him 12a. If the cost of one plantain and two mangoes be 4ps, how many of each sort did he buy?

50 The ages of A, B and C are in the proportion of 3 : 7 : 11, but 7 years hence the sum of their ages will be 105 years. Find their present ages.

51 Two passengers are charged for excess of luggage Rs 2 10a and Rs 5 8a respectively, had the luggage all belonged to one of them he would have been charged for excess Rs 8 12a, how much would they have been charged if none had been allowed free?

52 Two passengers have together 8 mds 25 sr of luggage and are charged for the excess Rs 3 4a and Rs 4 6a respectively. If all the luggage had belonged to one of them he would have been charged Rs 8 2a. Find the amount of luggage owned by each.

53 A footman who contracted for Rs 80 a year and a livery suit, was turned away at the end of 7 months and received only Rs 21. 10a 8p and his livery. What was its value?

54 In a jail, there are 260 male criminals and 120 female criminals. The former in the course of a year increased 5 per cent, while the latter decreased 10 per cent. Find the increase or decrease per cent in the whole number of criminals.

55 If 40 oxen consume the grass of a meadow growing uniformly in 12 days, and if 25 oxen consume the grass of the same meadow in 20 days, find in how many days 30 oxen will consume it.

56 A market woman bought 300 oranges at 3 for 2 p, and some more at 2 a pice. She mixed them together, and sold them at 4 for 3 p and gained 25 per cent. How many of the second sort did she buy?

57 A sum of money is distributed among A , B , C and D in the proportion of 1, 2, 3 and 4 so that D gets Rs 20 more than C . Find the whole sum distributed and the share of each.

58 A sum of money is divided among A , B and C , so that A has 20 per cent more than B , and B has 25 per cent more than C . If A 's share be Rs 50, find the whole sum distributed and the share of each.

59 Two casks, A and B are filled with different mixtures of wine and water, in the cask A the mixture is in the proportion of 2 : 5 and in the cask B it is in the proportion of 1 : 3. What quantities must be taken from the two casks to have a new mixture consisting of 8 gallons of wine and 21 gallons of water?

60 One vessel contains a mixture of milk and water in the proportion of 3 : 5, in another they are mixed in the proportion of 6 : 1. In what proportion should quantities be taken from the two vessels so as to form a mixture in which milk and water will be in the proportion of 7 : 3?

CHAPTER XIII

Interest, Present Worth and Discount

516 Interest is the payment made for the use of money lent for any length of time at a fixed rate.

The money lent is called the **Principal**. The sum lent or principal together with its interest is termed the **Amount**. The **Rate** is the money paid for the use of a certain sum for a certain time. The interest of Rs 100 or £100 for a year is called the **rate per cent per annum** (*per annum* means *for a year*). Unless otherwise stated, by rate per cent is always to be understood the rate of interest *per annum*.

Thus, if a man borrows a sum of money on the condition that for the use of every Rs 100 in the loan for one year he shall pay

an interest of Rs 8, he is said to borrow *at the rate of 8 per cent per annum*. Again, if the sum of money borrowed be Rs 900, and the interest upon it in a certain time is Rs 70, Rs 900 is called the *Principal*, Rs 70 the *Interest* and Rs (900+70) or Rs 970, the *Amount*.

517 Interest is of 11 kinds, simple and Compound

It is called *Simple Interest*, when the money advanced only pays interest for the whole time it is lent, and *Compound Interest*, when at the end of any assigned period as a year for instance, the interest which has accrued is added to the principal, and the whole then bears interest at the same rate for another equal period and so on.

I SIMPLE INTEREST

518 To find the Simple Interest on a given sum of money for a given time, when the rate of interest per *Re* or *£* for one month is given.

RULE Multiply together the principal, the given time, and the rate of interest, the product will give the required interest in the same denomination as the given rate of interest. Or apply *Rule of Three*.

Ex Find the Simple Interest on Rs 76 for 9 mo at $\frac{1}{2}\%$ per *Re* a mo.

The reqd int = $76 \times 9 \times \frac{1}{2}\%$ | Since the interest on *Re* 1 for 1 mo is $\frac{1}{2}\%$, the interest on Rs 76 for 1 mo is $\frac{1}{2}\%$,
 = 342a | $76 \times \frac{1}{2}\%$, and the interest on Rs 76
 or, 1 76 $\frac{21}{100}$ reqd int. for 9 mo is $76 \times 9 \times \frac{1}{2}\%$ or 342a =
 ∴ $\frac{3}{10}a = \text{int on Re 1 for 9 mo}$ Rs 21 6a

Examples CLVII

1 Find the Simple Interest on —

- (1) Rs 58 for 6 months at 6% per rupee per month
- (2) 86 8 2%
- (3) 465 9 $\frac{1}{2}\%$
- (4) 370 18 3%
- (5) 250 $2\frac{1}{2}$ years $\frac{3}{4}\%$
- (6) 550 2 years 4 mo 6%
- (7) £735 14 months $\frac{3}{4}\%$ per pound per month
- (8) 975 $4\frac{1}{2}$ years $\frac{1}{2}\%$ per pound

2 If a man gets 5a for the loan of 2s 6d for 1 year, what is the rate per cent per annum?

3 What is the rate of interest per cent per annum, if I pay Rs 25 5a for the use of Rs 3000 for 1 month?

4 A man gets 4a for the loan of Rs 2 8a for 2 months, find the rate of interest per cent per annum.

5 If 1a per week be paid for the loan of Rs 6 8a, what is the rate per cent per annum?

519 To find the Interest on a given sum of money at a given rate per cent for a given time

RULE Multiply the principal by the rate per cent, the product by the time in years, and divide the result by 100 Or apply Rule of Three

Ex Find the Simple Interest on Rs 2500 for 4 years at 5 per cent

$$\begin{aligned} \text{The reqd int} \\ = \text{Rs } \frac{2500 \times 5 \times 4}{100} \end{aligned}$$

$$\begin{aligned} &= \text{Rs } \frac{500}{100} \quad 2500 \\ \text{or } & \frac{500}{100} \end{aligned}$$

Rs 20 reqd int

Since the int on Rs 100 for 1 yr is Rs 5

$$\begin{aligned} \therefore & \text{Rs } 1 & \text{Rs } \frac{5}{100} \\ & \vdots & \\ & \text{Rs } 2500 & \text{Rs } \frac{2500 \times 5}{100} \\ & \cdot & \\ & \text{Rs } 2500 \text{ for 4 yrs is Rs } \frac{2500 \times 5 \times 4}{100} \end{aligned}$$

or Rs 500

520 In practice, however, we adopt the following forms —

Ex Find the Simple Interest on £240 12s 6d at 2½ per cent for 8½ years, and also the Amount

$$\begin{array}{rcl} \text{(i) } \begin{array}{r} \text{£} \quad \text{s} \quad \text{d} \\ 240 \quad 12 \quad 6 \quad \text{Principal} \\ \quad \quad \quad 2\frac{1}{2} \text{ rate p c} \\ \hline 481 \quad 5 \quad 0 \\ 120 \quad 6 \quad 3 \\ \hline 601 \quad 11 \quad 3 \\ \quad \quad \quad 8\frac{1}{2} \text{ no of years} \end{array} \end{array}$$

$$\begin{array}{r} 4812 \quad 10 \quad 0 \\ 451 \quad 3 \quad 5\frac{1}{2} \\ \hline \text{£} 52 \quad 63 \quad 13 \quad 5\frac{1}{2} \end{array}$$

20

s 12 73

12

d 8 81

4

q 3 25

$$\text{(ii) } 2\frac{1}{2} \times 8\frac{1}{2} - 100 = 7 - (8 \times 4)$$

$$\begin{array}{r} \text{£} \quad \text{s} \quad \text{d} \\ 240 \quad 12 \quad 6 \\ \quad \quad \quad 7 \\ \hline 8 \overline{) 1684 \quad 7 \quad 6} \\ \underline{4 \quad 1684 \quad 7 \quad 6} \\ 4 \overline{) 210 \quad 10 \quad 11\frac{1}{2}} \\ \underline{52 \quad 125 \quad 8\frac{1}{2} \quad 1\frac{1}{2}} \end{array}$$

$$\text{interest} = \text{£} 52 \quad 12\text{s} \quad 8\frac{1}{2}\text{d} \quad 1\frac{1}{2}\text{q}$$

$$\text{and principal} = \text{£} 240 \quad 12\text{s} \quad 6\text{d}$$

$$\therefore \text{the amount} = \text{£} 293 \quad 5\text{s} \quad 2\frac{1}{2}\text{d} \quad 1\text{a} \quad \text{Ans}$$

521 When the interest for months and days is required, it is found by Practice and the Rule of Three respectively, reckoning 12 months and 365 days to a year and 30 days to a month, but if calendar months be specified, and the interest has to be calculated from one given day to another, it is customary to include one only of the days named in counting the period of time

Ex 1 Find to the nearest penny the Simple Interest on £512

16s 8d at $4\frac{1}{2}$ per cent for 3 years 7 months 21 days, and also the amount

		£	s	d
		512	16	8
				<u>4½</u>
		2051	6	8
		320	10	5
6 mo	$\frac{1}{2}$	2371	17	1
				3
		7115	11	3
1 mo	$\frac{1}{12}$	1185	18	6½
15 days	$\frac{1}{24}$	197	15	1½
6 days	$\frac{1}{40}$	95	16	6½
		39	10	7½
		£86	37	10 0½
			20	
		£ 7	50	
			12	
		£ 6	00	

$$\begin{array}{r} \text{£}512 \quad 16s \quad 8d \times \frac{5}{8} \\ \hline 5 \\ 8)2564 \quad 3 \quad 4 \\ \hline \text{£}320 \quad 10 \quad 5 \end{array}$$

Interest = £86 7s 6d nearly,

and principal = £512 16s 8d

Amount = £599 4s 2d Ans

Ex 2 Find the amount of Rs 13765 10a at $4\frac{1}{4}$ per cent from April 6th to August 30th

	Rs	a	p
	13765	10	0
			<u>4¼</u>
	55062	8	0
	10324	3	6
Rs	653 86	11	6
	16		
a	13 87		
	12		
p	10 50		

	Rs	a	p
	13765	10	0 × ¼
			3
	4)41296	14	0
Rs	10324	3	6

Interest for 1 year = Rs 653 13a 10 5p
Now, the number of days from April 6th to August 30th = 24 + 31 + 30 + 31 + 30 = 146

∴ 365 days 146 days Rs 653 13a 10 5p int reqd

or 5 2 Rs 653 13a 10 5p int reqd

∴ int reqd = $\frac{2}{5}$ of Rs 653 13a 10 5p = Rs 261 3a 9p

∴ Amount reqd = Rs 13765 10a + Rs 261 3a 9p = Rs 14027 2a 9p

Examples CLVIII

1 Find the Simple Interest for one year on —

- | | |
|--|---|
| (1) Rs 6400 at $4\frac{1}{4}$ per cent | (4) Rs 7150 13a 4p at $2\frac{3}{8}$ per cent |
| (2) 984 12a at 5 per cent | (5) 578 8a at 5 per cent |
| (3) 1762 4a, at 5 per cent | (6) £2055 at $5\frac{1}{2}$ per cent |

- (7) £3197 5s at 5 per cent (9) £2814 7s 6d at 5 per cent
 (8) 572 at $4\frac{1}{4}$ per cent (10) 55 16s 8d at $3\frac{3}{8}$ per cent

2 Find the Simple Interest on —

- (1) Rs 525 for 5 yrs at $3\frac{1}{2}$ p c (7) £650 for 8 yrs at $4\frac{1}{4}$ p c
 (2) 9513 12a for $2\frac{1}{4}$ yrs at $6\frac{1}{4}\%$ (8) 450 10s for 7 yrs at $3\frac{1}{4}\%$
 (3) 3406 4a for $4\frac{1}{4}$ yrs at 4 p c (9) 237 10s for 15 mo at 4%
 (4) 12500 for 18 mo at $4\frac{1}{4}$ p c (10) 476 18s 6d for $4\frac{3}{4}$ yrs at $3\frac{1}{4}\%$
 (5) 4336 10a 8p for 219 dys at $4\frac{1}{4}\%$ (11) 3450 12s 7d for $8\frac{1}{2}$ yrs at $4\frac{5}{8}\%$
 (6) 5438 12a for $2\frac{1}{4}$ yrs at $3\frac{7}{8}\%$ (12) 1923 15s for 2yrs 8mo at $5\frac{1}{4}\%$

3 Find to the nearest *pie* or *penny* the Simple Interest on —

- (1) Rs 5438 12a for 43 days at $3\frac{1}{2}$ per cent
 (2) 568 5a for 3 yrs 113 days at $5\frac{3}{8}$ per cent
 (3) £317 10s $2\frac{3}{4}$ d for 3 yrs 73 days at $3\frac{5}{8}$ per cent
 (4) 550 14s 8d for 2 yrs 9 mo 25 days at $4\frac{3}{4}$ per cent
 (5) Rs 52605 1a 4p for 6 yrs 5 mo 21 days at $2\frac{7}{8}$ per cent
 (6) £460 3s 6d for 3 yrs $8\frac{3}{4}$ mo at $4\frac{3}{4}$ per cent
 (7) Rs 1841 14a from July 17th to Dec 5th at $5\frac{1}{4}$ per cent
 (8) 48655 11a 4p from Jan 1st to Aug 28th 1876, at $5\frac{3}{8}$ per cent
 (9) £473 3s 6d from April 14th to July 6th at $3\frac{1}{4}$ per cent
 (10) 164 15s 11d from 9 Nov 1867 to 3 Mar 1868, at $5\frac{3}{4}$ p c

4 Find the amount of —

- (1) Rs 5378 5a 4p for 4 yrs at $2\frac{1}{2}\%$ (2) Rs 3458 12a, for 3 yrs at 4%
 (3) £825 13s 8d for 3 yrs 5 mo at $4\frac{1}{4}$ per cent
 (4) Rs 2375 for 2 yrs 8 mo 29 days at 5 per cent
 (5) 18354 2a 8p from March 11th to Aug 4th at $4\frac{3}{4}$ per cent

522 Inverse Questions on Simple Interest

Every question in Simple Interest involves the consideration of **Principal**, **Rate per cent**, **Time** and **Interest** or **Amount**, and three of these quantities are always given, to find the fourth. There are then *four* cases according as the quantity to be found is (1) *Interest* or *Amount*, (2) *Principal*, (3) *Rate per cent*, (4) *Time*.

The RULES for finding the Interest or amount have already been explained. The next three cases will be considered in order.

“**Case II** *Having given the interest or amount, rate per cent and time, to find the principal*

(1) Let the *interest* be given

RULE Find the interest of Rs 100 or £100 at the given rate per cent. for the given time, then, state thus —

this interest given interest Rs 100 or £100 : principal reqd

Ex What principal will produce Rs 438 6a interest in 5 years at $3\frac{1}{2}$ per cent

Rs 100 produce, in 5 yrs at $3\frac{1}{2}$ p c, Rs $5 \times 3\frac{1}{2}$ = Rs $17\frac{1}{2}$ interest
 \therefore Rs $17\frac{1}{2}$ Rs 438 $\frac{3}{4}$ Rs 100 principal reqd

$$\therefore \text{required principal} = \text{Rs } \frac{100 \times 3507 \times 2}{8 \times 35} = \underline{\text{Rs } 2505} \quad \text{Ans}$$

(ii) Let the *amount* be given

RULE Find the amount of Rs 100 or £100 at the given rate per cent for the given time, then, state thus —

this amount given amount Rs 100 or £100 principal reqd

Ex What sum will amount to Rs 4053 7a in $4\frac{1}{2}$ yrs at 4 p c ?

Rs 100 in $4\frac{1}{2}$ yrs at 4 p c amounts to Rs $(100 + 4\frac{1}{2} \times 4)$ or Rs 119
 \therefore Rs 119 Rs 4053 $\frac{7}{8}$ Rs 100 principal reqd

$$\therefore \text{reqd principal} = \text{Rs } \frac{100 \times 6485}{16 \times 119} = \underline{\text{Rs } 3406 \text{ 4a}} \quad \text{Ans}$$

Case III *Having given the principal, time, interest or amount, to find the rate per cent*

RULE Find the interest on the given principal for the given time at 1 per cent, then, state thus —

this interest given interest 1 rate per cent reqd

that is,—rate per cent required is found by dividing the given interest by the interest at 1 per cent

Ex 1 At what rate per cent will £33 6s 8d amount to £38 4s 2d in $4\frac{1}{2}$ years ?

$$\text{Given Int} = \text{£}38 \text{ 4s } 2\text{d} - \text{£}33 \text{ 6s } 8\text{d} = \text{£}4 \text{ 17s } 6\text{d}$$

$$\text{Int on £}33 \text{ 6s } 8\text{d} \text{ at 1 p c for } 4\frac{1}{2} \text{ yrs} = \text{£}1 \text{ 10s}$$

$$\therefore \text{rate per cent reqd} = \text{£}4 \text{ 17s } 6\text{d} \div \text{£}1 \text{ 10s} = \underline{3\frac{1}{2}} \quad \text{Ans}$$

Ex 2 At what rate per cent will a sum of money double itself in $12\frac{1}{2}$ years ?

In $12\frac{1}{2}$ years the interest is equal to the principal,

$$\therefore \text{interest on £}100 \text{ for } 12\frac{1}{2} \text{ years} = \text{£}100,$$

$$\text{but interest on £}100 \text{ at 1 per cent for } 12\frac{1}{2} \text{ years} = \text{£}12\frac{1}{2},$$

$$\text{rate per cent reqd} = \text{£}100 \div \text{£}12\frac{1}{2} = \underline{8} \quad \text{Ans}$$

Case IV *Having given the principal, rate per cent, and interest or amount, to find the time*

RULE Find the interest on the given principal for one year, then, state thus —

one year's int given int 1 no of years reqd

that is,—the number of years is found by dividing the given interest by the interest for 1 year

Ex 1 In what time will Rs 4250 amount to Rs 6353 12a at $5\frac{1}{2}$ per cent ?

Given interest = Rs 6353 12a - Rs 4250 = Rs 2103 12a

Int on Rs 4250 for 1 year = Rs 4250 $\times 5\frac{1}{2}$ - 100

$$\therefore \text{no of years reqd} = \frac{\text{Rs } 2103\frac{1}{2} \times 100}{\text{Rs } 4250 \times 5\frac{1}{2}} = 9 \text{ Ans}$$

Ex 2 In what time will a sum of money treble itself at 8 p c ?

The time will be the same whatever sum of money be taken as the principal, suppose, the principal to be Rs 100,

given interest = 2 \times principal = Rs 200,

and interest on Rs 100 for 1 year = Rs 8

$$\text{no of years required} = \frac{\text{Rs } 200 - \text{Rs } 8}{8} = 25 \text{ Ans}$$

543 The following *formulae*, if committed to memory, will be of great use

If P, I, M, *n*, *r*, be respectively the Principal, Interest, Amount, time and rate,

$$I = \frac{Pnr}{100}, M = P + I = P + \frac{Pnr}{100} = P \left(1 + \frac{nr}{100} \right), P = \frac{100 \times I}{nr} = \frac{100 \times M}{100 + nr},$$

$$r = \frac{(M - P) \times 100}{Pn} = \frac{100 \times I}{Pn}, n = \frac{(M - P) \times 100}{Pr} = \frac{100 \times I}{Pr}$$

Examples CLIX

1 What principal will produce Rs 366 4a interest in $2\frac{1}{2}$ years at $3\frac{1}{2}$ per cent ?

2 What principal must be put out for $2\frac{1}{2}$ years at 4 per cent to amount to Rs 1325 8a ?

3 What sum will amount to Rs 3761 14a in $3\frac{1}{2}$ years at $4\frac{1}{2}$ per cent per annum ?

4 At what rate per cent will Rs 1368 12a amount to Rs 1642 8a in $6\frac{1}{4}$ years ?

5 At what rate per cent will the interest on Rs 357 8a amount to Rs 40 3a 6p in $4\frac{1}{2}$ years ?

6 At what rate per cent will the interest on 500 guineas amount to £103 9s $4\frac{1}{2}$ d in 3 years 7 months ?

7 What principal will amount to Rs 13577 2a in 2 year 7 months at $4\frac{1}{2}$ per cent ?

8 What principal will amount to £725 12s 6d in 2 years 9 months 18 days at $2\frac{1}{2}$ per cent ?

9 Find the principal whose interest amounts to Rs 578 5a 4p in 1 year 9 months 24 days at $3\frac{3}{4}$ per cent

10 In how many years will the interest on Rs 357 8a amount to Rs 40 3a 6p at $2\frac{1}{2}$ per cent ?

11 In what time will £563 13s 4½d amount to £901 17s 4½d at 3½ per cent ?

12 In what time will the interest on Rs 8125 6a 8p amount to Rs 7719 2a 4p at 4½ per cent ?

13 At what rate per cent will £7433 6s 8d amount to £9942 1s 8d in 7½ years ?

14 The interest of a sum of money at the end of 6½ years is ¼ths of the sum itself, what rate per cent was charged ?

15 In how many years will a sum of money amount to half as much again as itself at 7½ per cent ?

16 If Rs 79 12a be charged for the loan of Rs 7435 for 87 days, what is the rate per cent ?

17 In how many years will a sum of money double itself at 6½ per cent per annum ?

18 At what rate per cent will the interest on Rs 13687 8a become Rs 142 5s from July 5th to Nov 20th ?

19 What sum of money laid out at 4 per cent will give 2a interest a day ?

20 What principal in 15 years at 4 per cent will amount to the same sum as Rs 45000 in 9 years at 6 per cent ?

21 In what time will Rs 10755 amount to Rs 15594 12a if in 4 years Rs 1762 8a amount to Rs 1974 ?

22 At what rate will Rs 500 amount to Rs 700 in a time in which £120 produces £15 at 4 per cent Simple Interest ?

23 In what time will £140 12s 6d amount to £175 10s at a rate of interest at which £638 doubles itself in 25 years ?

24 What sum will amount to £678 5s in 1½ years at a rate in which £502 13s 4d amounts to £578 1s 4d in 3½ years ?

25 What will Rs 4906 4a amount to in 8 years, the rate of interest being that at which Rs 50000 amounts to Rs 54020 in 1 year 219 days ?

26 What is the rate of Simple Interest, if in 8 years the amount of £425 becomes the same as the amount of £502 18s 4d in 3½ years at 5½ per cent ?

27 What will Rs 18375 amount to at 4½ per cent in a time in which £1033 6s 8d amounts to £1103 1s 8d at 2½ per cent ?

28 A certain sum amounted to Rs 3666 10a 8p at 4 per cent Simple Interest in a time in which Rs 9120 amounted to Rs 10488 at 6 per cent. What was the sum ?

29 What sum at 3½ per cent will give a guinea interest per day ?

30 The sum of Rs 3270 is borrowed at the beginning of the year at a certain rate of interest and after 9 months Rs 4000 more is borrowed at double the previous rate. At the end of the year the

interest on both loans is Rs 131 12a What is the rate of interest at which the first sum was borrowed ?

31 A lends Rs 300 to B for 2 years, and Rs 75 to C for 4 years, and receives altogether from both Rs 60 as interest Find the rate of interest, Simple Interest being calculated

32 The interest on Rs 800 at 4 per cent for a certain time, and that on Rs 1000 for 2 years more at 5 per cent are together Rs 346 For what periods are the interests calculated ?

33 A received from B Rs 500 on the 8th of April, at 10 per cent per annum, Simple Interest Finding, however, that it was a bad debt, B agreed to accept Rs 480 on the 1st of September of the same year in repayment of the debt How much did B lose ?

34 A lends Rs 500 to B, and a certain sum to C, at the same time, at 8 per cent Simple Interest If in 4 years he altogether receives Rs 210 as interest from the two, find the sum lent to C

35 A lends a certain sum to B, and a sum larger than the first by Rs 800 to C, B agrees to pay 5 per cent interest and C 7 per cent Both return the sums with interests at the end of 5 years If C's amount exceeds B's by Rs 1240, what sum did each borrow ?

II COMPOUND INTEREST

524 In Compound Interest, the interest of each period is added to its principal and the amount forms a new principal for the next period The period is always understood to be a year, unless the contrary is stated

525 To find Compound Interest on a given sum of money at a given rate per cent for any number of years

RULE At the end of each year add the interest of that year, found by (Art 519), to the principal at the beginning of it, this will be the principal for the next year, and continue the process in the same way as far as may be required by the question Add together the interests so arising in the several years, and the result will be the compound interest for the given period

Ex 1 Find the Compound Interest on Rs 4508 6a for 3 years at $4\frac{1}{2}$ per cent

	Rs	a	p
1st Principal	4508	6	0
			4 $\frac{1}{2}$
	18033	8	0
	1127	1	6
Rs	191 60	9	6
	16		
	29 69		
	12		
	8 34		

	Rs	a	p
Principal	4508	6	0
1st year's int	191	9	8 34

2nd Principal	<i>Rs</i> 4699	<i>a</i> 15	<i>p</i> 8 34	2nd year's int	<i>Rs</i> 199	<i>a</i> 11	<i>p</i> 11 84445
			4½				
	18799	14	9 36				
	1174	15	11 085				
<i>Rs</i> 199 74		14	8 415				
	16						
<i>a</i> 11 98							
	12						
<i>p</i> 11 84445							
3rd Principal	<i>Rs</i> 4877	<i>a</i> 11	<i>p</i> 8 16445	3rd year's int	<i>Rs</i> 208	<i>a</i> 3	<i>p</i> 9 797839125
			4½				
	19598	14	8 7378				
	1224	14	11 0461125				
<i>Rs</i> 208 23		13	7 839125				
	16						
<i>a</i> 3 61							
	12						
<i>p</i> 9 797839125							
				Total int	<i>Rs</i> 599	<i>a</i> 9	<i>p</i> 6 nearly

Or thus, by Decimals — It is not really necessary to take more than *four* decimal places in the calculation

Now, $Rs\ 4508\ 67 = Rs\ 4508\ 375$, and $4\frac{1}{2}$ per cent $= \frac{1}{20} + \frac{1}{40}$

4 p c $= \frac{1}{20}$	<i>Rs</i> 4508 3750	1st Principal
1 p c $= \frac{1}{40}$	180 3350 }	1st year's interest
	11 2709 }	
	4699 9807	2nd Principal
	187 9992 }	2nd year's interest
	11 7500 }	
	4899 7301	3rd Principal
	195 9692 }	3rd year's interest
	12 2493 }	
	<i>Rs</i> 5107 9686	Amount
	4508 375	1st Principal

Interest reqd $Rs\ 599\ 5936 = Rs\ 599\ 9a\ 6p$ nearly

526 When the Compound Interest is required for any number of entire years and a part of 1 year, it is done either by considering the fractional part as a new period and finding the interest at a rate equal to the same fraction of the given rate, or by finding the interest of the next full period and taking the required part of it

Ex Find the amount of £45 12s 6d for $3\frac{1}{2}$ years at $3\frac{1}{2}$ per cent, Compound Interest

Now, £45 12s 6d = £45 62s, and $\frac{1}{2}$ of $3\frac{1}{2}$ p c $= 1\frac{1}{4}$ p c

(i)	3 p c = $1\frac{1}{2}\%$	£45 6250	Principal
		1 3688	1st year's interest
	$1\frac{1}{2}$ p c = $2\frac{1}{2}\%$	2281	
		£47 2219	Amount in 1 year
		1 4166	2nd year's interest
		2361	
		£48 8746	Amount in 2 years
		1 4662	3rd year's interest
		2443	
	1 p c = $1\frac{1}{2}\%$	£50 5851	Amount in 3 years
	$\frac{1}{2}$ p c = $\frac{1}{2}\%$	5058	
	$\frac{1}{4}$ p c = $\frac{1}{4}\%$	2529	last $\frac{1}{2}$ year's interest
		1264	
		£51 4702	Amount in $3\frac{1}{2}$ years
		= £51 9s 4 $\frac{1}{2}$ d	Ans

(ii)		£50 5851	Amount in 3 years
		1 5176	4th year's interest
		2529	
		£52 3556	Amount in 4 years
		8852	last $\frac{1}{2}$ year's interest
		£51 4704	Amount in $3\frac{1}{2}$ years

527 If the interest be payable **half-yearly**, the result may be obtained by finding the interest for double the number of years at half the given rate per cent, or if **quarterly**, by finding the interest for four times the number of years at one fourth of the given rate per cent

Ex Find the amount at Compound Interest on Rs 871 12a for $1\frac{1}{2}$ years at $4\frac{1}{2}\%$ per annum, payable half yearly

Here, there are 3 periods of half-a-year each, and the rate per cent *per period* is $\frac{1}{2}$ of $4\frac{1}{2}\%$ or $2\frac{3}{8}\%$

$2\frac{3}{8}$ p c = $1\frac{1}{2}\%$	Rs 871 12a = Rs 871 7500	Principal
$\frac{1}{2}$ p c = $\frac{1}{4}\%$	17 4350	
$\frac{1}{4}$ p c = $\frac{1}{8}\%$	2 1794	int for 1st period
	1 0897	
	Rs 892 4541	Amount in 1st period
	17 8491	
	2 2311	int for 2nd period
	1 1156	
	Rs 913 6499	Amount in 2 periods
	18 2730	
	2 2841	int for 3rd period
	1 1421	
	Rs 935 3491	Amount in 3 periods
		or $1\frac{1}{2}$ years
	= Rs 935 4a 7p nearly	Ans

(3) Rs 6500 in $3\frac{1}{2}$ yrs at 3 p c (4) £4321 10s in $2\frac{3}{4}$ yrs at $6\frac{1}{2}$ p c

(5) Rs 17605 in $2\frac{1}{2}$ yrs at 4 p c (6) £2533 6s 8d in $2\frac{1}{4}$ yrs at 3 p c

5 Find the difference between the Simple and the Compound Interest on Rs 17505 for 3 years at $5\frac{1}{4}$ per cent

6 Find the difference between the Simple and the Compound Interest on £3333 6s 8d for $3\frac{1}{2}$ years at $3\frac{1}{2}$ per cent

7 The population of a city is 765240 and its annual increase is at the rate of 2.7 per cent, what will be the number of its inhabitants at the end of 5 years?

8 A person at the beginning of each year lays aside Rs 2800, and employs the money at $3\frac{1}{2}$ per cent Compound Interest, how much will he be worth at the end of 5 years?

529 Inverse Questions on Compound Interest

Case I *Having given the amount or interest, time and rate per cent, to find the principal*

RULE Find the amount or interest of Re 1 or £1 for the given time, and then state thus —

amount or interest } { given amount }
of Re 1 or £1 } { or interest } Re 1 or £1 principal reqd

that is, — the principal is found by dividing the given amount by the amount of 1 or the given interest by the interest on 1

Ex What sum of money will amount to Rs 4134 6a in 2 years at 5 per cent per annum, Compound Interest?

Amount of Re 1 for 2 years at 5 p c = Rs (1.05)² = Rs 1.1025,

and Rs 4134 6a = Rs 4134 375

Principal reqd = Rs (4134 375 ÷ 1.1025) = Rs 3750 Ans

Case II *Having given the principal, amount and time, to find the rate per cent*

RULE Divide the given amount by the given principal, and then extract that root of the quotient which is denoted by the number of years, this will be the amount of Re 1 or £1 for 1 year, whence the rate per cent will be known

Ex At what rate per cent Compound Interest, will Rs 3750 produce as interest Rs 1740 6a in 4 years?

Amount = Rs 3750 + Rs 1740 6a = Rs 5490 6a = Rs 5490 375 and
Rs 5490 375 ÷ Rs 3750 = 1.4641, also $\sqrt[4]{1.4641} = 1.1$

∴ the amount of Re 1 for 1 year = 1.1,

∴ the interest of Re 1 for 1 year = 1, and ∴ rate = 10 per cent

Note It is obvious that this method can be adopted only when the number of years is some *power* of 2, or of 3, or the product of some *power* of 2 and some *power* of 3

Case I (I) *Having given the principal, amount and rate per cent, to find the time*

RULE Divide the amount of Rs 1 or $£$ 1 in the required number of years, (which may be obtained by dividing the amount by the principal), by the amount of Rs 1 or $£$ 1 for 1 year. If the quotient be greater than the divisor, divide it by that divisor, and repeat the operation till a quotient smaller than the divisor is obtained. If the last quotient be 1, the number of times the operation of division is performed will represent the number of years, but if not, add to this the time in which Rs 1 or $£$ 1 rises to the quantity represented by the last quotient. This will give the required number of years

Ex 1 The amount of Rs 10000 put out at Compound Interest for a certain number of years at 4 per cent per annum is Rs 11248 10s 2 88p, find the time

$$Rs\ 11248\ 10s\ 2\ 88p = Rs\ 11248\ 64$$

$$\begin{aligned} \text{Amt of } Rs\ 1 \text{ in the reqd no of years} &= Rs\ (11248\ 64 - 10000) \\ &= Rs\ 1\ 248\ 64 \end{aligned}$$

$$\text{Amount of } Rs\ 1 \text{ for 1 year} = Rs\ 1\ 04$$

$$\text{Now } 1\ 248\ 64 - 1\ 04 = 1\ 08\ 16, \quad 1\ 08\ 16 - 1\ 04 = 1\ 04 \text{ and } 1\ 04 - 1\ 04 = 1$$

$$\text{Hence the reqd time} = \underline{3} \text{ years } \text{Ans}$$

Ex 2 In what time will $£$ 12500 amount to $£$ 15185 9s 0 96d at 6 per cent Compound Interest?

$$£\ 15185\ 9s\ 0\ 96d = £\ 15185\ 454$$

$$\begin{aligned} \text{Amt of } £\ 1 \text{ in the reqd no of years} &= £\ (15185\ 454 - 12500) \\ &= £\ 1\ 2148\ 3632 \end{aligned}$$

$$\text{Amount of } £\ 1 \text{ for 1 year} = £\ 1\ 06$$

$$\text{Now, } 1\ 2148\ 3632 - 1\ 06 = 1\ 1460\ 72, \quad 1\ 1460\ 72 - 1\ 06 = 1\ 08\ 12 \text{ and } 1\ 08\ 12 - 1\ 06 = 1\ 02$$

Here, the last quotient 1 02 is less than the divisor, therefore the reqd no of years = 3 years + a fraction of a year

Now, to find the fractional part of a year in which $£$ 1 rises to $£$ 1 02, at 6 per cent Simple Interest

$$\text{The interest on } £\ 1 \text{ for reqd time} = £\ 1\ 02 - £\ 1 = £\ 02 = £\ \frac{1}{50}$$

$$\text{The Simple Interest on } £\ 1 \text{ for 1 year} = £\ \frac{6}{100} = £\ \frac{3}{50}$$

$$\text{and } \frac{1}{50} \text{ is } \frac{1}{3} \text{ of } \frac{3}{50}, \quad \text{the reqd time} = \underline{3\frac{1}{3}} \text{ years } \text{Ans}$$

Ex 3 The difference between the Simple and the Compound Interest on a certain sum of money for 3 years at 5 per cent per annum is Rs 133 7a, find the sum

Simple Int of *Rs* 1 for 3 yrs at 5 p c = $Rs \frac{5 \times 3}{100}$ or *Rs* 15

Amount of *Rs* 1 at the end of 3rd year at Comp Int = *Rs* (1 05)³
 = *Rs* 1 157625
 the int of *Rs* 1 = *Rs* 157625

∴ the diff between the Simple and the Compound Interest of *Rs* 1
 = *Rs* (157625 - 15) = *Rs* 007625, and *Rs* 133 7a = *Rs* 133 4375
 . *Rs* 007625 *Rs* 133 4375 *Rs* 1 sum reqd
 sum reqd = *Rs* (133 4375 - 007625) = *Rs* 17500 Ans

Examples CLXI

1 What sum of money lent at Compound Interest at 7½ per cent per annum will amount to *Rs* 49691 14a in 3 years?

2 What sum placed out at 3 per cent Compound Interest amounts in 3 years to £364242 6s 8d?

3 Find the principal which will produce £21 0s 4d in 3 years at 5 per cent Compound Interest

4 What sum of money will amount to *Rs* 48027 10a 6p in 2½ years at 5 per cent Compound Interest?

5 At what rate per cent will £100000 amount to £108243 4s 3½d in 4 years at Compound Interest?

6 At what rate per cent will £500 amount to £607 15s 0½d in 4 years at Compound Interest?

7 The amount of *Rs* 4000 for a certain time at 5 per cent Compound Interest is *Rs* 4520 4a, find the time

8 Required the time in which £20000 amounts to £24261 2s 4½d at 8 per cent Compound Interest

9 In what time will £15000 gain £1390 18s 1½d by Compound Interest at 3 per cent per annum?

10 The difference between the Simple and the Compound Interest on a certain sum for 3 years at 4½ per cent is *Rs* 86 13a 2p, find the sum

11 The difference between the Simple and the Compound Interest on a certain sum for 3 years at 3½ per cent per annum is £3 8s 4½d, find the sum

12 What will £400 amount to at 3 per cent Compound Interest in a time in which *Rs* 600 amounts to *Rs* 678 0a 7½p at 5 per cent Compound Interest?

13 A sum of money placed out at Compound Interest amounts to *Rs* 2420 in 2 years and to *Rs* 2662 in 3 years. Find the sum and the rate of interest

14 *A* placed out a sum of money for 3 years at 5 per cent Simple Interest, and *B* placed out an equal sum at the same rate and for the same time but at Compound Interest, and thereby gained £26 13s 4d more than *A*. What money was placed out by each?

15 A banker borrows money at $3\frac{1}{2}$ per cent per annum, and pays the interest, at the end of the year, he lends it out at 5 per cent per annum payable quarterly, and receives the interest at the end of the year by this means he gains Rs 2000 a year, how much money does he borrow?

III PRESENT WORTH AND DISCOUNT

530 Suppose *A* sold a horse to *B* for Rs 1025 payable 6 months hence the debt here cannot be claimed *at present*, for it will fall due only 6 months hence. But when such a debt is paid before it is due, a sum smaller than the actual debt must be paid by the debtor and will be accepted by the creditor as payment in full, with no loss to either party. In the present case, let us see what is that smaller sum of money which *B* pays and *A* accepts as his full due. Suppose the interest at which money is lent to be 5 per cent per annum Rs 1000, when put out to interest at 5 per cent will in 6 months amount to Rs 1025. Therefore *B* may pay Rs 1000 at present, and *A* may receive it as his full due instead of Rs 1025 which he is entitled to, at the end of 6 months. For, if *A* at once puts out to interest Rs 1000 at 5 per cent, it will in 6 months amount to Rs 1025 which is the actual debt. We call Rs 1000, the smaller sum accepted as the present payment the **Present Worth**, and Rs 25 the money deducted, the **Discount**. Hence,

531 The **Present Worth** or **Present Value** of a sum of money due at the end of a given time is that sum which with its interest for the given time amounts to the sum due.

Thus, if Rs 350 in 6 months at 4 per cent amounts to Rs 357, it follows that Rs 350 paid now is, equivalent to Rs 357 paid at the end of 6 months, that is, the *Present Worth* of Rs 357 due at the end of 6 months is Rs 350.

532 **Discount** is the abatement or allowance made when a sum of money is paid before it is due. But a sum of money due at the end of a given time is discharged now by the payment of its present worth true discount therefore is the difference between the sum due and its present worth.

Thus, in the above example, Rs (357-350) or Rs 7 is the *Discount* on Rs 357 due 6 months hence.

533 Since **Present Worth + Discount = Sum due**, (Art 532), and **Present Worth + int of Present Worth = Sum due**, (Art 531)

Discount on Sum due = interest of its Present Worth

534 *Discount on any sum is always less than the interest on the same sum for the same time*

Since the Present Worth is always less than the sum due, therefore the interest on the Present Worth is always less than the interest on the sum due. But, by *Def*—the interest on the Present Worth is the Discount. Therefore the Discount is always less than the interest on the sum due.

535 *To find the Present Worth and Discount of a given sum of money due at the end of a given time at a given rate per cent, we should first find what Rs 100 or £100 amounts to for the given time at the given rate, and then state thus —*

- (i) Amount of Rs 100 given sum Rs 100 Present Worth, and
 (ii) Amount of Rs 100 given sum Interest of Rs 100 Discount,
 or Discount = Sum - Present Worth

Ex Find the Present Worth and Discount of Rs 3552 8a due at the end of 4 months at $4\frac{1}{2}$ per cent

Int at $4\frac{1}{2}$ p c on Rs 100 for 4 mo = Rs $\frac{1}{2}$ of $4\frac{1}{2}$ = Rs $1\frac{1}{2}$

∴ Amount of Rs 100 = Rs $(100 + 1\frac{1}{2})$ = Rs 101 $\frac{1}{2}$

(i) Rs 101 $\frac{1}{2}$ Rs 3552 $\frac{1}{2}$ Rs 100 Present Worth

∴ Present Worth = Rs $\frac{3552\frac{1}{2}}{101\frac{1}{2}} \times 100$ = Rs 3500 Ans

(ii) Rs 101 $\frac{1}{2}$ Rs 3552 $\frac{1}{2}$ Rs $1\frac{1}{2}$ Discount

∴ Discount = Rs $\frac{1\frac{1}{2}}{101\frac{1}{2}} \times 3552\frac{1}{2}$ = Rs 52 8a Ans

or Discount = (Rs 3552 8a - Rs 3500) = Rs 52 8a

536 When Present Worth and Discount are reckoned at Compound Interest, we shall find the amount of Rs 100 or £100 at Compound Interest for the given time at the given rate, and proceed as before

Ex Find the Present Worth and Discount of Rs 2112 8a due at the end of 2 years, at 4 per cent per annum Compound Interest

At 4 per cent Compound Interest for 2 years Rs 1 amounts to Rs $(1.04)^2$ or Rs 1.0816

Rs 1.0816 Rs 2112 5 Rs 1 Present Worth reqd

∴ Present Worth = Rs $(2112 5 - 1.0816)$ = Rs 1953 125

= Rs 1953 2a

Hence Discount = Rs 2112 8a - Rs 1953 2a = Rs 159 6a } Ans

Examples CLXII

1 Find the Present Worth of —

- (1) Rs 9265 due 2 years hence at $4\frac{1}{2}$ per cent Simple Interest
 (2) 1321 8a $2\frac{1}{4}$ years $4\frac{1}{2}$

(3)	<i>Rs</i> 7933 12 <i>a</i>	due 3yrs 4mo	hence at 3 per cent	Simple Interest
(4)	3223 8 <i>a</i>	8 months		$3\frac{1}{2}$
(5)	£46 16 <i>s</i> 8 <i>d</i>	9 months		$3\frac{3}{4}$
(6)	370 4 <i>s</i> 8½ <i>d</i>	15 months		$4\frac{5}{8}$
(7)	437 14 <i>s</i> 9 <i>d</i>	5½ years		$3\frac{1}{2}$
(8)	<i>Rs</i> 11444 0 <i>a</i> 8 <i>p</i>	4 yrs 90 days		$2\frac{1}{4}$
(9)	£241 12 <i>s</i> 4 <i>d</i>	146 days		$4\frac{1}{3}$
(10)	£9724 1 <i>s</i>	4 years	5	Compound Interest

2 Find the Discount on —

(1)	<i>Rs</i> 41204 4 <i>a</i> 8 <i>p</i>	due 9 months	hence at 4 per cent	Simple Int
(2)	5600	16 months		5
(3)	£355 5 <i>s</i>	4 months		$4\frac{1}{2}$
(4)	670 3 <i>s</i> 9 <i>d</i>	9 months		3
(5)	520 17 <i>s</i> 6 <i>d</i>	3½ years		$4\frac{1}{2}$
(6)	<i>Rs</i> 2516 4 <i>a</i>	3 yrs 9mo 18days		$6\frac{1}{4}$
(7)	£621 3 <i>s</i> 4 <i>d</i>	245 days		$3\frac{1}{4}$..
(8)	298 0 <i>s</i> 10 <i>d</i>	11 months		4
(9)	<i>Rs</i> 32457 8 <i>a</i>	136 days		$5\frac{1}{4}$
(10)	£2450 18 <i>s</i> 9 <i>d</i>	3½ years	-3½	Compound Int

537 Inverse Questions on P W and Discount

When the *Sum due, its Present Worth or Discount* and the *Time* are given, to find the *Rate per cent* allowed, proceed precisely as in Interest (Art 529, Case II), and so too, when the other quantities are given, to find the time (Art 529, Case III). In such cases, consider the sum due as Amount, the Present Worth as Principal and the Discount as Interest

Ex 1 The discount on *Rs* 2957 8*a* due at the end of 2 yrs 8mo is found to be *Rs* 332 8*a*, at what rate per cent is the interest allowed?

The Present Worth of *Rs* 2957½ is *Rs* $(2957\frac{1}{2} - 332\frac{1}{2}) = \text{Rs } 2625$
Therefore the interest on *Rs* 2625 for 2½ yrs is *Rs* 332½

Now the Int on *Rs* 2625 at 1% for 2½ yrs = *Rs* $(2625 \times 2\frac{1}{2} - 100)$,
∴ Rate per cent reqd = $332\frac{1}{2} - (2625 \times 2\frac{1}{2} - 100) = 4\frac{3}{4}$ Ans

Ex 2 If the discount on £1321 10*s* at 4½ per cent be £121 10*s*, how long was the sum paid before it was due?

£1321½ - £121½ = £1200 is the Present Worth of £1321½

∴ £121½ is the interest on £1200,

Now the Int on £1200 for 1 year = £ $(1200 \times 4\frac{1}{2} - 100) = \text{£}54$

∴ the time reqd = $(121\frac{1}{2} \div 54)$ yrs = 2¼ years Ans

Examples CLXIII

1 What is the rate of interest in the following cases?—

- (1) When the Disc on Rs 1556 10s 8p due in 3 mo., is Rs 23 5s 4p.
- (2) When the Disc on Rs 784 14s due 8 mo hence, is Rs 30 3s
- (3) When the Disc on Rs 3888 14s due 18 mo hence, is Rs 220 2s
- (4) When the Disc on £574 3s 4d due 2 yrs 3 mo hence, is £32 10s
- (5) When the P W of Rs 3286 11s 4p due in 3 mo is Rs 3254 2s 8p
- (6) When the P W of £1336 11s 3d due in 3½ yrs is £1137 10s

2 When the sum due, if the—

- (1) Discount on Rs 13735 at 3½ per cent be Rs 335?
- (2) Discount on Rs 13371 8s at 4½ per cent be Rs 1621 8s?
- (3) Discount on Rs 238 14s 8p at 5 per cent be Rs 35 9s 4p?
- (4) Discount on £481 8s 2½d at 3½ per cent be £5 3s 2½d?
- (5) P W of £668 5s 7½d at 5 per cent be £568 15s?
- (6) P W of Rs 2753 5s 4p at 4 per cent be Rs 2581 4s?

3 The Discount allowed on a bill for £17192 8s paid 8½ years before due, is £3438 9s 7½d, what is the money worth?

4 Find the difference between the discount on £196 4s 4½d due 6 months hence at 8 per cent, and the interest on the same sum for the same time at the same rate

5 Find the difference between the interest on Rs 2466 10s 8p for 2½ years at 5½ per cent, and the discount on Rs 2839 12s due 2½ years hence at the same rate Explain the result

538 Miscellaneous Questions on P W and Discount

Ex 1 The discount on a sum of money due 1yr 4 mo hence at 4½ per cent is £48 9s Find the sum due, and also its present worth

Int on £100 for 1½ years at 4½ per cent = £4½ × 1½ or £6½

Discount on £(100 + 6½) due 1½ yrs hence at 4½ p c = £6½

Hence £6½ £48 9s £106½ sum due,

sum due = £(106½ × 48½ - 6½) = £813 9s
and Present Worth = £813 9s - £48 9s = £765 } Ans

Ex 2 If the interest on Rs 2531 4s at 5 per cent be equal to the discount on Rs 2573 7s for the same time at the same rate, when is the latter sum due?

Interest on Rs 2531 4s = Discount on Rs 2573 7s

Rs 2531 4s is the P W of Rs 2573 7s

Interest on Rs 2331 4a = Rs 2573 7a - Rs 2531 4a = Rs 42 3a,
 but Interest on Rs 2531 $\frac{1}{2}$ for 1 year = Rs 2531 $\frac{1}{2}$ \times $\frac{5}{100}$ = Rs 202 $\frac{5}{8}$
 \therefore the reqd time = $(42\frac{3}{8} - 202\frac{5}{8}) \text{ yr} = \frac{1}{3} \text{ yr} = 4 \text{ months}$ Ans

Ex 3 If the difference between the interest and the discount on a sum of money for 8 months at 4 per cent is 12a Find the sum

Interest on Rs 100 for 8 mo at 4 p c = Rs $\frac{8}{100}$

\therefore Rs 300 = Rs 8

Discount on Rs 300 = Rs 8

Rs 300 = Rs $\frac{800}{100} \times 8$

\therefore the diff bet Int and Dis on Rs 300 = Re $(8 - \frac{800}{100} \times 8) = \text{Re } \frac{84}{100}$

Hence Re $\frac{84}{100}$ 12a Rs 300 sum reqd

\therefore sum reqd = Rs $300 \times \frac{100}{84} \times \frac{84}{100} = \text{Rs } 1082 \text{ } 13a$ Ans

539 The difference between the interest and the discount on a certain sum of money for a given time at a given rate per cent is the interest on the discount for the same time at the given rate

Since the Sum due = the P W + the Discount,

Int on the Sum due = Int on the P W + Int on the Disc

But Int on the P W = the Discount on the Sum due,

\therefore Int on the Sum due = the Disc on the Sum due + Int on the Disc,

\therefore Int on the Sum due - the Disc on the Sum due = Int on the Disc

Ex 1 The interest on a certain sum of money is Rs 280 for a certain time, and the discount on the same sum for the same time and at the same rate is Rs 218 12a, find the sum

Rs 280 - Rs 218 12a or Rs 61 4a is the interest on Rs 218 $\frac{3}{4}$

Hence Rs 61 $\frac{1}{2}$ Rs 280, Rs 218 $\frac{3}{4}$ required sum

\therefore reqd sum = Rs $(218\frac{3}{4} \times 280 - 61\frac{1}{2}) = \text{Rs } 1000$ Ans

Ex 2 The discount on a certain sum of money due 9 months hence is Rs 200, and the interest on the same sum for the same time is Rs 207 8a, find the sum and the rate per cent per annum

Rs $(207\frac{1}{2} - 200)$ or Rs $7\frac{1}{2}$ is the interest on Rs 200 for 9 mo

\therefore Rs $7\frac{1}{2} \times \frac{4}{3}$ for 1 year

Int on Rs 100 or the rate reqd = Rs $\frac{1}{2} \times \frac{15}{100} \times \frac{4}{3} = \text{Rs } 5$ Ans

Again, Int on the sum for 9 mo at 5 p c = Rs 207 8a

for 1 year = Rs $207\frac{1}{2} \times \frac{4}{3}$ or Rs 830

Hence Rs 5 Rs 830 Rs 100 sum reqd

\therefore sum reqd = Rs $100 \times \frac{830}{5} \times \frac{5}{100} = \text{Rs } 5533 \text{ } 5a \text{ } 4p$ Ans

Ex 3 If Rs 8 be allowed as discount off a bill of Rs 80 due 6 months hence, how much should be allowed off a bill of the same amount due 15 months hence?

Rs 8 is 6 months' discount off Rs 80,

\therefore Rs 8 is 6 months' interest on Rs $(80 - 8)$ or Rs 72,

\therefore Rs 4 is 3 months' Rs 72,

$Rs\ 20$ is 15 months' interest on $Rs\ 72$,
 $Rs\ 20$ discount $Rs\ 72 + 20$ or $Rs\ 92$,

Hence $Rs\ 92$ $Rs\ 80$ $Rs\ 20$ discount reqd
 discount reqd = $Rs\ (80 \times 20 - 92) = Rs\ 17\ 6s\ 3\frac{1}{2}p$ Ans

Ex 4 If $Rs\ 24$ be allowed as discount off a bill of $Rs\ 132$ due 6 months hence, what should be the bill from which the same sum is allowed as 3 months' discount?

$Rs\ 24$ is 6 months' discount off $Rs\ 132$,
 $Rs\ 24$ interest on $Rs\ 132 - 24$ or $Rs\ 108$,
 $Rs\ 24$ is 3 months' $Rs\ 108 \times 2$ or $Rs\ 216$
 $Rs\ 24$ discount on $Rs\ 216 + 24$ or $Rs\ 240$

Hence the amount reqd = $Rs\ 240$ Ans

Ex 5 If $Rs\ 12$ be allowed as 6 months' discount off a bill of $Rs\ 132$, and at the same rate of interest $Rs\ 40$ be allowed off a bill of $Rs\ 240$, for how long a period had the little bill to run?

$Rs\ 12$ is 6 months' discount off $Rs\ 132$,
 $Rs\ 12$ interest on $Rs\ (132 - 12)$ or $Rs\ 120$,
 $\therefore Rs\ 20$ $Rs\ 200$ or $Rs\ (240 - 40)$
 $Rs\ 40$ is 12 months' $Rs\ (240 - 40)$,
 $Rs\ 40$ is 12 months' discount on $Rs\ 240$

Hence the time reqd is 12 months Ans

Examples CLXIV

- 1 On what sum of money due at the end of 1 year 4 months does the discount at $4\frac{1}{2}$ per cent amount to $Rs\ 484\ 8s$?
- 2 On what sum of money due at the end of 3 years 9 months does the discount at 4 per cent amount to $\pounds 105\ 8s$?
- 3 The discount on a bill due 9 months hence at 4 per cent per annum is $\pounds 6\ 15s$, what is the amount of the bill?
- 4 If the interest on $Rs\ 8825$ at 4 per cent be equal to the discount on $Rs\ 11119\ 8s$, when is the latter sum due?
- 5 If the interest on $Rs\ 5333\ 5s\ 4p$ for 2 years is equal to the discount at the same rate on $Rs\ 5600$ due 2 years hence, what is the rate of discount?
- 6 If the interest on $\pounds 4550$ at 3 per cent be equal to the discount on $\pounds 5573\ 15s$ at the same rate, when is the latter sum due?
- 7 At what rate per cent will the interest on $\pounds 3729\ 7s\ 6d$ in 4 years, be equal to the discount on $\pounds 4661\ 14s\ 4\frac{1}{2}d$ for the same time?
- 8 The difference between the interest on a certain sum for 3 years at 5 per cent and the discount on the same sum due 3 years hence at the same rate is $Rs\ 152$. Find the sum.
- 9 The interest on a certain sum of money for a certain time is $Rs\ 36$, and the discount for the same time is $Rs\ 30$, find the sum.

10 The interest on a certain sum for a certain time is Rs 70 and the discount for the same time is Rs 54 11a, find the sum

11 The discount on a certain sum due 2 years hence is Rs 638 8r, and the interest on the same sum for the same time is Rs 718 5a, find the sum, and the rate per cent per annum

12 On what sum will the difference between the interest and discount for 6 months at 4 per cent be 2a 144p?

13 The interest on a certain sum for 4 years is £35, and the discount on the same sum due 4 years hence is 15s less. Find the sum and the rate per cent

14 A tradesman marks his goods with two prices, one for ready money and the other for credit of 6 months, what ratio should the two prices bear to each other, allowing interest at $7\frac{1}{2}$ per cent per annum? If the credit price of an article be Rs 332, what is the cash price?

15 The discount on Rs 275 for a certain length of time is Rs 25, what is the discount on the same sum (i) for twice that length of time, and (ii) for half that length of time?

16 The interest on Rs 5225 for a certain time is Rs 343 12a, find the discount on the same sum for the same time

17 If 13s 4d be allowed as discount off a bill of £10 due 6 months hence, how much should be allowed off a bill for the same amount due 8 months hence?

18 If Rs 10 be allowed as discount off a bill of Rs 50, and at the same rate of interest Rs 25 be allowed as discount off a bill for Rs 150, due at the end of 8 months, for how long a period had the first bill to run?

19 If £6 be allowed as discount off a bill of £56 for 8 months find the amount of another bill, off which £30 is allowed as discount for 20 months

20 If Rs 5 be allowed as discount off a bill of Rs 125 due a certain time hence, what would be the discount allowed off, if the bill had twice as long to run?

21 If £2652 5s be due 3 years hence, what sum will be due at the end of 1 year, if Compound Interest be allowed at 3 per cent?

22 Five volumes of a work can be bought for a certain sum payable at the end of a year, and six volumes of the same work can be bought for the same sum in ready money, what are the rates of discount and interest?

23 A bookseller sells 20 copies of a book for a certain sum. How many will he give for the same sum, allowing credit for 5 years, if money is worth 5 per cent?

24 The discount on a sum due 3 years hence is $\frac{1}{2}$ of the

interest on the same sum for the same period Find the rate of interest

25 If Rs 10 be allowed as discount off a bill for Rs 70, due sometime hence, what should be the present worth of a bill for Rs 1300 which has only half the time to run ?

26 A man bought a horse for 30 guineas and sold him immediately for £38 10s payable at the end of 6 months If the use of money be reckoned at $6\frac{1}{4}$ per cent per annum, what is now his gain per cent ?

27 A offers for an estate Rs 37800, and B offers Rs 45400 to be paid at the end of 4 years Which is *now* the better offer and by how much, allowing 5 per cent interest ?

28 What sum must be paid now in order that a person may receive Rs 2500 at the end of every year for the next three years, the rate of interest being $3\frac{1}{2}$ per cent ?

29 Rs 1250 is due at the end of 3 months and Rs 900 at the end of 7 months, what sum at the present time is equivalent to both these sums, calculating interest at $4\frac{1}{4}$ per cent ? In what time will the result amount to Rs 1250 + Rs 900 at the same rate of interest ?

30 I buy a garden for Rs 35000, and sell it the same day for Rs 42000 to be paid in two equal instalments at the end of 3 and 6 months respectively How much do I gain by the transaction, reckoning interest at 6 per cent per annum ?

CHAPTER XIV.

Mercantile Transactions.

540 **Commercial Allowances** It is customary with merchants to allow both in buying and selling certain deductions (such as *Tare* and *Tret*) from the weight of goods that are sold by weight for the casks, bags, &c, which contain the goods

Tare is the allowance of a certain weight which the seller makes to the buyer on account of the weight of casks, bags, &c, which contain the goods

Tret is the allowance made to purchasers for waste or refuse matter after the tare is deducted

The total weight of goods without deductions for *tare*, *tret* or *waste* is called the **gross weight**, the weight free from all deductions is called the **net weight**

Ex Required the net weight of 18 chests of tea, weighing 28 cwt 0 qr 2 lbs, tare being allowed at 4 lbs per chest and tret 16 lbs per cwt

	28 cwt	0 qr.	2 lbs	gross
4 lbs × 18		2	16	tare
16 lbs = $\frac{1}{4}$ of 1 cwt	27	1	14	
	3	3	18	tret
	23	1	4	net weight

541. Discounting Bills A Bill is a discount of agreement to pay a particular sum of money at the end of a certain time

(i) A **Bill of Exchange** or **Hundi** is a written instrument in which one person orders another to pay to him, or to some other person, a sum of money at a specified time Thus —

Rs 500

Calcutta, 30th May, 1897

Two months after date pay me or order Five hundred

Rupees, value received

To X Y

A B

Lucknow,

N-W P

Here the Bill is drawn by A B and sent to X Y, who on accepting it, writes his name across the Bill, and engages to pay to A B or order Rs 500 in 2 months after 30th May, 1897

(ii) A **Promissory note** or **Note of Hand**, is a written instrument in which one person promises to pay another or his order a sum of money at a specified time Thus —

Rs 800

Calcutta, 30th May, 1897.

Three months after date, I promise to pay A B or order Eight hundred rupees, value received

X Y

Here, X Y engages to pay A B or his order Rs 800 at the end of 3 months from 30th May, 1897

When a bill is due 2 months hence, it is called payable at two months' Sight

542 If now the holder of the Bill or the Note wishes to cash it before it is due, he takes it to a *banker* or *bill broker*, and if he be satisfied of the credit of the parties to the bill, he **discounts** it, that is, he pays the sum specified on the bill or the note, deducting discount for the time it has still to run But the practice prevailing among the bankers is to deduct the *Interest* of the sum specified, instead of the *interest of the Present Worth* of that sum, which is the **True discount** (Art 533) And as the Present Worth of a sum, due at some future time is less than the sum itself, the *true discount* is less than the *banker's* or *mercantile* discount, called the **Commercial** or **Practical Discount** Hence the difference between the true and mercantile discount is the banker's gain

Thus, it has already been seen that the *True Discount* on Rs 357 due at the end of 6 months at 4 per cent interest is Rs 7 The *Commercial Discount*, however, is the interest on Rs 357 for 6 months at 4 per cent, and is therefore $Rs\ 357 \times \frac{4}{100} \times \frac{6}{12} = Rs\ 7\ 2a\ 3p$ nearly

543 A Bill of Exchange or a Promissory Note always runs 3 days beyond the time specified in it, and these three days are called the **Three days of Grace** Although originally a mere custom, it has now all the force of law

Thus, a Bill drawn on 5th January at 3 months is **nominally** due on 5th April, but **legally** on 8th April Moreover, calendar months are always reckoned, so that a Bill at 3 months, whether drawn on 30th or 31st Jan is **nominally** due on the 30th April, and **legally** on the 3rd May In calculating the number of days, it is usual to omit the day on which the Bill is drawn

Ex 1 Find the banker's disc on Rs 2505 due in 5 yrs at $3\frac{1}{2}\%$

The Banker's discount is the interest on the given sum

Interest or Banker's discount on Rs 100 for 5 yrs at $3\frac{1}{2}\%$ p c = Rs $3\frac{1}{2}$
 interest on Rs 2505 = Rs $3\frac{1}{2} \times \frac{2505}{100} = \underline{\text{Rs } 438\ 6a}$ Ans

Ex 2 A bill of Rs 770 is drawn on 8th March at 6 months, and discounted on 3rd June at 5 per cent Find the banker's gain

The Bill is nominally due on the 8th of September, and therefore legally due on the 11th Also it is discounted on the 3rd June, and the number of days between 3rd June and 11th Sept is 100

Interest on Rs 100 for 100 days at 5 p c = Rs $5 \times \frac{100}{100} = \text{Rs } 1\frac{2}{3}$

Rs $101\frac{2}{3}$ Rs 770 Rs $1\frac{2}{3}$ true discount

∴ true discount = Rs $\frac{770 \times 73 \times 100}{73 \times 7400} = \text{Rs } 10\ 6a\ 5\ 8p$,

and Rs 100 Rs 770 Rs $1\frac{2}{3}$ banker's discount

∴ banker's discount = Rs $770 \times \frac{100}{73} \times \frac{1}{100} = \text{Rs } 10\ 8a\ 9\ 2p$

∴ banker's gain = Rs $10\ 8a\ 9\ 2p - \text{Rs } 10\ 6a\ 5\ 8p$
 $= \underline{\text{Rs } 2a\ 3\ 4p}$ Ans

Examples CLXV

1 Find the banker's discount on Rs 4635, due in $2\frac{1}{2}$ years, at $2\frac{1}{2}\%$ per cent

2 A bill of Rs 12800, due in 1 year 10 months, is discounted by a banker at $4\frac{1}{2}\%$ per cent, what will the holder receive?

3 Find the difference between the banker's and the true discount on Rs 7249 6a due in 9 months, at $4\frac{1}{2}\%$ per cent

4 Find the difference between the bankers and the true discount on £300 2s 6d, due 4 months hence, at $6\frac{1}{2}\%$ per cent

5 How much less than the true present worth will a banker give for a bill of Rs 9504, due in $7\frac{1}{2}$ months, interest at 5 p c?

6 What will a banker gain by discounting a bill of Rs 7310 4a, due $2\frac{1}{2}$ years hence, at $5\frac{1}{2}\%$ per cent?

7 What will a banker retain on discounting a bill of Rs 12750 drawn on the 4th of March at 10 months, and discounted on the 14th of August at 5 per cent?

8 A bill is drawn for Rs 337 8a on July 17th at 2 months, and discounted Aug 11th at $3\frac{1}{2}$ per cent, how much does the holder receive?

9 What deduction does a banker make in discounting a bill for Rs 77163 6a drawn Oct 10th at 9 months and discounted March 15th at $6\frac{1}{2}$ per cent?

10 What does a banker give as the present worth of a bill for £562 2s 6d drawn on Sept 4th at 5 months and discounted the same day at $6\frac{1}{2}$ per cent? How much is the result less than the true present worth?

11 How much does a banker give as the present worth of a bill for Rs 52534 4a drawn Nov 6th at 10 months, and discounted by him on Feb 21st at $4\frac{1}{2}$ per cent?

12 On 31st Oct a bill was drawn at 6 months for Rs 3097 10a 8a and discounted Jan 27th at 7 per cent, what was charged for discount, and how much did the banker gain?

13 Required the true discount on £2454 2s 10d drawn on May 10, 1864, at 1 year, and discounted on April 3, 1885, at $9\frac{1}{2}$ per cent.

14 A bill was drawn on May 14th at 2 months, and was discounted on July 2nd at $8\frac{1}{2}$ per cent. If the banker's gain was 1s 4d, for what sum was the bill drawn?

15 A person, discounting a bill 8 months before it is due, at 6 per cent interest, receives £2 7s 6d less than the amount of the bill. What was the bill drawn for?

16 A bill was drawn on March 11 for 30 days, and was discounted on March 18 at 4 per cent, and the banker's gain was Rs 5. What was the bill drawn for?

17 A bill for £126 5s was drawn on March 9 at 5 months, interest being calculated at 5 per cent, the discount on the bill was £1 5s. On what date was the bill discounted?

18 A banker discounted a bill 9 months before due, and found, that he would have lost $\frac{1}{4}$ of what he deducted as discount had he reckoned true discount. Find the rate at which interest was calculated.

544 Commission and Brokerage Commission is the charge made by an Agent for buying or selling goods, property, &c for another, and is usually a percentage on the value of the goods or property bought or sold.

Brokerage is the charge made by a Broker for buying or selling goods, shares, &c, for another, and is usually a percentage on the full amount of the transaction.

545 Cash Discount Discount, without reference to time, is an allowance which merchants and tradesmen make to such of their

customers as are willing to pay ready money This allowance is usually percentage on the amount of the account

546 Since Commission, Brokerage, &c is a percentage on the given sum of money, to find their amount proceed thus —

Multiply the sum by the number expressing the rate per cent and divide by 100 Or apply Rule of Three

Ex 1 An agent sells goods to the value of Rs 5835, on which he receives a commission of $3\frac{1}{2}$ per cent, how much does his commission amount to?

$$\text{The commission} = \text{Rs } (5835 \times 3\frac{1}{2} \div 100) = \text{Rs } 218\ 8125 = \underline{\text{Rs } 218\ 1^a}$$

Ex 2 Find the brokerage on £5000 at $1\frac{1}{2}$ per cent

$$\text{The brokerage} = £5000 \times 1\frac{1}{2} \div 100 = \underline{£75} \text{ Ans}$$

547 Insurance Insurance is a contract by which one party undertakes to pay a specified sum, at the death of a person, or against any loss to houses or goods by fire, or to ships or their cargoes at sea, in consideration of the protected party paying year by year, or once for all, a certain percentage of that sum

The parties who take the risk are called the Insurers or Underwriters, and the protected party is called the Insured The consideration money paid by the insured is called the Premium the instrument containing the contract is called the Policy of Insurance, and the stamp duty on the policy is called the Policy duty

548 When a man insures, so as to recover not only his property, but the premium and all other expenses connected with its insurance, it is said to be covered

549 Insurance is calculated in the same way as Commission

Ex 1 What sum should be paid for insuring a vessel and cargo worth Rs 22250, at $3\frac{1}{2}$ per cent?

$$\text{The premium reqd} = \text{Rs } 22250 \times 3\frac{1}{2} \div 100 = \underline{\text{Rs } 723\ 2a} \text{ Ans}$$

Ex 2 A cargo is valued at £5270 6s, the premium on insurance is at the rate of 5 guineas per cent policy duty at 4s per cent, and commission $\frac{1}{8}$ per cent, what sum must be insured to cover the cargo and the expenses of insurance, and what premium must be paid?

<p> Deduct from £100 Premium £5 5s 0d Policy duty 4s 0d Commission 8s 9d <hr/> £5 17s 9d </p>	<p> £5 17s 9d <hr/> £94 2s 3d </p>	<p> In case of loss, for every £100 received from the underwriters, £5 17s 9d is for expenses of insurance, and the remaining £94 2s 3d is for cargo, hence to recover both cargo and expenses of insurance we must insure £100 for every £94 2s 3d of cargo, therefore </p>
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£94 2s 3d £5270 6s £100 sum to be insured,

or £94⁸⁰ £5270¹⁶ £100 sum to be insured

$$\therefore \text{sum to be insured} = £5270\frac{16}{100} \times 100 \times \frac{100}{94\frac{80}{100}} = \underline{£5600} \text{ Ans}$$

Also, the expenses of insurance are at the rate of £5 17s 9d for every £94 2s 3d of cargo, therefore

£94 2s 3d £5270 6s £5 17s 9d expenses of insurance,
 ∴ expenses reqd = £(5270₁₀ × 5¹⁷₈₆ - 94₈₀) = £329 14s

And £94 2s 3d £5270 6s £5 5s premium to be paid
 ∴ premium reqd = £(5270₁₀ × 5⁵₈₀) = £294 4s

Examples CLXVI

1 What does a factor receive for selling goods to the amount of Rs 3758 5a 4p at a commission of 1¹/₂ per cent?

2 What is the ready money payment of an account amounting to Rs 3597 6a, allowing a discount of 2¹/₂ per cent?

3 What is the brokerage upon a money transaction of £273 15s at 3s 4d per cent?

4 What is the brokerage on Rs 7681 4a at Re 1 10a 8p per cent?

5 For what sum should goods, worth £4384 or 3d be insured at £2 6s 8d per cent that the owner may recover, in case of loss the value of both goods and premium?

6 A commission agent sells 1436 barrels of flour at Rs 15 per barrel, what commission does he receive at 4¹/₂ per cent?

7 The brokerage on a certain sum at 3s 4d per cent amounts to £1 5s 7¹/₂d, find the sum

8 A man insured his house for Rs 16800 at Rs 3 12a per cent, so that in case of fire he might recover both the value of the house and the premium. Find the value of the house

9 What sum must be paid to insure a cargo worth Rs 25850, the premium being Rs 17 8a, policy duty Re 1, and brokerage Re 1 4a per cent respectively?

10 A ship is insured for ³/₄ of its value at ¹/₂ per cent, and the insurance amounts to Rs 1560. What is the value of the ship?

11 At what rate per cent is discount allowed when a tradesman deducts Rs 40 6a, 6p from a bill of Rs 897 14a 8p?

12 An agent sells goods to the value of Rs 796536 4a on which he receives a commission of 3¹/₂ per cent, while his office and other expenses amount to 22¹/₂ per cent of his commission. How much clear profit does he make, and how much does he remit to his principal?

13 A broker at the public sales buys 5 chests of indigo weighing 18 cwt 3 qrs 22lbs net, at Rs 2 14a 8p per lb; find the brokerage at ¹/₂ per cent

14 A ship worth £15325 is to be insured, so that its value

and all the expenses connected with its insurance may be covered. The premium is $2\frac{1}{2}$ guineas per cent, policy duty $4s$ per cent, and brokerage $\frac{1}{4}$ per cent, what is the amount of the whole expenses paid on insurance?

15 What sum must be paid on the insurance of a cargo of the value of Rs 34575 4a, so that in case of loss the cargo and all expenses of insurance may be recovered? The premium is at the rate of Rs 47 4a per cent, policy Rs 2 per cent, and agent's commission $\frac{1}{2}$ per cent

550 Invoice Account-Sales &c

An Account-Sale is a statement drawn out by a Commission Agent or Broker shewing the sales he has made of goods in behalf of another party. It is of the following form —

ACCOUNT-SALE of 15 chests of tea, per S S *San Salvador* from London, on account of Messrs Hare and Blackett, Canton

H B	cwt	qr	lbs	cwt	qr	lbs	£	s	d
No	3 chests	23	2	15 gross	2	0	12 tare		
	5	38	1	24	3	1	15		
	4	32	3	6	2	3	18		
1 to 15	3	24	0	18	2	2	11		
		119	0	7	0	3	17 tret		
Deduct		11	3	7	11	3	7		
Net		107	1	0 at £11	12s	4d	per cwt	1245	17 9
					cwt	qr	lbs		
Overtaker	7	2	19	1	1	19	tare		
Deduct	1	3	5	0	1	14	tret		
Net	5	3	14 at £11	10s			per cwt	67	11 3
							Gross proceeds	1313	9 0
CHARGES									
	Insurance on £1320 at 2 per cent				£	s	d		
	Policy Duty				26	8	0		
	Freight on 120 cwt at $11\frac{1}{2}d$ per cwt				3	18	6		
	Trimage and dock dues				5	15	0		
	Other charges				40	6	6		
	Insurance of fire				2	17	3		
	Sale charges				2	1	5		
	Commission on £1313 9s 0d at $2\frac{1}{2} p c$				3	9	7		
	on £1320 or 0d at $\frac{1}{2} p c$				32	16	9		
					6	12	0		
	Net proceeds due to Messrs Hare and Blackett				124	5	0		
	London				1189	4	0		
	The 31st July 1897								
	R S BUDGETT & Co								

When a merchant (Henry Smith) of London ships goods, &c, on account and risk of a merchant (Charles Brown) of Calcutta, he sends with the goods a paper containing a memorandum of goods

with prices to the buyer This is called an **Invoice** It is of the following form, and differs in many respects from the simpler Invoices given in Page 305 —

INVOICE of 36 boxes of coffee shipped on board the *S S Golconda*, William Burkett, master, for Calcutta on account and risk of Charles Brown

C B	No	cwt	qrs	lbs	cwt	qr	lbs	£	s	d	
1 to 4	1	24	3	18 gross	1	2	25 tare				
	2	18	2	25	1	0	9				
	3	36	1	7	2	0	27				
	4	30	2	13	1	3	14				
		120	2	7	6	3	19				
		6	3	19							
	103	2	16	net at 10½d per lb				507	17	0	
CHARGES											
	Bond and Custom house entry							£	s	d	
	Cost of empty boxes							0	18	6	
	Small Charges							3	4	0	
	Bills of Lading							1	15	6	
	Brokerage on £508 0 0 at 1¼ p c							0	3	4	
	Commission on £521 10 9 at 2½ p c							7	12	5	
	Insurance on £525							13	0	10	
	Commission on £525							2	10	0	
	Policy duty							½ p c	2	12	6
								1	3	1	
	Errors Excepted							27	6	8	
	London							548	17	5	
	The 10th Jan, 1896										
	HENRY SMITH										

551 Bill of Lading, Bill of Exchange, &c

When a merchant in London ships goods to a merchant in Calcutta, he takes from the captain of the ship a receipt of the goods made over to his charge This is called a **Bill of Lading**, and the Captain will not deliver the goods to any one who does not hold the Bill of Lading

The Shipper or Consignor also draws a **Bill of Exchange** on the Consignee ordering him to pay to his Agent or to a certain person in his behalf the value of the shipment It is of the following form —

No 512 *Exchange for £548 17s 5d*

At Thirty days after sight of this our First of Exchange (Second and Third of the same tenor and date unpaid) pay to the Mercantile Bank of India Limited, the sum of Five hundred and forty-eight

London, Jan 10, 1896

pounds, seventeen shillings, and five pence, value received against shipment per S S *Golconda*, which place to account of

HENRY SMITH

TO MESSRS CHARLES BROWN & CO,
Calcutta

This is accompanied with an advice containing the rate at which Exchange is to be calculated

Now, the Shipper may, if he chooses, send this Bill, together with the Bill of Lading and Policy of Insurance to his Agent in Calcutta for presentation to the *Drawee*, but instead of doing this, as it will incur some delay in getting the money, the usual custom is that he goes to a Banker and asks him to discount the Bill, *i.e.*, pay over the value of the Bill (less commission) on the strength of the documents. If the banker agrees, he takes the Bill and pays cash to the Shipper, at a rate of exchange to be agreed upon between them. The banker now becomes the master of the goods.

The Banker forthwith despatches the Bill by the first and fastest mail steamer to his Agent in Calcutta (sending also two duplicates called the Second and Third of Exchange, by other steamers in case of accident). On its arrival in Calcutta, the Banker's Agent presents the Bill to the Drawee for acceptance. The Bill is then said to be sighted, and if the business is all in due form, the Drawee accepts it, *i.e.*, signs it and writes on it the date at which it matures, *i.e.*, becomes due. The Bill is then returned to the Banker's Agent and the Drawee binds himself to pay the amount of the Bill on due date. When the Bill is attached to the Bill of Lading and Policy of Insurance, it is called a **Documentary Bill**, if not, a **Clean Bill**.

The Bill, which one merchant draws on another for certain goods sold to him is called a **Draft**, and it becomes a **Bill** when the buyer accepts or puts his signature to it.

552 Exchange Exchange is the rule by means of which it is ascertained what sum of money of one country is equivalent to a *given* sum of another, according to some *settled* rate of commutation.

Thus, by *Exchange* we find the value of £5 in Rs, and *vice versa*.

553 The Course of Exchange is used to express the sum of money of any place given in exchange for a *fixed* sum of that of another, and the **Par of Exchange** denotes the sum of money of any place, which is of the same *intrinsic* and *real* value as the fixed sum.

Thus, in the *Par of Exchange*, £1 is equal to Rs 10, but in the *Course of Exchange* £1 sterling, a fixed sum is equivalent to a variable number of Indian Rupees, more or less, according to circumstances.

The **Arbitration or Comparison of Exchanges** is the-

method of determining upon the rate of Exchange, called the Par of Arbitration, between the first and last of a given number of places, where the Course of Exchange between the first and second, the second and third, &c., of these places is known. It is called Simple or Compound Arbitration, as three or more places are concerned.

554 Exchanges between merchants are effected by means of written instruments, called Foreign Bills of Exchange or briefly Foreign Bill, and a Bill on London entitles the holder to obtain gold in London for the value of the amount mentioned in the Bill.

Foreign Money Table.

(a) GOLD CURRENCY

France, Belgium, } Switzerland }	1 franc	= 100 centimes	} = 9½d nearly
Italy	1 lira	= 100 centesimi	
Spain	1 peseta	= 100 centesimos	
Greece	1 drachme	= 100 lepta	
Servia	1 dinar	= 100 paras	
Bulgaria	1 leva	= 100 stotinkis	
Romania	1 ley	= 100 banis	
Austria	1 florin	= 100 kreutzers	= 1s 11½d
German Empire	1 marc	= 100 pfennige	= 11½d
Holland	1 florin	= 100 cents or 20 stivers	= 1s 8d
Norway, Sweden, } Denmark }	1 krone	= 100 ore	= 1s 1d
Portugal	1 milreis	= 1000 reis	= 4s 6d
United States } Canada }	1 dollar \$	= 100 cents	= 4s 2d

(b) SILVER CURRENCY

India } Ceylon }	1 rupee	= 16 annas	} = 1s 4d
China	1 tael	= 100 cents	
Russia	1 rouble	= 100 cash	= 6s 6d
Turkey	1 pound	= 100 copecks	= 3s 1½d
Japan	1 yen	= 100 piastres	= 18s 0½d
		= 100 sen	= 4s 1d

555 From the nature of Exchange we see that the operations necessary to calculate it, are only applications of the Rule of Proportion, but the easiest method is the application of the Chain Rule.

Ex 1 What Indian money must be paid for £6643 17s 6d, the course of exchange being 1s 10½d per rupee

(1) 1s 10½d £6643 17s 6d Re 1 sum in Indian money,
or 22½d 1594530d Re 1 sum reqd
∴ sum reqd = Rs (1594530 - 22½) = Rs 70868 Ans

(ii) Reqd no of Rs = £6643 17s. 6d or £6643 $\frac{3}{8}$

1s 10 $\frac{1}{2}$ d or £ $\frac{3}{32}$ = Re 1

no of Rs reqd = $6643\frac{3}{8} - \frac{3}{8} = \frac{53151}{8} \times \frac{32}{3} = 70868$

Ex 2 If the exchange between Amsterdam and Paris be 54d for 1 crown, and between Amsterdam and London be 33s 9d Flemish for £1, what is the par of exchange or the arbitrated price between Paris and London?

Let x be the exchange value of the crown in d

Reqd no of $d = 1$ crown,

1 crown = 54d

$$x = \frac{54 \times 240}{405} d = 32d$$

33s 9d or 405d = £1 or 240d Hence 1 crown at Paris = 32d in London

Ex 3 Convert Rs 23000 into English money, when English money is at a premium of 15 per cent, the par of exchange being 2s per rupee

At par, 2s = Re 1, at 15 p c premium, 2s = Re 1 + Re $\frac{15}{100}$ = Rs $\frac{23}{20}$

\therefore Re 1 = $\frac{20}{23} \times 2s = \frac{40}{23}s$, and \therefore Rs 23000 = $\frac{40}{23} \times 23000s = \underline{\underline{£2000}}$ Ans.

Ex 4 A New York merchant remits 27940 florins to Amsterdam by way of London and Paris, at a time when the exchange of New York on London is 4 88s dollars for £1, of London on Paris is 25 fr. 40 c for £1, and of Paris on Amsterdam is 212 francs for 100 florins, $\frac{1}{8}$ per cent brokerage being paid in London and in Paris

Dollars reqd = 27940 florins,

100 = 212 francs,

100 = 100 $\frac{1}{8}$ fr, with brokerage,

2540 = 100 $\frac{1}{8}$ £,

1 = 4 88s dollars,

Dollars reqd

$$= \frac{27940 \times 212 \times 801 \times 801 \times 4885}{100 \times 100 \times 2540 \times 8 \times 8 \times 1000}$$

$$= 11420 \ 317$$

$$= 11420 \text{ dollars } 32 \text{ c } \text{ Ans}$$

Ex 5 Find the par of exchange between the U S gold eagle, weighing 258 grains $\frac{9}{10}$ fine and the sovereign of which 1869 weigh 40 lbs of gold $1\frac{1}{2}$ fine (4 eagle = 10 dollars)

£'s reqd = 1 eagle,

1 = 258 grs \$ standard,

10 = 9 grs fine,

11 = 12 grs Brit standard,

5760 = 1 lb

40 = £1869,

$$\begin{aligned} \text{£'s reqd} &= \frac{258 \times 9 \times 12 \times 1869}{10 \times 11 \times 5760 \times 40} \\ &= 2 \ 054838 \end{aligned}$$

$$\therefore, 1 \text{ eagle} = \underline{\underline{£2 \ 054838}}$$

$$1 \text{ dollar gold} = \underline{\underline{£2 \ 054838}}$$

$$\text{and } \underline{\underline{£1 = \frac{10000000}{2054838} \$ = 4 \ 8665 \ \$}}$$

Ex 6 Find the relation between the sovereign and the Napoleon, as determined from the intrinsic value of the two coins — 40 lbs British standard gold, $1\frac{1}{2}$ fine, is coined into 1869 sovereigns, and 15432 grains French standard gold, $\frac{9}{10}$ fine, is coined into 155 Napoleons (1 Napoleon = 20 fr

Napoleons reqd = £1,
 1869 = 40 lbs British standard,
 12 = 11 lbs fine,
 1 = 5760 grs
 9 = 10 grs French standard,
 15432 = 155 Napoleons,

∴ Napoleons reqd

$$\frac{40 \times 11 \times 5760 \times 10 \times 155}{1869 \times 12 \times 9 \times 15432}$$

 = 1.261106
 ∴ £1 = 1.261106 Napoleons
 = 25 22212 francs
 = 25 fr 27 c Ans

Examples CLXVII

1 Reduce £1857 14s 3d to rupees, &c at the rate of 11 11½d for 1 rupee

2 How many francs will be given in Paris for £688 14s 8d, when the course of exchange is 25 fr 42½ c for £1?

3 If £1 = 24 fr 57 c, express in francs and centimes £107 16s 0½d, and £256 3s 4½d

4 How many dollars must be given for a letter of credit on London for £2346 10s, when the exchange is 489 cents for £1?

5 What is the arbitrated rate of exchange between Hamburg and Paris in francs per 100 marcs, when the course of exchange between London and Paris is 25 45 francs for £1, and between London and Hamburg 2048 pfennige for £1?

6 A person in London owes another at Petersburg 500 roubles, exchange at 40d sterling per rouble, but remits to Paris at 24 francs per pound sterling, thence to Lisbon at 500 reis for 3 francs, thence to Amsterdam at 20 stivers per crusado of 400 reis and thence to Petersburg at 25 stivers per rouble, find the arbitrated rate between London and Petersburg and the gain or loss by the circuitous mode of remittance

7 The rate of exchange between London and Petersburg is 31½d for one rouble, between Vienna and Petersburg is 95½ florins for 60 roubles, and between Paris and Vienna is 93½ florins for 200 francs, find the arbitrated rate between London and Paris in francs for £1 sterling

8 If London exchanges with Holland at a gain of 6½ per cent when the rate of exchange is at 35s 6d per £ sterling, what is the par of exchange?

9 A person on leaving England exchanged his English money for French at the rate of 25 francs for a sovereign, and on arriving at Munich received 135 Bavarian gulden for 300 francs, what was his loss (i) in English money, (ii) in French money, supposing a gulden to be worth 15 8½d.

10 The rates of exchange being £1 = 25 4 francs, 375 francs = 105 kreutzers, 60 kreutzers = 1 florin and the cost of travelling in

Germany being $1\frac{1}{2}$ florins per German mile which is equal to $4\frac{1}{2}$ English miles, find the expense, in English money, of travelling 381 English miles in Germany

11 If £1 be worth 1175 Dutch guilders, 101 thalers worth 175 guilders, and 2 thalers worth 735 francs, how many francs should be received for £40?

12 Exchange 5220 dollars for English money when it is at a premium of $7\frac{1}{2}$ per cent, given that at par 1 dollar = 4s 2d

13 Exchange Rs 19000 for English money when it is at a discount of 5 per cent, given that at par $Rs\ 1 = 2s$

14 A merchant in Calcutta wishes to remit Rs 4100 to London, a rupee being equivalent to 1s 3d. For what sum in English money must he draw his bill when bills on London are at a premium of $2\frac{1}{2}$ per cent?

15 Calculate the par of exchange between the dollar and the shilling, when British standard silver is valued at 60s 7d per oz, having given that 1 dollar weighs 412½ grains, and is $\frac{1}{10}$ fine, and 1 lb Troy standard silver, $\frac{7}{8}$ fine, is coined into 66 shillings

16 When British standard silver is valued at 61s 7d per oz, find how many francs are equal to 20s, having given that 1 lb, Troy standard silver, $\frac{7}{8}$ fine, is coined into 66 shillings, and 15432 grains French standard silver, $\frac{9}{10}$ fine, is coined into 200 francs

17 A person in London owes another at St Petersburg 2460 roubles 50 copecks, which must be sent to him through Paris. He pays the needful sum to his broker at a time when the exchange between London and Paris is 25 fr 35 c for £1, and between Paris and St Petersburg 339 centimes for 1 rouble. The broker delays remitting until the rates are 25 fr 62½ c for £1, and 337 centimes for one rouble. What does the broker gain or lose by the delay?

18 Find the value of £1 in marcs and pfennige of Germany, having given that 15432 grains of fine gold is coined into 139½ 20 marc pieces, that 1 lb of standard gold is coined into 46½ sovereigns and that standard gold is $\frac{1}{12}$ fine

556 Annuities An Annuity is a fixed sum paid periodically under certain stated conditions, the payment may be made either once a year or at more frequent intervals

Annuities are called **certain** when they are payable for a fixed number of years, when they are payable during the lifetime of a person, or of the survivor of a number of persons, they are called **contingent** or **life annuities**, and when they continue for ever, they are called **perpetual annuities**. An annuity is said to be **in possession**, when it is payable at present, but when the payment is not to begin until after the lapse of a certain number of years, it is called a **deferred annuity** or **reversion**

If the annuity is to continue for ever, it is called a **perpetuity**. An annuity is said to be worth as many years' purchase as there are pounds in the value of the annuity of £1

557 A freehold estate is an estate which yields a perpetual annuity called the *rent*, and thus the value of the estate is equal to so many years' purchase or so many years' rent

Ex 1 If a freehold estate be worth 20 years' purchase, find the rate of interest

If the annual rent be £1, the value of the estate is £20

rate of interest = $(\frac{1}{20} \times 100)$ or 5 per cent *Ans*

Ex 2 If the rate of interest be 4 per cent, how many years' purchase is an estate worth?

The interest would amount to £100 in (£100 - £4) or 25 years

∴ the estate is worth 25 years' purchase *Ans*

Ex 3 An estate is bought at 25 years' purchase for £15000, two-thirds of the purchase money remaining on mortgage at 3 per cent. The cost of repairs averages £100 per annum. What interest does the purchaser make on his investment?

$\frac{2}{3}$ of £15000 or £10000 remains on mortgage, so he invests only £(15000 - 10000) or £5000. Also the estate being bought at 25 years' purchase, he gets yearly $\frac{1}{25}$ of £15000 = £600

Now he has to pay $\frac{3}{100} \times £10000$ or £300 for interest on mortgage.
∴ his total expenses are £300 + £100 = £400

Hence, on £5000 he gets an annual income of £(600 - 400) or £200

∴ $\frac{£200}{£5000} \times 100 = 4$ per cent rate per cent reqd

∴ rate per cent reqd = 4 *Ans*

Examples CLXVIII.

1 A freehold estate is sold at $28\frac{1}{2}$ years' purchase. What rate of interest is received on the investment?

2 How many years' purchase, *i.e.*, how many years' rental should be paid for freehold property to clear $3\frac{1}{2}$ per cent?

3 A person purchased a freehold estate for Rs 40000, what is the annual rent, if it is worth 25 years' purchase?

4 A freehold estate worth Rs 2800 a year is sold for Rs 70000, find the rate of interest.

5 An estate is bought at 20 years' purchase for Rs 200000, three-quarters of the purchase money remaining on mortgage at 4 per cent. The cost of repairs averages Rs 1500 per annum. What interest does the purchaser make on his investment?

6 A College purchases 47 ac 3 ro 9 po of land for £2824, and 33 ac 1 ro 20 po of land for £1974, the incidental expenses of

transfer, &c amount on the first lot to £60, and on the 2nd lot to £40, the first lot realizes a yearly rent of £2 4s an acre, the second lot a yearly rent of 50s an acre. How many years' purchase was given for all the land?

7 An estate is bought at $25\frac{1}{2}$ years' purchase for £3400, three-fifths of the purchase money remains on mortgage at $4\frac{1}{2}$ per cent. The cost of repairs, &c averages £9 6s 8d. What interest does the purchaser make on his investment?

8 The annual rent of a freehold estate bought for £8817 5s 3d, is £345 15s 6d, at how many years' purchase was it bought?

CHAPTER XV

Stocks and Shares

558 **Stocks** Stock is the term applied to money lent to the Government of a country, or to a Trading Company, at some specified rate of interest.

When the Government of a country wants money for any purpose, such as to carry on any costly war or to construct a large railway, it generally *borrow*s or *contracts* a *Loan*. If the money is borrowed from the nation itself, it is called the **National Debt**, but if borrowed from people of other nations, it is called a **Foreign Loan**, or more briefly, a **Loan**. The borrowing of money by the Government is effected by giving to the *Lenders* in exchange for their money, **Government Bonds** or **Acknowledgments**, implying that the Nation is indebted to them for the sums advanced, whilst it reserves to itself the option of the *Time* of paying off the *Principal* on the express condition to pay the interest on it regularly at fixed periods in the mean time.

Thus, if the Government of India were to borrow to the amount of 5 crores of rupees at 4 per cent, and A had lent Rs 1000 of this sum, A would be said to have Rs 1000, 4 per cent Stock, and would receive a document entitling him to receive the Interest, (*viz*, Rs 40) upon this stock from year to year, until the Government chose to repay the Principal and put an end to the debt.

559 The source from which the Interest is paid is called the **Public Funds** or simply the **Funds**, being however, only an imaginary Property, representing the credit of the country itself, which is pledged to the payment of the debts contracted by its Government, the Interest is paid *half yearly*, and the document, entitling the possessor to receive it, may be *sold* and *transferred* from one party to another, just as any other kind of property.

560 The debts of the British and the Indian Governments are

in the form of Bonds or Promissory Notes Money lent to the Government of India is said to be invested in Government of India Securities, but when lent to the Government of England, it is said to be invested in the Funds

561 The following are the most important of the English Stocks —

(1) The Consols (abbreviated from *Consolidated Annuities*) are so called from the consolidation or amalgamation of several loans into one loan, bearing the uniform interest of 3 per cent, now reduced to $2\frac{1}{2}$ per cent

(2) The Reduced Consols are so called because their rate of interest was *reduced* from a higher to a lower rate at a subsequent date.

(3) The New 3 per cent Stock have originated from the conversion of a higher stock to a 3 per cent stock

Besides these, there is a small amount of New $2\frac{1}{2}$ per cent Stock, and still smaller amounts of New $3\frac{1}{2}$ per cent, and New 5 per cent stock

562 It is usual to name the several kinds of stocks according to the annual rates of interest they bear, as 'the $2\frac{1}{2}$ per cent consols,' 'the Turkish 5 per cents,' 'the 4 per cents,' 'the 3 per cents,' 'the 4 per cent promissory note,' and so on. The prices of the various stocks published in the money market column in newspapers, are said to be quoted

Thus if the 4 per cents be *quoted* in the money market at 96, the meaning of this is, that for Rs 96 or £96 of money a person can purchase Rs 100 or £100 stock, which will entitle the owner to receive a half yearly dividend of Rs 2 or £2 from Government

563 The fluctuations in the price of stock are not caused by any variation in the rate of interest which is paid, for it is fixed once for all by the Government at the time the money is borrowed. But commercial or political changes or expectations at home and abroad constantly disturb the price of stock, even two or three times in the *same* day, according to circumstances. Hence, if a person *sells out* his stock from the Funds, he will be able to obtain more or less sterling money for each of his bonds, according to the interest it bears and also according to the circumstances of the times, which may influence the *stability* of the national credit, and if he *buys into* or *invests capital in* the Funds, the sum of ready money advanced by him for each bond will be regulated by the same circumstances

Thus, if at the time *A* wished to sell his stock, money was elsewhere making 5 per cent, it is plain that no one would give him £100 for the right to receive 4, but since £80 of common or *sterling* money (as it is called) would now bring £4 interest, he would be able to sell his £100 stock for £80, and the 4 per cents. would be said to be selling at £80

564 The students should most carefully note the difference *between the paper or nominal value of stock and the cash or actual price*, and also *between the sum invested and the amount of stock held*. Thus, if the 4 per cents are at 96, a man who invests Rs 19200 will be able to purchase Rs 20000 stock. And, therefore, while the amount of stock held by the man in this case is Rs 20000, the actual or cash value of that stock is only Rs 19200. It is also worthy to note the expression *so many in the so many per cents* — for instance '£5000 in the 4 per cents'. The meaning of this expression is *not that the man invested £5000 in the 4 per cents*, but that he holds *stock of the nominal value of £5000 in the 4 per cents*. If we suppose these 4 per cents to be at 98, the sum invested at the time of purchase would be £4900.

565 **Shares** Trading Companies raise money in a different way. Suppose several persons interested in a particular line of business, such as the construction of a railway, meet together and propose to start a company. These men, called **Promoters**, first decide what amount of money or Capital will be required for the purpose, and elect among themselves a few who are most fit to manage, and direct the affairs of the company. These men are called the **Directors** and the association a **Joint stock company**. The directors then issue a prospectus, stating the amount of capital required and the rate of profit expected if the project succeeds, they divide the capital into a large number of equal parts or shares and invite the public to subscribe towards them. A man can take as many shares as he pleases, and thus become a **Shareholder**. Generally the whole amount of a share is not paid up at once. The directors make several **Calls** upon the shareholders for paying up each time a part of the amount. Some times the whole capital is not required but only a part of it, and each shareholder has to pay a proportionate part of his share. The part which is thus paid is called the **Paid up Capital**. When all the shares are fully paid up, and when therefore, the capital considered necessary for the project is raised, the Company usually *converts its shares into stock*, because, in the case of Stock, transactions can be carried on with reference to *any portions of it*, whereas in the case of Shares, fractional parts of those Shares cannot be transferred. When the concern begins to yield profits, they are divided among the shareholders at so much per share or generally at so much per cent, at regular intervals usually six months, after all the expenses are paid and a certain amount is set aside to meet contingencies. This is called the **Reserve Fund**. What each shareholder receives as his portion is called his **Dividend**. Hence it appears that the rise or fall in the market value of shares depends very much on the dividend which is declared.

Suppose a company's subscribed Capital is 20 lacs divided into 20,000 shares of Rs 100 each, and the paid up capital 16 lacs or Rs 80 per share. If the *net* profits at the end of six months be Rs 25000, the dividend would be Rs 25000 for 20000 shares or Re 1

4a per share, and the percentage on the paid-up capital would be Re 1 4a on Rs 80 in the half year, i.e., $3\frac{1}{2}$ per cent per annum

566 Preference Stock When the originally subscribed capital of a Company proves insufficient to carry on the project, it does not, in such cases, issue more shares of the kind issued at first, but borrows money at a fixed rate of interest promising to pay this interest before the profits are divided among the shareholders. Capital raised in this way is called **Preference Stock**, as distinguished from the capital subscribed at the beginning, which is called **Ordinary stock**

567 Debentures The deeds of mortgage or *bonds* given by joint stock Companies, Municipalities and similar other Corporated bodies for *borrowed money*, are called **Debentures**

568 Par, Premium, &c When the market values of stocks are the same as their paper or nominal values, they are said to be *at par*. when greater, *at Premium* or *above par*, when less, *at Discount* or *below par*

Thus, if Rs 100 stock sells for Rs 100, it is said to be *at par*, if at Rs 105, *at 5 Premium* or *above par*, and if at 96, *at 4 Discount* or *below par*

569 Brokerage Purchases and sales of Stocks, Shares, &c are usually made through *Agents*, called **Stock-Brokers** or **Brokers**, who generally charge $\frac{1}{2}$ per cent (i.e. 2a on Rs 100, or 2s 6d on £100) upon the amount of Stock bought or sold. Hence, the *brokerage* must be *added* to the price of Stock which is *bought* and *subtracted* from the price of that which is *sold* through them

Thus, if the market value of Rs 100 stock be Rs 102 8a, the buyer will have to pay Rs 102 10a, while the seller will get only Rs 102 6a. Unless the brokerage is mentioned, it need not be noticed in working Examples in Stocks

570 All Examples in Stocks depend upon the *Rule of Proportion* and may therefore be solved by the *Rule of Three*

Ex 1 What sum of money will purchase Rs 24000 stock in the 3 per cents at $89\frac{1}{2}$?

Here, Rs 100 stock costs Rs $89\frac{1}{2}$ in money,

∴ Rs 100 stock Rs 24000 stock Rs $89\frac{1}{2}$ reqd sum of money;
∴ the reqd sum of money = Rs $(240 \times 89\frac{1}{2}) = \underline{\text{Rs } 21480}$ Ans

Ex 2 What sum must be invested to purchase £2300 stock in the 3 per cents at $90\frac{3}{4}$, brokerage $\frac{1}{2}$ per cent?

Here, £100 stock costs, with brokerage, $\underline{\text{£}(90\frac{3}{4} + \frac{1}{2})}$ or $\underline{\text{£}90\frac{1}{2}}$;

∴ £100 stock £2300 stock $\underline{\text{£}90\frac{1}{2}}$ reqd sum of money,
∴ reqd sum of money = $\underline{\text{£}(23 \times 90\frac{1}{2})} = \underline{\text{£}2081 \text{ } 10s}$ Ans

Ex 3 A person has one lac of rupees stock in a Tea Company;

whose shares are at 135 per cent premium He sells out, what amount of money will he receive, brokerage being $\frac{1}{8}$ per cent ?

Here, Rs 100 stock sells for Rs $(235 - \frac{1}{8})$ or Rs $234\frac{7}{8}$ in money,
 \therefore Rs 100 stock Rs 100000 stock Rs $234\frac{7}{8}$ reqd sum of money,
 reqd sum of money = Rs $(1000 \times 234\frac{7}{8}) = \underline{\text{Rs } 234875}$ Ans

Ex 4 How much will be received from the sale of £2450 10s stock in the $3\frac{1}{2}$ per cents at $96\frac{1}{2}$, brokerage $\frac{1}{8}$ per cent ?

Here, £100 stock realises £ $(96\frac{1}{2} - \frac{1}{8})$ or £ $96\frac{3}{8}$ in money,
 \therefore £100 stock £2450 10s stock £ $96\frac{3}{8}$ reqd sum of money,
 reqd sum of money = £ $(24\ 50\ 5 \times 96\ 37\ 5) = \underline{\text{£}2361\ 13\ 4\frac{3}{4}}$

Ex 5 How much stock can be purchased by investing Rs 28500 in the 3 per cents at 75 ?

Here, Rs 75 in money will purchase Rs 100 stock,
 Rs 75 Rs 28500 Rs 100 reqd amount of stock,
 reqd stock = Rs $(28500 \times 100 - 75) = \underline{\text{Rs } 38000}$ Ans

Ex 6 Find the quantity of stock purchased by investing £2353 in the 3 per cents at $90\frac{1}{8}$, brokerage $\frac{1}{8}$ per cent

Here, £ $(90\frac{1}{8} + \frac{1}{8})$ or £ $90\frac{1}{4}$ will be required to purchase £100 stock
 £ $90\frac{1}{4}$ £2353 £100 stock stock reqd
 reqd stock = £ $(2353 \times 100 - 90\frac{1}{4}) = \underline{\text{£}2600}$ Ans

Ex 7 What amount of India 5 per cent stock at $111\frac{1}{8}$ must be sold to realise Rs 17728 8a through the agency of a broker ?

By selling Rs 100 stock, Rs $(111\frac{1}{8} - \frac{1}{8})$ or Rs $111\frac{1}{4}$ can be realised,
 Rs $111\frac{1}{4}$ Rs 17728 $\frac{1}{2}$ Rs 100 stock reqd quantity of stock ;
 reqd stock = Rs $(17728\frac{1}{2} \times 100 - 111\frac{1}{4}) = \underline{\text{Rs } 15900}$ Ans

Ex 8 A person invests £14340 in consols when they are at $89\frac{1}{2}$ and sells out when they are at $93\frac{3}{8}$, what is his gain ? (brokerage as usual)

Here, the stock which costs £ $(89\frac{1}{2} + \frac{1}{8})$ or £ $89\frac{5}{8}$ is sold for £ $(93\frac{3}{8} - \frac{1}{8})$ or £ $93\frac{1}{4}$, therefore on every £ $89\frac{5}{8}$ there is a gain of £ $(93\frac{1}{4} - 89\frac{5}{8})$ or £ $3\frac{3}{8}$

£ $89\frac{5}{8}$ £14340 £ $3\frac{3}{8}$ gain required ;
 gain reqd = £ $(14340 \times 3\frac{3}{8} - 89\frac{5}{8}) = \underline{\text{£}580}$ Ans

Ex 9 A person buys Railway Stock at $89\frac{3}{4}$, and sells out at $103\frac{1}{2}$, and clears Rs 3850, how much money did he invest ?

Here, what cost him Rs $89\frac{3}{4}$ he sells for Rs $103\frac{1}{2}$, and therefore, on every Rs $89\frac{3}{4}$ invested, he gains Rs $(103\frac{1}{2} - 89\frac{3}{4})$ or Rs $13\frac{1}{4}$

\therefore Rs $13\frac{1}{4}$ Rs 3850 Rs $89\frac{3}{4}$ money invested,
 \therefore reqd money = Rs $(3850 \times 89\frac{3}{4} - 13\frac{1}{4}) = \underline{\text{Rs } 25130}$ Ans

Note In Examples like the above, the rate of interest is not considered, for no question of Income enters in the statement

Examples CLXIX

1 How much money must be given for the purchase of —

- (1) Rs 4500 in the $3\frac{1}{2}$ per cents at 88 $\frac{1}{2}$?
- (2) £5550 in the 4 per cents at $97\frac{1}{8}$?
- (3) £439 12s 5d in the $3\frac{1}{2}$ per cents at $92\frac{1}{8}$?
- (4) Rs 46494 in the 4 per cents at 81 ?
- (5) £650 in the 3 per cents at $90\frac{1}{2}$, brokerage as usual ?
- (6) Rs 32577 12a in the $3\frac{1}{2}$ per cents at $91\frac{1}{8}$, brokerage $\frac{1}{8}$ per cent ?
- (7) Rs 8833 5a 4p in the 5 per cents at $3\frac{1}{2}$ premium, brokerage $\frac{1}{8}$ p c
- (8) £1229 3s 4d in the 4 per cents at $\frac{1}{2}$ discount, brokerage $\frac{1}{8}$ p c

2 How much money can be obtained from the sale of —

- (1) Rs 16000 in the $3\frac{1}{2}$ per cents at $88\frac{1}{8}$?
- (2) Rs 12505 4a in the 3 per cents at $64\frac{1}{2}$?
- (3) Rs 7569 6a in the 5 per cents at $6\frac{1}{2}$ premium ?
- (4) £15850 16s 8d in the $3\frac{1}{2}$ per cents at $98\frac{1}{8}$, brokerage as usual ?
- (5) Rs 73515 in the 4 per cents at $96\frac{1}{2}$, brokerage $\frac{1}{8}$ per cent ?

3 Find the quantity of stock purchased by investing —

- (1) Rs 8280 in the $3\frac{1}{2}$ per cents at 92
- (2) £821 5s in the 4 per cents at $82\frac{1}{2}$
- (3) £6451 3s 6d in Bank Stock at $217\frac{1}{2}$
- (4) Rs 125466 13a 4p in India Stock at 252
- (5) Rs 23172 6a in the 3 per cents at $95\frac{1}{2}$, brokerage as usual
- (6) £1101 6s 8d in M Railway Stock at $8\frac{1}{2}$ above par
- (7) Rs 6307 12a in R Railway Stock at $27\frac{1}{2}$ below par
- (8) £506 9s 2d in the $3\frac{1}{2}$ per cents at $89\frac{1}{2}$, brokerage $\frac{1}{8}$ per cent.

4 What amount of stock must be sold, when the quotation is $12\frac{1}{8}$ above par, to realise £11306 11s ?

5 If I lay out Rs 13597 8a in the purchase of the 3 per cents at $92\frac{1}{8}$ and afterwards sell it at $94\frac{1}{2}$, what profit shall I make ? (brokerage as usual)

6 A person expended £2653 10s in the purchase of the New $3\frac{1}{2}$ per cents at $97\frac{1}{2}$, and after a time sold out at $96\frac{1}{2}$, find his loss, the usual brokerage being charged on each transaction

7 A person bought some $2\frac{1}{2}$ per cent stock at $65\frac{1}{2}$, and sold it when the price had risen to $69\frac{1}{2}$, thereby gaining Rs 1256 4a, how much money did he lay out ?

8 If a person invest Rs 126540 in the 4 per cents at $99\frac{1}{2}$, at what price must he sell to gain Rs 15817. 8a ?

9 How much stock in the 4 per cents must be bought at 96, in order that by selling out at par Rs 250 may be gained ?

10 How much must a person invest in the 3 per cents when they are at 10 per cent below par, that by selling out when they are at 5 per cent premium he may gain Rs 1250 ?

11 I buy £5000 stock in the 3 per cents at 85, and sell out when the funds have fallen $3\frac{1}{2}$ per cent What do I lose by the transaction ?

12 A person sold Rs 60000 3 per cent stock at 95, thereby realising Rs 800 more than his investment At what price had he bought in ?

13 A person invests Rs 15000 in the $3\frac{1}{2}$ per cents at 98 He sells his stock when the funds rise 4 per cent, and invests the proceeds in the same stock when they sink again to 98 How much more stock does he now hold ?

571 When a transfer of capital is made from one kind of stock to another, it is evident that there will be an equitable claim for *more* or *fewer* bonds of the second stock, according as the rate of interest of such bonds is *less* or *greater* than that of the first, thus, a number of bonds or *quantity of stock* in the 4 per cents, will produce the same interest as a *greater* quantity of stock in the 3 per cents, and consequently be of the same value to the possessor in point of income Hence, all questions on the transfer of stock from one kind to another belong to the Rule of Three Inverse

Ex 1 What annual income will be derived from Rs 42750 of 4 per cent paper ?

Here, Rs 100 paper yields Rs 4 interest per annum,

\therefore Rs 100 Rs 42750 Rs 4 annual income reqd

\therefore income reqd = Rs $(4275 \times 4) = \underline{\text{Rs } 1710}$ Ans

Ex 2 What income will be derived from investing £3220 in the $3\frac{1}{2}$ per cent stock at $80\frac{1}{2}$?

Here, for every £80 $\frac{1}{2}$ we get £100 stock, and the int on £100 stock is £3 $\frac{1}{2}$, therefore for every £80 $\frac{1}{2}$ of money we get £3 $\frac{1}{2}$ interest,

\therefore £80 $\frac{1}{2}$ £3220 £3 $\frac{1}{2}$ reqd annual income,

reqd income = £ $(3220 \times 3\frac{1}{2} - 80\frac{1}{2}) = \underline{\text{£ } 140}$ Ans

Ex 3 What sum must a person invest in the 3 per cent stock at $94\frac{3}{8}$ to have an annual income of Rs 240, brokerage $\frac{1}{8}$ per cent ?

To get Rs 3 annually he will have to invest Rs $(94\frac{3}{8} + \frac{1}{8})$ or Rs $94\frac{1}{2}$

Rs 3 Rs 240 Rs $94\frac{1}{2}$ reqd investment,

\therefore reqd investment = Rs $(80 \times 94\frac{1}{2}) = \underline{\text{Rs } 7560}$ Ans

Ex 4 Find the alteration in income occasioned by transferring Rs 32000 stock from the 3 per cent stock at 106½ to the 4 per cent stock at 114½, the brokerage being ½ per cent on each transaction

The price of Rs 100 stock (excluding brokerage) in the 3 p c is Rs $(86\frac{3}{8} - \frac{1}{8})$ or Rs $86\frac{1}{4}$., and the price of Rs 100 stock (including brokerage) in the 4 p c is Rs $(114\frac{1}{8} + \frac{1}{8})$ or Rs 115

. Rs 100 Rs 32000 Rs 86¼ proceeds of sale of the 3 p c

∴ proceeds of sale = Rs $(320 \times 86\frac{1}{4})$ = Rs 27600

Now, Rs 115 Rs 27600 Rs 4 income from the 4 per cents

. income from the 4 per cents = Rs $(27600 \times 4 - 115)$ = Rs 960

Also Rs 100 Rs 32000 Rs 3 income from the 3 per cents

∴ income from the 3 per cents = Rs (3×320) = Rs 960

Hence, the income remains the same Ans

Ex 5 Find the price of the 4 per cent stock, that an investment of £4680 may produce an income of £180, brokerage as usual

£180 £4 £4680 cost of £100 stock (including brokerage),

∴ cost (including brokerage) = £ $(4680 \times 4 - 180)$ = £104

Hence the price of £100 stock = £103½ Ans

Examples CLXX

1 What annual income will be derived from the following ?—

(1) £3300 of 5 p c stock (2) Rs 90150 of 3½ p c stock

(3) £4326 10s of 2½ p c. stock (4) Rs 82250 of 3½ p c paper

2 What yearly income will arise from the following investments?—

(1) Rs 55800 in the 4 p c at 93 (2) £4788 in the 3½ p c at 105

(3) Rs 35190 in the 3½ p c at 90 (4) Rs 21755 in the 4½ p c at 89½

(5) Rs 23500 in the 3½ per cents at 97½, brokerage as usual

(6) Rs 276438 12a in the 3 per cents at 91½, brokerage ½ per cent

3 How much money must a person invest in the under-mentioned stocks in order to secure the following incomes ?—

(1) Rs 600 in the 3 per cents at 85

(2) £73 10s in the 3½ per cents at 85

(3) Rs 1465 in the 3 per cents at 90½, brokerage ½ per cent

(4) 200 guineas in the 3½ per cents at 96½, brokerage as usual

4 What half yearly dividend is derived from an investment of Rs 3000 in 3½ per cent stock at 98½, after deducting income-tax at the rate of 3p in the rupee ?

5 What half yearly dividend is due upon an investment of

£5000 in 3 per cent stock at $87\frac{3}{8}$, after deducting $7d$ in the pound for income tax?

6 A man invests £4031 10s in the 3 per cents at $94\frac{1}{2}$, what will be his net income after an income-tax of $10d$ in the pound has been deducted, $\frac{1}{8}$ p c brokerage being allowed?

7 What must be the price of the 3 per cents, so that by investing £16425 a man may have a clear income of £515 5s after an income tax of $11d$ in the pound has been deducted?

8 What sum must a person invest in the 4 per cents at $91\frac{1}{2}$, in order to have a clear income of Rs 2300 after paying an income tax of $6p$ in the rupee?

9 A person having £10000 in the 3 per cents sells out at 65 and invests the produce in the 4 per cents at $82\frac{1}{2}$ find the change in his income

10 A person transfers Rs 11000 from the 4 per cents at 92 to the 5 per cents at 110, what is the difference in his income?

11 A person invested £9075 in the 3 per cents at $90\frac{1}{2}$, and on the stock rising to 91 transferred it to the $3\frac{1}{2}$ per cents at $97\frac{1}{2}$ What increase does he thereby make in his annual income?

12 Find the alteration in income occasioned by shifting £11000 from the 3 per cents at $101\frac{3}{8}$ to $4\frac{1}{2}$ per cent debentures at $137\frac{3}{8}$, the usual brokerage being charged on each transaction

13 £5151 is invested in 5 per cent stock at 101, the stock rising to 105, it is sold out, and the proceeds invested in stock at 102, which gives $4\frac{1}{2}$ per cent interest Find the change in income

14 By selling out Rs 45000 in the India 5 per cent stock at $112\frac{1}{2}$, and investing the proceeds in the Egyptian 7 per cent stock, a person found his income increased by Rs 1687 8a What was the price of the Egyptian stock?

15 The difference between the incomes derived from investing a certain sum in 6 per cent stock at 126, and in 9 per cent stock at 210, is Rs 225 What is the amount invested?

16 A person laid out Rs 25500 in a $3\frac{1}{2}$ per cent stock at 91 and after receiving the half-year's dividend he sold out at $90\frac{7}{8}$, how much did he gain?

17 An income of £126 is obtained by investing £3591 in the 3 per cents, what is the price of the stock?

18 A person invests Rs 4095 in 4 per cent stock, and obtains an income of Rs 173 5a 4p, what is the price of the stock?

19 A person bought the M Railway Stock at $88\frac{3}{8}$, and after receiving the half-year's dividend at the rate of $4\frac{1}{2}$ per cent per annum sold out at $93\frac{1}{8}$ and made a profit of Rs 1425, how much stock did he buy?

20 If a person invest £25350 in the 3 per cent consols at 72½, at what price must he sell out after receiving the dividend to make a profit of £250?

21 A person invests Rs 9625 in the 3 per cents at 77, and when the funds have fallen 1 per cent he transfers his capital to the 4 per cents at 95, find the alteration in his income

22 A person invested £5330 in the 3 per cents at 91, and when they had risen 1½ per cent he sold out and invested the money in the India stock at 102½, how much India stock does he hold?

23 A person laid out Rs 7492 8s in the purchase of 5 per cent stock at par, and after receiving the half yearly dividend he sells out at 4 premium and invests the proceeds in the C Railway shares at 87½, how much Railway stock does he hold?

24 A man invests £4297 10s in the 3 per cents at 95½. He sells out one third when the funds have fallen to 94, £1600 stock when they have risen to 96½, and the remainder at par. What sum does he gain? If he invests the proceeds in the shares of a gold mine paying 3 per cent at 67 5, what would be the difference in his income

572 Miscellaneous Questions on Stocks

Ex 1 At what rate will a person receive interest, who invests his capital in the 4 per cents when they are at 103½? brokerage ¼ %

Since £(103½ + ¼) or £104 produces an interest of £4 annually,

∴ £104 £100 £4 rate per cent required

∴ rate per cent required = £(100 × 4 ÷ 104) = £3½ Ans

Ex 2 Find the price of the 3 per cents when Rs 2600 stock can be purchased for Rs 2353, brokerage ⅛ per cent

Rs 2600 Rs 100 Rs 2353 price of Rs 100 stock (including brokerage),

∴ price (including brokerage) = Rs (2353 - 26) = Rs 90½

∴ price of Rs 100 stock = Rs (90½ - ⅛) = Rs 90⅞ or Rs 90 6s Ans

Ex 3 A person investing in the 3½ per cents pays ⅛ per cent for brokerage, and obtains 4 per cent on his money. At what price does he buy in?

Ex 4 £3½ · £100 price of 3½ per cent stock (including brokerage)?

the price (including brokerage) = £(100 × 3½ ÷ 4) = £81½

the price of £100 stock = £(81½ - ⅛) = £81¼ Ans

Ex 4 Which is the better stock to invest, £10000 in the 3 per cents at 90½, or the 4 per cents at 101?

From the 1st investment, income on £90½ = £3, or on £1 = £1⅞.

∴ ∴ 2nd ∴ ∴ ∴ £101 = £4, or on £1 = £1⅞

Now, comparing the fractions $\frac{6}{101}$ and $\frac{4}{101}$,

since 4×181 is $> 6 \times 101$,
the 2nd fraction $>$ the 1st, and \therefore the 2nd investment is the better *Ans*

Otherwise thus —

$\pounds 90\frac{1}{2}$ invested in the 3 per cents yields yearly $\pounds 3$,
and $\pounds 90\frac{1}{2}$. . . 4 per cent at 101 yields
($\pounds 101$ $\pounds 90\frac{1}{2}$ $\pounds 4$ *Ans*) or $\pounds (90\frac{1}{2} \times 4 - 101) = \pounds 310\frac{1}{2}$
 \therefore the 4 per cents is the better investment *Ans*

Ex 5 At what price must I invest in the 4 per cents so that after paying 6d in the pound income tax I may receive $4\frac{1}{2}$ per cent on my money?

The tax being 6d in the \pounds , it is $\frac{2}{10}$ or $\frac{1}{5}$ of the gross income, and \therefore the net income $= (1 - \frac{1}{5})$ or $\frac{4}{5}$ of the gross income

$\pounds \frac{9}{10}$ $\pounds 4\frac{1}{2}$ $\pounds 1$ gross income, \therefore gross income $= \pounds 1\frac{1}{9}$

Now, $\pounds 1\frac{1}{9}$ $\pounds 4$ $\pounds 100$ price of 4 per cent stock,

\therefore price reqd $= \pounds (100 \times 4 - \frac{100}{9}) = \pounds 86\frac{2}{9}$ *Ans*

Ex 6 A person finds that if he invests his money in the 4 per cents at 92 his income will be less by $\pounds 21$ than if he invests it in the $4\frac{1}{2}$ per cents at par, find the sum to be invested

From the 1st he gets $\pounds \frac{4}{92}$ or $\pounds \frac{1}{23}$ per \pounds ,

and from the 2nd $\frac{\pounds 4\frac{1}{2}}{100}$ or $\pounds \frac{9}{200}$ per \pounds

from the 2nd investment he gets $\pounds (\frac{9}{200} - \frac{1}{23})$ or $\pounds \frac{1}{2300}$ more per \pounds

$\pounds \frac{1}{2300}$ $\pounds 21$ $\pounds 1$ sum to be invested,

sum reqd $= \pounds 21 \times \frac{2300}{1} = \pounds 13800$ *Ans*

Examples CLXXI

1 What interest per cent per annum is obtained from investing money in the following stocks?—

- (1) The 5 per cents at 105 $\frac{1}{2}$ (2) The 3 per cents at 91
(3) 3 per cents at 91 $\frac{1}{2}$ (B $\frac{1}{8}$) (4) 6 $\frac{1}{2}$ per cents at 129 $\frac{1}{2}$ (B $\frac{1}{8}$)

2 At what price must a person purchase—

- (1) The 3 per cent consols to obtain 3 $\frac{1}{2}$ per cent for his money?
(2) 3 $\frac{1}{2}$ per cents to get 4 per cent on his money?
(3) 3 $\frac{1}{2}$ per cents to get 4 $\frac{1}{2}$ per cent on his capital?

3 Find the price of the 3 $\frac{1}{2}$ per cents when Rs 23437 8a stock can be purchased for Rs 22500, the usual brokerage being charged

4 Find the price of the $3\frac{1}{2}$ per cents when £5050 stock can be purchased for £4311 8s 9d

5 A man invested in the 3 per cents, if, after deducting an income tax of 6d in the pound, he obtained $3\frac{1}{2}$ per cent interest on the money invested, at what price did he buy?

6 Which stock is the better to invest in, the 4 per cents at 102, or the $3\frac{1}{2}$ per cents at 96?

7 Whether is it better to invest in the 3 per cents at $89\frac{1}{2}$, or in the $3\frac{1}{2}$ per cents at 95? (Brokerage $\frac{1}{8}$ p c)

8 How much 3 per cent stock has been sold out at 89, if the owner's income is increased by Rs 120 a year by investing the proceeds in 4 per cent stock at 92?

9 What was the price of consols (3 per cent), when, after paying an income tax of 5d in the pound, a man received a profit of $3\frac{1}{2}$ per cent on his money?

10 How much 3 per cent stock at par must a man sell in order to purchase enough 4 per cent stock at $124\frac{1}{2}$ to produce an income of Rs 3995, the usual brokerage being charged on each transaction?

11 What amount of stock must be sold out of the 3 per cents at $87\frac{1}{2}$ to pay the present worth of Rs 16458 12a, due 10 months hence at $3\frac{1}{2}$ per cent?

12 A person has an annual income of £191. 5s from stock in the 3 per cents, if he were to sell out at $92\frac{1}{2}$ and invest the money in the 5 per cents at 105, how much of the latter stock would he hold, and what would be the alteration in his income?

13 A person holds Rs 46750 stock in the 5 per cents, what sum must he lay out in the purchase of the $4\frac{1}{2}$ per cents at $102\frac{1}{4}$ so that his income from both sources may be Rs 8435?

14 When the $2\frac{1}{2}$ per cents are at $83\frac{3}{4}$, what ought to be the price of the $3\frac{1}{2}$ per cents to give the same rate of interest?

15 Which is the better investment,—3 per cent stock at $87\frac{3}{8}$, or shares at £233 each, on each of which a dividend of £7 13s 4d is paid annually? How much more money must be invested in one rather than in the other to produce an annual income of £460?

16 A man sells out his stock in the $3\frac{1}{2}$ per cents at $97\frac{1}{4}$, and re-invests the money in $4\frac{1}{2}$ per cent Debentures at $124\frac{1}{8}$, thereby increasing his income by £6 3s, find the amount of his stock in the $3\frac{1}{2}$ per cents the usual brokerage being charged on each transaction.

17 If the 4 per cents give 4 per cent clear, after paying an income-tax of 7 pies in the rupee, what must be the price of the 4 per cents?

18 By selling out £5000 in the 4 per cents at 90, and investing the proceeds in the India 5 per cent stock, a person finds that his income is increased by £6 13s 4d. What is the price of the India stock?

19 A person invests Rs 4000 in the $4\frac{1}{2}$ per cents at $94\frac{1}{2}$ and a certain sum in the $3\frac{1}{2}$ per cents at 77. If his total income is Rs 800, how much stock does he hold in the $3\frac{1}{2}$ per cents and how much has he invested therein?

20 A person laid out £1001 in the 3 per cents at $89\frac{3}{4}$, and having received a half year's dividend, he sold out, and then found that his capital had increased by 72 guineas, find the price at which the stock was sold.

21 What sum must I have invested in the $3\frac{1}{2}$ per cents at 91, if, after investing £4000 in the 3 per cents at 75, and paying an income-tax of 7d. in the pound on my total gross receipts, I find my net income to be £524 5s?

22 A person invests a certain sum in the 4 per cents at 96, and an equal sum in the $3\frac{1}{2}$ per cents at 75. His total income from both these sources is Rs 1250. How much does he invest in each stock?

23 If I invest my money in shares paying Rs 7 per share when the Rs 100 share is at $122\frac{1}{2}$, I find that I get Rs 355 a year more than if I invest it in the $5\frac{1}{2}$ per cent bonds at 105, find my capital.

24 A person sells out of the $3\frac{1}{2}$ per cents at $92\frac{3}{4}$ and realises Rs 18550. If he invests $\frac{2}{5}$ ths of the produce in the 4 per cents at 96, and the remainder in the 3 per cents at 90, find the alteration in his income.

25 By investing a certain sum of money in the $3\frac{1}{2}$ per cents at 72 a man gets Rs 35 less in income than he would get by investing the same sum in the $4\frac{1}{2}$ per cents at 90, find the sum invested.

26 What sum must a person invest in the 3 per cents at 90 in order that by selling out Rs 20000 stock, when they have risen to $93\frac{1}{2}$, and the remainder when they have fallen to $84\frac{1}{2}$, he may gain Rs 125 by the transaction? If he invest the proceeds in the 4 per cents at par, what will be the difference in his income?

27 I invest Rs 40000 partly in the 3 per cents at 80, and partly in the 4 per cents at 96, and then I find that on the whole I receive 4 per cent interest on the sum invested. What sums have I invested in the two stocks?

28 A and B invest an equal sum in the 3 per cents at 90, and the 4 per cents at 125. A purchases an equal amount of each kind of stock, and B divides his money equally between the two stocks. If the difference between their incomes is Rs 70, what did each invest?

29 A person transferring his stock from the 3 per cents at 90 to the 4 per cents increases his annual income by 10 per cent Find the price of the 4 per cents

30 Railway Stock is sold at 108 and with the proceeds Government Securities is bought at $91\frac{1}{2}$, after a time the Government Securities is sold at $95\frac{3}{4}$ and the original stock repurchased at 109, leaving a profit of Rs 1090 on the transaction find the amount of Railway stock sold

31 A person possesses £3200 3 per cents which he sells at $99\frac{1}{2}$, he invests the proceeds in Railway Shares at £56 per share, which shares pay 5 per cent interest on £45, the amount paid on each share How much is his income altered by the transaction?

32 A person after paying an income-tax of 7d in the pound has a clear income of £262 2s 6d derived from stock in the $4\frac{1}{2}$ per cents, he sells out $\frac{2}{3}$ of this stock at $95\frac{1}{2}$ and invests the money in the I Railway Stock at $112\frac{1}{2}$, which pays $5\frac{1}{2}$ per cent per annum, what is now his clear income after paying the income-tax as before?

33 When an income is less than £400, income tax is not chargeable upon the first £120, a man having £13300 in the 3 per cents sells out £300 at par and invests it in a mortgage returning 4 per cent, his income is now 1s 6d less than formerly, what is the amount of the income-tax?

34 A invests a certain sum of money in the 3 per cents at 90, and B invests an equal sum in the $3\frac{1}{2}$ per cents at 104 If A's income exceeds B's by Rs 400, what is the investment of A?

35 A invests a certain sum in the 3 per cents at 80, and B invests half the sum in the 4 per cents If A's income is to B's income, as 8 is to 5, find the price of the 4 per cents

36 A person invests Rs 10000 partly in the 4 per cents at 96, and partly in the $4\frac{1}{2}$ per cents at 117 What amount does he invest in each stock, if he receive the same income from each?

37 A man invests Rs 16000 partly in the 3 per cents at 75, and partly in the 4 per cents at 80 If his total income is Rs 760, how much does he invest in each stock?

38 A Railway Company could declare a dividend of 6 per cent if there were no Preference shares But Rs 80,00,000 being Preference shares, guaranteed at 7 per cent, the ordinary shareholders receive only $5\frac{1}{2}$ per cent What is the amount of Ordinary Stock?

39 A person invested Rs 6000 in the 3 per cents at 75, and after a certain number of years sold his stock, and found that during that time his sum had amounted to Rs 10000 at Simple Interest If the price of stock was 80 when he sold, find how many years he was a stock-holder

40 A person invests Rs 12000 partly in the 3 per cents at 72,

and partly in the 4 per cents at 84. He sells the former at 64 and the latter at 96, and thus realises the sum invested. How much does he invest in each stock?

41 A person invests Rs 86666 10s 8d partly in the 3 per cents at 80, and partly in the $4\frac{1}{2}$ per cents at 102. If he receive Rs 360 more a year from the second stock than from the first, how much does he invest in each kind of stock?

42 A person holds £5000 partly in the 3 per cents at 60, and partly in the 4 per cents at 75. If his income from both the sources is £160, what sum does he hold in the 4 per cents?

43 A person invests £2000 partly in the 4 per cents at 96, and partly in the 3 per cents at 90. If his total income be £10 less than what it would have been had he invested the whole in the 5 per cents at 125, find how much he has invested in each stock.

44 A person holds £5000 partly in the 3 per cents at 90, and partly in the 4 per cents at par. He sells the former at 80, and the latter at 20 per cent premium, and thereby realises £100 more than the sum invested. How much does he hold in the 3 per cents?

45 If the 3 per cents be at 95, and the Government offer to receive tenders for a loan of £5,000,000, the lender to receive £5,000,000 stock in the 3 per cents together with a certain sum in the $3\frac{1}{2}$ per cents, what sum in the $3\frac{1}{2}$ per cents ought the lender to accept?

CHAPTER XVI

Evolution and Surds.

I EVOLUTION

573 Application of Geometry to Arithmetic. By Euclid I 47, we know that in a right angled triangle the square on the side opposite the right angle is equal to the sum of the squares on the sides containing the right angle. Hence, the square of the measure of the side opposite the right angle is equal to the sum of the squares of the measures of the sides containing the right angle. Therefore, we have

$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2, \quad (i)$$

$$\begin{aligned} (\text{Perpendicular})^2 &= (\text{Hypotenuse})^2 - (\text{Base})^2, \\ &= (\text{Hyp} + \text{Base})(\text{Hyp} - \text{Base}), \quad (ii) \end{aligned}$$

$$\text{Similarly, } (\text{Base})^2 = (\text{Hyp} + \text{Perp})(\text{Hyp} - \text{Perp}) \quad (iii)$$

Thus, if any two of the three sides in a right-angled triangle be given, we can easily determine the third.

Ex 1 The sides of a right-angled triangle are 56 yds and 42 yds, find the hypotenuse

$$56^2 + 42^2 = 3136 + 1764 = 4900, \text{ and } \sqrt{4900} = 70$$

\therefore the required hypotenuse = 70 yds Ans.

Ex 2 The diagonal of a rectangular space is 890 ft, and its length 792 ft, find its width

The length and breadth form with the diagonal a right angled triangle, of which the diagonal is the hypotenuse and the length the base, to find the perpendicular

$$\therefore (\text{Perp})^2 = (890 + 792)(890 - 792) = 1682 \times 98 = 3364 \times 49,$$

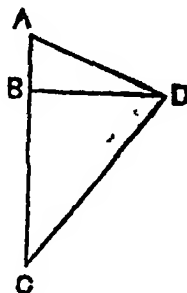
Hence perpendicular or width = $58 \times 7 \text{ ft} = \underline{406 \text{ ft}}$ Ans

Ex 3 The tip of a reed was 8 inches above the surface of a lake, but forced by the wind, it gradually advanced, and was submerged at a distance of 28 in. Find the depth of water

Let $CA = CD$ represent the reed, BD the surface, CB the depth, $AB = 8$, $BD = 28$, to find CB

In the right-angled triangle CBD , we have $BD = 28$, and the difference of CD and $CB = 8$

Since $BD^2 = CD^2 - CB^2 = (CD + CB)(CD - CB) = 8(CD + CB)$, $CD + CB = 28^2 - 8 = 98$ and $CD - CB = 8$, hence $CB = \frac{1}{2}(98 - 8) = 45$ Thus the depth of water = 45 in Ans



Ex 574 Application of Algebra to Arithmetic The student would do well to remember the following formulæ —

$$(i) a^2 - b^2 = (a + b)(a - b)$$

$$(ii) a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$(iii) a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Ex 1 Find the value of

$$\frac{687 \times 687 - 313 \times 313}{687 - 313}$$

$$\text{Exp} = \frac{(687)^2 - (313)^2}{687 - 313} = \frac{(687 + 313)(687 - 313)}{687 - 313} = 687 + 313 = 1000$$

Ex 2 Find the value of $\frac{(07)^2 + (05)^2}{(007)^2 + (005)^2}$

$$\text{Exp} = \frac{(70)^2 + (50)^2}{(7)^2 + (5)^2} = \frac{10^2(7^2 + 5^2)}{7^2 + 5^2} = 10^2 = 100 \text{ Ans}$$

Ex 3 Simplify $\frac{3\frac{1}{2} \times 3\frac{1}{2} \times 3\frac{1}{2} - 1}{3\frac{1}{2} \times 3\frac{1}{2} - 1}$

$$\begin{aligned} \text{Exp} &= \frac{(3\frac{1}{2})^3 - 1}{(3\frac{1}{2})^2 - 1} = \frac{(3\frac{1}{2} - 1)\{(3\frac{1}{2})^2 + 3\frac{1}{2} + 1\}}{(3\frac{1}{2} - 1)(3\frac{1}{2} + 1)} = \frac{(3\frac{1}{2})^2 + 3\frac{1}{2} + 1}{3\frac{1}{2} + 1} \\ &= \frac{3\frac{1}{2}(3\frac{1}{2} + 1) + 1}{3\frac{1}{2} + 1} = 3\frac{1}{2} + \frac{1}{4\frac{1}{2}} = 3\frac{1}{2} + \frac{1}{4\frac{1}{2}} = 3\frac{3}{8} \quad \text{Ans} \end{aligned}$$

Ex 4 Simplify $\frac{2\frac{1}{3} \times 2\frac{1}{3} \times 2\frac{1}{3} + 1}{2\frac{1}{3} \times 2\frac{1}{3} - 1}$

$$\begin{aligned} \text{Exp} &= \frac{(2\frac{1}{3})^3 + 1}{(2\frac{1}{3})^2 - 1} = \frac{(2\frac{1}{3} + 1)\{(2\frac{1}{3})^2 - 2\frac{1}{3} + 1\}}{(2\frac{1}{3} + 1)(2\frac{1}{3} - 1)} = \frac{2\frac{1}{3}(2\frac{1}{3} - 1) + 1}{2\frac{1}{3} - 1} \\ &= 2\frac{1}{3} + \frac{1}{1\frac{1}{3}} = 2\frac{1}{3} + \frac{3}{4} = 3\frac{1}{2} \quad \text{Ans} \end{aligned}$$

Examples CLXXII

1 The two sides of a right angled triangle are 28 and 195 feet respectively, what is the length of the hypotenuse?

2 The hypotenuse of a right-angled triangle is 75 15 ft, and one of the sides is 60 12 ft, what is the length of the remaining side?

3 Find the side, and also the diagonal of a square having the same area as a rectangle 43 ft 5 in long and 34 ft 7 in broad

4 One side of a rectangle is 8076 yds and the diagonal is 8749 yds, find the other side

5 If the perpendicular sides of a right-angled triangle are 13 02 and 5 2 feet, what is the third side?

6 If the town A is 72 miles West of B and 135 South of C, what is the distance from B to C?

7 Two men travelled from the same town—the one North, 28 miles per day, the other West, 36 miles per day, how far were they distant from each other after travelling 6 days?

8 Close by the side of a river rises a precipice to the height of 261 feet, and a line, reaching from its top to the opposite bank of the river, measures 582 feet, what is the breadth of the river?

9 Suppose the top of a straight ladder, 18½ ft long, to rest against a building at the height of 13¾ ft from the ground, at what horizontal distance from the bottom of the building is the foot of the ladder placed?

10 A certain number of boys spent Rs 90 4a each spending as many four-anna pieces as there were boys, what was the number of boys?

11 A ladder 25 ft long has its foot placed in a street, and its top resting against a wall, on one side of the street, at a height

of 15 ft from the ground. If the ladder be turned over to the other side, its top reaches to a point 20 ft. high on the opposite wall. Find the breadth of the street.

12 M and N start together at B , to walk to another point C , 1332 yds north from B , M takes the direct road BC , N goes first to A , a point west of B , and then straight to C , his journey being 2738 yds. How far is the distance from B to A ?

13 Find the values of —

$$(1) \frac{874 \times 874 - 126 \times 126}{874 - 126},$$

$$(2) \frac{7654 \times 7654 - 2346 \times 2346}{7654 - 2346}.$$

$$(3) \frac{425 \times 425 - 175 \times 175}{425 + 175},$$

$$(4) \frac{(014)^2 - (008)^2}{(0014) - (0008)^2}.$$

$$(5) \frac{(54)^4 - (53)^4}{(0054)^2 - (0053)^2}$$

$$(6) \frac{1\frac{1}{4} \times 1\frac{1}{4} \times 1\frac{1}{4} - 1}{1\frac{1}{4} \times 1\frac{1}{4} - 1}$$

$$(7) \frac{4\frac{1}{2} \times 4\frac{1}{2} \times 4\frac{1}{2} - 1}{4\frac{1}{2} \times 4\frac{1}{2} - 1}$$

$$(8) \frac{3\frac{1}{4} \times 3\frac{1}{4} \times 3\frac{1}{4} + 1}{3\frac{1}{4} \times 3\frac{1}{4} - 1}.$$

14 In extracting the square root of 0.003 you have by mistake "pointed" thus 0.0300, &c, and proceeded with the operation and marked off the decimals accordingly. Without extracting the root of 0.003 over again, there is a certain quantity, which if multiplied into your erroneous result will give a correct value of $\sqrt{0.003}$, find the first three decimal places of this multiplier.

15 In a certain lake the tip of a bud of lotus was seen a span above the surface of the water. Forced by the wind it gradually advanced and was submerged at a distance of two cubits. Compute the depth of the water.

II SURDS

575 When the quantity whose root is to be extracted is not a *complete* square, cube, &c, we have seen that there will be a remainder left however far we may continue the operation, and the root can therefore be found only *approximately*, that is, such a quantity has no *exact* root, and its representation is termed a **Surd** or **Irrational Quantity**.

For instance, the square root of 2 expressed by $\sqrt{2}$, is evidently not a whole number, because the square of no *whole number* whatever is 2, neither can it be a *vulgar fraction*, because the square of every vulgar fraction properly so called is itself a vulgar fraction, and it cannot be a *recurring decimal*, because all such quantities are equivalent to finite vulgar fractions, in other words, the square root of 2 may be found as nearly as we please, but not exactly; and it is termed an **incommensurable** quantity, because it admits of no exact measure which is any *finite* quantity whatever either integral or fractional.

576 The surds of most frequent occurrence are those designated by the sign $\sqrt[n]{}$ or $\sqrt{}$, or by the index $\frac{1}{n}$, and termed **Quadratic surds** and in general, when any *quantity* is represented in the form of a surd by means of a *fractional index* it is always understood that the numerator of the index denotes the power to which the *number* is intended to be raised, and that the denominator expresses the root afterwards to be extracted

Thus, $27^{\frac{2}{3}}$ will represent the cube root of the square of 27, and is therefore equivalent to the cube root of 729, which is 9 that is $27^{\frac{2}{3}}$, though expressed in the *form* of a surd, is in reality a rational quantity and conversely

577 Hence, the fundamental operations on surds must be performed upon their approximate values obtained as before, but these operations may frequently be shortened, as will appear in the following instances —

$$\begin{aligned} \text{Since } \sqrt{8} &= \sqrt{4 \times 2} = \sqrt{4} \times \sqrt{2} = 2 \times \sqrt{2}, \text{ or } = 2\sqrt{2}, \\ \text{we have, in Addition, } &\sqrt{8} + \sqrt{2} = 2\sqrt{2} + \sqrt{2} = 3\sqrt{2}, \\ \text{in Subtraction, } &\sqrt{8} - \sqrt{2} = 2\sqrt{2} - \sqrt{2} = \sqrt{2}, \\ \text{in Multiplication, } &\sqrt{8} \times \sqrt{2} = 2\sqrt{2} \times \sqrt{2} = 4, \\ \text{in Division, } &\sqrt{8} \div \sqrt{2} = 2\sqrt{2} \div \sqrt{2} = 2, \end{aligned}$$

where the extraction of only *one* root is sufficient for the operations of Addition and Subtraction, and the product and quotient are rational quantities

578 The Involution and the Evolution of such quantities may frequently be effected in the same way

Thus, the square of $2\sqrt{5}$ = the product of the square of 2 and the square of $\sqrt{5} = 4 \times 5 = 20$, which is a rational number and conversely

Again, by multiplying each of the terms of the numerator and denominator by $\sqrt[3]{100}$, we have,

$$\begin{aligned} \frac{\sqrt[3]{(512)} + \sqrt[3]{(03375)}}{\sqrt[3]{(80)} - \sqrt[3]{(01)}} &= \frac{\sqrt[3]{(512)} + \sqrt[3]{(3375)}}{\sqrt[3]{(8000)} - \sqrt[3]{1}} = \frac{8 + 15}{20 - 1} \\ &= \frac{23}{19} = 5\frac{1}{19}, \text{ a rational quantity} \end{aligned}$$

579 It has been said that the *values* of surds may be found as nearly as we please, and this will clearly be done by continuing the extraction to the number of places of decimals in the root which we may find necessary for the purpose

Thus, since $\sqrt{2} = 1.41421$ &c., we have,
 $\sqrt{2} = 1.4$ nearly
 $= 1.41$ more nearly,
 $= 1.414$ still more nearly
 $= 1.4142$ still more nearly
 \therefore &c

and consequently its magnitude may be compared with that of any other numerical quantity either rational or irrational, although its absolute magnitude can never be exactly ascertained

580 To find the value of such expressions as

$$\frac{1}{\sqrt{2}-1}, \frac{4}{\sqrt{3}-\sqrt{2}}, \frac{\sqrt{5}}{\sqrt{5}+\sqrt{2}}, \frac{\sqrt{3}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}, \text{ \&c}$$

the student must notice the following statements, which are true whatever numbers the letters may stand for —

$$\begin{array}{ll} \text{(i)} \sqrt{x} \times \sqrt{x} = x & \text{(ii)} \sqrt{r} \times \sqrt{y} = \sqrt{r \times y} \\ \text{(iii)} \sqrt{r^2 \times y} = r \times \sqrt{y} & \text{(iv)} (x+y)(x-y) = x^2 - y^2 \\ \text{(v)} (m+n)(x+y) = m \times x + m \times y + n \times x + n \times y \end{array}$$

Ex 1 Find the value of $\frac{1}{\sqrt{2}-1}$

The first thing to be done is always to free the denominator from surds. If we multiply both numerator and denominator by $\sqrt{2}+1$, the denominator becomes $2-1$ by (iv)

$$\begin{aligned} \text{Thus, } \frac{1}{\sqrt{2}-1} &= \frac{(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)} = \frac{\sqrt{2}+1}{2-1} = \frac{\sqrt{2}+1}{1} = 1 + \sqrt{2} \\ &= 1 + 1 \cdot 4142 = \underline{2 \cdot 4142} \quad \text{Ans} \end{aligned}$$

Ex 2 Find the value of $\frac{4}{\sqrt{3}-\sqrt{2}}$

Multiply both numerator and denominator by $\sqrt{3}+\sqrt{2}$, and we get

$$\begin{aligned} \frac{4}{\sqrt{3}-\sqrt{2}} &= \frac{4(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})} = \frac{4(\sqrt{3}+\sqrt{2})}{3-2} = 4(\sqrt{3}+\sqrt{2}) \\ &= 4(1 \cdot 732 + 1 \cdot 414) = 4 \times 3 \cdot 146 = \underline{12 \cdot 584} \quad \text{Ans} \end{aligned}$$

Ex 3 Find the value of $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

$$\begin{aligned} \frac{\sqrt{3}+\sqrt{2}}{\sqrt{5}-\sqrt{2}} &= \frac{(\sqrt{3}+\sqrt{2})(\sqrt{5}+\sqrt{2})}{5-2} = \frac{\sqrt{15}+\sqrt{6}+\sqrt{10}+2}{3} \\ &= \frac{\frac{1}{3}(3 \cdot 873 + 2 \cdot 445 + 3 \cdot 162 + 2)}{3} = \frac{1}{3} \times 11 \cdot 480 \\ &= \underline{3 \cdot 826} \quad \text{Ans} \end{aligned}$$

Ex 4 Find the value of $\frac{2+\sqrt{3}}{\sqrt{3}+1}$

$$\begin{aligned} \frac{2+\sqrt{3}}{\sqrt{3}+1} &= \frac{\frac{1}{2}(4+2\sqrt{3})}{\sqrt{3}+1} = \frac{\frac{1}{2}(\sqrt{3}+1)^2}{\sqrt{3}+1} = \frac{\frac{1}{2}(\sqrt{3}+1)(\sqrt{3}+1)}{\sqrt{3}+1} = \frac{1}{2}(\sqrt{3}+1) \\ &= \frac{1}{2}(1 \cdot 732 + 1) = \frac{1}{2} \times 2 \cdot 732 = \underline{1 \cdot 366} \quad \text{Ans} \end{aligned}$$

Examples CLXXIII

1 Find the value (to 5 places of decimals) of —

- (1) $\frac{1}{\sqrt{3}-1}$ (2) $\frac{3}{\sqrt{6}-2}$ (3) $\frac{2}{\sqrt{5}+\sqrt{3}}$ (4) $\frac{\sqrt{5}}{\sqrt{2}+\sqrt{3}}$ (5) $\frac{3}{\sqrt{5}+\sqrt{2}}$
 (6) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-1}$ (7) $\frac{\sqrt{3}+\sqrt{5}}{\sqrt{5}-\sqrt{2}}$ (8) $\frac{\sqrt{3}+\sqrt{7}}{\sqrt{11}-\sqrt{5}}$ (9) $\frac{\sqrt{5}+1}{\sqrt{5}-1}$
 (10) $(\sqrt{6}+\sqrt{3}+\sqrt{2}+2)(\sqrt{6}-\sqrt{3}+\sqrt{2}-2)$ (11) $(\sqrt{3}+2)^2+(\sqrt{3}-2)^2$
 (12) $\frac{\sqrt{(05-005)}}{\sqrt{(05)}-\sqrt{(005)}}$ (13) $\frac{(\sqrt{5}+\sqrt{3})(\sqrt{5}+\sqrt{2})' \sqrt{3}+\sqrt{2})}{(\sqrt{5}+\sqrt{3}+\sqrt{2})^2}$
 (14) $\sqrt{\left\{ \frac{2+\sqrt{(12)}-\sqrt{(27)}}{2+\sqrt{(48)}-\sqrt{(27)}} \right\}}$ (15) $\frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} + \frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}$

2 Which is the greater quantity —

- (1) $\sqrt{2}$ or $\frac{2}{3}$? (2) $\frac{2}{3}$ or $\frac{1}{2}(19)$? (3) $\sqrt{3}$ or $\frac{1}{2}(15)$?

3 What fraction of $\{\sqrt{(4050)} \times 0\dot{0}2 - 2\dot{0} + \sqrt{(1458)}\} = \sqrt{(02)}$
 is $\sqrt{(6008-3042)} + \sqrt{(116\dot{6} \times 04\dot{6})}$?

4 Find the value of $\frac{15+\sqrt{(009)}}{1-\sqrt{(9)}}$ correct to 3 places of decimals

CHAPTER XVII.

The Metric System and Decimal Coinage

I THE METRIC SYSTEM

581 Soon after the French Revolution of 1789, a Commission was appointed consisting of several eminent mathematicians, for the purpose of preparing a new system of weights and measures

The system they recommended was established in France, under the name of **The Metric System**. Since then it has been introduced to a greater or less extent into almost all the countries of Europe

582 In the formation of the multiples and sub multiples the decimal system is followed exclusively, the Greek prefixes to any unit denoting multiples and the Latin prefixes denoting sub multiples, thus,

Greek prefixes			Latin prefixes		
Deka	means	10	Deci	means	10th
Hecto	..	100	Centi		100th
Kilo		1000	Milli		1000th
Myria		10000			

583 The unit of length is the *Metre*. It is also the *fundamental* unit, because from it every other unit of weight or measure is derived and hence the name *Metric System*. A metre was defined to be the *ten millionth* part of the distance from the Equator to the Pole measured along the surface of the ocean; but subsequent calculations have proved this to be incorrect. 1 Metre = 39.371 English inches nearly.

10 Millimetres (m m) = 1 Centimetre	10 Metres = 1 Dekametre
10 Centimetres (c m) = 1 Decimetre	10 Dekametres = 1 Hectometre
10 Decimetres (d m) = 1 Metre	10 Hectometres = 1 Kilometre
	10 Kilometres = 1 Myriametre.

Note Long distances are generally quoted in kilometres, 1 kilometre = $\frac{5}{8}$ English mile nearly.

584 The unit of surface is the *Square Metre*. In measuring land it is usual to take for unit a *square decametre*. This is called an *Are*. It is equal to 119.6036 English sq yds nearly.

10 Centiares (sq m) = 1 Deciare	10 Ares = 1 Decare
10 Deciares = 1 Are	10 Decares = 1 Hectare
= 1 sq Dekametre	= 1 sq Hectometre

Note Large surfaces are generally quoted in Hectares, 1 Hectare = 2 $\frac{1}{2}$ English acres nearly.

585 The unit of volume is the *Cubic Metre*. In measuring wood this is called a *Stere*. It is equal to 35.317 English cub feet nearly.

10 Decisteres = 1 Stere	10 Steres = 1 Dekastere
-------------------------	-------------------------

586 The unit of Capacity is the *Litre*. It is equal to a cubic decimetre = 61.028 English cub inches nearly and contains about 22 gallons.

10 Centilitres = 1 Decilitre	10 Litres = 1 Dekalitre
10 Decilitres = 1 Litre	10 Dekalitres = 1 Hectolitre
= 1 Cub Decimetre	10 Hectolitres = 1 Kilolitre
	= 1 Cub Metre

Note 1 Litre = $1\frac{1}{4}$ English pint nearly.

587 The unit of Weight is the *Gramme*. It is the weight of a cubic centimetre of distilled water at its greatest density, i.e., when the temperature is about 4°C. 1 Gramme = 15.434 grains English, nearly.

10 Milligrams = 1 Centigram	10 Hectograms = 1 Kilogram
10 Centigrams = 1 Decigram	10 Kilograms = 1 Myriagram
10 Decigrams = 1 Gram	10 Myriagrams = 1 Quintal
10 Grams = 1 Dekagram	10 Quintals = 1 Millier
10 Dekagrams = 1 Hectogram	

Note 1 Kilogramme = 2 $\frac{1}{2}$ English lbs Avou nearly. 1 Quintal = 100 Kilogrammes = 1.97 English cwt. 1 Millier = 1000 Kilogrammes = 19.7 English cwt.

588 The unit of money is the **Franc** It is about 9 $\frac{1}{4}$ d. English The sub multiples of the franc are decime seldom used) and the centime, which are respectively the tenth and hundredth part of the franc, so that

$$\begin{aligned} 10 \text{ Centimes (c)} &= 1 \text{ Decime} \\ 10 \text{ Decimes or } 100 \text{ Centimes} &= 1 \text{ Franc (fr)} \end{aligned}$$

Accounts however, are kept in France in francs and centimes only, thus 58 76 francs is read 58 francs 76 centimes

The franc is a silver coin composed of 9 parts of silver and 1 part of copper, and weighs 5 grams The Napoleon is a gold coin = 20 francs A sou (a copper coin) = 3 centimes

Ex 1 Express 9 hectom 9 dekam 7 metres in *metres*, and 5 litres 3 decil 7 centil in *centilitres*

$$(i) \text{ 9 hectom 9 dekam 7 metres} = 997 \text{ metres}$$

$$(ii) \text{ 5 litres 3 decil 7 centil} = 537 \text{ centilitres}$$

Ex 2 Find the circumference of a wheel which will turn 1130 times in 2 kilom 33 dekam 91 decim

$$2 \text{ kilom } 33 \text{ dekam } 91 \text{ decim} = 2339 \text{ 1 metres, and } 2339 \text{ 1} - 1130 = 2 \text{ 07}$$

$$\text{the circumf reqd} = 2 \text{ 07 metres} = \underline{2\text{m } 7 \text{ centim}} \quad \text{Ans}$$

Ex 3 Make out the following bill —

$$\begin{aligned} &5 \text{ kilog } 27 \text{ dekag of moist sugar at } 1 \text{ fr } 50 \text{ c per kilog } 12 \text{ kilog } \\ &6\text{gr of lump sugar at } 1 \text{ fr } 75 \text{ c per kilog, } 10 \text{ kilog } 15 \text{ dekag of tea at } \\ &7 \text{ fr } 50 \text{ c per kilog and } 12 \text{ kilog } 9 \text{ hectog of coffee at } 4 \text{ fr } 15 \text{ c per kilog} \\ &5 \text{ kilog } 27 \text{ dekag at } 1 \text{ fr } 50 \text{ c per kilog} = (1 \text{ 50} \times 5 \text{ 27 fr} = 7 \text{ 905 fr} \\ &12 \text{ kilog } 9 \text{ gr at } 1 \text{ fr } 75 \text{ c per kilog} = (1 \text{ 75} \times 12 \text{ 009 fr} = 21 \text{ 01575} \\ &10 \text{ kilog } 15 \text{ dekag at } 7 \text{ fr } 50 \text{ c per kilog} = (7 \text{ 05} \times 10 \text{ 15) fr} = 71 \text{ 5575} \\ &12 \text{ kilog } 9 \text{ hectog at } 4 \text{ fr } 15 \text{ c per kilog} = (4 \text{ 15} \times 12 \text{ 9) fr} = \underline{53 \text{ 535}} \quad . \\ &\qquad\qquad\qquad 154 \text{ 01325.} \end{aligned}$$

$$= \underline{154 \text{ fr } 1 \text{ 325 c}} \quad \text{Ans}$$

Ex 4 A man walked a distance of 4 $\frac{1}{2}$ miles in 72 minutes, find his rate per hour of walking in French measure

$$\text{In 72 min he walks } 4\frac{1}{2} \text{ miles or } 4\frac{1}{2} \times 1760 \text{ yds}$$

$$\text{1 hour } \frac{60}{72} \text{ of } 4\frac{1}{2} \times 1760 \text{ yds or } 337600 \text{ in}$$

$$= \frac{337600}{39 \text{ 371}} \text{ metres} = \underline{6035 \text{ m}} \text{ nearly}$$

Examples CLXXIV

1 Express in terms of a *kilom*, and also of a *centimetre* —

$$(1) \text{ 17 kilom } 3 \text{ dekam } 3 \text{ millim} \qquad (2) \text{ 500 hectom } 37 \text{ millim}$$

2 Express in terms of a *sq metre*, and also of a *sq millimetre* —

$$(1) \text{ 533 sq hectom } 3 \text{ sq m } 2 \text{ sq millim} \qquad (2) \text{ 725 sq decim}$$

3 Express in terms of a *gram* and also of a *kilogram* —

- (1) 1 hectog 16 gr 75 millig (2) 53 dekag 45 centig

4 Express 763 sq hectom 5 sq m in terms of an *are*

5 In 530 000465 kilolitres, how many *decalitres*?

6 Express in terms of a *franc*, and also of a *centime* —

- (1) 24 fr 14 c (2) 480 fr 5 $\frac{1}{2}$ c (3) 5 fr 8 c (4) 555 dec 3 $\frac{1}{2}$ c.

7 How many fields each containing 2 hectares, 47 ares can be made out of a farm of 313 hectar, 69 ares?

8 A certain number of sawyers cut out 390 sterres of wood, on the average each sawyer cuts 32 sterres 5 decisterres. How many sawyers were employed?

9 Make out the following bill —

27 m 8 centim of calico at 95 c per m, 12 m 12 centim of lining at 70 c per m, 3 dozen pairs of gloves at 2 fr 75 c per pair, and 19 m 4 decim of flannel at 2 fr 45 c per m. If a person give a ten pound note to pay this bill, how much change in English money should he receive, £1 being = 25 fr 25 c?

10 Divide 80 hectares of land between *A*, *B* and *C*, so that *A* shall have 2 hectar 76 ares more than *B*, and *C* 11 hectar 12 ares more than *B*.

11 If 6 horses in 4 days eat 144 kilog 96 gr of hay, how long will 675 kilog 450 gr serve 15 horses?

12 From Paris to Lyons is 507 kilometres. The express trains take 10 $\frac{1}{2}$ hours for the journey, and the first class fare is 56 80 francs, find the rate of the trains in miles per hour, and the fare per mile in English money, taking £1 = 25 francs.

13 A bankrupt's effects are worth 3535 fr 98 c, his debts amount to 20037 fr 22 c more than his effects, what will a creditor to the amount of 350 fr lose?

14 A person's income is reduced from 56085 fr 50 c to 52720 fr 37 c by income-tax. (1) How much tax does he pay in the franc? (2) How much tax does he pay in the £, supposing £1 = 25 francs?

15 A needle woman earns on Monday 3 fr 40 c, on Tuesday 3 fr 75 c, on Wednesday nothing, on Thursday 4 fr 5 c, on Friday 14 c less than what she earned on Monday, on Saturday 1 fr 5 c less than what she earned in Thursday, find her average daily earnings during the week.

16 A person selling a kilog of sugar at 1 fr 80 c gained 12 $\frac{1}{2}$ per cent on the cost price, find the cost price of a millier of the sugar.

17 At what rate per cent will 4850 francs amount to 5820 francs in 3 years and 4 months?

18 After the payment of an income-tax of 15 c in the franc, a gentleman's income is 365318 fr 10 c. Express his income-tax as a fraction of his net income.

19 A person uses 53 48 kil of thread in weaving 231 5 metres of linen, 1 15 metres broad, how many metres of linen 95 metres broad can be woven with 37 75 kil of the same thread?

20 How many Napoleons, five-franc pieces, francs, half-francs, and sous, are there in 4072 fr 45 c, there being an equal number of each of the first two coins, and half that number of each of the last three coins?

21 A woman lays out at one shop 23 fr 5 c in calico, 34 fr 45 c in flannel, 3 fr 50 c in tape, 4 fr 20 c in thread, &c, 65 c in needles. At another shop 5 fr in tea, 4 fr 25 c in sugar, 7 fr 5 c in cheese, 9 fr 45 c in candles, 6 fr 3 c in soap. (1) How much did she lay out at one shop more than at the other? (2) What change ought she to receive out of 5 Napoleons? What change ought she to receive in French, and also in English money, out of a £5 note? £1 being = 25 fr 50 c

22 A exchanges with B 200 sacks of wheat, each of 157 kilog at 46 fr 25 c a sack, for 55 tons of iron at 21 fr 50 c the 100 kilog. What English money should pass between them to make it a fair bargain? £1 being = 25 fr 12 c

23 Divide 85 hectar 50 ares between A, B and C, so that A's share B's share 11 6, and that C shall have 9 hectar more than the united shares of A and B

24 A man made the following bill 29 dozen pairs of gloves at 3 fr 25 c a pair, 116 pairs of stockings at 2 fr 24 c a pair, 84 pairs of stockings at 2 fr 6 c a pair, 4 pieces of calico, each containing 39½ m at 95 c a m, 1 piece of Irish linen, 84 m 6 centim, at 3 fr 22 c a m, 5 pieces of flannel each containing 42 m 27 centim, at 3 fr a m, amount to 2634 fr 3 c, find the amount of his error.

25 In how many years will 450 fr amount to 576 fr at 3½ per cent simple interest?

26 What will it cost to paint a room 7 m 25 centim long, 5½ m broad, and 4 m 75 centim high at 4 fr 5 c a sq m? Find the cost in English money, a Napoleon being worth 15s 9¾d

27 A person invests 3000 fr in the 5 per cents at 87 73, what amount of stock will he hold?

28 A rectangular piece of ground is 32 m 7 decim long, and 19 m 5 centim broad. Find the cost of enclosing it with a path 1 m 5 decim broad at 3 fr 5 c a sq m. (1) When the path is outside the piece of ground. (2) When the path is part of the ground.

29 A speculator bought 264 hectares 6 aies of land for 387074 fr 25 c, he sold ⅔ of it at 1465 fr the hectare, at what price per hectare must he sell the rest, in order to gain 9177 fr 12 c by his purchase?

30 If the driving wheel of a locomotive be 1 dekam 25 decim in circumference, and it makes 2½ revolutions in 1 second, how long will it take to travel 100 miles, if 1 mile = 1⅔ kilom?

31 Two masons' wages, for 7 days' work of the 1st, and 5 days' work of the 2nd, amount to 45 fr 90 c, and for 5 days' work of the 1st and 7 days' work of the 2nd, to 48 fr 66 c Find the daily wages of each

32 Which is the best stock to invest 100000 fr, in the 5 per cents at 121 90, or the 3 per cents at 84? What is the difference of income in English money, if 24 fr 96 c = £1?

33. Make out the following bill —

2 lbs of sugar at 14 cents 9 mills a lb, 14½ lbs of cheese at 17½ cents a lb, 7 lbs 4 oz of butter at \$ 245 a lb, 5½ lbs of tea at 39½ cents a lb, 40 eggs at 27 cents a dozen, 12 lbs of currants at \$ 15 12 cents a cwt Find the value in English money, if \$ 1 = 4s 2d

34 Suppose a person to invest 6000 fr on the 1st of Jan 1866 at 5 per cent per annum, and also to invest the same amount on the 1st of Jan of each of the next 3 years, what will his investments amount to at the end of the 4th year, supposing him not to touch any of the money, and to receive compound interest?

35 A person gives 5 Napoleons to pay a bill of £3 12s 6d, when the exchange is £1 = 25 fr 12 c (1) How much English money ought he to receive back? (2) How much French money?

36 If 3 kilog of tea be worth 5 kilog of cocoa, and 7 kilog of cocoa be worth 11 kilog of coffee, what will 5 kilog of tea be worth, if 1 kilog of coffee is worth 6 fr 30 c?

37 If 1 kilog 7 dekag of moist sugar be worth 99 dekag of lump sugar, and 22 dekag of lump sugar be worth 13 dekag 5 gr of coffee, and 2 kilog of coffee be worth 9 hectog 63 gr of tea, what is the value of 100 kilog of moist sugar, if a kilog of tea be worth 9 fr 50 c?

38 A can plant with cabbages $\frac{1}{8}$ of an hectare in 10 hrs 15 min, B can plant the same space in 8 hrs 45 min What space can they together plant in 2½ hours? How many square yards?

39 It costs 217 fr 25 c to carpet a room 17 7 metres long, and 8 4 metres broad, what will it cost to carpet another room 13 1 metres long and 6 3 metres broad with the same material?

40 It takes 14375 sq bricks to pave a path 184 m long, and 4 m 50 centim broad, find the length of the side of each brick

41 If 38 kilog 25 grs of sugar cost 134 fr 7½ c, how much sugar worth 8 as much per dekag can be bought for 26 fr 81½ c?

42 What number of dollars will amount to 1059 38 dollars in 5 years at 4½ per cent simple interest?

43 A man buys a steam plough for 30807 fr 84 c, he ploughs 170 hectar, 4 ares of his own farm, and $\frac{2}{3}$ of that quantity for a neighbour He estimates the expenses at 10 per cent on the cost of the steam-apparatus, and wages and fuel, &c, at 1210 fr, what is the cost of ploughing an hectare?

44 If 20 men can dig a trench 160 m long 2 m wide, and 1 m 2 decim deep in 8 days, what will be the depth of a similar trench 90m long, 1 m 80 centim broad, which 24 men can dig in 4½ days?

45 A person bought for 164 fr 64 c a certain number of boxes of oranges, each box contained 3 times as many oranges as there were boxes, and each orange cost twice as many centimes as there were boxes. Find the number of boxes, and also the number of oranges.

46 A Prussian buys and pays for the following articles of a Paris merchant, 15 5 metres of cloth at 14 fr 75 c the metre, 6 25 metres of velvet at 21 fr 5 c the metre, 19 metres of silk at 6 fr 25 c the metre, 28 50 metres of merino at 4 fr the metre, half a dozen pairs of stockings at 25 fr the dozen pairs, 5 pairs of gloves at 17 fr the dozen pairs, in consequence of ready payment $\frac{3}{100}$ of the amount of the bill is deducted, he offers for payment 200 thalers, how many Napoleons ought he to receive from the merchant, 6 thalers, 20 silver groschen being = 25 francs? (1 Thaler = 30 silver groschens)

II DECIMAL COINAGE

589 In the Proposed Decimal System of Coinage, the pound or sovereign, which is of the same value as the pound sterling at present, is taken as the unit of money, with sub multiples *florins*, *cents*, and *mils*, which are respectively the tenth, hundredth and thousandth part of the pound, so that

10 Mils (m) = 1 Cent (c)
10 Cents = 1 Florin (fl)
10 Florins = 1 Pound (£1)

In this system, in a decimal of £1, the figure in the first place of decimals represents *florins*, in the second *cents*, and in the third *mils*.

Ex 1 Reduce £15 7c 8m to *mils*, and 25684 mils to £ fl &c.
 $\begin{array}{r} \text{£15 } 7\text{c } 8\text{m} = \text{£15 } 078 \\ = \underline{15078} \text{ mils} \end{array} \quad \begin{array}{r} 25684 \text{ mils} = \text{£25 } 684 \\ = \underline{\text{£25 } 6\text{fl } 8\text{c } 4\text{m}} \end{array}$

Ex 2 Reduce £4 18s 10 $\frac{3}{4}$ d to £ fl &c, and £5 6fl 2c 5m to £ s d

$$\begin{array}{r} 4 \overline{) 3} \\ 12 \overline{) 1075} \\ 20 \overline{) 188958333} \\ \text{£4 } 944791666 \\ = \underline{\text{£4 } 9\text{fl } 4\text{c } 47916\text{m}} \end{array}$$

$$\begin{array}{r} \text{£5 } 6\text{fl } 2\text{c } 5\text{m} = \text{£5 } 625 \\ \quad \quad \quad 20 \\ \quad \quad \quad \text{s } 125 \\ \quad \quad \quad \underline{12} \\ \quad \quad \quad \text{d } 60 \end{array}$$

the sum = £5 12s 6d

Ex 3 Find the value of £12 3fl 5m + £25 46c - £18 9fl + £7 6fl 9c 5m + £23 6m - £37 7fl 8c 9c 5m - £9 9fl 9c 6m

$$\begin{array}{r} \text{£12 } 3\text{fl } 5\text{m} = \text{£12 } 305 \\ \text{£23 } 6\text{m} = \text{£23 } 006 \\ \text{£25 } 46\text{c} = \text{£25 } 46 \\ \text{£7 } 6\text{fl } 9\text{c } 5\text{m} = \text{£7 } 695 \\ \quad \quad \quad \text{£68 } 466 \\ \quad \quad \quad \underline{\text{£66 } 685} \end{array}$$

$$\begin{array}{r} \text{£37 } 7\text{fl } 8\text{c } 9\text{m} = \text{£37 } 789 \\ \text{£18 } 9\text{fl} = \text{£18 } 9 \\ \text{£9 } 9\text{fl } 9\text{c } 6\text{m} = \text{£9 } 996 \\ \quad \quad \quad \underline{\text{£66 } 685} \end{array}$$

£1 781 = £1 7fl 8c 1m Ans

times $(5-2)$ Therefore, A 's present age is $4\frac{1}{2}$ times the difference of A 's and B 's ages, and his former age was $1\frac{2}{3}$ times the difference of A 's and B 's former ages

But it should be borne in mind that the *difference* of the ages of two persons is *always the same*, though the *ratio* of the ages is *always varying* Hence, we have

$$\begin{aligned} A's \text{ former age} &= (1\frac{2}{3} - 4\frac{1}{2}) \text{ or } \frac{1}{6} \text{ of his present age,} \\ \text{but } A's \text{ former age} &= A's \text{ present age} - 34 \text{ years,} \\ \therefore A's \text{ present age} - 34 \text{ years} &= \frac{1}{6} \text{ of } A's \text{ present age,} \\ (1 - \frac{1}{6}) \text{ of } A's \text{ present age} &= 34 \text{ years,} \\ \text{or } \frac{5}{6} \text{ of } A's \text{ present age} &= 34 \text{ years,} \\ \therefore A's \text{ present age} &= (34 \times \frac{6}{5}) \text{ years} = 54 \text{ years,} \\ \text{and } B's \text{ present age} &= \frac{2}{3} \text{ of } 54 \text{ years} = 42 \text{ years} \end{aligned} \quad \left. \vphantom{\begin{aligned} A's \text{ former age} &= (1\frac{2}{3} - 4\frac{1}{2}) \text{ or } \frac{1}{6} \text{ of his present age,} \\ \text{but } A's \text{ former age} &= A's \text{ present age} - 34 \text{ years,} \\ \therefore A's \text{ present age} - 34 \text{ years} &= \frac{1}{6} \text{ of } A's \text{ present age,} \\ (1 - \frac{1}{6}) \text{ of } A's \text{ present age} &= 34 \text{ years,} \\ \text{or } \frac{5}{6} \text{ of } A's \text{ present age} &= 34 \text{ years,} \\ \therefore A's \text{ present age} &= (34 \times \frac{6}{5}) \text{ years} = 54 \text{ years,} \\ \text{and } B's \text{ present age} &= \frac{2}{3} \text{ of } 54 \text{ years} = 42 \text{ years} \end{aligned}} \right\} \text{Ans}$$

Ex 2 Five years ago, the ages of A and B were as 4 : 5, and five years hence they will be as 13 : 15, find their present ages

In $(5+5)$ or 10 years, the ratio would be changed from 4 : 5 to 13 : 15. Now $5-4=1$, and $15-13=2$. Reducing the latter ratio to one in which also the difference between the terms $=1$, we get the ratio $\frac{1}{2} : \frac{1}{5}$

Now, if A 's age five years before had been 4 years, it would have taken only $(\frac{1}{2}-4)$ or $\frac{5}{2}$ years to become $\frac{1}{2}$

$$\begin{array}{rcccl} \frac{5}{2} & 10 & 4 & A's \text{ age 5 years ago,} & \\ \text{or } A's \text{ age 5 years ago} & = & 10 \times 4 \times \frac{2}{5} \text{ years} & = & 16 \text{ years} \end{array}$$

$$\begin{array}{l} A's \text{ age} = (16+5) \text{ or } 21 \text{ years,} \\ \text{and } B's \text{ age} = (\frac{5}{4} \times 16+5) \text{ or } 25 \text{ years} \end{array} \quad \left. \vphantom{\begin{array}{l} A's \text{ age} = (16+5) \text{ or } 21 \text{ years,} \\ \text{and } B's \text{ age} = (\frac{5}{4} \times 16+5) \text{ or } 25 \text{ years} \end{array}} \right\} \text{Ans}$$

Ex 3 Divide the number 237 into three parts such that 3 times the first may be equal to 5 times the second and to 8 times the third

Since 5 times the 2nd $=$ 3 times the first, the 2nd $= \frac{3}{5}$ of the 1st

Similarly, the 3rd $= \frac{3}{8}$ of the 2nd $= \frac{3}{8} \times \frac{3}{5}$ of the 1st

Hence the three parts are as 1, $\frac{3}{5}$ and $\frac{9}{40}$ of $\frac{3}{5}$, or as 40, 24 and 15

$$\begin{array}{l} \text{Now } 40+24+15=79, \quad \left. \begin{array}{l} \text{the 1st} = \frac{40}{79} \text{ of } 237 = 120, \\ \text{the 2nd} = \frac{24}{79} \text{ of } 237 = 72, \\ \text{and the 3rd} = \frac{15}{79} \text{ of } 237 = 45 \end{array} \right\} \text{Ans} \end{array}$$

Ex 4 Divide Rs 210 among A , B and C , so that Rs 20 more than $\frac{2}{5}$ of A 's share, Rs 22 more than $\frac{3}{7}$ of B 's share, and Rs 2 more than $\frac{1}{8}$ of C 's share may be all equal

$$\begin{aligned} \frac{2}{5} \text{ of } A's \text{ share} + Rs 20 &= \frac{2}{5} (A's \text{ share} + Rs 50), \\ \frac{3}{7} \text{ of } B's \text{ share} + Rs 22 &= \frac{3}{7} (B's \text{ share} + Rs 1\frac{1}{3}), \\ \frac{1}{8} \text{ of } C's \text{ share} + Rs 2 &= \frac{1}{8} (C's \text{ share} + Rs 1\frac{3}{4}) \end{aligned}$$

$$\begin{aligned} \text{Also } (A's \text{ share} + Rs 50) + (B's \text{ share} + Rs 1\frac{1}{3}) + (C's \text{ share} + Rs 1\frac{3}{4}) \\ = Rs (210+50+1\frac{1}{3}+1\frac{3}{4}) = Rs 270\frac{1}{4} \end{aligned}$$

$$\begin{array}{l} \text{Now, if } A's \text{ share} + Rs 50 = 1, \quad B's \text{ share} + Rs 1\frac{1}{3} = \frac{1}{5}, \\ \text{and } C's \text{ share} + Rs 1\frac{3}{4} = \frac{1}{8} \end{array}$$

$$\text{Also } 1 + \frac{1}{5} + \frac{1}{8} = \frac{13}{40}, \quad \text{and } Rs 270\frac{1}{4} \div \frac{13}{40} = Rs 130$$

$\therefore A's \text{ share} + Rs\ 50 = Rs\ 130$, or $A's \text{ share} = Rs\ (130 - 50) = Rs\ 80$
 $B's \text{ share} + Rs\ 1\frac{1}{4} = Rs\ 130 \times \frac{1}{2}$, or $B's = Rs\ (\frac{130}{2} - 1\frac{1}{4}) = Rs\ 70$
 $C's \text{ share} + Rs\ 1\frac{1}{2} = Rs\ 130 \times \frac{1}{3}$, or $C's = Rs\ (\frac{130}{3} - 1\frac{1}{2}) = Rs\ 60$

Ex 5 A person begins to speculate with a certain sum of money, in his first transaction he loses $\frac{1}{4}$ th of the sum, in his second he gains 10 per cent on his investment, in his third he loses $\frac{1}{4}$ th of the sum invested, in his fourth he gains $66\frac{2}{3}$ per cent. If he then has Rs 10000, with what sum did he start?

Suppose his capital in the beginning to be 1, then
 after 1st transaction, he has $(1 - \frac{1}{4})$ or $\frac{3}{4}$ of his capital

2nd $\frac{1\frac{1}{4}}{100}$ of $\frac{3}{4}$ or $\frac{9}{80}$

3rd $\frac{1}{4}$ of $(1 - \frac{1}{4})$ or $\frac{3}{8}$

4th $\frac{166\frac{2}{3}}{100}$ of $\frac{3}{8}$ or $\frac{3}{4}$

by the question, $\frac{3}{4}$ of his capital = Rs 10000

his capital = $Rs\ 10000 \times \frac{4}{3} = Rs\ 13333\frac{1}{3}$ Ans

Ex 6 How many francs are equivalent to £1, when gold purchased in London at 77s 10½d per ounce standard, is sold in Paris at 4 per mille (i.e. per 1000) premium on the fixed price? (An ounce Troy being = 31.1 grammes, and 1000 grammes of English standard gold being worth 3151 francs)

Since 31.1 grammes = 1 oz, 311 grammes = 10 oz,

Hence 1 gramme = $\frac{10}{311}$ oz, and 1000 grammes = $\frac{10000}{311}$ oz

Now 1000 grammes are bought in London for $77\frac{1}{2}s \times \frac{10000}{311}$,

and 1000 grammes are sold in Paris for 3151 frs $\times 1.004$,

$77\frac{1}{2}s \times \frac{10000}{311} = 20s\ 3151 \times 1.004$ frs reqd no of francs

\therefore the reqd no of francs = $\frac{3151 \times 1.004 \times 20 \times 8 \times 311}{623 \times 10000}$

= 25 27 nearly Ans

Ex 7 I bought 128 yards of cloth for £100, and am now obliged to sell it at a loss of as much money as I shall receive for a dozen yards. At what do I sell it per yard?

In the sum realised by selling 128 yds, my loss is the selling price of 12 yds, but loss = cost price - selling price. Hence the cost price of the cloth = selling price of (128 + 12) or 140 yds, i.e. 140 yds are sold for £100, the cost price. Therefore the selling price per yard = $\frac{100}{140} = \frac{5}{7} = 14s\ 3\frac{1}{2}d$ Ans

Ex 8 A farmer bought a flock of sheep. He lost 5 of them and sold the remainder for Rs 4 a head more than the prime cost, and gained Rs 50 by the transaction. Had he sold them only at Rs 2 a head more than the prime cost he would have lost Rs 70. How many did he buy?

For each sheep sold the man gets Rs (4 - 2) or Rs 2 less in the second case than in the first. And in this way for all the sheep sold,

he gets Rs (50+70) or Rs 120 less in the second case than in the first
Hence, we have

Rs 2 Rs 120 .1 no of sheep sold

∴ no of sheep sold = 60, and ∴ no in the flock = $60 + 5 = 65$ Ans

Miscellaneous Examples VIII

1 A person by disposing of goods for Rs 1820 loses at the rate of 9 per cent, what ought they to have been sold for to realise a profit of 7 per cent?

2 If a person owes Rs 1000 on the 1st May, at what date ought he to pay Rs 750, so that he may retain the remaining Rs 250 till the 1st October?

3 I buy goods for Rs 600, and sell them immediately for Rs 680, giving three months' credit, what is gained per cent reckoning interest at 8 p c per annum?

4 A gentleman sells a lac of rupees out of the 4 per cents at 16 discount and invests the proceeds when exchange is at 2s 1d in 3 per cent consols at 96 What income does he derive therefrom?

5 If the difference between the Simple and the Compound Interest on a sum of money for 2 years at 5 per cent be £5 18s 9½d, what is the sum?

6 A banker, in discounting a bill due in 3 months at 4 per cent, charges 5s 1½d more than the true discount, find the amount of the bill

7 The debts of a bankrupt amount to Rs 21345 4a and his assets consist of property worth Rs 9167 10a 8p and an undiscounted bill of Rs 5130 due 4 months hence, simple interest being reckoned at 4 per cent How much in the rupee can he pay his creditors?

8 A person remits Rs 15480 to England at the rate of 1s 6d per rupee, and the money is invested in the purchase of 3 per cent consols at 79½ At what price should he sell out, in order to realise a gain of £24, after having paid ¼ per cent commission on each of the last two transactions?

9 A company guarantees to pay 5½ per cent on shares of £100 each, another guarantees to pay 3½ per cent on shares of £10 each, the price of the former is £115 10s and of the latter £7 15s Compare the rates of interest which the shares return to the purchasers

10 What income shall I obtain in England, from Rs 78000 in Indian Government Bonds at 5½ per cent, when my agents in Calcutta charge me 3 per cent for drawing and remitting it, and the exchange on England is 1s 10½d for the rupee?

11 A mixture of milk and water contains 32 seers and there is only 1 seer of water in it, how much water must be added to this mixture that in every 32 seers of the second mixture, there shall be $3\frac{1}{8}$ seers of milk?

12 When English money bears a premium of 5 per cent in America, how much sterling should be given for 750 dollars each worth 4s 6d at par?

13 Two kinds of wheat are sold at the same price, in which 20 per cent is gained on one kind, and 20 per cent lost on the other. What percentage will be gained or lost if they be mixed equally, and then sold at the same price?

14 A merchant sells a mixture in the ratio of 1 : 3 of two different kinds of teas, at Rs 2 4a per lb and gains $33\frac{1}{3}$ per cent on the original cost. If he had mixed the teas in the ratio of 1 : 2 and sold the compound at Rs 2 8a per lb he would have gained $33\frac{1}{3}$ per cent on his selling price. Find the prime cost of each of the two kinds of teas.

15 A merchant owes £650. He gives a bill for £202 due 3 months hence, a second for £204 due 6 months hence and pays the remainder of the debt in ready money. Supposing the rate of interest to be 4 per cent per annum, what ought he to pay in ready money?

16 A company has a capital of £5000000, of this $\frac{2}{3}$ is ordinary stock and the remainder is 5 per cent preference. The working expenses and reserve absorb 48 per cent of the gross receipts. Find the annual earnings, so that the company should just be able to pay the interest on the preference stock. What is the additional amount requisite for paying 1 per cent on the ordinary stock?

17 If at compound interest the second year's interest is Rs 2310, and the third year's Rs 2425 8a, what was the first year's interest?

18 A debt is paid on 23rd June by a bill dated at 6 months. Supposing the bill to be discounted on October 14th, the real discount would be Rs 35. Find the amount of the bill, the rate of discount being 5 per cent.

19 A gold chain is made of 3 parts gold to 1 copper, and the cost of manufacture is 10 per cent on the value of the gold. A sovereign consists of 11 parts gold to 1 copper, and 120 oz Troy of the mixture makes 467½ sovereigns. What should be the price of a chain which weighs 2½ oz, the value of the copper being neglected?

20 A man has an income of £200 a year, an income tax is established of 7d in the £, while a duty of 1½d per lb is taken off sugar, what must be his yearly consumption of sugar that he may just save his income-tax?

21 The income tax being 4d in the £, a person has to pay

£69 15s 11d less than when the tax was 7d in the £, although his income has increased by £295. What was his income at first?

23 Two passengers have together 150 seers of luggage and are charged for the excess above the weight allowed 8a 4p and Rs 1 or 8p respectively. If the luggage had all belonged to one of them, he would have been charged Rs 2 13a. Find how much luggage each passenger is allowed without charge.

23 A merchant having lost his cargo in the sea, which he had insured, the broker offered him a sum of money for his loss which the merchant refused as being 10 per cent below the estimated value of his loss, the broker then offered £379 15s more than at first, and the amount of the second offer was 5½ per cent in excess of the estimated value. What was that value, and what sum did the broker first offer?

24 Calculate the profit made by a book seller, assuming that he pays 11s 4d for a 16 shilling book, receives 25 copies at 2s 4d, and deducts 10 per cent for commission.

25 An inclined roof rests upon two walls, one of which is 13½ feet high and the other 7½ ft., the distance between the two walls is 8 ft., what is the area of the roof, supposing its length to be 20 yards?

26 The discount and interest on a certain sum for the same time are Rs 22 and Rs 24 respectively, find the sum.

27 If I buy the 3 per cents at 76½, and the 3½ per cents at 95½, which is the better investment? If I had invested £6762 19s 3½d in each, and the former rose and the latter fell ½, how much should I lose or gain?

28 A tradesman bought a quantity of goods, and sold ¾ of them at a profit of 6 per cent, the price rising he got 10 per cent profit on the remainder, and on the whole gained £114, what sum did he lay out?

29 A man has Rs 41000 which he invests in the 3 per cent stock at 87 and 5 per cent stock at 103. What sums must he invest in the respective stocks to make 3½ per cent on the whole?

30 I buy goods for Rs 5040 and incur 10 per cent expenses, what must I charge in order to make 10 per cent profit on my capital after allowing 10 per cent discount?

31 Certain Railway shares pay an annual dividend of £3 10s. A person having bought 12 shares, at such a price that they yielded 5½ per cent on his investment, sold them when the price had risen £5, and invested the proceeds in 3½ per cent stock at 85. Find the alteration in his income.

32 The capital of a Railway Company consists of £19,000,000 4 per cent debentures, £37,000,000 5 per cent preference stock, and £20,000,000 ordinary stock. The receipts are £8 per mile per day,

the length is 2500 miles, the working expenses are 55 per cent of the receipts. What dividend can it pay on the ordinary stock?

33 A gentleman, deriving his income from an investment which pays 5 per cent, spends $\frac{1}{3}$ of his income and pays an income-tax of $8d$ in the \pounds , next year the investment pays $5\frac{1}{2}$ per cent, he spends 15 per cent more, the tax is reduced to $6d$ in the \pounds , and he saves $\pounds 7$ more. What was his income?

34 I bought 50 horses and sold 15 of them at a gain of 20 per cent, 25 at a gain of 16 per cent and the rest at cost price. Had I sold all at a gain of 18 per cent, I should have gained Rs 560 more. Find the cost price of each horse.

35 Two kinds of tea are mixed together and the mixture sold at a gain of 15 per cent. If each kind were to be sold separately at the same price, 20 per cent would be gained on the first kind, and 5 per cent lost on the second. Find the proportion of the mixture.

36 A boy bought 200 eggs, and gained 25 per cent by selling $\frac{2}{3}$ of them at 2 a penny, and the rest for 1s $5\frac{1}{2}d$ more than what they cost him. At what rate did he buy them?

37 Two boys buy oranges at 7 for $3d$ and 5 for $2d$ respectively. They buy equal numbers. Compare their rates of profit, if the former sells his oranges at 9 for $4d$ and the latter at 7 for $3d$.

38 A Railway bridge crosses a straight canal obliquely. The bridge on one side is 36 ft lower down the canal than on the other, and the breadth of the canal is 48 ft. Find the length of the bridge.

39 A tradesman lost 4 per cent by selling an article for 15s, what should he have sold it for so as to gain 10 per cent?

40 A grocer buys 2 cwt of tea, the first cwt he sells at 5 per cent profit, and the second which costs $\pounds 1$ more, at 12 per cent profit. The difference in retail price being $4d$ per lb, what is the cost price of each?

41 The gross receipts of a Railway Company in a certain year are apportioned thus — 40 per cents to pay the working expenses, 54 per cents to give the shareholders a dividend at the rate of $3\frac{1}{2}$ per cent on their shares, and the remainder Rs 283500 is reserved. Find the paid-up capital of the Company.

42 A man, buying goods by means of false scales, defrauds to the extent of 15 per cent and 15 per cent in selling, find his whole gain per cent.

43 Eight years ago the ages of A and B were as 5 : 6, and eight years hence they will be as 9 : 10, find their present ages.

44 A owes B Rs 2725, and offers to pay him at a certain rate of discount instantly, instead of at the end of 2 years, when the debt will be due, B can place out the money which he will receive at 5 per cent interest, and by that means will gain by the transaction.

Rs 25 Reckoning simple interest throughout, find at what rate the discount is calculated

45 *A*'s present age is to *B*'s as 8 : 7, and 10 years ago the proportion was 11 : 9 Find the present age of each

46 £1000 sterling is due from London to Portugal, when the exchange is 61½ *d* per mille Whether is it better, for Portugal, to draw directly on London, or circuitously, at an expense of 1½ per cent, through Holland and France, —exchange between Britain and Holland 11 90 florins per £ sterling, between Holland and France 10 florins for 21 francs, and between France and Portugal 480 rees for 3 francs ?

47 The original cost of a pipe of port is *Rs* 550, and it is sold to *A* at a certain loss per cent, then *A* sells it to *B* at the same losing rate, but *B* sells it to *C*, at a profit of 12 per cent for the original cost What was the loss per cent at which the wine was sold to *A* and *B* ?

48 If the Compound Interest of *Rs* 2500 for 2 years be *Rs* 204, what is the rate per cent per annum ?

49 What fraction of $\mathcal{N}(0135)$ is $\mathcal{N}(004)$?

50 A narrow rectangular field *ABCD* has its length *AB* 160 yds and breadth *BC* 31½ yds To what point *E* in the side *AB* must a straight line from *C* be drawn, so that *AECD* may contain an acre ?

51 I bought paper at the rate of 3s 7½ *d* for 5 quires, and sold it so as to gain as much on the cost of 32 quires as 3 quires were sold for At what rate did I sell it per quire ?

52 My age is 62, and my son's age 30, how long ago was my age 5 times that of my son ? and how many years hence (if we are both alive) will my age be a third of 5 times his age ?

53 Divide *Rs* 54339 into three sums, such that their amounts by Compound Interest at 5 per cent per annum, for 20, 23 and 27 years respectively, shall be equal

54 Given that 1 oz Troy equals 31.1 grammes, that 10 grammes of French standard gold are worth 31 francs, and that the worth of a given weight of English standard gold is to that of the same weight of French standard as 3151 to 3100 —To what number of Troy ounces of English standard gold is the franc equivalent, and what is the fixed number of francs equivalent to £1 ?—the English mint price for standard gold being £3 17s 10½ *d* per ounce

55 How many francs are equivalent to £1, when gold purchased in London at 77s 10½ *d*, per ounce standard is sold in Paris at 14½ per mille (i.e. per 1000) premium on the fixed price ? And how many, when gold is at 1 per mille discount ?

56 A bankrupt owes £4594 1s, he will pay a dividend of

3s 6d in the pound three months hence, a second of 3s six months hence, and a third of 1s nine months hence. What is the present value of his assets, money being worth 5 per cent per annum?

57 A person having to pay Rs 10850 two years hence, invests a certain sum in the 3 per cent Government Securities (to accumulate interest till the debt is paid) and also an equal sum the next year. Supposing the price of the Government Securities to remain throughout at 73, what must be the sum invested on each occasion so that with its interest, there may be just sufficient to pay the debt at the proper time?

58 A person borrows Rs 6180 in two separate sums, at the respective rates of $3\frac{1}{2}$ and 5 per cent per annum, and he repays the two loans at the end of 10 months, with interest amounting to Rs 225. Required the amount of each loan.

59 A person finds that if he invest a certain sum in Railway shares paying £6 dividend a share, the price of the share being £132, he will obtain £54 a year more than if he invested in the 3 per cents at 93. How much has he to invest?

60 A contractor sends in a tender of £5000 for a certain work, a second sends in a tender of £4850, but stipulates to be paid £500 every 3 months, find the difference of the tenders, supposing the work in both cases to be finished in 2 years, and money to be worth 4 per cent Simple Interest.

61 (1) Given that the square of 15334 is 235131556, find that of 153347 without going through the operation of squaring.

(2) Given that the square root of 1038361 is 1019, find the square root of 103876864.

62 A landlord has an estate that brings him in £3000 a year, but this gross income is liable to deductions for rates and repairs to the extent of 12 per cent. He sells it at 26 years' purchase on the gross income, and invests the price in the 3 per cents at 97½. What difference is caused in his income?

63 A man sold at 48 and 95 respectively £500 ordinary stock in the A Railway paying a dividend at the rate of $1\frac{1}{2}$ and £800 preference stock in the B Railway paying a dividend of 4 per cent. He then invested $\frac{1}{4}$ of the money in the Tramway Company where the £24 share paying interest at 6 per cent was at £6 premium. £150 in the C Railway which paid no interest and the remainder in Bank shares at par, what rate of interest must he receive from the Bank in order to increase his annual income by £12 5s?

64 A corn merchant having bought 1300 quarters of wheat, sold one fifth of it at a profit of 5 per cent, one third at a profit of 8 per cent, and the remainder at a profit of 12 per cent, but had he sold all at a profit of 10 per cent, his gain would have been £16 13s 8d more. What did the wheat cost him?

65 The discount on Rs 825 for a certain length of time is Rs 75, what is the discount on the same sum (i) for twice that length of time, and (ii) for half that length of time?

66 A person has Rs 24180 to invest, the $5\frac{1}{2}$ per cent Government Loan being at 108, and the 6 per cent Municipal Loan of Rs 1000 being at 1020, find how he must divide his capital between the Government and Municipal Loans, that he may obtain the same income from each

67 Divide £444 among *A*, *B*, *C* and *D* in such a manner that £10 more than $\frac{1}{5}$ of *A*'s share, £20 less than $\frac{1}{5}$ of *B*'s share and £32 10s more than $\frac{1}{4}$ of *C*'s share, and £65 more than $\frac{1}{2}$ of *D*'s share may all be equal

68 What sum of money will amount in Rs 6996 9s 7d in 2 years, reckoning compound interest for the first year at 4 per cent, and for the second at $3\frac{1}{2}$ per cent per annum?

69 A certain article of consumption is subject to a duty of 6s per cwt, in consequence of a reduction in the duty the consumption increases one-half, but the revenue falls one-third. Find the duty per cwt after the reduction

70 Divide Rs 429 into four parts such that their simple interest for 4, 6, 7 and 10 months, and at 3, 4, 5 and 6 per cent per annum respectively, shall be all equal

71 *A* sells to *B* a horse, which had cost him Rs 300 at $12\frac{1}{2}$ per cent profit, *B* sold it to *C* at a profit of 5 per cent on what it cost him. What would *A* have gained per cent, had he sold the horse to *C* for the money which *C* had paid *B* for it?

72 A farmer gave for a horse a bill of £73 due in 1 month, and sold him at once for a bill of £87 at 4 months. Required the farmer's gain per cent, reckoning interest at $4\frac{1}{2}$ per cent

73 A man has a certain amount of 5 per cent stock. He sells out one-third of it at 104, and invests the proceeds in the 4 per cents at 98. He sells out from the 4 per cents when they have risen 2 per cent and then repurchases the same amount of 5 per cent stock at 102 as he sold out originally. His gain being £202, find the amount of 5 per cent stock originally held by him

74 A tradesman selling goods for a certain price to be paid 6 months hence offers to give $\frac{1}{10}$ th more of the same goods for the same price in ready money. What was the rate of discount?

75 *A* and *B* each lends £250 for three years, *A* lends at $4\frac{1}{4}$ per cent simple interest, and *B* at $4\frac{1}{2}$ per cent per annum, compound interest. Find the difference in the amount of interest they receive

76 If by selling cloth at Rs 14 4a for 5 yards my gain would be $6\frac{2}{3}$ per cent, what should I gain or lose per cent by selling it at Rs 18 12a for 7 yards?

77 A banker sells 400 English sovereigns at 25 35 fr, and buys Spanish piasters at 5 40 fr. Two months afterwards he sells again his piasters at 5 70 fr and buys 400 sovereigns at 25 20 fr. What is his profit, and at what rate does he place his money?

78 A merchant fits out 3 ships in succession to run the American blockade. He reckons the total outlay on each ship after the first to be 25 per cent more than on the one that preceded it. The first and third get into port, and he gains 160 per cent on their cost, the second is taken. Find his loss or gain per cent on the whole.

79 A man left £30000 stock in 3 per cent Government Securities to be divided among his three sons in the proportion of their ages, which were 15, 8 and 7 years respectively, afterwards when these securities stood at 10 discount the eldest son sold out, and invested the proceeds in 6 per cent Bank shares at par. By how much did the annual income of the eldest then exceed that of the youngest son?

80 A man invests £864 in the following manner. One half he invests in the 5½ per cents at 8 premium and the other half in Bank shares at 116 premium. After one year he sells out both his 5½ per cents and also his Bank shares, the former being now at 12 premium, and the latter 130 premium, and invests the whole of the proceeds in the 4½ per cents at 90½. His annual income is now £1 less than it was before. What rate of interest did the shares pay?

81 Find the length of an edge of a cube of pure gold equal in value to the annual revenue of Great Britain (70 millions sterling), given that gold is 19 26 times as heavy as water, that a cubic foot of water weighs 1000 oz and that the value of fine gold is £4 5s per oz.

82 Prove the truth of the following extract from the *Times* Newspaper — (See Ex 54, above)

The question of gold at Paris is about 1 per mile discount (according to the last tariff), which at the English Mint price of £3 17s 10½d per oz for standard gold, gives an exchange of 25 14½, and, the exchange at Paris on London at short being 25 07½, it follows that gold is about 0 28 per cent dearer in Paris than in London.

MISCELLANEOUS EXERCISES

1 The number of emigrants from the United Kingdom in ten years was 1,697,579,—649,742 being English, 158,226 Scotch, 31,983 foreigners How many were Irish?

2 A library contains 3275 volumes, and each volume on the average contains 493 pages, and each page 39 lines How many lines are there?

3 How many times has a batsman been out, if he has made 1073 runs and his average is 29?

4 What was the cost of each pig, if after buying 23 pigs I have £23 7s 2d left out of £50?

5 A waggon loaded with 217 equal parcels weighs 2 tons 2 cwt and the waggon itself weighs 18 cwt 3 qrs, find the weight of each parcel

6 What amount will be left out of £50, after paying the following bills £9 17s 4d, £4 12s 6½d, £5 10s 9¼d, £27 5s 2¼d?

7 How often does £789303 6s 6¾d contain £1654 14s 5¼d?

8 A sum of money was distributed among *A*, *B* and *C* The shares of *A* and *B* together amounted to Rs 120, those of *A* and *C* to Rs 160, and those of *B* and *C* to Rs 184 Find the share of each

9 I sold a horse for Rs 600, and thereby gained one-fifth of my outlay, what was my outlay?

10 How many ponies must a person buy at £8 10s each, so that after allowing 3s 6d for the food of each for a week, he may then gain £440 11s 3d by selling each of the lot at £9 4s 7d?

11 The area of the British Colonies is 8,869,096 square miles and the population is 292,680,168 How many people are there to the square mile? How much would a tax of 2½d per head on the whole population amount to in £ s d?

12 If the greater of two numbers be 19 times 508, and their difference be 15 times 112, find the sum and the product of the numbers

13 If 7423971 be the dividend, 12130 the quotient, and 411 the remainder, what is the divisor?

14 Reduce £5672841 16s 9¾d to farthings

15 If £15942 16s 6d were distributed equally among 216 persons, how much would each receive?

16 If the circumference of a coach wheel measures 17 ft- $7\frac{1}{2}$ in, how often will it turn round in travelling a distance of 8 miles 264 feet?

17 How often is 3 tons 27 lbs 13 oz contained in 228 tons 18 cwt 3 qrs 13 lbs 12 oz?

18 Find the greatest number which will divide 13956 and 14565, and leave a remainder 7 in each case

19 What is the price of a chair, if after buying 15 chairs my money has been reduced from Rs 250 to Rs 132 13a?

20 A man whose weekly earnings are Rs 8 12a, saves $\frac{1}{2}$ of that sum every fortnight, when will he have saved Rs 224?

21 If 2948 bricks are used per yard to build a tunnel which is 6285 yards long, and 2175 bricks per yard to build another tunnel which is 6082 yards long state *in words* how many more are used for the one tunnel than for the other

22 Multiply 562347892 by 207099011 in three lines

23 The product of two numbers is 15580656, and one of them is 6552 Find the other

24 Find the L C M of 298717, 197457 and 207583

25 Reduce 32100546829 farthings to £ s d

26 A tax collector collected Rs 10 1a, Rs 210 12a, Rs 64 0a 2p and Rs 18 11a 8p, but his pocket burst, and the money was scattered. He picked up Rs 303 8a 4p. Did he lose any money? If so, how much?

27 A farmer pays 30s an acre rent on a farm of 215 acres. His working expenses amount to £589 17s 4d. His receipts are £1305 6s 8d. Find his net income

28 How many hens have I bought at 1s 8 $\frac{1}{2}$ d each, if I have £2 16s 9 $\frac{1}{2}$ d left out of a £5 note?

29 How many minutes were there in the first six months of 1888? Find the G C M of 23992091 and 8209897

30 A man spends £84 12s 3d in each of the first 5 months of the year. If he does not wish to spend more than £826 8s 9d in the whole year, what must be his average monthly expenditure for the remainder of the year?

31 There are 57 boxes of rice each containing 809309 grains, and 76 other boxes each containing 719294 grains. write *in words* how many grains of rice there are altogether

32 Find the G C M of 230299, 3083035 and 1093729

33 What is the least number that must be added to 56438971, that the result may be exactly divisible by 4064 ?

34 What is the least number that can be exactly divided by $1\frac{1}{2}$, $2\frac{1}{2}$, 5, 6 and $1\frac{1}{3}$?

35 Reduce 32087698.41 square inches to acres

36 I buy 60 gallons of wine at £1 3s 6d a gallon and £1 10s is gained by selling it at £1 2s 6d a gallon How much water has been added ?

37 Simplify $\frac{1}{2}$ of $(\frac{1}{3} + \frac{1}{4} + \frac{1}{5}) + 7 \times (\frac{1}{35} + \frac{1}{40}) - \frac{1}{12} - (\frac{11\frac{1}{2}}{15\frac{1}{2}} - 4)$

38 The live stock on a farm consists of a certain number of horses worth 60 guineas each, an equal number of pigs worth £2 10s each, 3 times as many cows worth £18 10s each and 15 times as many sheep worth £1 15s each The whole value of the live stock is £1030 15s How many are there of each kind ?

39 A man leaves Rs 50000 to his wife, and the remainder of his property to be divided equally among 4 children It is found that each child has $\frac{1}{4}$ of the whole How much did he leave altogether ?

40 A rupee is worth 2s 0 $\frac{1}{2}$ d, and a dollar 4s 4 $\frac{1}{2}$ d, find the least number of rupees which makes an exact number of dollars

41 If the distance of the Sun from the Earth be 92 $\frac{1}{2}$ million miles, and light travels from one to the other in 498 seconds, find the velocity of light in miles per hour

42 Divide 198 ac 3 ro 16 po 3 yds 72 in by 187

43 Four bills, amounting to £27 10s 3 $\frac{1}{2}$ d, £13 4s 7 $\frac{1}{2}$ d, £43 or 6d and £5, are paid out of £100 What money is left ?

44 Twenty-five years ago a man was four times as old as his son, whose present age is 33 How old is the father now ?

45 Divide a lac of rupees between A, B and C in the proportion of 2, 3, 4, and the same amount between D, E and F in the proportion of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$

46 How many payments of 14s 11 $\frac{3}{4}$ d amount to £89 2s 6 $\frac{1}{4}$ d, and how many plots each containing 2 ro 16 po 8 sq yds are there in 10 ac 36 po 15 sq yds ?

47 Find by Practice the cost of warming a building for 11 days 17 hrs 28 min, if the cost is £4 10s per day

48 A man runs 4 $\frac{7}{8}$ times round a course 930 $\frac{1}{4}$ yds long What fraction of 3 miles 301 $\frac{1}{2}$ yds does he run ?

49 Simplify—

$1\frac{11\frac{1}{2}}{15\frac{1}{2}}$ of $\frac{1}{16}$ of £31 $\frac{3}{4}$ + 6 $\frac{3}{4}$ of £3 or 9d - 4 $\frac{1}{2}$ of £3 2s

50 If I pay Rs 23 13s 6d as income tax on a rental of Rs 715 5s, what should I pay on a rental of Rs 5107 8s?

51 What prime numbers divide 222222, and what is the least number that must be added to it that the sum may be divisible by 47, 53 and 59?

52 Divide 2875962 by $5 \times 3 \times 11$ by short division, and find the true remainder

53 Reduce 5762309875407 inches to miles

54 Simplify $\frac{3\frac{1}{2} \times 1\frac{1}{2} + 4\frac{1}{2} - 3\frac{1}{8}}{1\frac{1}{2} - 7\frac{1}{2} - 28\frac{1}{2} + \frac{1}{2}} + \frac{3\frac{1}{2}}{4\frac{1}{2}} \times 3\frac{1}{2} \times 5\frac{1}{2} - 17\frac{1}{2}$

55 Divide 23 by 0015, and £021 by (176s - 32d)

56 What will be the cost of painting a room 20 ft 3 in long, 18 ft 6 in broad and 10 ft 4½ in high, containing two windows, whose dimensions are 7½ ft by 4½ ft each, at the rate of 2s 9d per sq yd?

57 23 cwt 3 qrs 7 lbs are bought at £2 10s 8d per cwt and 72 cwt 2 qrs 8 lbs at £2 7s 10d per cwt. Find by Practice the amount expended and give the average price per lb

58 A bankrupt is indebted to A, B, C and D—A's debt is twice B's, B's three times C's, C's half of D's. How much should each receive of assets to the amount of Rs 45680?

59 If 2 men and 5 women can do a piece of work in 8 days of 9 hrs each, how long will it take 3 men and 6 women to do a piece of work twice as great, working 8 hours a day, the work of a man being double that of a woman?

60 Find the simple interest at 4 per cent per annum on Rs 595 9s for 4 years and 17 weeks, reckoning 52 weeks to a year

61 What must I pay for a bill of exchange on London for £73 15s 6d, the exchange being at the rate of 1s 10½d for the rupee?

62 Find the value of—

387 of £8 16s 3d + 6½ of ¾ of 7s 8½d + 1½ of 1d

63 If the carriage of 30 mds through 36 miles cost Rs 125, what weight ought to be carried 48 miles for Rs 66 10s 8d?

64 Find the least number of weeks in which an exact number of half guineas can be earned, the wages per week being 16s 4d

65 If a person receive 4½ per cent interest on his capital by investing it in the 4 per cent Government stock, what is the price of stock, and how much can be purchased for Rs 12000?

66 A person buys 64 animals—cows and horses—for Rs 6000. Each cow costs Rs 60, and each horse Rs 100. Find the number of horses purchased

67 What would a banker gain by discounting on September 21st a bill of £318 3s dated July 31st at 4 months, at 5 per cent ?

68 A man sells a horse for Rs 246 and loses $26\frac{1}{2}$ per cent on what the horse cost him, what was the original cost ?

69 Four merchants *A*, *B*, *C* and *D* trade together, *A*'s stock of £400 was in trade 12 months, *B*'s stock of £450 for 9 months, *C*'s stock of £480 for 8 months and *D*'s of £405 for 6 months. The whole profit, being £1000, was in trade 12 months. How much ought each to receive ?

70 A person in India wishes to invest Rs 24000 in the 3 per cent consols at 90, the rate of exchange is 1s 10d for a rupee, brokerage in England is $\frac{3}{4}$ per cent, how much stock would he realise, and what would be his yearly income ?

71 Show that the sum of the squares of six thousand and twenty one and eight thousand and twenty eight is equal to the square of ten thousand and thirty five

72 If when a number is divided successively by 11, 19, 23 (as in short division) the remainders are 8, 3, 14 respectively, find what the remainder would be if the same number were divided by 4807

$$73 \text{ Simplify } \frac{1}{8} \text{ of } \left(\frac{1}{10} - \frac{1}{11} \right) - \frac{\frac{1}{2} - \frac{1}{3} - \left(\frac{1}{6} + \frac{1}{11} \right)}{\left(\frac{1}{4} + \frac{1}{6} \right) - \left(\frac{1}{5} - \frac{1}{11} \right)} \times \frac{\frac{1}{2} + \frac{1}{3} - \left(\frac{1}{4} - \frac{1}{6} \right)}{\left(\frac{1}{2} + \frac{1}{3} \right) - \frac{1}{4} - \frac{1}{6}}$$

74 124 men dig a trench 110 yds long, 3 ft wide and 4 ft deep, in 15 days of 11 hours each, another trench is dug by half the number of men in 7 days of 9 hours each, how many cubic feet of water is the latter capable of holding ?

75 A cistern, without a lid, whose floor and walls are an inch and a half thick is 5 ft 3 in long, 3 ft 7 in wide and 2 ft $5\frac{1}{2}$ in high in its external dimensions. Find its internal surface, and the cost of painting the same at 4d per sq foot

76 In a certain firm *A* invests half as much again as *B*, and $\frac{2}{3}$ as much again as *C*, and *A*'s capital is Rs 12000 more than *C*'s. They gain Rs 1330, what is each man's share of the profits ?

77 If 4 men earn as much in a day as 7 women, and one woman as much as 2 boys, and if 6 men, 10 women and 14 boys working together for 8 days earn £22, what will be the earnings of 8 men and 6 women working for 10 days ?

78 A tank is 300 yds long and 150 yds broad with what velocity per second must water flow into it through an aperture 2 ft broad and $1\frac{1}{2}$ ft deep, that the level may be raised 1 foot in 9 hours ?

79 A person having invested a sum of money in the 3 per cents receives annually therefrom £233 after deducting the income tax of 7d in the pound. Find the amount of stock, and also what it can be sold for when the $3\frac{1}{2}$ per cents are at 109 $\frac{3}{4}$

80 The length of the Eastern Bengal Railway being 110 miles and the capital employed in its construction £1500000, what must be the gross annual traffic receipts per mile in order that a dividend of 5 per cent may be paid to the shareholders after allowing 45 per cent of the gross receipts for current expenditure?

81 *A* had some rupees, the number being composed of three 7s, preceded by 16, and followed by 216. *B* took some away, and the number which remained was expressed by the same digits in the same order with the exception of the digit 2. How many did *B* take?

82 If the remainders on dividing a certain number (by short division) by 5, 9, 7, 11 successively be 3, 5, 8, 4, find the remainder when the number is divided by 3465.

83 Simplify $2\ 123456 + 1\ 23456 + 1\ 23456 - 1\ 123456 - 1\ 123456$

84 Multiply 3720789426 by 0086341532 to five places.

85 Find by Practice the cost of 3 tons 17 cwt 3 qrs 23 lbs 11 oz of iron at Rs 9 13s 7d per cwt.

86 If 21 men can in 12 days make 1260 thousand bricks working 12 hours a day, in how many days of 8 hours each, can 59 men make 177 lacs of bricks?

87 The cost of carpeting a room whose length is 22 ft 8 in with carpet at 3s 4½d per sq yd is £7 7s 4d, find the breadth of the room.

88 A merchant buys two pipes of wine, one for £112, the other for £120, and he also buys a third pipe, on mixing the three, he sells his wine at 50s per dozen, gaining 25 per cent on his outlay, what was the price of the third pipe? (The number of dozens in a pipe is 56)

89 The wholesale price of books is 25 per cent lower than the retail price, and 13 books are counted into the dozen. What percentage is obtained by the retailer?

90 I pay Rs 51000 to a bank for a bill of exchange payable in London. The rate of exchange is 1s 10½d for the rupee and the bank charges me 2 per cent on the amount payable in England. How much will my agent in London receive?

91 When 582167 is divided by a certain number, the quotient is 762 and the remainder 761. What is the number?

92 A number diminished by $\frac{2}{3}$ of itself, when divided by 809, gives a quotient 327 and a remainder 456. What is the number?

93 What decimal multiplied by 125 will give the sum of $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, 0.9375 and 2.46?

94 The owner of $\frac{7}{8}$ of a ship sells $1\frac{1}{2}-4\frac{2}{3}$ of his share for £21 $\frac{1}{4}$, what was the value of $\frac{3}{4}$ of $\frac{2}{3}$ of $2\frac{1}{2}$ of the ship?

95 Find the value of 375 of a guinea + 54 of 8s 3d + 027 of £2 15s and reduce the result to the fraction of a guinea and a half

96 If the value of the rupee varies from 1s 9d to 1s 9 $\frac{1}{2}$ d and the franc from 9 $\frac{1}{2}$ d to 10d, find the maximum number of francs which it is always safe to give for Rs 500

97 A person after paying 7d in the £ for income-tax on his income, has £1632 18s 10d remaining, what had he at first?

98 A farm contains 190 ac 3 ro 25 po 13 $\frac{3}{4}$ sq yds, of which 81 ac 3ro 10 po 27 $\frac{1}{2}$ sq yds is pasture What fraction of the whole farm is not pasture?

99 Extract the square root of 047619—06857142, and the cube root of 206 425071

100 A merchant buys goods for £568 4s and sells half of them at a gain of 1d in the shilling on the cost price, one third of them at a gain of 2d in the shilling and the remainder at a gain of £15 15s 8d How much per cent does he gain on the whole transaction?

101 Find the G C M and the L C M of 157 days 7 hrs 4 min 7 sec, and 243 days 2 hrs 11 min 49 sec

102 Multiply 32856 by 121711 in *three* lines only

103 Find the value of 36 of 954 of 428571 of 3s 1d

104 Divide 372808976 by 1369840056, retaining six places

105 A and B own a ship in shares which are in the ratio of 2 to 3 They dispose of parts of their shares to C, so that A, B and C hold the ship in equal shares What is the ratio of the payments that C must make to A and B?

106 Tea at 4s 3 $\frac{1}{2}$ d per lb is mixed with tea at 3s 7 $\frac{1}{2}$ d per lb so that the mixture contains 72 per cent of the former Find the weight of a chest of this mixture which is worth £6 16s 10d

107 How many planks of teak, 12 ft long and 7 $\frac{1}{2}$ in broad, will be required to floor a room 7 $\frac{1}{2}$ yds long and 5 yds wide, a space 7 $\frac{1}{2}$ ft long and 5 ft broad being left unplanked?

108 A barter sugar with B for rice which is worth 1 $\frac{1}{8}$ annas a seer, but on weighing his sugar uses a false maund weight B discovers this, and to make the exchange fair, raises the price of his rice to 2 $\frac{1}{2}$ annas a seer Find the real weight of the false maund which A uses

109 The diameter of a fore wheel of a carriage is 1 $\frac{1}{2}$ ft and that of the hind-wheel is 3 ft, how far will the carriage have travelled when the fore-wheel has made 100 more revolutions than the hind-wheel? (the circumference of a circle diameter 3 1416 1)

110 A person bought 10 Bank of Madras shares at Rs 1540 each and for $5\frac{1}{2}$ years got interest on his investment at the rate of 5 per cent. He then sold his shares at a loss of $22\frac{1}{2}$ per cent. How much did he make by the transaction and what rate per cent per annum had he for his money?

111 If when a number is divided continuously by 6, 7, 8, 9, 10, (as in short division) the remainders are 4, 2, 5, 1, 8 respectively, find the remainder when the number is divided by 30240.

112 A vulgar fraction has for its numerator 183, and its nearest approximate value in hundredths is $\frac{43}{100}$, what is the denominator?

113 A can copy a certain manuscript in 17 hours by writing at the rate of 3 lines per minute, B can copy the same in 24 hours. After 476 lines have been copied by A, in what time can B finish it?

114 If a cubic foot of marble weigh 2716 times as much as a cub foot of water, find the weight of a block of marble 9 ft 6 in long, 2 ft 3 in broad, and 2 ft thick, supposing a cub foot of water to weigh 1000 ounces.

115 If 48 men working 8 hrs a day for one week can dig a trench 235 ft long, 40 wide and 28 deep, in what time can 12 men working 10 hours a day form a Railway cutting 156060 cub yards? (A week = 6 working days)

116 A train starts from A at 12 o'clock and runs towards C, which is 100 miles distant, at the rate of 30 miles an hour, at the same time the mail cart starts for C, from B, which is half-way between A and C, and runs at 10 miles an hour, at what distance from C will it be overtaken by the train?

117 A and B enter into partnership. A supplies the whole of the capital amounting to Rs 45000 upon condition that the profits are to be equally divided, and that B pays A interest on half the capital at 10 per cent per annum, but receives Rs 120 per mensem for carrying on the concern. Find their total yearly profits, when B's share is equal to one half of A's share.

118 What sum of money put out at Compound Interest will in 2 years amount to £1944 8s, interest being at the rate of 10 per cent per annum, and being paid *half-yearly*?

119 A gentleman receives 10 per cent upon his investment in India. When the exchange is at 1s 10 $\frac{1}{2}$ d he disposes of his investment at a premium of 60 per cent and transfers his capital to 6 per cent English Securities at par, which yield him an annual income of £1770. Find his original income in India per mensem.

120 A certain number of men and women subscribe a sum of money, the number of women being four times the number of men.

Each man subscribes as many annas as there are men altogether, and each woman as many pies as there are women altogether. The total amount subscribed being Rs 756, find the number of men and women

121 Find the least number which, when multiplied into 253125000, will make the product a perfect cube

122 A farmer has 1134 sheep and 630 lambs. He forms them into flocks, keeping sheep and lambs separate, and having the same number of animals in each flock. If these flocks are as large as possible, how many animals are there in each?

123 Calculate to four places $\frac{(11\ 29 + 1\ 306 + 0009) \text{ of } (4\ 13 + 6)}{(5\ 23 + 7\ 98) \text{ of } (3\ 1 + 6\ 283)}$

124 If one lb Troy of gold be coined into $46\frac{2}{3}$ sovereigns and standard gold contains $10\frac{1}{2}$ parts of pure gold to $1\frac{1}{2}$ parts of copper, find the weight of pure gold in one sovereign in grains. Also express the weight of copper in a sovereign as the decimal of one lb Avoir

125 If the work done by a man, a woman and a child be in the ratio of 3, 2, 1, and there be in a factory 24 men, 20 women, and 16 children, whose weekly wages amount to Rs 204, what will be the yearly wages of 27 men, 40 women and 15 children?

126 A room is 19 ft 5 in long and 16 ft 7 in broad and the cost of painting the walls at 7a 6p per sq yd is Rs 43 3a. Required the height of the room

127 Divide a guinea between A, B, C and D, so that B's share is $\frac{1}{2}$ more than A's, C's $\frac{1}{3}$ more than B's and D's $\frac{1}{4}$ more than C's

128 If the rate of exchange is 1s $8\frac{1}{2}$ d per rupee, what must be paid in India for a bill for £850 on England?

129 A person finds that if he invest a certain sum in Railway shares paying £6 per share when the £100 share is at 132, he will obtain £10 16s a year more for his money than if he invest in 3 per cent consols at 93. What sum has he to invest?

130 A certain sum put out at Compound Interest amounts in two years to £270 4 and in three years to £281 216. Find the sum and the rate per cent

131 By what must 152207 be multiplied so that the product may consist of 8 digits, each digit being 1?

132 The population of India was by a former census found to be 190531440, of whom 139421250 were Hindoos. Find the ratio of the Hindoos to the whole population

133 A person settling his bills paid $\frac{1}{3}$ of his money to one

man, $\frac{3}{4}$ of the remainder to another, and $\frac{1}{2}$ of the rest to a third. If he had Rs 33 remaining, what had he at first?

134. Find the square root of $74538-68-85-203 \times 117$

135 Two equal sums were divided, the one among 141 men and the other among a certain number of women, each man received Rs 12 8s and each woman 12s less, how many women were there?

136 Twelve years ago, A had Rs 13000 and B had Rs 9100. A has been more fortunate than B , and gained 3 per cent where B gained 1 per cent. B has now doubled his capital. How much has A got?

137 Bought a quantity of tea, and sold part of it, at the rate of Rs 10a 6p per lb, losing thereby 1 per cent. What was gained per cent on the remainder which was sold at Rs 11a 6p?

138 What amount of stock must be purchased in the English 5 per cents at $111\frac{1}{2}$ to produce the same yearly return as 3 lacs of rupees (a rupee = 15 10 $\frac{1}{2}$ d) invested in the Government 4 $\frac{1}{2}$ per cents at $101\frac{1}{2}$.

139 A horse was sold at a loss for Rs 60. Had it been sold for Rs 81 the gain would have been $\frac{1}{3}$ of the former loss. Find the cost of the horse.

140 What is the smallest number of articles costing £5 9s 4 $\frac{1}{2}$ d each, which can be purchased for an integral number of sovereigns?

141 Multiply 6372 and 630072 by 567, and explain why the difference of the two products is divisible by 11, and by the squares of 7 and 10, and by the fourth power of 9.

(i) What change would occur in these factors if the two zeros instead of being in the centre of the digits 6, 3 and 7, 2 occupied some other position still being together?

142 Reduce the expression $\left(\frac{3\frac{1}{2}}{7} + \frac{2}{10\frac{1}{2}} - \frac{5}{12} \text{ of } \frac{4}{7}\right) - \frac{1}{7}$ to its simplest form, and find the value of £ $\frac{2}{3}$ + $\frac{1}{4}$ of a guinea - $\frac{1}{8}$ of 2s 6d.

143 Divide 12150138 by 2023 and 000072072 by 000012. Find the fraction corresponding to 101, and divide 2736 by 3109.

144 What is the least number of dollars at 4s 2d each, which is equal to an exact number of sovereigns?

145 If the income tax be 6 pies in the rupee for the first half of the year and 3 per cent in the second, what is the gross income of a gentleman whose net annual receipts amount to Rs 1454 1a?

146 A can do $\frac{1}{4}$ of a piece of work in $\frac{2}{3}$ of the time that it would take B to do $\frac{1}{2}$ of it, and B can do $\frac{2}{5}$ of the same work in $\frac{1}{2}$ the time C would occupy in doing another piece of work half as large again as the first. If C can finish the first named piece of work in 6 hours, how long would A and B together be in doing it?

147 A debt of Rs 700 is discharged by a payment of Rs 180 in cash, and a bill for Rs 533 due 6 months hence. At what rate is discount calculated?

148 Exchange on London is at the rate of 1s 7d on demand, and 1s 7½d at 60 days' sight. What is the rate per annum for money in the latter case?

149 A man buys wine at 4s a gallon, he mixes it with water, and by selling the mixture at 3s a gallon gains 20 per cent on his outlay. How much water did each gallon of the mixture contain?

150 What sum must a person invest in the 3 per cents at 90, in order that by selling out £1000 stock when they have risen to 93½, and the remainder when they have fallen to 84½, and investing the whole proceeds in the 4 per cents at par, he may increase his annual income by £9 5s?

151 The yearly expense of a village school was Rs 300. This is paid partly by a rent of 29½ bighas of land, at Rs 3 8a per bigha, partly by a tax on the village of Rs 100, partly by Government allowance of Re 1 4a per scholar, and the rest was made up by the scholars, of whom there were 45, what did each pay?

152 How many times is ¼ of 13½ of 2s 2½d contained in 2½ of 3s 4d + 4½ of 1s 1½d + $\left(\frac{27\frac{1}{2}}{20\frac{1}{2}} \text{ of } 28\frac{1}{2} + \frac{3\frac{1}{2}}{7\frac{1}{2}} \text{ of } 1\frac{1}{2} \right)$ of 16s 8d?

153 Find by Practice the cost of replacing a cistern, to weigh 8 cwt 2 qrs 14 lbs at the rate of £2 os 6d per cwt, if the plumber allows £1 11s 6d per cwt for the lead of the old one which weighs 6 cwt 1 qr 10 lbs.

154 If one pound of standard gold of 22 carats (i.e. parts in 24) fine be worth £46 14s 6d, find the value of a gold mohur of weight 7 dwts 23 grs of fineness 993 in 1000.

155 Find the least number of ounces of pure silver worth Rs 2 14a 6½d per oz that, with the proper proportion of alloy, can be coined into an exact number of rupees.

156 A work can be completed in 36 days by 30 men working 6 hours a day, in what time would 18 men and 60 women working 9 hours a day complete it, supposing that 3 men can do as much as 5 women, and that in the longer days a man does only ¾ per hour of what he does per hour in the shorter days?

157 A contractor agrees to supply 10½ lacs of bricks for a particular work. His bricks cost him 3½ rupees per 1000 to make, and of these 12½ per cent are rejected. How many bricks must he make in order to fulfil his contract, and what price per 1000 must he put on those supplied in order to gain 25 per cent on his outlay?

158 A merchant sells 60 mds of rice at a profit of 8 per cent and 94 mds at a profit of 10 per cent, if he had sold the whole at a profit of 9 per cent he would have received 17*α* less than he actually did, how much per maund did he pay for the rice?

159 If a cloth 4 yds long and 15 in wide, cost Rs 3 2*α*, how much should you give for a cloth 19 yds long, 12 in wide, and every square inch of which is worth $\frac{1}{4}$ ths of the value of a square foot of the former?

160 In a field of cabbages the distance between the rows of cabbages is 2 feet, the distance between the cabbages in a row is 9 inches, how many cabbages are there in an acre?

161 A person goes to a bookseller's shop with a certain sum of money, and after buying 20 books at Rs 2 4*α* each, finds that $\frac{7}{8}$ of his money remains. How much had he when he entered the shop?

162 A person with a monthly income of Rs 264 spends as much in 4 months as he earns in three. After 12 years he divides his savings among his three children in such a manner that the eldest has twice as much as the second, and thrice as much as the youngest. How much did each receive?

163 Simplify:—

$$(i) \frac{2 \times \sqrt{(1+\frac{1}{2})} \div \sqrt{(1-\frac{1}{2})}}{5 \times \sqrt{(1+\frac{1}{3})} \times \sqrt{(1-\frac{1}{2})}}$$

$$(ii) \sqrt{\left(\frac{00195\dot{3} \times 00027}{00016} \right)}.$$

164 How many rings, each weighing 4 dwts 18 grs can a goldsmith make from a mixture of 1 lb 10 oz 1 dwt 18 grs of pure gold with 7 oz 7 dwts 6 grs of alloy?

165 Find, by Practice, the time of building a wall 27 yds long by 6 ft high, of which one square yard is built in 3 hrs 18 min 45 sec

166 In the Centigrade thermometer the freezing point is zero and the boiling point is 100°, in Fahrenheit's the freezing point is 32°, and the boiling point is 212°, what degree Centigrade corresponds to 68° Fahrenheit?

167 If the rate of wages vary as the price of rice, and if 57 men working for 35 days receive Rs 405 3*α* 9*φ* when rice is sold at the rate of 136 measures for Rs 39, find the price of rice per measure when 70 men working for 19 days receive Rs 353 4*α* 6*φ*

168 Tea shares, original value Rs 1000, are selling at Rs 1250. They pay a dividend of 5 per cent on the original value, what income would Rs 57975 invested in them give?

169 A ship's hold is 99 ft long, 40 ft broad and 5 ft deep, how many bales can be stowed in it each 3 ft 6 in long, 2 ft 8 in broad and 2 ft 6 in deep, leaving a gangway of 4 ft broad?

170 A person has 200 shares in a Railway Company for which

he paid Rs 1000 per share. When the shares are paying 2 per cent he sells them all at Rs 460 per share, and invests the proceeds in the Government 3 per cents at 92. Find the alteration in his income.

171 A steamship whose speed averages 14 miles an hour reaches a certain port in 12 days, how many days afterwards will a sailing vessel arrive, which starts at the same time and sails on an average 8 miles an hour?

172 At an examination $\frac{1}{10}$ th of a class gains $\frac{7}{8}$ ths of the maximum number of marks, $\frac{1}{10}$ th gains $\frac{1}{2}$ th, $\frac{1}{10}$ th gains $\frac{1}{4}$ th and the rest $\frac{1}{4}$. The average number of marks gained by the whole class is 165, what is the maximum?

173 A bag contains 160 coins consisting of half-crowns, shillings sixpences and four penny pieces, and the values of the sums of money represented by each denomination of coin are the same, how many of each are there?

174 A and B engage in trade, their capital being as 3 : 2. At the end of 3 months A takes out a sum equivalent to $\frac{1}{3}$ of B's capital, and at the end of another 3 months B puts in a sum equivalent to what A took out. If A's profits are at the end of the year £110 more than B's find the amount of the profits of each.

175 The hands of a clock which gains uniformly at the rate of 15 sec a day were set at sunset on the evening of the first of the month at 6 o'clock. The true time of sunrise on the 3rd, was known to be a quarter to six but the clock indicated a quarter past 6. Find the error made in setting the clock on the 1st.

176 A person pays an income tax of 4d in the £ during the first half of the year and of 3d in the £ during the second half, and finds that owing to an increase in his income he pays the same amount of tax for the second as for the first half of the year. If his gross income for the year is £700, find his net income.

177 The length and breadth of an enclosure which is in the form of a parallelogram are respectively equal to 47 yds 2 ft 4 in and 22 yds 2 ft 11 in, what should be the breadth of another equiangular enclosure (which is also a parallelogram) if its length is 63 yds 1 ft 5 in, and its area is $\frac{4}{5}$ ths of the former?

178 What must be the price of a £50 Railway share which pays a dividend of $2\frac{1}{2}$ per cent, if the purchaser is to make 4 per cent upon his outlay?

179 A merchant sells tea to a tradesman at a profit of 60 per cent, but the tradesman becoming a bankrupt pays only 2s 6d in the £. How much per cent does the merchant gain or lose by his sale?

180 A merchant made a mixture of wine at 28s a gallon, with brandy at 42s a gallon, and he found that by selling the mixture

at 35s a gallon, he gained 15 per cent on the price of the wine, and 20 per cent on the price of the brandy. In what ratio were the wine and brandy mixed together?

181 Find the product of 18988 and 808, divide it by 0235 and subtract the square of 808 from the quotient.

182 A merchant buys in Calcutta 210 bags of rice at Rs 10 12s per bag of 164 pounds. He sends them by rail 320 miles at $6\frac{1}{2}$ pies per ton per mile, but during the journey $7\frac{1}{2}$ pounds are stolen from each bag. Find at how many measures per rupee he must sell the remainder in order to clear Rs 95 15s by the transaction (One measure = $3\frac{1}{2}$ pounds).

183 A building worth Rs 9000 was burnt, of which $\frac{1}{3}$ belonged to A, $\frac{1}{3}$ to B and the rest to C, what loss will each sustain, supposing that Rs 5400 of the value of the house were insured?

184 If 10 compositors who can set 3 letters in 5 seconds finish 27 pages in an hour and a half, how many compositors who can set 5 letters in 6 seconds, will complete 56 pages in an hour?

185 The cost price of a book is 12s 6d the cost of the sale 8 per cent on this, the profit 22 per cent, find the retail price of the book.

186 There is a piece of work which 8 men working in pairs would accomplish in 20 days, and working singly in 30 days. One man is engaged on the work for 25 days, at the end of which time 7 men arrive to his assistance, and continue at the work for 4 days, when they are strengthened by the arrival of 7 men more. In what time would the work be finished, supposing all the men to continue at work, and operate in pairs whenever practicable?

187 A cask of 144 $\frac{1}{2}$ gallons is bought for £50 and kept 10 years, during which time $\frac{1}{8}$ of a gallon evaporated yearly, at what rate per gallon must the contents be sold so as to clear 20 per cent on the amount of the original outlay at 4 per cent per annum, Simple Interest?

188 Supposing a gallon of water to contain 277 $\frac{1}{2}$ cubic inches, find what number of gallons of water would cover a square mile to the depth of 5 inches.

189 The diameter of the fore wheel of a carriage is $\frac{2}{3}$ of that of the hind-wheel and the former makes 528 revolutions in passing over $\frac{3}{4}$ of a mile. How many revolutions does the hind-wheel make in passing over a mile? and what is the diameter of each wheel?

190 A person borrows £500 at 5 per cent. per annum and subsequently £400 at $3\frac{1}{2}$ per cent, if the amount of both sums 6 months after the latter was borrowed is £957, find the time for which interest is paid on the former sum.

191 Find the value of

$$\frac{2 \text{ 3475 of 1 ton} + 4 \text{ 6875 of 1 cwt} + 5 \text{ of 3 qrs} - 1 \text{ 4 lbs}}{1 \text{ 05 of 1 ton}}$$

192 Simplify —

$$(i) \frac{17}{15} \cdot \frac{\sqrt{(2-\frac{2}{3})} \times \sqrt{(3+\frac{4}{3})}}{3\sqrt{(2-\frac{2}{3})} - \sqrt{(3+\frac{4}{3})}} \quad (ii) \frac{10\frac{1}{11}}{3 \times 3 \text{ 39}} - \frac{\sqrt{(75\frac{1}{11})} - 6\frac{1}{11} \text{ of } 7\frac{2}{11}}{5\frac{1}{11} - \sqrt{(2\frac{2}{11})}}$$

193 From a vessel containing 50 seers of milk, 10 seers are taken away, the vessel is then filled up with water, and 10 seers of the mixture drawn off, how many seers of milk are left?

194 Find the square root of 012345679, and also of 002 to 4 places of decimals

195 If a pound of pure silver be worth 62 shillings, the shilling containing 222 parts of pure silver in 240, what will be the value in shillings of a rupee weighing 180 grains, the rupee containing 925 parts of pure silver in 1000?

196 A, B and C can together do a piece of work in 12 days, A can do it in 28 days and B in 26 days. All three work together for 7 days, when A leaves off work. In how many days will B and C complete the work?

197 A rectangular parish, 6 fur long and 4 fur broad, is enclosed, a belt of plantation, 200 ft wide is carried the whole way round, a main road, 60 ft wide, runs across the land in the direction of its length and a cross road, 41 ft wide in the direction of its breadth. How many acres of field are there?

198 If the cost of making bread be one rupee per bushel of wheat, what is the price of wheat, when the two anna loaf is twice as large as it is when wheat is Rs 5 a bushel?

199 A publisher wishes to net 14s for each copy of a work, what price should he put upon it so as to be able to allow the trade 30 per cent discount?

200 A merchant receives a bill on London at 3 months of Rs 3000, which he keeps till maturity and then exchanges at the rate of 1s 10½d per rupee, when he finds he receives as much as he would have done had he discounted the bill when drawn at 4 per cent, what was the rate of exchange when the bill was drawn?

201 Add together $3\frac{5}{8}$ of $2\frac{1}{2}$ of $7\frac{1}{10}$ of £1, $9\frac{1}{2}$ of $3\frac{8}{11}$ of 1s and $8\frac{1}{4}$ of $4\frac{1}{2}$ of 1d, and divide the sum by $\frac{1}{12}$ of $\frac{1}{12}$ of $3\frac{1}{2}$ of 1s the quotient an abstract or a concrete number?

202 Simplify —

$$\frac{1\frac{1}{10} + \frac{1}{1}}{2 \text{ 6}} \text{ of } 5\frac{1}{2} - \left\{ 1\frac{1}{17} \text{ of } \left(1 - \frac{4^3}{9^2} \right) + \frac{8}{11} \text{ of } 16 \text{ of } \left(1 + \frac{5}{12} \right) \right\}$$

203 If 8 men or 12 boys can do a piece of work in 20 days, how long will it take 3 men and 5 boys to complete a piece of work twice as great?

204 A grain of gold beats out into a leaf of 54 sq in, and a cubic foot of gold weighs 1211 lbs Avoir. How many leaves together are as thick as a sheet of paper when 175 sheets of paper placed together are an inch high?

205 Two straight rods, each 1 foot in length, divided into 11 and 12 equal parts respectively, are placed side by side with their ends together. What fraction of an inch would the distance of the third division of the first be from the third division of the second?

206 In 1861 three towns had populations of 17650, 19600, and 18760 respectively. In 1871 the population of the first had decreased 18 per cent, that of the second had increased 21 per cent, while the population of the third had increased by 4690, find the average population of the three towns in the year 1871.

207 On what sum of money will the Compound Interest for 2 years be the same as the Simple Interest on £943 for 10 years, reckoning interest at the rate of 5 per cent per annum?

208 If the true discount on a bill of £14641 be £4641 at 10 per cent. Compound Interest, how many years has the bill to run?

209 Five thousand copies are issued of a book the price of which is Rs 3 per copy, the cost of printing is 8a per copy, binding Rs 2 per dozen, and of carriage, advertising, &c Re 1 per dozen, the publisher disposes of them to the retail bookseller charging 25 copies as 24, and 30 per cent less than the selling price and upon the whole receipts takes 10 per cent commission for himself. What are the gains respectively of author, publisher, and bookseller on this edition?

210 For two thirds of the distance up a ghaut the rise is 1 foot in 24 (measured along the road) and for the remaining third the rise is 1 in 16. The top of the ghaut is 1400 feet above the bottom, what is its length?

211 - How many acres are contained in three countries, of which the first comprises 723100 square miles, the second 12342, and the third 89704 square miles?

212 The gallon contains 277.27 cubic inches, and a cubic foot of water weighs 62.42 lbs. Find the weight of a pint of water to two places of decimals.

213 Divide £954. 9s between A, B and C, so that A's share may be to B's share 3 5, and B's share C's share 10 11.

214 A, B, C and D working together can perform a piece of work in 8 days. A and B or B and D together take twice as long

as A , B , C and D together to perform the same work A works during the whole of the day, B during three fourths, C during a half and D during $\frac{1}{4}$ th of the day In how many days will the work be finished ?

215 An annual tax of Rs 2255 is laid upon a district containing four villages— A , B , C , D —and the rate to be paid by each of the villages A , B and C is to the rate to be paid by D , as 3 to 2, what are the annual payments due from the villages ?

216 If 10 men or 15 boys can reap 20 bighas of corn in 6 days working 14 hours a day, how many boys must be employed to assist 3 men to reap 6 bighas in $1\frac{1}{2}$ days of 8 hours a day ?

217 A shop keeper buys $\frac{1}{2}$ cwt of tea at 4s 2d per lb, and mixes it with tea which costs him 2s 11d per lb How much of the latter must he add to the former that he may sell the mixture at 3s 8d per lb and gain 20 per cent on his outlay ?

218 Find the cost price of an article, which, if sold at 7 per cent profit realises 5s more than if it were sold at 17 per cent loss

219 A owes B £500, in liquidation of which debt he gives him a bill of £300 due 10 years hence, another bill due 4 years hence, and £133 6s 8d in cash What is the value of the latter bill, interest being at the rate of 5 per cent per annum and allowing true discount ?

220 A man had £10000 of 3 per cent stock which he sold out at 72 He placed one half into the 4 per cents at 80 and the other half into the 5 per cents at 90 The former having fallen to 76, he transferred his stock from them to the $5\frac{1}{2}$ per cents at 95 State the difference between his present and his original income

221 Two men A and B start together, and when A has gone a mile, B has gone $\frac{3}{20}$ of $1\frac{2}{5}$ of $\frac{1\frac{1}{2}}{5\frac{2}{3}}$ of $\frac{\frac{3}{2}+1}{\frac{2}{3}-\frac{1}{4}}$ of 713 of

$\frac{\frac{2}{3}}{1-\frac{2}{3}} + \frac{1}{3} + 1$
 of a mile, which is in advance of the other ?
 $1 - \frac{1}{7}$ of $\left(\frac{\frac{2}{3}}{1-\frac{2}{3}} + \frac{1}{3} \right)$

222 Express the difference between 378 of 13s 10½d and 378 of 16s 6d as the fraction of

426 of $\frac{3\frac{3}{8}}{35}$ of $\frac{3}{111}$ of $\frac{147 \times 44}{111}$ of £1 17s 6d

223 The Hindoo year consists of 365 days 6 hours 12½ minutes the Mahomedan of 354 days 8 hours 48 minutes After what length of time would the accumulated difference between them amount to the tropical year of 365 days 5 hours 48 minutes 497 seconds ?

224 If the rates of running of A , B are as 14 11, and if A give B a start of 36 yards, how far must the winning post be for A , to get in by the same distance?

225 What is the cost of making a ditch, 64 yds 1 ft 4 in long, 2 yds 2 ft 2 in wide and 3 yds 0 ft 6 in deep at $1\frac{1}{2}d$ per cubic foot? How long would 4 men, 3 women and 2 boys together take to make it, working 8 hrs a day, if 2 men equal 3 women and 4 women equal 3 boys, and a boy earns $1\frac{1}{2}d$ per hour?

226 A can do $\frac{1}{2}$ of a piece of work in 24 days, and B can do as much work in 3 days as A can do in 4 days, they work together for 4 days A then leaves and C joins B and they work together for 6 days, then A returns, and the three finish the work in 11 days, how long would it have taken C to finish the whole piece of work?

227 Two watches are both set right at noon on the 15th of June, 1859, one gains $1\frac{1}{2}'$ in a day, the other loses $1'$, when will they be together again, and what o'clock will it be by each of them?

228 Two ships sail from the same port, one of them sails west, 50 miles, and the other sails north, 48 miles. Find the distance between them in miles to 4 places of decimals.

229 If the daily wages of a labourer rise from four and three quarters to six anns, what percentage of the increase in the price of food and other commodities will cause his position to be unaltered?

230 A person invests £6200 in the three per cents at $89\frac{1}{8}$, and pays income-tax of $10d$ in the £, on the stock rising to 92, he sells out and invests the proceeds in £50 Railway shares at par which yield an annual dividend of 3 per cent, clear of income tax. Find the alteration in the income.

231 Add together 062435 of £100 + 7 4375 of 10s + 1 356 of 7s 6d + 2 784 of $2\frac{1}{2}d$, and reduce the result to the fraction of £29 10s $7\frac{1}{2}d$.

232 The area of a rectangular enclosure being 33 sq poles 1 yd 6 ft 108 in, and the length 9 poles 1 ft 6 in, what is the width?

233 If the rate of interest for money is 6 per cent, what should be the rate of exchange for bills payable at sight in England when the rate for those payable 6 months after sight is 1s 11d per rupee?

234 How much per cent must be added to the cost price of goods that a profit of 20 per cent may be made after throwing off a discount of 10 per cent from the labelled price?

235 Given that gold is worth £3 17s 10d per oz and silver 4s 10d per oz, and that the weights of equal volumes of gold and silver are as 19 11, find the length (in inches to 3 places of decimals) of an edge of a cube of silver equal in value to a cubic inch of gold.

236 If the volume of a sphere $= \frac{4}{3} \times 3.1416 \times$ the cube of the radius, find how many spherical balls each $\frac{1}{2}$ inch in diameter can be made out of a cubic inch of clay, and how much clay will remain?

237 The top of a tank is a rectangle, whose sides are 9 feet and 15 feet, it is of the same horizontal section throughout its depth. What must be its depth in order, that it may contain 12960 gallons of water, one gallon containing 277 774 cubic inches?

238 A person buys saddlery in London for £31 and pays £3.5s for freight and insurance to Calcutta. On the arrival of the goods he pays 7 per cent duty on the declared value, which was the London price turned into rupees at the current rate of exchange. If he had bought them in Calcutta he would have paid 40 per cent above the declared value. How much did he gain by buying the goods in London, the rate of exchange being Rs 10 13a for a sovereign?

239 A dealer buys 10 horses at Rs 400 each, 8 horses at Rs 500 each and 4 horses at Rs 600 each. He keeps the horses for 6 months, during which each costs Rs 15 per month, and then sells them clearing $12\frac{1}{2}$ per cent on his original outlay after paying all his expenses. Find the selling price.

240 A man who can walk down a ghaut at the rate of $4\frac{1}{2}$ miles and up at the rate of $3\frac{1}{2}$ miles an hour, descends, and returns to his starting point after walking for 2 hours 4 minutes. How far did he walk?

241 Multiply 3 mi 5 fur 17 po 5 yds 1 ft 10 in by 7, and 1574 hhds 62 gals 3 qts 0 pt 2 gills by 27. Reduce 23 ac 1 ro 27 po 24 sq yds 6 sq ft 103 sq in to square inches.

242 The length of a room is treble its breadth. The cost of flooring, at Rs 3 12a per sq yd, is Rs 281 4a, and that of painting the four walls, at 3a per sq foot, is also Rs 281 4a. What is the height of the room?

243 A's rate of working is to B's as 4 to 3, and B's is to C's as 2 to 1. How long will it take C to do what A would do in 6 days?

244 An Indian officer, whose annual pay was estimated in rupees, lost £41 12s 6d in one year by a fall in the value of the rupee from 1s 11½d to 1s 10½d, what was his salary estimated in rupees?

245 A warehouse consists of seven floors, the rent of each floor is 875 times that of the floor below, the rent of the middle floor is £120 1s, compare the rents of the highest and lowest floors, and find that of the lowest.

246 If a certain amount of work is done by 9 men, 12 women and 13 boys in 11 days, how long will the same work take if 18 men,

3 women and 5 boys are set to do it assuming that the ratio of a man's work to a woman's is as 5 to 3, and a woman's work to a boy's as 4 to 3?

247 The travelling expenses of 7 tourists for 5 weeks amounted to Rs 752 8a, a second party of 18 made the same tour in 6 weeks, their average weekly expenditure per man being $\frac{1}{4}$ of that of the first party. What were the total expenses of the second party?

248 An armourer undertakes to supply 2000 swords at 17s 3d each. He estimates that if 5 per cent fail to stand the required test and are worthless, the profit will be 15 per cent on his whole outlay. At the trial, 35 per cent of the swords prove worthless. How much does the armourer lose by the contract?

249 In a hundred yards race *A* can beat *B* by 4 yards, in a quarter of a mile race *C* can beat *A* by 11 yards, by how much can *C* beat *B* in a mile race, supposing that the average speeds of each man when running a hundred yards, a quarter of a mile, and a mile, are proportional to 9 8 7?

250 A field containing 26 ac 3 ro 10 po is let in equal allotments to 66 agricultural labourers at a rental of 3d a pole per annum, a reduction of 15 per cent being offered to those tenants who shall pay their rent on the day that it becomes due. When all the years' rent have been paid, the landlord finds that he has received the sum of £49 4s 9d. How many labourers paid to the day?

251 Find the sum, difference, product, and quotient of two dozen dozen and half-a dozen dozen.

252 If 8 lbs of coffee cost as much as 5 lbs of tea, and 7 lbs of coffee as much as 40 lbs of sugar, what is the price of each when 1 lb of tea, of coffee, and of sugar, together cost Rs 2 5a?

253 Two cog wheels, one with 15 teeth, the other with 28 teeth, work together. If the former turns round 16 times in $7\frac{1}{2}$ seconds, how many times will the latter turn round in 21 seconds?

254 *A*, *B* and *C* play at cricket. *A*'s runs are to *B*'s and *B*'s runs are to *C*'s, as 3 to 2. They get altogether 342 runs. How many does each get?

255 If 3 kilometres are as much under 2 miles as 5 kilometres are over 3 miles, what is the length of a kilometre?

256 The income tax is reduced from 10½d to 5d in the pound, but a man's gross receipts are at the same time reduced by 10 per cent owing to the stoppage of a mine. Find by what percentage his net income is altered.

257 Two settlers in New Zealand own adjoining farms of 3000 and 5000 acres respectively. They unite their farms, taking at the same time an additional partner, who pays them £8000, on the

understanding that a third share of the land shall in future belong to each How is the £8000 to be divided between the original owners ?

258 The debts of a bankrupt amount to £1067 5s 3d, and his assets consist of property worth £458 7s 8d, and an undiscounted bill of £256 10s due 4 months hence, simple interest being reckoned at 4 per cent How much in the pound will he pay ?

259 A journey of 560 miles was made by rail, steamer and coach The distance by coach was one-fourth, and the distance by sea three-fourths of that by rail The fare per mile by coach was double, and by sea four-fifths of that by rail What was the expense of the whole journey, railway fare being 1 571.28d per mile ?

260 One clerk has 24 428571 and a second clerk has 384 sheets to engross, they call in a third clerk and agree to divide the work equally among three, and to pay the third clerk at the rate of 24305 shilling per sheet, how much will he receive from each of them ?

261 Arrange the figures 194678 in the six different ways in which 194 are the *first* three figures in different order, and also in the six different ways in which 678 are the *last* three figures in different order, and add the twelve arrangements together

262 Multiply the difference between 12 cwt 3 qrs 17 lbs 10 oz and 5 cwt 2 qrs 23 lbs 11 oz by 528 Divide the sum of the same two quantities by 324

263 Gold is 19 times as heavy as water, and copper 9 times In what proportion should these metals be mixed that the mixture may be 15 times as heavy as water ?

264 Exchange at *six months' sight* is at 1s 10d per rupee By depositing in a bank for 6 months, certain interest can be had at the rate of 3 per cent per annum, at the end of the 6 months, exchange at *sight* is 1s 9½d per rupee What is the gain or loss per cent on remitting from India either (i) at *six months' sight*, or (ii) depositing at interest for 6 months, and then remitting at *sight* ?

265 Calculate the value of $\sqrt{\frac{(78) \times (00004)}{(013) \times (015)}}$

266 C does half as much in a day as A and B can do together, and B does half as much again as A If all three working together can mow 20 acres of grass in 16 days, how long would each, working by himself, take to mow 5 acres ?

267 If it cost Rs 497 4a to decorate a wall space measuring 69 ft 4 in by 6 ft 9 in, what will it cost for one measuring 22½ yds by 3½ yds, the style of decoration used in the second case being half as expensive again as in the first case ?

258 If the cost of provisioning a gunboat carrying 84 men be £598 10s when the ship is at sea for 95 days, what will it cost to provision for 33 days a ship carrying a crew of 110 men?

259 If the true discount on Rs 1000 at $3\frac{1}{2}$ per cent Simple Interest, be Rs 165 10s 8d, when is the sum due?

270 Arrange in order of magnitude — $\sqrt{(50)}$, $\sqrt[3]{(344)}$, $\sqrt[4]{(2402)}$

271 Reduce $\frac{1 + \frac{2}{3\frac{1}{2}}}{1 - \frac{2}{3\frac{1}{2}}} \left(1 + \frac{4}{9 - \frac{3}{1 - \frac{1}{2}}} \right)$ to a decimal

272 Find by Practice the cost of a fence 3 fur 11 po $3\frac{1}{4}$ yds long at £183 6s. 8d per mile

273 What is the whole cost of 5 pairs of gloves at 2s $11\frac{3}{4}$ d per pair, 24 yds of muslin at 1s $9\frac{1}{2}$ d per yard, $17\frac{1}{2}$ yds of ribbon at $8\frac{1}{2}$ d per yard, and 35 yds of flannel at 1s $7\frac{3}{4}$ d per yard?

274 A piece of work is done by three men A, B and C in 5 days in the following manner: A works the whole time, B only on the first and second days, C only on the third fourth and fifth days. The work might also have been done by B and C working together for 6 days without the assistance of A. If B and C working together for 2 days can do as much work as A can do alone in 3 days, find how long it would take A, B and C each to do the work separately.

275 In an examination A obtains 10 per cent less than the minimum number of marks required for passing, B obtains $11\frac{1}{2}$ per cent less than A, and C $41\frac{1}{2}$ per cent less than the number of marks obtained by A and B together. Does C pass or fail?

276 A rectangular swimming bath is 60 ft long and 40 ft broad, it can be filled by a supply pipe in 5 days, and if 6000 cub ft of water be thrown in, the rest can be filled in 3 days 18 hours. Find the depth of the bath.

277 What will £3255 4s 2d amount to in a year and a half, if put out at Compound Interest at 8 per cent per annum, the interest being added at the end of each half-year?

278 A person rows a distance of $1\frac{1}{2}$ miles down a stream in 20 min, but without the aid of the stream it would have taken him half an hour, what is the rate of the stream per hour? And how long would it take him to return against it?

279 Three sums of money are in the proportion of 2 3 5, and when each has been reduced by Rs 25, the remainders are in the proportion of 1 2 4. Find the sums of money.

280 A sum of £3750 was sold out of the 3 per cents at 95 and put out at Compound Interest for two years at 4 per cent, the amount being then invested in the $3\frac{1}{2}$ per cents at 104. Find the alteration in income

281 Show that the sum of the squares of three thousand and nine and four thousand and twelve is equal to the square of five thousand and fifteen. Write this sum in figures

282 In a book on Arithmetic an example was printed thus -

"Add together $\frac{1}{14\frac{2}{3}}$, $\frac{1}{19\frac{1}{4}}$, $\frac{1}{\quad}$, $\frac{1}{13\frac{3}{4}}$," the denominator of one fraction being accidentally omitted. The answer given at the end of the book was $\frac{1}{1\frac{1}{2}}$. Required the missing denominator

283 A 7 ft strip along two adjacent sides of a rectangular garden forms a border for flowers. If the dimensions of the garden be 42 ft \times 34 ft, what is the exact proportion of the strip to the whole area?

284 If a family, by using 6 gas-burners 5 hours a day, pay Rs 12 8a per quarter, when gas is at Rs 2 8a per 1000 cub ft, what will a family, using 8 burners 3 hours a day, pay per quarter, when gas is at Re 1 14a per 1000 cub ft?

285 A grocer buys 15 lbs of tea, he sells 8 lbs at 2s $7\frac{1}{2}$ d per lb, and the rest at 2s 9d per lb, and finds that he has made 15 per cent profit. What rate per cent profit would he have made if he had sold it all at 2s $9\frac{1}{2}$ d per lb?

286 Calculate the value of each of the following to 3 places of decimals -

$$(i) \frac{\sqrt{9}-\sqrt{9}}{\sqrt{4}+\sqrt{4}} \quad (ii) \frac{\sqrt{9}+\sqrt{9}}{\sqrt{4}-\sqrt{4}} \quad (iii) \frac{1}{\sqrt{(12)+2}} + \frac{1}{\sqrt{(12)-2}}$$

287 A rectangular field which is twice as long as it is wide costs 10138s per square yard to turf. If the whole cost is £191 17s $0\frac{1}{2}$ d, find the lengths of the sides of the field

288 A person buys a piece of land at £25 an acre, and by selling it in allotments finds that the value is increased by one-half, so that, after reserving 20 acres for himself, he clears £200 on his purchase money by the sale of the remainder. How many acres were there?

289 Gold is sold at the Mint at £3 17s 9d per oz and is mixed with alloy worth 5s 2d per oz in the ratio of 11 : 1. If sovereigns be coined of this mixture, each weighing 5 dwts 3 47 grs, what is the Mint profit per 100 sovereigns?

290 922 ft 77 in of colouring have to be done on the walls

and ceiling of a room whose form is that of a cube Find its dimensions

291 In a long division sum the dividend is 529565 and the successive remainders are 246, 222 and 542 Find the divisor and the quotient

292 What two numbers of 4 digits each can have 119 as their G C M and 13923 as their L C M ?

293 A sum of money is to be divided amongst 11 men and 18 boys, and 5 men are to receive as much as 9 boys When 3 men and 3 boys have received their shares, what fraction of the whole sum will remain ?

294 An engine while driving machinery burns coal at the rate of 1 ton 12 cwt 2 qrs in 8 hrs 40 min When the machinery is not in motion, the consumption of coal is only $\frac{1}{4}$ of this rate How much coal will the engine burn in 1584 hours, during $\frac{1}{3}$ of which time the machinery is at rest ?

295 Find correct to 7 places of decimals the value of

$$\frac{1}{9} + \frac{1}{39^3} + \frac{1}{59^5} + \frac{1}{79^7} + \&c$$

296 A can do a piece of work in 25 days, B can do it in 20 days and C in 24 days The three work together for 2 days, and then A and B leave, but C continues, and after $8\frac{1}{2}$ days, is rejoined by A, who brings D along with him, and these three finish the remainder of the work in 3 days more In what time would D alone have done the whole work ?

297 What sum will amount to £1591 13s 2 16d in 3 years at Compound Interest the rate of interest for the first, second and third years being 3, 2 and 1 per cent respectively ?

298 What must be the rate of interest in order that the discount on £387 7s 7½d payable at the end of 3 years may be £41 10s 1½d ?

299 If after adding 14 gals of water to a pipe of wine, and selling it at 12s 6d a gallon, I gain 25 per cent, find the prime cost of the wine

300 A boat is rowed down a river at the rate of a mile in 6 min, and up the river at the rate of a mile in 10 min, the crew working equally hard, find the velocity of the current

301 The officers of a regiment are 0.42 of its strength, but after 50 privates have been added the officers are 0.4 of the whole What is the number of officers ?

302 A pound of powder costs 3s and the charge of a gun is $2\frac{1}{2}$ drams, how many shots will 6s 9d worth of powder furnish, and what will be the cost of powder for 2560 shots?

303 A regiment of 1000 men are to have new coats, each coat is to contain $2\frac{1}{2}$ yds of cloth, $1\frac{1}{2}$ yds wide, and to be lined with shalloon $\frac{3}{4}$ in wide, how many yards of shalloon will be required?

304 If the driving wheel of a railway engine be 16 ft in circumference, and the velocity of the engine 25 miles an hour how many revolutions does the wheel make in 5 minutes?

305 A cistern is supplied by 4 taps which would fill it (if empty) in 3, 4, 5 and 6 hours respectively, and discharged by two others which would empty it (if full) in $2\frac{1}{2}$ and $3\frac{3}{4}$ hours respectively. Suppose the cistern empty, and all the taps open, how long would it be in filling?

306 A sailing vessel reaches Madras from Calcutta in 6 days, a steamer whose speed is to that of the sailing vessel as 3 to 2, starts at the same time, but meets with detentions that average 6 hours daily. Which will reach Madras first, and by how much?

307 If it is high water at noon on a certain day, find after how many days it will again be high water at noon, supposing the time of high water to be three-quarters of an hour later every day.

308 A fixed rent of £780 per annum is converted into a corn rent of one half wheat at 48s per quarter, and the other half barley at 30s per quarter. What will be the rent when wheat has advanced to 56s and barley to 32s per quarter?

309 At what time between 6 and 7 will the two hands of a watch be exactly 23 minute spaces apart?

310 A person invests a certain sum in the 3 per cent's when they are at 96 $\frac{1}{2}$, had he waited till they had fallen to 96 $\frac{1}{8}$ he would have obtained £16 more of stock. How much money did he invest, $\frac{1}{2}$ per cent being charged as brokerage in both cases?

311 In a certain sum the dividend is 31884740, the quotient 40930, find the divisor and remainder.

312 A manufactory turns out 50 tons of iron goods weekly, using up for that purpose 51 tons of iron at £6 15s per ton, 100 tons of coal at 11s 6d per ton, and £15 worth of other materials, rates and taxes amount to £219 annually, wages and incidental expenses to £75 per week. At what price per cwt must the iron goods be sold in order that the works may gain 8 per cent per annum on a capital of £35000?

313 Find by Duodecimals how many cubic feet are contained in a beam 25 ft 5 in long, 1 ft 7 in broad and 1 ft 2 in thick. If an

inch in thickness be taken off from each of the four sides, the length of the beam remaining the same, find how much wood has been removed

314 The average of 25 returns is 43 The first 9 average 52, the next 12 equal 37 Find the average of the last 4

315 If the par of exchange be two English shillings for the Indian rupee, but if an Indian bill of exchange for Rs 540 12a be negotiated in London for £51 10s, how much per cent below par is the rate of exchange?

316 A, B and C do $\frac{4}{5}$ ths of a piece of work together in 24 days, A does the same amount of work as B does in the same time, had A or B been absent, then the two others would have accomplished $\frac{5}{6}$ ths of the work in 28 days In what time can each separately do the work?

317 A bill for £202 16s was drawn on March 2nd, at 7 months, what will it be worth on May 12th, discount being calculated at $3\frac{1}{2}$ per cent?

318 A merchant bought a fifty-gallon cask of wine for Rs 741 Supposing it to have lost $3\frac{1}{2}$ gallons, at what price per dozen bottles (nine bottles holding a gallon) should he sell it in order to gain 15 per cent upon the whole original cost?

319 When the East India 4 per cent stock is at 12 discount and the 5 per cent ditto at $5\frac{1}{2}$ premium, find the difference in the rates of interest obtained by investing in these stocks respectively A person has a certain sum to invest, and finds that the latter stock will give him an annual income of £3 7s 6d more than the former, what is the sum?

320 A watch is 5 min fast at noon on the first of the month, on the 11th at true noon it shows 12 hrs 55 min Find its rate of gaining, and the true time to the nearest second, when the watch shows 5 min past noon on the 16th

321 Find the value in acres, &c of

75 of $\frac{13}{16}$ of $\frac{85}{225}$ of $\frac{13}{26}$ of $\frac{£3}{14s}$ of $\frac{5s}{3d}$ of $\frac{1 \text{ vd } 1 \text{ ft } 6 \text{ in}}{6 \text{ yds } 2 \text{ ft}}$ of $\frac{136 \text{ gals } 2 \text{ qts}}{178 \text{ gals } 3 \text{ qts}}$
of $\frac{1 \text{ lb } 6 \text{ oz } 17 \text{ dwts } 2 \text{ grs}}{2 \text{ lbs } 2 \text{ oz (Avoir)}}$ of $\frac{77 \text{ da } 4 \text{ hrs } 30 \text{ min}}{6 \text{ da } 12 \text{ hrs}}$ of 517 sq ft 72 sq in

322 A drawing room, 36 ft 10 in long and 23 ft 2 in wide, is surrounded with a cornice $3\frac{1}{2}$ in wide, the gilding of which costs £4 11s 10 $\frac{1}{2}$ d, how much was that per square foot?

323 Gun-metal is composed of 9 parts of copper to one of tin, and bell metal is equal to 80 parts of copper to 101 of tin,

56 of zinc and 43 of lead What quantity of zinc, lead and tin must be added to 24 tons of gun-metal to convert it into bell-metal ? Give the answer in tons

324 A man having Rs 50000 to invest, expends a proportion of $\frac{1}{2}$ pie in the rupee, in purchase of building land at Rs 63 4a 9p per acre, and a proportion of 7a 2p in the rupee in iron, at Rs 10 2a 5p per cwt How much of each did he obtain, and what will be the interest for $2\frac{1}{2}$ years, on the balance of his capital at $4\frac{1}{2}$ per cent ?

325 A cistern can be filled by two taps in 20 and 24 minutes respectively, when in good order, but after they have both been running together 15 minutes, it is found that the cistern is only $\frac{1}{2}$ full Determine the rate of leakage

326 A clerk's salary before the imposition of the income tax was £200 a year How much must it be now with an income tax of 16d in the pound, that he may still possess £200 a year net income ?

327 Supposing the price of the 3 per cent consols to be $97\frac{3}{8}$ brokerage $\frac{1}{8}$, what will be the Simple Interest of £260 for 2 years ?

328 A man bought a bigha of land for Rs 140 and intending to sell it, fixed such a price, that by selling it at $12\frac{1}{2}$ per cent under the intended price he would still have a gain of $12\frac{1}{2}$ per cent on the prime cost At what price he intended to sell the land ?

329 A rectangular pile of wood is 12 yds high and 10 broad, find the number of oblong pieces 18 ft long, 8 in broad and 4 in deep contained in it, supposing the cost of covering the pile with matting at 4d per square foot to be £87

330 What time must elapse between the time of placing Rs 250 in the Government Saving's Bank, and taking out the amount just as it goes over Rs 300, supposing Compound Interest at 5 per cent per annum ?

331 Find the sum of all the numbers that can be formed by the digits 2, 3, 4, 5 taken all together

332 A body of men in column of 625 ranks of 64 abreast, was drawn up in a solid square How many would there be in each face ?

333 If 560 flag stones, each $1\frac{1}{2}$ ft square, will pave a courtyard, how many will be required for a yard twice the size, each flag-stone being 14 in by 9 in ?

334 A man must get $\frac{1}{3}$ marks to pass an examination, he answers $\frac{1}{2}$ the questions, but to $\frac{1}{5}$ of his answers gets on an average only $\frac{2}{5}$ of the marks, and thus gets 25 marks too few to pass How many does he get altogether ?

335 If 5 men build a tower 30 ft high in 3 days, in how long a time can 4 men build a similar tower 64 ft high ? If each foot of

the tower above the 30th takes on an average twice as long to build as each foot below it, how long would they take?

336 Determine the value of each of the following expressions to 4 places of decimals —

$$(i) \frac{2+\sqrt{3}}{1+\sqrt{3}} \quad (ii) \frac{15+\sqrt{10}}{15-\sqrt{10}} + \frac{30-\sqrt{10}}{30+\sqrt{10}} \quad (iii) \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

337. If the 3 per cent stock is at £87 and the $3\frac{1}{2}$ per cent at £92, which is the most advantageous investment? And if I have £3000 stock in the least advantageous, and I wish to change it into the other, how many £100 shares of stock shall I obtain? Brokerage of $\frac{1}{2}$ per cent being charged on all transactions

338 A reduction of 40 per cent in the price of a half-crown tea enables me to buy for my money 6 lbs more than I could previously buy for a sovereign. What amount have I?

339 A person borrows Rs 1000, and at the end of each year pays Rs 250 to reduce the principal and to pay interest at 4 per cent on the sum which has been standing against him through the year. How much will remain of the debt at the end of 3 years?

340 By paying an income-tax of 9% in the pound, a man's income is reduced to £1212 15s. Find (i) his original income, and (ii) the sum of money which must be invested in the $4\frac{1}{2}$ per cent stock at 110 $\frac{1}{2}$ to produce that income, a brokerage of $\frac{1}{2}$ per cent being charged on the purchase of the stock.

341 A farmer bought 4 horses and 7 cows for Rs 2380, the prices of a horse and a cow being as 5 : 2, how much did he give for each?

342 Two rooms contain equal quantities of air. The area of the floor of one of them is 340 square feet and its height is 12 feet. Find the area of the floor of the second, whose height is 17 feet.

343 An article is first sold at a profit of 10 per cent, the purchaser then again sells it for Rs 2 4s 8d, and his gain is 15 per cent of the price at which he sells it. How much did the article originally cost?

344 The area of each of the 64 squares of a chess board is 4 2025 inches, and the outer rim of the board is $\frac{3}{4}$ of an inch wide, find the length of a side of the board.

345 If 7 men and 2 women earn Rs 510 in 8 weeks, and 4 men and 2 women earn Rs 465 in 12 weeks, what are the weekly wages of a man and a woman?

346 A gentleman divided £4 18s among 150 school children, giving the girls a shilling each, and the boys six-pence, how many boys and girls were there?

347 A man after a tour in Switzerland found that he had spent every day half as many shillings as the total number of days he had

been from home His tour costs £57 12s How many days did it occupy ?

348 On heating a piece of metal its volume is increased 24 per cent, by what percentage of the new volume does the metal decrease on cooling again to its original temperature ?

349 *A* bequeaths *B* a certain sum of money, which after paying a legacy duty of 10 per cent yields an income of £810 when placed at 3 per cent interest Find the amount bequeathed

350 If £31250 put out at Compound Interest amount in 3 years to £43904, what is the rate per cent ?

351 The total area of three estates is 1768 acres If the areas of the two smaller estates be respectively three-fifths and two thirds that of the largest, find the acreage of each

352 A debtor owing £11500 paid a composition of 6s 3d in the pound How much would a creditor to whom 2 of the whole owing receive ?

353 *A* and *B* ride a race of 31 miles on bicycles The driving wheel of *A*'s machine makes 3,410 revolutions per hour and has a circumference of 168 inches, that of *B* makes 3,520 revolutions per hour, and has a circumference of 162 inches, which will win, and by how much ?

354 A man who has a certain capital calculates that if he invest it in the $3\frac{1}{4}$ per cent stock at 91 his income will be £25 more than if he invest it in the 3 per cent stock at 88 What is his capital ?

355 *A*'s present age is to *B*'s present age as 8 : 7, 27 years ago their ages were as 5 : 4 Find *B*'s age

356 If *A*'s income be 150 per cent more than *B*'s, how much per cent is *B*'s income less than *A*'s ?

357 If the price of the 4 per cents just before the payment of a half yearly dividend be 93, what ought to have been the price three months previously, supposing no change in the value of money to have taken place during that interval ?

358 A cubical box contains 9261 cubic inches, find the expense of gilding it at 3 farthings a square inch, and reduce the result to the decimal of 10 guineas

359 In an examination in which the full marks were 6000, *A* got 12 per cent more than *B*, *B* 16 per cent more than *C* and *C* 20 per cent more than *D*, if *A* got 4872, find what percentage of the full marks were obtained by *D*

360 It is desired to put a cubical case, whose content is 4019 679 cub feet, through a square hatchway whose area is 37791 36 sq inches, shew whether this can be done

361 Divide $15 \times 48 \times 49 \times 50 + 16 \times 48 \times 49 \times 51 + 17 \times 48 \times 50 \times 51 + 18 \times 49 \times 50 \times 51 + 4 \times 48 \times 49 \times 50 \times 51$ by $48 \times 49 \times 50 + 48 \times 49 \times 51 + 48 \times 50 \times 51 + 49 \times 50 \times 51$

362 Express $\sqrt{\left(\frac{678 \times 901}{0234}\right)}$ correctly to the nearest integer

363 A man rode a bicycle from A to B , a distance of 54 miles at an average rate of 8 miles an hour, another man started from A on horseback half an hour after the bicyclist and arrived at B 15 min before him Find the ratio of their speeds

364 Add together $26 + \sqrt{(1225)} + \sqrt[3]{(015625)}$, and express the sum as the vulgar fraction of 36

365 Find the value of $\sqrt{(00139876)} - \sqrt[3]{(000030664297)}$

366 A woman bought eggs at the rate of 8 for 5d, and sold them so as to gain $1\frac{1}{2}d$ on a dozen, (1) what did she charge for each? (2) What did she gain per cent?

367 A merchant mixes 54 gallons of wine at Rs 24 per gallon, with 36 gallons at Rs 14 per gallon How much water must be added to the mixture in order that by selling it at Rs 20 per gallon he may gain 10 per cent?

368 If 12 labourers working for 10 hours a day can accomplish their task in 9 days, but after 5 days two are taken ill, find how much longer per day they must work in order to finish it, at the proper time, (1) when no fresh hands are employed, (ii) when two boys are added, a boy's work being $\frac{1}{3}$ that of a man's

369 A shilling weighs 3 dwts 15 grs, of which three parts out of forty are alloy and the rest pure silver if the value of silver rises 8 per cent, what must be the reduction in the weight of pure silver in a shilling?

370 If the 3 per cent consols be at $90\frac{1}{8}$, what sum must I invest in order to secure from them a yearly income of £466, after paying an income tax of 7d in the £, brokerage being $\frac{1}{8}$ per cent?

371 How many ninths of a shilling are there in $9\frac{1}{8}$ shillings? How many ninetieths? And what is that number whose ninth part is nine dozen and nine?

372 Simplify $-\left(\frac{2\frac{3}{4} + 3\frac{2}{3} + 3\frac{3}{4}}{4\frac{1}{2} + 5\frac{1}{2} + 10\frac{1}{2}}\right) \times \left(\frac{2\frac{4}{5} - 2\frac{7}{10}}{2\frac{1}{5} - 8\frac{7}{10}}\right) - \frac{281}{1405}$

373 M starts from C and travels towards D at the rate of 6 miles per hour, two hours afterwards N starts from C , and going 10 miles per hour reaches D 4 hours before M Find the distance from C to D

374 *A*, *B* and *C* are partners *A* receives $\frac{2}{3}$ ths of the profits, *B* and *C* dividing the remainder equally *A*'s income is increased by Rs 220 when the rate of profit rises from 8 to 10 per cent Find the capital of *B* and *C*

375 A sum of money amounts in 3 years at 5 per cent Compound Interest to Rs 9261, what would be its amount in 5 years?

376 How often between 11 and 12 are the hands of a clock an integral number of minute-spaces apart?

377 *A* and *B* walk a race of 25 miles, *A* gives *B* 45 minutes start, *A* walks uniformly a mile in 11 minutes and catches *B* at the 20th milestone, find *B*'s rate and by how much he lost in time and space

378 *A*, *B* and *C* are employed on a piece of work After 15 days *A* is discharged, one third of the work being done *B* and *C* continue at the work, and after 20 days more *B* is discharged, one-third more of the work being done *C* finishes the work in 30 days In what time would the work have been done, if *A* and *B* had continued to work?

379 The gold coinage of one nation contains 1 part of silver to 11 parts of gold without any alloy that of another nation, 1 part of alloy to 23 parts of gold It is found that $88\frac{1}{2}$ of the first weigh as much as 46 of the second The intrinsic value of silver is one sixteenth that of gold Determine the part of exchange

380 A man sold a horse for Rs 250 and thereby lost $\frac{1}{3}$ of his purchase money What part of his purchase money would he have gained had he sold the horse for Rs 400?

381 Find the greatest and least numbers of 6 digits which are exactly divisible by 789

382 Divide $\frac{1}{3}[3 + \frac{1}{3}\{3 + \frac{1}{3}\{3 + 1\}\}]$ by 125

383 A company of Sepoys proceed in 5 equal rows, and after some time arrange themselves in 7 equal rows Find the least number above 1000 which the company could contain

384 A clock loses 85 sec an hour when the fire is alight, and gains 51 sec an hour when the fire is out, but on the whole it neither loses nor gains, how long in the 24 hours is the fire burning?

385 An agent has to receive a rent paid in corn from a tenant, and to deliver it to the landlord At both payments he uses, so as to benefit himself, a false balance, such that 9lbs in one scale balances 10 lbs in the other Corn being worth 49s a quarter, the value of his plunder is £46 11s What is the corn rent?

386 *A* and *B* fire at targets, and have 55 cartridges each, *A* fires twice in 3 minutes, and *B* three times in 5 minutes how many times will *B* have to fire after *A* has finished?

387 A hollow cubical box, made of material which is $1\frac{1}{3}$ in thickness, has an interior capacity of 50 653 cub ft, determine the length of the outside edge of the box

388 The expense of painting the wainscot of a room is Rs 167 10a, at Re 1 2a per sq foot, the wainscot being 2 ft deep What will be the expense of carpeting the room with carpet 27 inches wide at Rs 2 4a per yard, the breadth of the room being to its length as 62 is to 87?

389 A and B run a race, their speeds are as 17 to 18 A runs $2\frac{1}{2}$ miles in 16 min 48 sec, B having run $2\frac{1}{3}$ miles, takes 34 minutes in finishing the course, determine the length of the course

390 What are the times between 3 and 4 when the hands of watch are equally distant from Fig 111?

391. Find all the numbers of 5 digits divisible by 9, which have unity for their 1st and 5th digits and 2 for their middle digit Enunciate and prove the principle upon which you proceed

392 How many revolutions will be made by a wheel, which revolves at the rate of 151 revolutions in 3 min, while another wheel revolving 241 times in 17 min makes 723 revolutions?

393 Simplify —

$$(1) 10! \left\{ \frac{2!}{7} + \frac{3!}{9} \right\} - \frac{4! - 2!}{6! + 2!} \times 4! \quad (11) \left\{ 3\sqrt{\frac{1+\frac{1}{3}}{\frac{1}{3}-\frac{1}{4}}} + 2\left(\frac{\frac{1}{2}+\frac{1}{8}}{\frac{1}{8}-\frac{1}{8}}\right) \right\}^2$$

394 In the centre of a room 21 ft square, there is a square Turkey carpet, the rest of the floor is covered with oil-cloth The carpet and oil cloth cost respectively Rs 8 4a and Rs 4 4a per sq yd, and the whole cost of both is Rs 352 4a Find the width of the oil cloth border

395 The masters of a school are $\frac{1}{16}$ of its whole number, but after 40 new boys have been added the masters become $\frac{1}{16}$ of the whole How many boys and masters were there before the new boys came?

396 A traveller meets two Arabs in the desert, one of whom has 8 loaves and the other 5, and the loaves are shared equally by the three In what proportion should the Arabs be paid?

397 M invests one-third of his property in Bank stock, one-sixth in Consols, and the remainder in Railway shares When he sells out he makes a profit of 5 per cent, 3 per cent, and 2 per cent respectively on the investments, and realises £6190 Required the amount of his property originally

398 The average of 8 results is 20, that of the first two is $15\frac{1}{2}$, and of the next three is $21\frac{1}{4}$, the sixth is less than the seventh by 4, and less by 7 than the eighth Find the last.

399 The telegraph posts on Railways are generally erected at intervals of 60 yards. Shew that if a traveller count the number of the posts, which pass his eye in two minutes, that number will nearly express in miles per hour the speed of the train.

400 The breadth of a room is two-thirds of its length and three halves of its height, and the contents are 5832 cubic feet. Find the dimensions of the room.

401 Decompose 831600 into its prime factors, and find the least multiplier of it, which will make the product a perfect cube.

402 A man having lived at the rate of Rs 5000 a year for 6 years, finds himself in debt, and reduces his expenditure to Rs 4500. He is out of debt in 4 years. What is his income?

403 A boat propelled by 8 oars which take 30 strokes per minute travels at the rate of $9\frac{1}{2}$ miles per hour, find the rate of a boat propelled by 6 oars which take 28 strokes per minute, the work done by each oar during one stroke in the latter case being a quarter as much again as in the former case.

404 The value of a certain length of a material, A , is $\frac{7}{11}$ of the value of $\frac{1}{5}$ as much again of another, B , and the weight of 17 yds 2 ft of A is $\frac{5}{11}$ of the weight of 15 yds 1 ft 9 in of B . If the value of 3 cwt 27 lbs of A be Rs 351, what is the value of 1 cwt 2 qrs 12 lbs of B ?

405 A and B barter. A has 27 tons of coal worth £1 2s 6d a ton, but insists on having £1 5s a ton, B has hops worth £2 14s a pocket, which he raises in price in proportion to A 's demand. A receives 6 pockets of hops. What cash does he get besides?

406 A rectangular field, whose diagonal measures 825 feet, has one of its sides $\frac{7}{5}$ the length of the other. Find the length of each side in yards, and the area in acres.

407 The sum which will pay A 's wages for $61\frac{1}{2}$ days will pay B 's wages for $81\frac{2}{3}$ days. For how many days will it pay the wages of A and B together?

408 Four persons possess respectively 14 mangoes, 24 oranges, 34 plantains and 44 peaches. Each of them gives one of his kind to each of the others, and it is then found that they are all equally rich. If the price of each plantain be 4 pies, find the price of each of the others.

409 A can excavate 142884 cubic yards per day, how many can B do per day, if A could do B 's daily quantity in $1\frac{1}{2}$ of the time that B would take to do A 's daily quantity?

410 If money invested in the 3 per cent consols give exactly

3 per cent after the payment of 1s. in the £ income tax, find the price of the consols, allowing $\frac{1}{4}$ per cent to a broker for purchase

411 A multiplication sum having been worked is partially rubbed out, the figures remaining —the multiplicand 7699, the first two (of four) figures in the multiplier 42, and the last two figures 47 in the product. Restore the complete work

412 Three men, whose strides are 2 ft 9 in., 3 ft and $3\frac{1}{2}$ ft respectively, walk 2 miles. Find how often they will step together

413 Of the books in a library, $\frac{1}{4}$ are literature, $\frac{1}{4}$ mathematics, $\frac{1}{8}$ history, $\frac{1}{8}$ philosophy, and the rest novels. What is the least number of novels that the library can have?

414 A coal merchant had 150 tons of coal, of which he sold 50 tons at Rs 27 per ton, and found that he was only gaining $7\frac{1}{2}$ per cent. At what rate must he sell the remainder, so that he may gain 10 per cent on the whole?

415 A speculator sells at a profit of 50 per cent, but his purchaser fails, and only pays 10s in the £. How much per cent does the speculator gain or lose by his venture?

416 The wheel of an engine 8 ft 2 in. circumference slips back along the rail 1 ft during each revolution, and goes round 12 times in 1", how long is it going 30 miles?

417 If 4 men dig a piece of ground 350 yds long and 144 yds broad, in $2\frac{1}{2}$ days, how long will it take 5 boys to dig one 600 yds long and 168 yds broad, the resistance of the soil in the two cases being as 4 : 5, and 4 men having the strength of 7 boys?

418 The cost of painting a room 9 ft 6 in. high, 15 ft 3 in. long and 10 ft broad is Rs 47 8a, what must be the height of another room whose length and breadth are respectively 18 ft 2 in. and 11 ft 7 in., if the painting of the walls at the same cost per sq yard amount to Rs 74 6a?

419 If the discount on a sum due at the end of $2\frac{1}{2}$ years be $\frac{80}{100}$ of the simple interest, at what rate is the interest calculated, and if the discount and interest together amount to £46 5s $5\frac{1}{2}$ d, what is the sum of money?

420 I bought goods at 23s 9d with 4 months' credit, and sold them forthwith at 25s 6d with such allowance of credit, as made my gain $6\frac{2}{3}$ per cent. How long credit did I give, reckoning interest at 4 per cent per annum?

421 650 horses are conveyed in transports to the seat of war at a cost for food of Rs 15420. A storm occurs just after $\frac{1}{4}$ th of the voyage is completed, in which 10 horses are killed. If the expense

of the food of each horse be $8a$ per day, what was the length of the voyage?

422 Brussels carpet is 2 ft wide, costs Rs 3 12a per yard, and will last 5 years, Kidderminster carpet is $2\frac{1}{2}$ ft wide, costs Rs 2 8a per yard, and will last 3 years, find the ratio of their cost, not reckoning interest on the outlay

423 Find the difference between 16 of 34 of £1 12s and $\frac{1}{10}$ of 36 of £9 12s, and find the value of

$$\frac{627 \times 0.5}{(\frac{1}{2} \text{ of } \frac{1}{4}) \times 836} - \frac{(\frac{1}{4} \text{ of } \frac{1}{10}) \times (\frac{2}{3} \text{ of } 21\frac{1}{2})}{(\frac{2}{3} \text{ of } \frac{5}{8}) + 14}$$

424 A, B, C rent between them a field of grass for 12 weeks for Rs 128 5a 4p, and put into it 3, 4, 5 score sheep respectively. After 2 weeks A added 2 score, and at the end of 9 weeks removed all, B withdrew 1 score after 3 weeks, and added 2 score at the end of the 5th week. What ought each to pay? At what rate per acre per annum is this rent, supposing the field to be rectangular, the shorter side 110 yards, and the longer side 440 yards?

425 Three merchants A, B, C engage in a speculation (of which A's share is 142857, B's 428571) and agree to deposit $\frac{1}{5}$ th of their respective contributions to the required capital at once, 230769 at the end of a month, and the remainder at the expiration of another month after the second instalment B dies, and A and C, make up the remainder, C paying three times as much as A, supposing 1857142 of the capital to be Rs 16900, what were A's and C's last deposits?

426 If, while the common rate of interest is $3\frac{1}{2}$ per cent I can get my tradesman's bills discounted as if it were 5 per cent, what is the advantage to me, in an expenditure of £450 of ready money payments, rather than leaving my bills unpaid for 12 months?

427, A is at Calcutta and B at Ichapore ($21\frac{1}{2}$ miles apart) A leaves for Ichapore at 9 A.M., and B for Calcutta at 9 30 A.M. They meet at 12-8 (noon), and find that B has walked $\frac{1}{2}$ mile an hour more than A. How many miles has each travelled?

428 The area of a verandah 3 ft broad around a room 26 ft long, is 300 sq ft. Find the breadth of the room.

429 Inside a rectangular vessel 9 ft by 7 ft is another rectangular vessel, whose sides are 6 inches apart from the sides of the larger vessel. The smaller vessel is filled with a certain fluid which costs Rs 900 at the rate of $\frac{1}{4}$ of an anna per cub inch. Find the depth of the vessel.

430 There are three pendulums. The first makes 35 beats in 36 seconds, the second 36 beats in 37 seconds, and the third

37 beats in 38 seconds. Supposing they commence together, find how many times they will beat simultaneously in 24 hours

431 In passing through a certain district, a person went 5 miles due north, then 6 miles due east, then 10 miles due south, and lastly straight to the place whence he started. Find the area of that part of the district round which he travelled

432 There is an oblong room, whose $B L = 3 \frac{5}{8}$, $L H = 3 \frac{2}{3}$, and H the thickness of its walls $3 \frac{3}{4}$. How many stones of 13 cub ft each would be required for the construction of its walls, given that there are two doorways each $4 \frac{1}{2}$ ft \times 6 ft and two windows, each $3 \frac{1}{2}$ ft \times 4 ft and that the area of the room is 135 sq ft?

433 A can do as much work in 17 days as B , C and D together in 4 days, B can do as much in 7 days as A , C and D together in 3 days, and C can do as much in 13 days as B and D together in 7 days. How long would it take A , B and C together to do what D could alone perform in 13 days?

434 The earth from a rectangular trench 8 ft deep, one of whose sides is $\frac{7}{5}$ of the other, and whose diagonal measures 25 yards, is required to fill up a square pit whose depth is 6 ft. Find the length of a side of the square and how many coolies it would take to fill up the pit in 16 days of 10 hours each, supposing that each cooly can carry $\frac{1}{4}$ cubic foot of earth at a time and make 9 trips per hour.

435 A train starts from P to go to Q , after it has travelled 1 hr 12 min, it meets with an accident which delays it 48 min, and diminishes its rate by 5 miles, hence it arrives 2 hours behind time. Had the accident happened 20 miles further on, it would have been only 1 hr 48 min late. Find the original rate of the train and the distance from P to Q .

436 A cistern can be filled by a pipe in 6 hours and emptied by another in 4 hours. They are alternately opened and closed for $1 \frac{1}{2}$ hours. Find the time in which the cistern when full can be emptied.

437 If 21 horses and 217 sheep can be kept 10 days for the same sum as it would cost to keep 9 horses and 60 sheep for 27 days, find how many sheep eat as much as 3 horses.

438 A man buys 27 sheep for Rs 90 and sells 12 of them at a loss of 3 per cent, at how much price must he sell the remainder per head, in order that he may gain $2 \frac{1}{2}$ per cent on the whole purchase?

439 Two men undertake to do a piece of work for Rs 7 8a. One could do it alone in 5 days, the other in 6 days. With the assistance of a boy they finish it in 2 days. How should the money be divided?

440 A book sent from England costs me (including 1s 6d

postage) 16s 1d But my bookseller allows me two-pence in the shilling discount on the published price What is the published price ?

441 Find the diagonal of a square whose side is one-fifth of a mile

442 A square area is bounded by 36 yds of wire fencing If the enclosed area be increased by 40 sq yds, still retaining the form of a square, how many additional yards of fencing will be required ?

443 A broker charges one eighth per cent commission on the money invested He is handed over Rs 14800 to invest in a certain stock at $92\frac{1}{2}$ and succeeds in obtaining the stock at $92\frac{1}{2}$, keeping the balance to himself, what is his total profit ?

444 A person, who pays 5d in the £ income tax, finds that a rise of interest, from 6 to $6\frac{1}{2}$ per cent, increases his income by £23 10s What is his capital ?

445 A man who has sold tea at Rs 2 8a per seer making a profit of 25 per cent, lowers his price so as to gain only 2a per seer, in what ratio must his monthly sale increase that he may make twice as much as before ?

446 The cost of converting wheat into bread is 20 per cent on the cost of the wheat If the price of wheat fall 20 per cent, and that of converting it into bread rises 20 per cent, for what should a loaf as large as an old 10d loaf be sold, so as to gain 20 per cent ?

447 There are three cubical boxes, the edge of the first is 12 in, that of the second 20 in, and that of the third 30 in Find the length of the edge of a cubical box which shall contain as much as all three

448 During a distillation the fluid that comes over in three consecutive hours contains 53, 50 and 48 per cent of alcohol respectively The ratio at which it comes over during these 3 hours are in the ratio of 1, 2 and 3 What is the percentage of alcohol in the whole mixture ?

449 Two monkeys, having stolen a pile of walnuts and filberts from a garden, are on the point of beginning their feast, when they see the injured owner of the nuts approaching with a stick At once they see that he will take $2\frac{1}{2}$ minutes to reach them There are twice as many filberts as walnuts, and one monkey finishes the latter at the rate of 15 per minute in $\frac{1}{4}$ of the time and runs away, the other manages to eat the filberts just in time If the first monkey had stopped to help the other till all were finished, find when they would have got away, (i) if they eat filberts at equal rates, (ii) if the first monkey eats filberts at the same rate as he eats walnuts

450 A goldsmith mixes together silver, copper, zinc, gold and brass, and makes an idol weighing 30 lbs, which he sells for Rs 720

ncluding a profit of 15 per cent and 5 per cent for interest on his outlay. For every one pound of gold he has $\frac{1}{3}$ of silver, $\frac{1}{2}$ of copper, $\frac{1}{6}$ of zinc and $\frac{1}{8}$ of brass, supposing that a lb of brass costs Rs 2, a lb of silver 11 times as much as brass, a lb of copper $16\frac{2}{3}$ per cent less than that of brass, and a lb of zinc $16\frac{2}{3}$ per cent less than that of copper, what was the cost price of the gold in the mixture?

451 Write in figures twelve thousand twelve hundred and twelve

452 One-fifth of the difference of two numbers = 58, and one-fifth of their sum is less than their difference by 92. Find the numbers

453 Reduce 3 roods 7 po 28 sq yds 2ft 36 in to the decimal of 1 ac .31 po 27 sq yds 2 ft 36 in

454 Reduce $(575 + 16 \times 15)$ of 4 viss to the decimal of $1\frac{19}{56}$ cwt, a viss being equal to 3 lbs 2 oz Avoir

455 The materials of an old building were sold for Rs 1500 on condition that they should be removed within 30 days under a penalty of Rs 10 per day for every day beyond 30 days. The purchaser employed 40 men at $3\frac{1}{2}$ annas per day to do the work, and after selling the materials for Rs 2365, he cleared Rs 190 by his bargain. Find the number of days the men were at work.

456 In a rectangular area 100 yds long and 50 yds broad, there are two paths crossing one another, each parallel to one side of the rectangle, and each 4 yds broad. Find the cost of paving the area with stone at 12a per sq yd, and of covering the paths with gravel at 6a per sq yd.

457 A bankrupt has goods worth Rs 9750, and had they realised their full value, his creditors would have received 13a in the rupee, but $\frac{2}{5}$ ths were sold at 17.5 per cent and the remainder at 23.75 per cent, below their value. What sum did the goods fetch, and what dividend was paid?

458 Two clocks begin to strike 8 together. The one strikes in 14 seconds and the second strikes in $10\frac{1}{2}$. Find the interval between their fifth strokes.

459 If 3 men, 2 women and 6 children, or 4 men and 7 children can dig 125 cub ft of earth in 6 days, how many cub ft would 4 men, 4 women and 4 children dig in 20 days, 2 women being supposed to do as much as 3 children?

460 Two trains set out from A and travel towards B, which is 400 miles distant from A. The one leaves A at 2 A.M. and travels at 23 miles an hour. The other leaves A at 2.30 A.M. and overtakes the former 75 miles from B. Find the rate of the latter train.

PROBLEMS

1 A pays £9 3s 4d more rates than B , their incomes being equal, living in different towns they are rated at 2s and 1s 4d in the pound respectively what is their income?

2 In running a 3 mile race on a course $\frac{1}{2}$ of a mile round, A overlaps B at the middle of the 7th round By what distance will A win at the same rate of running?

3 A person bought a French watch, bearing a duty of 25 per cent, and sold it at a loss of 5 per cent, had he sold it for Rs 30 more, he would have cleared 1 per cent on his bargain What had the French maker for the watch?

4 A four wheeled carriage travels round on a circular railway The circumferences of the two wheels of the carriage and of the two circles of rails, are proportional to 6, 7, 7000, 7014 Find the number of revolutions made by each of the four wheels in a complete circuit

5 A and B run a race A starts at the rate of 400 yards a minute, and in every successive minute diminishes his pace by a yard a minute B increases his pace by the same, and overtakes A in 4 minutes What was B 's pace at starting?

6 Three tramps meet together for a meal the first has 5 loaves, the second 3, and the third, who has his share of the bread, pays the other two 5s 4d, how ought they to divide the money?

7 A person puts £1197 out at 30 per cent per annum, interest, and spends at the end of the year £300 more than the annual interest on £1197, and thus at the end of a certain time has nothing left If he had spent £300 less than the annual interest, how much would he have had at the end of the same time?

8 A person borrows two equal sums at the same time at 5 and 4 per cent respectively, and finds that if he repays the former sum with interest on a certain date 6 months before the latter, he will have to pay in each case the same amount, viz, £1100 Find the amount borrowed and the time for which interest is paid

9 The Fort Barracks are lighted with gas from 100 burners Find the cost of lighting them per night of 10 hours at the rate of Rs 5½ for 1000 cub ft of gas, assuming that for the first 3 hours each burner consumes 1 cubic inch per second, and during the remainder of the night the light is so reduced that the consumption of gas by each burner is only $\frac{1}{4}$ th of that quantity per second

10 The distance by Railway from Madras to Salem is 206½ miles A passenger Train travelling 20 miles an hour leaves Madras at 7 A M, and a Special Train at 10 A M the same day At what rate must the latter travel, so as just to overtake the former at Jollarepet Junction (132 miles from Madras), and at what hour must

a Goods Train leave Salem for Madras travelling 15 miles an hour, so as to reach Jollarpett at the same time as the other Trains ?

11 A person having to pay Rs 10572 two years hence, invests in the 4 per cent Transfer Loan to accumulate interest till the debt shall be paid, and also an equal sum the next year. Supposing the investment to be made when paper is at 86½, and the price to remain the same, what sum must be invested on each occasion that these be just sufficient to pay the debt at the given time ?

12 A owes Rs 356 and Rs 743 to be paid in 10 months and 2½ years respectively. At what time should A pay the whole in a lump-sum to clear off the debt, so that neither may lose by the transaction ?

13 A has stock in the 3 per cent consols which produces him £300 per annum. He sells out one half at 92 and invests the proceeds in the south Devon Railway where a £50 share is worth £23. What dividend per cent per annum ought the South Devon Railway to pay so that he may increase his income £50 per annum by the operation ?

14 A person spending annually Rs 2400 saves Rs 27 8a of it quarterly by ready payments. If by this means he increases his annual saving 20½ per cent, what is his annual income ?

15 A person going from Pondicherry to Ootacamund travels 90 miles by steamer, 330 miles by rail and 30 miles by horse transit. The journey occupies 30 hours 50 min and the rate of the train is 3 times that of the horse transit and 1½ times that of the steamer. Find the rate of the train.

16 A tradesman finds that if he asks for his goods 15 per cent above the wholesale price, he can sell his whole stock in 4 months, whereas if he asks 20 per cent he requires 6 months to sell the same amount. Which will he find the more profitable system at the year's end ?

17 A cistern has two pipes, A and B, which singly could fill it in 9 hours and 10 hours, respectively. It has also two taps C and D, which singly could empty it in 12 hours and 8 hours, respectively. Suppose that when the cistern stands half full of water, A and D are turned on for 3 hours, that then B is also turned on for the next two hours, and that then A and D are turned off, and C is turned on for the next 8 hours, after which all are shut, and the cistern is found to contain 95 gallons more than its half content. Find the content of the cistern. Find also how much per hour the cistern would lose or gain, if all the pipes were set open at once.

18 Of the whole cost of constructing a Railway, $\frac{1}{5}$ is held in shares, and the remainder, £400000, was borrowed on mortgage at 5 per cent. Find what amount of gross annual receipts,—of which 40 per cent will be required for the working expenses of the line, and 8 per cent for a reserved fund,—which will yield to the shareholders a dividend of $4\frac{1}{2}$ per cent on their investments.

19 An investment was made by a certain person in the 4 per cents, when they were selling at 20 discount, twelve months afterwards, when they were selling at 16 discount he sold out, what interest did he get upon his investment?

20 A person borrows £1261 at 5 per cent compound interest, which he wishes to pay off in 3 equal yearly instalments commencing at the end of the first year, what ought he to pay yearly to effect this?

21 A bath can be filled by the cold water pipe in 9 minutes, and by the hot water pipe $11\frac{1}{4}$ minutes. A person leaves the bathroom after turning on both pipes simultaneously, and returns at the moment when the bath should be full. Finding however that the waste pipe has been open, he now closes it. In $3\frac{1}{4}$ minutes more the bath is full. In what time would the waste-pipe empty it? -

22 A square field contains 22 acres 2 roods. how long will it take a man to run round the boundary, running at the rate of 12 miles an hour? If the field be increased by 9 acre., so as to form a rectangle whose shorter side is the former side of the square, at what rate does a man run who runs round it in 1 min 39 sec longer than was occupied in running the square field?

23 A cubical block of metal of 7 84 inches side weighs 25 lbs per cub inch. A hole of square sectional area is to be cut completely through the metal perpendicular to a face of the cube, in order that the weight of the metal left may be 100 lbs. Find to three places of decimals the side of the square section.

24 A rectangular cistern $10\frac{1}{2}$ ft in length, $6\frac{1}{4}$ ft in breadth and $3\frac{1}{2}$ ft in depth, contains $140\frac{5}{8}$ cubic ft of water. What is the least number of bricks, each 9 in long, by $4\frac{1}{2}$ in wide, by 3 in thick, that must be thrown into the cistern to make the water rise to the top, a brick being found to absorb water, to the extent of one fifth of its volume?

25 A man bought a house, which cost him 4 per cent upon the purchase money to put into repair, it then stood empty for a year, during which time he reckoned he was losing 5 per cent upon his total outlay. He then sold it again for £1192, by which means he gained 10 per cent upon the original purchase money. What did he give for the house?

26 Divide £3010 into three sums, so that if the first be put out at simple interest for 3 years at 4 per cent, the second for 5 years at 3 per cent, and the third for 2 years at $2\frac{1}{2}$ per cent, the amount of the second shall be double that of the first, and the amount of the third treble that of the second.

27 How many persons can be accommodated in a concert room which is 117 ft long and 90 ft wide, allowing a space of 2 ft 3 in by 1 ft 6 in for each sitting, and a gangway 3 ft wide the whole length of the room?

28 At what distance from London will a train which leaves London for Rugby at 2.45 P.M., and goes at the rate of 41 miles an hour meet a train which leaves Rugby for London at 1.45 P.M., and goes at the rate of 25 miles an hour, the distance between London and Rugby being 80 miles?

29 If the manufacturer makes a profit of 20 per cent., the wholesale dealer a profit of 25 per cent., and the shopkeeper a profit of 20 per cent., what was the cost of the manufacture of an article bought at a shop for 17s 6d?

30 The external length, breadth and depth of a rectangular tin vessel are 14, 10 and 9 inches respectively, and the thickness of the tin $\frac{1}{2}$ an inch. When the vessel is empty it weighs 1500 oz., and when filled with water 2046 oz. Find the weight of a cub foot of water.

31 A boat's crew row over a course of a mile and a quarter against a stream which flows at the rate of 2 miles an hour in 10 minutes. The usual rate of the stream is half-a-mile an hour. Find the time which the boat would take in the usual state of the river.

32 A soldier has 5 hours' leave of absence. How far may he ride on a coach which travels 10 miles an hour, so as to return to the camp in time, walking at the rate of 5 miles an hour?

33. An express train starts from a station A at one o'clock for a station B at 30 miles an hour, and in 15 minutes is followed by an ordinary train at 20 miles an hour. A train from B to A at 25 miles an hour after travelling one hour, meets the express, and in 20 minutes more meets the ordinary train. At what time did the train leave B ?

34. Two men, A and B , start from Cambridge, at 4 and 5 o'clock A.M. respectively, to walk to London, a distance of 50 miles. B passes A at the 20th milestone, and reaches London at 5 P.M. When will A arrive there?

35 Two ships are built. Twice as many ship-carpenters are employed about the first as about the second. The first is built in 9 months, the second in 8 months. The wages of each man of the first set are 7d per hour, and they work 12 hours a day; the wages of each of the second set are 6d per hour and they work 10 $\frac{1}{2}$ hours a day. The cost of the first in carpenter's wages was £6000, what was that of the second?

36 If when 25 per cent. is lost in grinding wheat, a country has to import ten million quarters, but can maintain itself on its own produce if only 5 per cent. be lost, find the quantity of wheat grown in the country.

37 Two trains start at the same time, the one from Calcutta to Delhi the other from Delhi to Calcutta. If they arrive in Delhi and Calcutta respectively 1 hour and 4 hours after they passed each other, show that one travels twice as fast as the other.

38 A franc being worth $9\frac{3}{4}d$, find the sum of money which can be paid by an exact number of either shillings or francs, the number of francs exceeding the number of shillings by 27

39 A poor fellow ignorant of Arithmetic, had Rs 3 4a in his purse, he talked of spending $\frac{1}{2}$ of it on cakes, $\frac{1}{3}$ of it on fruit, and $\frac{1}{4}$ of it on a knife Show (i) his mistake, and (ii) how he might have spent his money in proportion to the fractions he intended

40 A person sells 40 horses at a gain of 10 per cent, and 50 horses at a gain of 20 per cent Had he sold all of them at a uniform profit of 15 per cent he would have got Rs 40 less. Find the cost price of each horse

41 A ship's crew sailed with provisions for 30 days, after being at sea 20 days they encountered a storm in which they lost 6 men, and 2 days after the storm they took on board 18 men who had been wrecked and were without provisions, they then found that to make their supplies last so long as was intended each man's daily allowance must be reduced to $\frac{5}{8}$ of what had been before, how many men were on board when the ship set sail?

42 A man has his money in a bank, which pays him 5 per cent interest He buys a £100 share in a company, of which £20 is to be paid up at once At the end of the first year, a dividend is declared at the rate of 10 per cent on the paid-up capital At the end of the second year, a further call of £20 per share is made, and no dividend is declared At the end of the third year, a dividend is declared at 12 per cent on the paid-up capital How much will he have gained or lost by his speculation at the end of the three years?

43 The transverse section of a river channel is a semi circle whose diameter is half a-mile, how much water passes through it in 7 hours, supposing the average velocity of the stream to be 2 miles an hour? [Area of a circle = $3\frac{1}{2} \times (\text{radius})^2$]

44 If the wholesale dealer sell to a retailer at 10 per cent profit, and the retailer sell to the consumer at 50 per cent profit, what proportion of the price paid by the consumer is profit?

45 If when corn is 15s 9d a quarter, and hay $5\frac{1}{2}d$ per stone, 7 horses can be kept 8 days for £4 1s 3d, how many weeks can 16 horses be kept for £95, when corn is 2s a bushel, and hay 70s a ton, supposing that 126 lbs of hay are consumed with 1 bushel of corn?

46 In a certain manufactory, 158 men of ordinary ability, and working the same number of hours each day, execute a certain piece of work, but if the abilities of 2 of them had been, respectively, $\frac{1}{2}$ and $\frac{1}{3}$ less than ordinary, and the abilities of 2 others $\frac{2}{3}$ and $\frac{3}{4}$ more, the work could have been finished $\frac{2}{3}$ of an hour sooner How many hours a day did the men work?

47 If gold be beaten out so thin that an oz Avon will form

a leaf of 20 sq yds, how many of these leaves will make an inch thick, the weight of a cubic foot of gold being 10 cwt 95 lbs ?

48 Two houses are built, the first is twice as long in building as the second, half as many men again are employed in building the first, their wages per hour are one third higher, and they work 10 hours a day and 6 days a week, whilst the others work only 8 hours a day and 5 days a week, the cost of the second in workman's wages was Rs 1000. What was that of the first ?

49 A man fastens his cow by a rope to a stake in the hedge of an oblong grass field. What must be the length of the rope if the cow be allowed 1 acre to graze upon ? Given that the area of a circle $= 3\frac{1}{2} \times (\text{radius})^2$

50 A sold a horse to B, who sold it to C at a profit of 5 per cent, who sold it for Rs 1071, gaining thereby 20 per cent. What did B give for the horse ? What gain per cent was the last price on the first price ?

51 Suppose that 15 men would be necessary to excavate 966 cub yds in 8 days of $10\frac{1}{2}$ hours each. —How many men did a contractor engage for 12 days of $7\frac{1}{2}$ hrs, to excavate 575 cub yds, if he found it necessary to engage 4 additional men during the last 4 days, in order to complete the work in 12 days ?

52 The ready-money price of a dress at a tradesman's, who allows mercantile discount in the ordinary way for ready money, is Rs 85 8a and the credit price is Rs 90. What ought the credit price to be, in order that, while charging the same ready-money price, he may allow twice the rate of discount ?

53 I sell a horse for Rs 306, and lose, 15 per cent on what I gave for him, whereas I ought to have gained 25 per cent, how much did I sell him under the price I had fixed on ?

54 A baker fixes the price of a quartern loaf, when the cost of converting wheat into bread is 5 per cent on the cost of the wheat, with a view to gaining 20 per cent on the cost of his bread. Shew that if wheat falls 16 per cent and the cost of converting it into bread rises 20 per cent, he ought to sell seven loaves for the price at which he before sold six, and that if he does not alter his prices he will gain on his outlay 40 per cent instead of 20 per cent.

55 A bar of metal, one square inch in section, can just suspend a weight of one ton, also a cubic inch of the metal weighs 4 oz. Find the length of the longest wire of uniform section into which a cubic foot of the metal can be drawn, so as not to break when suspended at one end.

56 A, B and C enter into partnership, A and B place their capital in the hands of C, who is to have the sole management, but who does not invest any money in the business. It is agreed that when the net annual value of the profits are not less than 5 per cent

on the capital, A and B shall have 4 per cent on their money, and of the rest C shall receive one-half, and the other half shall be divided between A and B in proportion to their investments. If A embark in the concern half as much again as B , and C receive Rs 2500 when the profits are 5 per cent, find the amount invested by A and B , and the share of each partner when the net profits are Rs 35000

57 If 40 men, working 24 days of 8 hrs can build a wall $5\frac{1}{2}$ ft high, 34 feet thick and 36 yds long, how many days of 10 hrs will 480 boys require, to raise a wall $3\frac{1}{2}$ times the size? It being known that whilst 12 boys can build a cubic yard, 5 men can build

$\frac{1}{2}$ of $\frac{21}{106}$ of a pillar 12 feet high, with a square base whose breadth is $1\frac{1}{2}$ feet

58 A and B who are at the opposite extremities of the diameter of a circular area 135 miles in circumference, start to go round it at the same time in the same direction, A at the rate of 11 miles in 2 hours and B at the rate of 17 miles in 3 hours. How many rounds will each take before the one will overtake the other, and how long will the chase continue?

59 In a mixture of wine and water, the ratio of the wine to the water is 2 : 3. A gallon more wine being poured into the mixture, there is now as much wine as water, of how many gallons did the original mixture consist?

60 On the Eastern Counties Railway an ordinary train takes 50 minutes from Ely to Cambridge, an express train a quarter of an hour less. Supposing an express to leave London at 10 57 A.M. and arrive in Cambridge just as an ordinary train is leaving which arrives in London at 4 20 P.M., find the times respectively taken by these trains.

61 In a theatre the number of seats in the stalls is $\frac{1}{2}$ of the number of seats in the boxes, and are together twice as numerous as the seats in the pit. The number in the pit = 4 times the number of seats in the gallery, and taken together afford as many places as the boxes. The number in the gallery is 100, find how many persons the house will accommodate, and state the number of seats in each portion.

62 A , B and C are sent to empty a cistern by means of two pumps of the same bore. A and B go to work first, making 37 and 40 strokes respectively per minute, but after 5 minutes they make each 5 strokes less a minute, but after 10 minutes more A gives way to C , who works at the rate of 30 strokes per minute. The cistern is emptied in 22 minutes altogether, and the men are paid 12s 7d for their labour. What should each receive?

63 A ship having a crew of 26 persons carries provisions for 21 days, after having been at sea for 11 days they pick up a party from a wreck, and it is then found that the provisions will be exhausted in 5 days, how many persons were taken from the wreck?

61 In a company paying 10 per cent on all its capital, a person buys a £100 share at 92 premium, and afterwards he takes up a new share allotted to him at par, and he finds that he makes 6 per cent on his money, what was the amount of the new share?

65 Y sells Z a horse, Z gives Y a bill at 3 months for Rs 555. At the expiration of 1 month Y gets the bill discounted at 6 per cent, what ready money should he receive?

66 How many pipes each $1\frac{1}{2}$ ft in length will be required for draining a square field containing 11 ac 4 po with parallel drains, $5\frac{1}{2}$ yds being the distance between each two drains and also the distance between the extreme drains and the fences?

67 What is the distance between the hour and the minute hands of a watch at 17 minutes past 4?

68 A bath is supplied with water from two pipes, one of which can fill it in $12\frac{1}{2}$ minutes, the other in 15 minutes, there is also a discharging pipe which would empty it, when filled, in 10 minutes. The first pipe is open alone for 4 minutes and then the first and second open together for one minute, if now the third pipe be opened as well, how long will it take to fill the bath?

69 A certain reef of quartz when crushed yields 0011 per cent of gold. If the working expenses amount to 62.5 per cent of the gross receipts, and the profit on each 100 tons crushed amounts to £52 10s, find the number of grains in sovereigns.

70 Two watches A and B whose rates are uniform, at noon yesterday indicated 11-55' and 12-2' respectively. A indicated the correct time at 9 P.M. yesterday, and B at 6 A.M. this morning. When did A and B last agree, and what time did they then indicate? When will they agree next?

71 A merchant sells 49 quarters of wheat at a profit of 7 per cent and a certain number of quarters at a profit of 11 per cent. The price of a quarter of wheat being £3 12s 6d, he would have lost £2 10s 9d if he had sold the whole at a profit of 9 per cent. Find the total number of quarters of wheat sold by him.

72 A cistern the cubic contents of which are 360 cubic feet, has two pipes which can empty it in 3 and 4 hours respectively. It has also a third pipe with an orifice of 1 sq ft through which water flows into the cistern at the rate of 1 yard per minute. If all the three pipes be opened together when the cistern is full, in what time will it be emptied?

73 A and B start at the same time on a journey. A walks at the rate of 4 miles an hour and B of 3 miles an hour. When A has gone half way, B gets a ride, and goes at twice the rate of A , until he has ridden a distance equal to $\frac{2}{3}$ of the whole journey beyond the spot at which he passes A . B then walks the remainder

of the journey, A having walked it all Will A or B arrive first ? And what fraction of the whole journey will the other still have to travel ?

74 A and B exchange goods, A gives 13 cwt of hops, the retail price of which is 56s, but in barter he rates them at £3 B gives 10 barrels of beer, the retail price of which is 1s a gallon, but the value of which he raises in proportion to the increased price of the hops How much must B give in money ?

75 If gold be at a premium of 20 per cent, and a person buy goods marked 135 dollars, and offers gold to the amount of 135 dollars, what change ought he to receive in notes, 5 per cent being abated for ready payment ?

76 A grocer buys twice as much black tea as green, giving 2s a pound more for the green than the black, he retails it when mixed at 5s a pound, and makes 25 per cent on his outlay What did he give for each sort of tea ?

77 Ash saplings after five year's growth are worth 1s 3d, and increase in value 1s 3d each year afterwards Each is allowed 40 sq yds of ground, and they are cut after 20 years' growth, what will then be the value of an acre ?

78 A man and a boy are to work on alternate days at a piece of work, which would have occupied the boy alone 13 days If the boy take the first day, the work will be finished half a day later than if the man commences Find how long they would take to do it working together

79 A man has three vessels, P , Q , R , holding 1, 2, 4, gallons respectively, P is empty, Q is full of water, R is full of wine He fills P from Q , replenishes Q from R , and empties P into R When he has performed this operation twice, what will be the proportion of the wine in Q to the water in R ?

80 Two boats start to row a race at 3 o'clock The race is over at 6 $\frac{3}{4}$ minutes past 3, the losing boat being 40 yards behind at the finish At 4 minutes past 3 this boat being 700 yards from the winning post Find the speed of each boat in miles per hour

81 An up train 88 yds long, travelling at the rate of 35 miles an hour, meets a down train 88 yds long, at 10 o'clock and passes it in 6 seconds At 15 m 3 sec past 10 the up train meets a second down-train, 132 yds long, and passes it also in 6 seconds At what time will the second train run into the first ?

82 A tree grows each year one inch less than it did the previous year, and it grew a yard during its first year, the value of the tree at any time is equal to the number of pence in the cube of the number of yards in its height find what the tree is worth when it has done growing

83 A train, going at the rate of 60 miles an hour overtakes another train, 160 yds long, going 45 miles an hour in the same direction on a parallel line, and passes it in half a minute. In what time would the first train pass another, of its own length, going in the opposite direction at the rate of 30 miles an hour?

84 The apparent intensity of a light varies inversely as the square of its distance. Find the point (1) between two lamps 50 yards apart, (11) in a straight line beyond one of them, at which one appears twice as bright as the other.

85 If three concentric rings be drawn in one plane and the diameter of the external ring be 139 feet, find the radii of the others to three places of decimals so that the whole area may be divided into three parts by the rings in the ratio 4, 5, 11 [The area of a circle = $3.14159 \times (\text{radius})^2$]

86 Four coins of equal weight are made of gold and silver in the ratio 1, 2, 2, 3, 3, 4, 4, 5 respectively. They are melted together and recast. In what ratio will the gold and silver be in the new coins?

87 From a cask of wine $\frac{1}{2}$ is drawn off and the cask is filled up with water, $\frac{1}{2}$ of the mixture is then drawn off and the cask is again filled with water, after this process has been repeated 4 times altogether, what fraction of the original quantity of wine will be left in the cask?

88 Two passengers are charged for excess of luggage Rs 1 12s 8p and Rs 4 3s 4p respectively. Had the luggage all belonged to one person he would have been charged Rs 9 8s for excess. How much is allowed free the charge for excess being 4p per seer?

89 A person shooting at a target at a distance of 500 yards, hears the bullet strike the target 4 seconds after he fired. A spectator equally distant from the target and the shooting point, hears the shot strike $2\frac{1}{2}$ seconds after he heard the report of the gun. Find the velocity of sound.

90 A merchant buys China tea at 3s 6d per lb. To improve the flavour, he adds 2 oz of Assam to every lb of China tea, and finds that the mixture costs him 4s a lb. How much per lb did he give for Assam?

91 If a certain number of workmen can do a piece of work in 25 days, in what time will $1\frac{2}{3}$ of that number of men do a piece of work twice as great, supposing that 2 of the first set can do as much work in an hour as 3 of the second set can in $1\frac{1}{2}$ hours, and that the second set work half as long a day as the first set?

92 A train running at the rate of 40 miles an hour meets a person walking along the line in the opposite direction at the rate of 4 miles an hour, and passes him in $5\frac{1}{2}$ seconds. Find the

length of the train Had the person been going in the same direction as the train, in what time would it have passed him ?

93 Rs 180 is to be divided among 60 men, women and children The sums of the men's shares, women's shares and children's shares are in the proportion of 5 4 3, but their individual shares are as 3 2 1 respectively Find the number of men, of women, and of children

94 A spirit merchant buys two sorts of spirit in equal quantities, one at 1s per gallon more than the other, he mixes them and sells the mixture at 16s 6d per gallon, gaining 10 per cent on his outlay What was the price paid per gallon by the merchant ?

95 A man spends Rs 1200 in buying horses and cows, at the respective rates of Rs 54 and Rs 48 each He sells them at a uniform rate of Rs 51 each, and gains 2 per cent How many horses did he buy ?

96 A crew, who can row at 4 miles an hour in still water, rows down a certain distance and up again in 1 hour 36 min Find the distance in miles, if the velocity of the current be $1\frac{1}{4}$ miles

97 Two trains, running on parallel rails in opposite directions at 40 and 30 miles per hour respectively, are observed to pass one another in 6 seconds When they are running in the same direction, a man in the faster train finds that he passes the slower train in 24 seconds What is the length of each train ?

98 On a piece of work 3 men and 5 boys are employed, who do half of it in 6 days After this one more man and one more boy are put on, and $\frac{1}{2}$ more is done in 3 days How many more men must be put on that the whole may be completed in 1 day more ?

99 A man has £1583 17s 11d in the 3 per cent stock and £982 12s 6d in the $3\frac{1}{2}$ per cent stock, he transfers a certain sum from the former to the latter when the stocks are at 91 and 98 respectively and thus makes the income derived from each the same How much has he finally in the 3 per cent stock ?

100 A person has a certain sum to invest, he divides it equally between the $3\frac{1}{2}$ per cents at 84 and the 4 per cents at 88 and finds that the latter stock gives him an annual income of £3 7s 6d more than the former, what is the sum ?

101 Two clocks begin striking the hour of noon together on a certain day, the interval between every two strokes being 1" and 2" respectively They gain 1" and 2" respectively in every 24 hours Shew after what length of time they will end striking the hour of noon together, both shewing the correct time at the fall of the last stroke

102 The height of a room is $\frac{1}{2}$ of the sum of its length and breadth, and the cost of papering its walls at 6d per square foot is Rs 31 4a Find its height

103 A mixture consists of milk and water mixed in the ratio of 8 : 1, 5 seers of water were then added, and it was found that the quantity of water is $\frac{1}{4}$ of the quantity of milk. Find the quantity of milk in the mixture.

104 In a certain meadow there is a crop of 525 stone of grass, which grows uniformly. If 11 oxen turned in would consume all the grass in 48 days, but 6 oxen would require 98 days, what weight of grass would each ox eat in a day?

105 Four apples are worth as much as 11 plantains, 8 mangoes as much as 5 apples, 2 oranges as much as 7 plantains, and 9 plantains sell for a 2-anna piece. I wish to buy an equal number of each of the four fruits, and to spend an exact number of 2 anna pieces, find the least sum I require.

106 Three boys begin to fill a cistern. One of them brings a pail at the end of every 3 minutes, another a quart every 5 minutes, and the third a gallon every 7 minutes. If the cistern holds 53 gallons, in what time will it be filled?

107 Two trains on the same railway are running past each other in opposite directions, one at 40 and the other at 30 miles per hour. Each has an engine and tender, the first has 12 carriages, and the second 17. If the length of an engine and tender be 40 ft and the length of a carriage 32 ft, and the coupling spaces be each 5 ft, how much time will elapse from the moment that the engines meet till the last carriages of each train have passed each other?

108 An empty cistern had two supplying pipes *A* and *B*, and two taps *C* and *D*. *A* would fill the cistern in $42\frac{1}{2}$ min and *B* in 46 min, and *D* can carry off per minute half as much again as *C*. After *A* and *B*, running together, have supplied a certain quantity, *C* is allowed to run with them, and takes 51 min to empty the cistern, but had *D* been turned on along with *C*, the two would have taken only $5\frac{1}{2}$ min to empty it. In what time would the cistern have been emptied if *D* had been turned on instead of *C*? And how much of the cistern was filled when *C* was set upon?

109 A tradesman imported a quantity of foreign goods for which he had to pay a duty of 15 per cent, but the demand for them having gone off he is obliged to sell at a loss of $5\frac{1}{4}$ per cent, a month earlier he could have sold them for Rs 1242 more, and then would have cleared $2\frac{1}{4}$ per cent on the transaction, what price did he pay for the goods?

110 The paper duty was 1s 7d a lb, the weight of a newspaper $\frac{1}{4}$ lb, the paper manufacturer made 20 per cent profit on his outlay, and the proprietor of the newspaper 10 per cent on his sale. What reduction should be made in the price of the newspaper, now that the duty on paper is abolished, allowing to each tradesman the same rate of profit?

111 In buying rice on three occasions I found the prices per bag were as 1 2 3 respectively. The price of 30 bags, 10 bought on each day, was Rs 480. Find the price per bag on each day.

112 A field of 7 acres is sown with wheat, barley, and maize, the areas of the crops being respectively as $2\frac{1}{2}$ $3\frac{1}{2}$ $4\frac{1}{2}$. If the values of an acre of each be also respectively in the same ratios, and an acre of wheat be worth £7, what is the worth of all the crops in the field?

113 A clock and a watch together cost Rs 24. If the value of the watch were to rise 30 per cent and that of the clock 40 per cent, both would together cost Rs 32. What is the price of each?

114 A person went to a tavern with a certain sum of money; there he borrowed as much as he had about him and spent 1s out of the whole, with the remainder he went to a second tavern where he borrowed as much as he had left and also spent 1s, and he then went to a third tavern borrowing and spending as before, after which he had nothing left. How much had he at first?

115 An oz of gold is worth £3 18s, and an oz of silver is worth 7s. A mass of gold and silver weighing 170 oz is worth £166. What is the worth of another mass, in which the numbers respectively of gold and silver in the first mass are interchanged?

116 A cord 95 ft long, when fully stretched, reaches from the top of a pole 57 ft high, standing vertically, to a point in the ground. A shorter pole is placed vertically between the foot of the longer pole and this point at a distance of 48 ft from the point, and it is found that the cord just reaches its top. Find the height of the longer pole over the shorter pole, it being given that the length of the cord between them is $\frac{1}{10}$ of the length of the entire cord.

117 A person bought 4 railway tickets to go 60 miles. Two were for the 1st class, one for the 2nd, and the fourth a half first class ticket for a child. The cost of a second class ticket was $\frac{2}{3}$ of that of a first class, and the whole sum was Rs 15 13s 4d. Find the price of each ticket, and the rate per mile for the first class.

118 A and B set out from the same place in the same direction. A travels uniformly 18 miles per day, and after 9 days turns and goes back as far as B has travelled during those 9 days, he then turns again, and pursuing his journey overtakes B at the end of 22 $\frac{1}{2}$ days after the time they first set out. Find the rate of B in miles per day.

119 A person bought goods on the continent, the cost of freight and insurance was 15 per cent, and that of duty 10 per cent on the original outlay, he was obliged to sell them at a loss of 15 per cent, but if he had made Rs 80 more of them he would have gained 1 per cent. What was the original outlay?

120 It is agreed that the rent of a farm shall consist of a fixed sum together with the value of a certain number of maunds of wheat, when wheat is Rs 3 8s per maund, the rent is Rs 312 8s, when

wheat is Rs 3 12a per maund, the rent is Rs 325, what will the rent be when wheat is Rs 5 a maund?

121 Riding a journey of 27 miles into town, I meet the coach which left town at the same moment that I started from home (7 o'clock), at the 18th milestone from town. Supposing that it travels 10 miles an hour, determine the hour when we meet, and the time when, (proceeding at the same rate as before) I shall reach Calcutta.

122 A, B and C are partners, A receives $\frac{1}{2}$ profits, and B twice as much as C, find the capital of C, A's income being diminished Rs 400 by a fall of $\frac{1}{2}$ per cent in the rate of profit.

123 A person increased his capital annually by one third part, and at the end of 4 years, one year's interest thereon at 4 $\frac{1}{2}$ per cent amounted to Rs 2700. What capital did he start with?

134 Divide Rs 2025 among A, B, C, D, E, so that A's share B's share 1 2, C's share B's share 5 4, D's share C's share 6 5, and E's share D's share 4 3.

125 The length of a room is 20 ft, the cost of carpeting the floor at 6s 6d per sq yd is £11 11s 1 $\frac{1}{2}$ d, and the cost of papering the walls (inside) at 10d per sq foot is £36, the area of the walls (outside) the room is 1056 sq ft, find the thickness of the walls.

126 To what uniform depth must a piece of ground 414 yds long, 37 yds wide be excavated, that the earth taken out may form an embankment of 25530 cub yds, supposing the earth to be increased one ninth in volume by removal?

127 A train which travels at the uniform rate of 30.8 ft a second leaves Madras at 7 A.M., at what distance from Madras will it meet a train which leaves Arconum for Madras at 7.20 A.M. and travels one third faster than it does, the distance from Madras to Arconum being 42 miles?

128 When the income tax was 7d in the pound, a person had to pay £63 more than when it was 4d in the pound, although his income had diminished in the interim by £22 $\frac{1}{2}$. What was his income at first?

129 It costs Rs 90 to mat the floor of a room whose dimensions are as 6 5 at 12a per sq ft, and it takes Rs 155 more to paper its walls *outside* at 10a per sq foot. If the height of the room be 8 ft, and there are four doorways in it, the area of each of which is 4 sq ft, find the thickness of the wall.

130 The price of a mixture of two metals, weighing 14 oz is Rs 34. If the proportions of the two metals in the mixture be interchanged, its price is Rs 36. Supposing the difference between the prices of 1 oz of each metal to be Re 1, find the price per cwt of each of the metals.

131 If a publisher, in selling a book for cash, rates it at 25 per cent below publishing price, and then charges for 13 copies as 12, how long credit could he allow, so that, on the principle of true discount at 4 per cent per annum, the sum to be received for a book should be just 29 per cent below publishing price?

132 A boatman rowing against the tide passes a body floating with the tide, and in 9 minutes afterwards is a mile distant from it, in 35 minutes more he rows $2\frac{1}{2}$ miles, and then returns. At what rate per hour does he return, supposing the tide to flow uniformly in one direction?

133 A dealer buys 18 cwt 3 qrs at 1s 3d per lb, which, to obtain a fair profit, he should retail at $8\frac{1}{3}$ per cent above cost price. But, while he professes to sell at the rate of 3 lbs for 3s 10d, he serves his customers, to his own advantage, with a false balance, in which 10 lbs weighs $10\frac{1}{2}$ lbs, and at the same time he uses a false lb of 6860 grains. How much does he make beyond the fair profit?

134 Divide 33 cwt 2 qrs 22 lbs into three such parts that 6 times the first, 9 times the second, and 10 times the third may be equal amounts.

135 The interval between the firing of two guns, at a railway station was 6 minutes, and a passenger in a train, approaching the station at a uniform rate, heard the second report 5 min 51 sec after hearing the first. Now, suppose that the sound of the train's approach to have become audible at the station when the train was 2 miles off, how soon after that did the train pass the station,—sound travelling 1125 feet per second?

136 A boatman rows 5 miles with the tide in the time he would take to row 3 miles against it, but if the hourly velocity of the current were $\frac{1}{2}$ a mile more, he would move twice as rapidly with the tide as against it. What is his power of rowing in still water?

137 The only three creditors of an insolvent, whose assets amounting to £200 can only pay 5d in the £, agree among themselves to take dividends in the proportion of the number of £ s and d, respectively contained in the amounts due to them. The dividends thus taken are in the proportion of 12 7 6. What are the amounts of their debts?

138 The sum of £2000 is due in 4 years, but it is paid by instalments as follows —£275 at the end of 2 years, £460 at the end of the 3rd year, £500 at the end of the 4th year and £600 at the end of the 5th year. What amount should be paid at the end of the 6th year, in order to clear off the balance, simple interest being reckoned at 5 per cent per annum.

139 A metal is composed of 11 parts gold and 1 copper, another of 5 parts gold and 8 silver, and a third of 12 parts silver and 1 copper. These are mixed together in the ratios of 8 9 10, and

the compound weighs 1 cwt 5 lbs, how much gold, silver and copper respectively are contained in the mixture?

140 A barrack has to be built to hold 50 beds which are to be arranged in single file, with their lengths parallel to the shorter side of the room, each bed being 6 ft 2 in long and 3 ft 1 in broad. A space of 2 ft 3 in is to be left between each bed, and a passage 3 ft 6 in wide is left on all sides between the walls and the bed. Given that each individual requires 1000 cub ft of air per night, and that each bed with its occupant takes up 50 cub ft of space, required the dimensions of the room

141 Two carriages are in motion in parallel lines and in the same direction along a straight road, the distance between their fore wheels is 600 yds. The circumferences of the fore-wheels are to each other as 7 : 6, that of the first being 3 yds 1 ft, the fore-wheels of the first and second carriages turn round respectively 14 and 15 times every minute, find (i) when they were alongside of each other, (ii) when they were 100 yds apart

142 The area of a rectangular space is an acre, and its length and breadth are in the ratio of 5 : 2, around this space on the inside is a path whose breadth at any point is $\frac{1}{10}$ of the distance to the opposite path, find how many bricks will pave the path, allowing 45 bricks for a square yard

143 A wine merchant buys 3 kinds of wine and mixes them in this proportion : 1 pipe of the first kind, the price of which is Rs 800 a pipe, 3 pipes of the second, the price of which is Rs 900 a pipe, 2 of the third kind. He keeps this mixture for 12 months and then sells it at Rs 10.45 a pipe, clearing 10 per cent, after allowing 4 per cent for interest of capital. What was the original price of the third kind of wine?

144 The Governors of Queen Anne's Bounty advance £845 on mortgage of a living on the following conditions : the principal to be repaid in 30 years by equal annual instalments and interest at the rate of $3\frac{1}{2}$ per cent to be charged on the part unpaid. If the sum due in any particular year be £43 18s 9 $\frac{1}{2}$ d, find how many previous annual payments have been made

145 If 9 oxen are kept for the same money as 7 horses (for any given time), and a team of oxen are one-fifth as long again in ploughing 97 acres as the same number of horses are in ploughing 90 acres and a field costs as much whether ploughed by oxen or by horses viz, £7 5s 6d, the same men being required in both cases, and paid by the time, what is due to them?

146 A shed rests upon two walls, one 10 ft, the other 8 ft 7 in high, and covers a space of ground 8.4 ft in breadth by 15 ft in length. Find the price of roofing it at Rs 9 a square yard

147 A bill of £61 due 4 months hence would with interest

amount to £63 or 8*d* at the end of the year, find its present value at the same rate of interest. If at a shop where a year's credit is allowed, I by paying ready money obtain 13 articles to the dozen, at what rate per cent is the discount allowed?

148 *A* with a capital of Rs 60000 began business on the 1st day of January, and wishing to extend his trade, he took in *B* as partner, with a capital of Rs 50000 on the 15th March following, and on the 27th May they admitted *C* as a third partner, who brought Rs 70000 into the concern. On taking stock at the end of the year, they find the profits of the firm to be Rs 24850. How must this sum be divided amongst the partners?

149 From a place *A*, a messenger goes to a place *B*, distant 21 miles from *A*, and immediately returns, going at the rate of 4 miles an hour, and simultaneously with the messenger's departure from *A*, another messenger starts from *B* at the rate of 3 miles an hour, goes to *A*, and immediately returns. Find the distance between the two points at which they cross each other.

150 A company is formed in which the liability of each partner is limited to the amount of his shares. There are 500 shares of £10 each, after 3 calls have been made of £2 on a share, it is found that the concern is a failure, and its affairs are wound up. At this period its assets amount to £10217 or 0*d* and its liabilities to £15763 17*s* 6*d*. How much will the company be able to pay in the pound after all the remaining calls are paid up?

151 If 36 oxen in four weeks eat up the grass on a field of 12 acres and what grows upon it during the time, and 21 oxen eat up the same in 9 weeks, how many oxen will it maintain for 18 weeks, supposing the grass to grow uniformly during the time?

152 The premium of gold at Paris is $7\frac{1}{2}$ per mille, which, at the English mint price of £3 17*s* 10*d* per oz for standard gold, gives exchange 25 35 $\frac{1}{2}$, and the exchange at Paris on London at short, being 25 33 $\frac{1}{2}$, it follows that gold is about 0.09 per cent dearer in Paris than in London.

153 Incomes below £150 a year being subject to 5*d* in the pound income tax, and incomes above £150 to 7*d* in the pound, find what income above £150 a man must have, that he may be just 7*d* a year poorer than a man who has £149 10*s* a year.

154 *A* has £90000 stock in the 3 per cent South Sea Annuities, and is offered by Government the choice of being paid off at par at the end of the year, or of receiving £110 of a new 2 $\frac{1}{2}$ per cent stock for each £100. He chooses the former alternative, and, on being paid off, is able to invest his money in the 3 per cent consols at 92. Find the amount of his stock in consols and the excess of his income above what it would have been if he had agreed to the proposed conversion.

155 If 28 men can excavate 750 cub yds in 4 days working $6\frac{1}{2}$ hrs a day, what uniform length of day will 24 men require, to excavate 615 cub yds in $3\frac{1}{2}$ days, supposing that any 5 of the latter party can do as much in 4 hours as any 6 of the former can do in $3\frac{1}{2}$ hours, and that 2 men will be withdrawn from the latter party after 2 $\frac{1}{2}$ days' work?

156 Eight billiard balls revolve with equal velocities in eight concentric circular grooves. They start from a position in which they are all in the same radius of the outermost circle. The innermost ball occupies 5 sec in traversing its groove once. After what time will they all be again in the position in which they started, the radius of the circular grooves being proportional to the nos 1, 3, 5, 7, 9, 11, 13, 15. When first will the 1st, 3rd, 5th, 7th, be in the same position in which they started?

157 A vessel is full and contains 36 gallons of water. Two pipes are opened, through one of which the water runs off continuously at the rate of 100 gallons per hour, whilst the vessel is irregularly supplied by the other which is first open for 2 min, when the water flows in at the rate of 36 gallons per hour and then closed for 3 min and so on. Find when the vessel will be emptied to parts of a second.

158 A packet sails from Dover to Calais, and starts at the rate of 15 miles in 16 of an hour, but, after sailing (18—27) hours at this rate, the wind drops, and, for the next (35—2) hours she makes only 5 of the way she did, however, the wind again freshening, her rate of sailing per hour is in consequence increased by $\frac{5}{71428}$ of the difference between her former ones, supposing her rate thence forward to continue uniform, and the distance between the two ports to be 2203571428 miles, find what time was occupied in the passage.

159 *A* and *B* start at the same time walking to and fro a course of 1 mile, *A* walks a mile in 13 minutes, *B* in 18 minutes, find the series of points where *A* crosses *B* and the series of points where *A* overtakes *B*, distinguishing those where *A* is walking towards the starting point from those where he is walking from it.

160 If a man can beat a boy $\frac{2}{3}$ of a mile in a race of 4 miles, how much start must the boy have in order to reach the tenth mile stone at the same time as the man? And supposing the man's rate is 5 miles an hour, how long would it take the boy to go $8\frac{1}{2}$ miles?

161 A baker's outlay for flour is 70 per cent of his gross receipts, and other trade expenses are 20 per cent. The price of flour falls 50 per cent and the other trade expenses are thereby reduced 25 per cent. What reduction should he make in the price of a five anna loaf allowing him still to realise the same amount of profit from it?

162 If I am allowed $1\frac{1}{4}$ per cent discount on a amount

charged to me for goods and give my acceptance at 5 months for the net sum, and if by selling the goods forthwith for a bill of £162 12s 2d payable in 7 months, my present gain is $11\frac{1}{2}$ per cent; what is the amount originally charged to me, interest being reckoned at 5 per cent per annum?

163 Three persons invest sums of money in a business in the ratio of 3 2 1, but agree to divide the profits in the ratio of 4 2 1, the profits of the concern are 10 per cent on the whole money invested, what interest will each partner make of his money?

164 Two hands (as in a clock face) revolve round a common centre 20 and 30 times respectively in 24 hours, starting together from a given position in the same direction. How often during 24 hours will they be in a similar condition after complete revolutions? Also after what times will they be $\frac{1}{3}$ ths of a revolution apart?

165 A and B have invested Rs 1500 and Rs 2450 respectively in the same business for a year, it is expected that the rate of profit will be doubled the following year, and A calculates that if he doubles his capital the profits of the business for the two years will amount to Rs 1039 8s, what does he expect at his share of this?

166 Two circular gold plates, each an inch thick, the diameters of which are 6 in and 8 in respectively, and melted and formed into a single circular plate, also an inch thick, find its diameter. Having given that the area of a circle equals $\pi \times (\text{radius})^2$.

167 A boy runs away from home towards Dover at 8 A.M., taking the longer of two roads (one of which is 25, the other 28 miles) and walking $3\frac{1}{2}$ miles per hour. At 10 A.M. he is missed, and immediately his father pursues on horseback, riding 8 miles per hour, but taking the wrong road. On reaching Dover he spends half an hour in bating his horse and making enquiries, and then starts back by the other road, find where he meets the truant. He takes him up behind, and the horse so laden goes $6\frac{1}{2}$ miles per hour, at what o'clock will they reach home?

168 A, B and C row from Cambridge to Ely, and it is observed that the rate of the current is $\frac{1}{6}$ of the difference between A's and C's rates of rowing, now supposing C's rate of rowing per hour to be $\frac{5}{6}$ of B's, $0\frac{2}{3}$ of A's to be $0\frac{38461}{5}$ of 13 miles and $0\frac{625}{16}$ of B's to be $0\frac{572916}{16}$ of A's, supposing also that B left Cambridge 6 hour after A and C 6 after B, find how far B and C were asunder, when A was 7 miles from Ely, $0\frac{0625}{16}$ of the distance between Cambridge and Ely being $\frac{125}{16}$ of a mile.

169 A and B are bound to perform certain works in 3 and 4 hours respectively. Compare their quantities of work and rates of working, (1) if upon exchanging works they could perform them in

2 and 6 hours respectively, (ii) if supposing them to do the works in common they could finish them in 4 hours

170 Part of the journey from A to B is performed by coach which charges carriage in proportion to bulk, and the rest by rail which charges according to weight, a parcel of 15 lbs and whose length, breadth and depth are 9 in, 7 in and 5 in respectively costs 10s $1\frac{1}{2}$ d, while another of 30 lbs and whose dimensions are 10 in, 8 in, and 6 in, costs 19s, find the carriage of a parcel of 47 lbs, equal in bulk to the sum of the other two

171 A boat whose speed was $9\frac{1}{2}$ miles an hour sailed from A to B a distance of 65 miles, and a second boat, which left A $2\frac{1}{2}$ hours after the first arrived at B 5 min before the first. Compare the rates of sailing

172 A person sets out to walk from A to B at the rate of 4 miles an hour. After he has walked $1\frac{3}{4}$ miles he is overtaken by the coach which started a quarter of an hour after him. At a distance of 13 miles from A he meets the coach returning from B where it has stayed for half an hour. Find the distance from A to B .

173 A landlord has an estate which after deducting 10 per cent for repairs and rates brings him a net income of £2250. If he were to sell the estate at 21 years' purchase on the gross income and invest the money obtained in the 3 per cents at 92, what would be his income after deducting 10d in the £ for income tax?

174 A purchases equal amounts of stock in the 3 and 5 per cents. The half-yearly interest on the latter is payable 3 months after that on the former. If A upon receiving the interest on the first stock, lend it to B for 3 months, on condition of being paid back a sum equal to the interest on the second stock at what rate per cent per annum, does A lend the money?

175 The volume of a certain figure increases in the same proportion as either one of its three axes increases, the other two remaining the same. Given that the weight of such a figure of tin whose axes are 8 in, 6 in, 3 in, is 318.4797 oz, find the weight of another such figure of lead whose axes are $1\frac{1}{2}$ ft, 9 in, 2 in, supposing that tin and lead are respectively 7.299 and 11.352 times as heavy as water.

176 A man invests Rs 5120 in a stock (A), he sells out when the stock has risen 50 per cent and invests the proceeds in a stock (B). He sells out of (B) when its value has fallen 25 per cent, and invests successively in stocks (C), (D), (E), (F). He gains by (C), (E) as he gained by (A), and loses by (D), (F) as he lost by (B). Find his ultimate gain.

177 A homoeopathist takes a fluid ounce of medicine, and mixes it with 9 times its quantity of water. He then takes a fluid ounce of the mixture and mixes it with 9 ounces of pure water. How many times must he repeat this operation that the strength of the

medicine may be the same as if he had put the fluid ounce originally in the ocean, supposing that the ocean contains 10^{13} fluid ounces?

178 A cistern, holding 1200 gallons, is filled by 3 pipes A , B and C in 24 minutes. The pipe A requires 30 min more than C to fill the cistern, and 10 gals less run through C per minute than through A and B together. What time would each pipe take to fill the cistern by itself?

179 The shorter of two roads between A and B is 15 miles and goes over a hill the summit of which is 3 miles from B and the longer is 36 miles on level ground. A thief runs away from A by the shorter road at the rate of 3 miles per hour up hill and 4 miles per hour down-hill. Three-fourths of an hour afterwards, a Constable, whose speed up-hill is 4 miles per hour and down-hill 5 miles, starts in pursuit of the thief but takes the wrong road. The thief, an hour after he turns into the longer road at B , sees the Constable $1\frac{1}{2}$ miles ahead of him, and at once turns back and retraces his steps increasing his speed by $1\frac{1}{2}$ miles per hour on the level ground, his speed up and down the hill being the same as before. If the speed of the Constable be to the original speed of the thief as 4 : 3 on the level ground, find where the Constable will overtake the thief?

180 A contractor employs a fixed number of men to complete a work. He may employ either of two kinds of workmen: the first at 26s 6d per week each, the second at 18s 6d per week each, the work of one of the former is to that of one of the latter as 5 to 4. If he finishes it as quickly as possible, he spends £270 more than he would have done if he had finished it as cheaply as possible but takes 4 weeks less time. What would it have cost if he had employed equal numbers of the two kinds of workmen?

181 Ash saplings after 5 years' growth are worth 1s 3d, and increase in value 1s 3d each year afterwards. For their growth each requires twice as many sq yds as the number of years it is intended to grow before cutting. A plantation is arranged so that each year the same number may be ready for cutting. Find the greatest annual income which can be obtained per acre allowing 20 per cent for expenses.

182 On a Railway line, telegraph posts are fixed at a distance of 80 yards from one another. A watchman from the 100th telegraph post, expecting a train, observes it at the 122nd post, and at the same time sees a lamb at the next post to where he is, proceeding towards the train at the rate of 3 miles an hour. He at once runs and overtakes the lamb at the 102nd post, and without loss of time driving the lamb off the line, returns to his place, walking at half his former speed. Just as he reaches his place, the train comes up to the 102nd post. What is the speed of the train?

183 The 2 P.M. goods train from Madras, travelling at the rate of 18 miles an hour, meets with an accident in the way and

reaches Arconum (48 miles from Madras) $\frac{5}{8}$ of an hour late. The passenger train from Erode, which ought to have left Arconum at 5 P.M., is consequently delayed, and leaves Arconum for Madras 6 minutes after the arrival of the 2 P.M. train, and meets the 6 P.M. mail train from Madras midway between Madras and Arconum. If the rates of the passenger and the mail trains be as 2 : 3 find where the mail train will overtake the 2 P.M. goods train, leaving stoppages out of consideration.

184 What is the least number of years for which interest must be reckoned on £145 7s 6d at 4 per cent, so that the interest may be an exact number of £s?

185 On a certain day mackerel were being sold at a certain price per dozen, on the next day as many fish could be bought for 1s as dozens for £1 the day before. The whole price of 20 fish, 10 bought one day, and 10 the other, was 2s 2d. Find the price of a fish on each day.

186 A is to give B 3 marbles every time B wins the game, and to receive 4 every time B loses. They begin with 15, 18 respectively, and leave off with 5. Shew that this may have happened after 5, 12, 19, &c games.

187 A and C can do a piece of work in 20 days, B and D can do the same in 25 days, A and D in 30 days. A works 13 days, B 10 days, C 8 days, D 15 days and E 2 days and thus complete the work. If E is twice as good a workman as D , find the time in which the 5 men together can do another eight times as great as the former.

188 A travels from P to Q , and B from Q to P . The point where they meet is nearer Q than P by 160 yds. If A , after reaching Q , turns back and goes towards P , and B , after reaching P , turns back and goes towards Q , the point where they next meet would be 640 yds. from P . Find the distance from P to Q , and compare the rates of motion of A and B .

189 A works 7 days and takes rest for 3 days, again works 7 days, takes rest for 3 days, and so on. If he earns 3s for every working day, (i) what will he earn in 47 days? (ii) when will he earn £3 3s 3d?

190 A and B embark in a trade for 5 years, B is to have $\frac{1}{8}$ of the net annual profits for the first half of the time, and half of them for the remainder. After $3\frac{1}{2}$ years, the annual profits, by a lowering of the tariff, were increased in the proportion of 6 : 5, and at the same time, became liable to a reduction of 7d in the £ by the laying on of the income tax. At the termination of the partnership, B 's share of the total net profits amounted to Rs 9870, find the annual profits before the duties were reduced.

191 I have two godowns which I want to fill with cases of tea all of equal size. Exclusive of passage room, the larger godown

is 28 ft 9 in long, 17 ft wide, and 9 ft 9 in high. The other is 21 ft 1 in long, 9 ft 11 in wide, and 8 ft 8 in high. Now I want to have my cases made of the largest possible dimensions each way, so as not to lose unnecessary space. What must be their length, breadth and depth? And how many cases must I make?

192 A certain piece of work can be done by A and B in 25 days, C and E can do the same in 35 days, and A and E 7 times as much in 350 days. They all work at it, and with the assistance of a fifth person D , complete the work. A working for 12 days, B 13 days, C 15 days, D 5 days and E 14 days. If the energies of B and D be in the ratio of 1 to 4, find in what time A , B , C and D together can do another work, 9 times as great as the former.

193 Divide Rs 420 among 5 persons, so that A may have $\frac{1}{5}$ th of what all the others have together, B $\frac{1}{4}$ th of what the other four have together, C $\frac{1}{3}$ th of what the other four have together, D $\frac{1}{2}$ rd of what the other four have together, and E the rest.

194 A and B set out at the same time from the foot of a hill to go to the summit. Four hours and 45 min after starting, A meets C who is going down the hill at a point R , but had A waited 1 hour at P till B arrived there and then proceeded as before, he would have met C 3 hours after leaving B , and reached the summit 4 hrs 15 min after meeting C , and 2 hrs 25 min before B . If B is travelling 6 miles an hour, find the rates of A and C .

195 The men employed by a certain farmer work 12 hrs, the women 9 hrs and the boys 8 hrs each day, for labouring the same number of hours, each man receives a half more than each woman, and each woman a third more than each boy. The entire sum paid to all the women each day is double of the sum paid to all the boys, and for every 5s earned by all the women each day, 12s are earned by all the men. Find the number of each class employed, the entire number being 59.

196 A tank, 50 yds square at the top, has flights of uniform steps on all sides leading to the bed which is 40 yds square. If the breadth and the depth of the steps be each one yard, find the volume of water in the tank when two steps are completely visible on all sides. If the water be let out by means of a sluice the area of whose section is 10 sq ft and the velocity of water 2 miles an hour, find in what time the tank will be emptied.

197 Two sums of money which are in the ratio of 7 : 5 were borrowed on the same day at the respective rates of 4 and 5 per cent per annum, if the latter were paid 6 months before the former, the difference in the amounts of both sums would be Rs 455. If the interest on the former sum exceeded that on the latter by Rs 55, how long did the latter loan continue?

198 A person whose rates of walking up hill, on level ground, and down-hill are in the ratios 2 : 3 : 4, walks a certain distance,

the parts of which up hill, level, and down-hill are as 3 4 5 If he is able to walk the whole distance in 49 hours, in what time would he have walked it, if the road had been (i) level throughout, (ii) half up-hill and half down hill

199 In the making of pins 3 men who file the points can keep it work exactly 5 men who put on the heads, no man in either of these two sets can do the work of the other set Suppose one of the first set to stay away for a week, by what fraction are the earnings of each of the remaining men diminished, supposing them to work by the piece, and divide their earnings equally?

200 In a Railway train the total lengths of all the first class carriages, all the second-class carriages and all the third class carriages are the same, the length of a first class carriage being 12 ft, of a second class carriage 18 ft, and of a third class carriage 24 ft Each first-class carriage contains the same number of first-class passengers, each second class carriage the same number of second-class passengers, and each third class carriage the same number of third class passengers, the number of passengers in a first, second, and third class carriage being different The total fares received from the first class, second class, and third class passengers are the same If Rs 5, Rs 3, and Rs 2, be the fares of a 1st, 2nd, and 3rd class passenger respectively find the *least* amount of the total fares

201 A vessel is sent to the Arctic Ocean to catch whales, and the captain who is entitled to $\frac{1}{18}$ of the entire produce of oil receives £735 at the end of a 4 years' voyage Supposing the oil to be worth 60 guineas per ton (each whale on an average yielding 6 tons), and the crew to have been $\frac{2}{3}$ of the entire period on the look out, what was the average number of days between the capture of each whale?

202 If 56 Indian workmen, each earning 6s per day, can do the same piece of work in 25 days that takes 20 English workmen, each earning 3s 6d per day, 15 days to complete, taking the value of the shilling at 12s, determine which class of workmen it is more profitable to employ If a piece of work done by Indian workmen cost Rs 3000, what would be the cost of the same work done by English workmen?

203 There is a serpent's hole at the bottom of a post and on the top of the post is perched a peacock The post is 9 cubits high The peacock observing the serpent gliding to its hole when at a distance from it equal to 3 times the height of the post, flies down obliquely and seizes it when both have gone over an equal distance At what distance from the serpent's hole did they meet?

204 An ordinary train leaves one terminus at 6 A M and reaches the other terminus at 8 P M, losing as much time in stoppages as it would take to travel 40 miles without stopping An express train follows the ordinary 2 hours later and reaches the second terminus 3 hours earlier, losing only half as much time in stoppages as the

ordinary train' If the distance between the 'termini' be 240 miles, find the rates of the trains

205 The time which an express train takes to travel 480 miles is to the time taken by an ordinary train as 11 to 20. The ordinary train takes as much time in stoppages as it would take to travel 70 miles without stopping, while the express train loses only $\frac{1}{11}$ of the time lost by the ordinary train in stoppages and it also travels 13 miles an hour quicker. Find the rate of each train

Additional Examples

- 1 Find the least number which when divided by the L C M of 3, 5, 7, 9 and 11 will leave a remainder 1 [Ans 3466]
 - 2 Find the least number which will be exactly divisible by the L C M of 7, 9, 11 and 13 [Ans 9009]
 - 3 A number when divided by 5, 7, 9 and 11 leaves respectively the remainders 3, 5, 7 and 9, find the remainder when it is divided by $5 \times 7 \times 9 \times 11$ [Ans 3463]
 - 4 A number when divided by 4, 6, 8, 10 and 12 leaves the remainders 2, 4, 6, 8 and 10, find the remainder when it is divided by their L C M [Ans 118]
 - 5 A number when divided by 8, 10, 12, 14 and 16 leaves in every case the remainder 1, find the remainder when it is divided by their L C M [Ans 1]
 - 6 A number when divided by 16, 20, 24 and 28 leaves the remainders 13, 17, 21 and 25, find the remainder when it is divided by their L C M [Ans 1677]
 - 7 Find the greatest number which will divide 14490 and 31530 so as to leave the remainder 6 in each case [Ans 852]
 - 8 Find the greatest number which will divide 11296 and 13528 so as to leave remainders 11 and 23 respectively [Ans 185]
 - 9 In finding the G C M of two numbers the last remainder is 35, and the quotients in order are 1, 2, 1, 3. Find the numbers [Ans 385, 525]
 - 10 Find the greatest number which is such that when 142408, 153599 and 166402 are divided by it, the remainders are all the same [Ans 31]
 - 11 In a long division sum the quotient consists of two figures, if the dividend is 40051, and the two remainders are 173 and 294, find the divisor and quotient [Ans 479, 83]
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EXAMINATION PAPERS

CALCUTTA ENTRANCE PAPERS

1885

1 Of what number is $2\frac{1}{2}$ the $\frac{5}{6}$ th part? By what fraction must $\frac{1\frac{1}{2}}{11\frac{1}{2}}$ of $\frac{3}{4} + \frac{2\frac{1}{2} - 1\frac{5}{8}}{\frac{1}{4} + 1\frac{5}{8}} - \frac{8\frac{1}{2}}{7\frac{1}{2}}$ be divided in order to give a quotient $= \frac{2}{3}$?

2 Simplify $\frac{12 \text{ of } (0104 - 002) + 36 \times 002}{12 \times 12}$, and express your result as a fraction of 6

Reduce $\frac{5}{8}$ of $16s\ 4\frac{1}{2}d$ to the decimal of £1 9s $10\frac{1}{2}d$

3 What circulating decimal multiplied by $\frac{2\frac{3}{5}}{4\frac{1}{2}}$ will give 2 for a product?

If 428571 of a barrel of beer be worth 72 of £2 10s, what is the value of 625 of the remainder?

4 Find the price of 10 lbs 11 oz 16 dwts 16 grs of gold at £3 17s $10\frac{1}{2}d$ per oz. Extract the square roots of $9\frac{3}{4}$ and $\frac{1}{12\frac{1}{5}}$ to 4 places

5 If 27 men can perform a piece of work in 15 days, how many men must be added to the number that the work may be finished in three fifths of the time?

I buy a horse for £40 and sell it for £45 at a credit of 8 months. What do I gain per cent, reckoning money worth 6 per cent per annum?

6 Which is the better investment, bank stock paying 10 per cent at 319 or 3 per cent consols at 96?

What will be the cost of £1,500 3 per cent consols at $89\frac{5}{8}$, brokerage being $\frac{1}{8}$ per cent? What rate of interest will such investment obtain?

1886

1 Divide $\frac{1\frac{2}{3} - 1\frac{1}{4}}{1\frac{1}{8} - 1\frac{1}{3}} + \frac{1\frac{5}{8} - 1\frac{7}{8}}{1\frac{8}{9} - 1\frac{9}{10}}$ by $\frac{1}{11} - \frac{1}{12} + \frac{1}{13} - \frac{1}{14}$

2 Simplify $\frac{3\frac{125}{216}}$ of $\frac{2\frac{1}{2}}{125} - \frac{2\frac{2}{5}}{15}$ of $\frac{187\frac{5}{8}}{342}$

- 3 Reduce £1 11s 10½d to the fraction of £7 18s 6½d
 What fraction of £10 must be added to £16 10s 3d to make it £20
- 4 What decimal of 9 mds 20 sr is $\frac{2}{3}$ of 7 mds 5 sr
 Reduce 5½ sq yds to the decimal of an acre
- 5 Find the value, by Practice, of 2 tons 15 cwt 35 lbs at £13 6s 8d per ton
- 6 What sum of money at 4 per cent simple interest will secure the same income as Rs 25475 at 4½ per cent?
- 7 If a rupee is equivalent to 1s 6½d, what is the price of a sovereign in rupees? If, after buying 250 sovereigns at this price, I sell them again when the rupee is equivalent to 1s 6d, how much shall I gain or lose by the transaction?

1887

- 1 Simplify (a) $(4\frac{2}{3} - 1\frac{1}{3}) \times (3\frac{1}{2} - \frac{1}{8}) - (13\frac{1}{2} + 7\frac{1}{2})$ of $\frac{3\frac{1}{2}}{11}$
 (b) $\frac{183 + 20416 + 3 - 3\frac{1}{2}}{10025 + 0625 - 1\frac{1}{8}}$
- 2 Express $\frac{2}{3}$ of 7s 6d + 12s of 5s - 54s of 9s 2d as a decimal fraction of £10
- 3 (a) Find by Practice the value of 5 tons 5 cwt 2 qrs 17½ lbs at £3 6s 8d per ton
 (b) Find the income on which the income-tax at 5p per rupee is Rs 52 1a 4p
- 4 If 50 men can do a piece of work in 12 days, working 8 hours a day, how many hours a day would 60 men have to work in order to do another piece of work twice as great in 16 days?
- 5 If Rs 450 amount to Rs 540 in 4 years at simple interest what sum will amount to Rs 637 8a in 5 years at the same rate?
- 6 Extract the square root of 177 1561, and of 2 to 3 decimal places

1888

- 1 Simplify $\frac{7(\frac{1}{2} \text{ of } 3\frac{1}{2} - \frac{1}{2} \text{ of } 2\frac{1}{2})}{\frac{2}{3} \times 1\frac{1}{2} \times 1\frac{1}{2} - \frac{1}{3} - 2\frac{1}{2}} - \frac{\frac{1}{2} + 1\frac{1}{2} - 1\frac{1}{2}}{\frac{1}{2} \times \frac{1}{2} - \frac{1}{2} \times 1\frac{1}{2}}$
- 2 Divide 16016 by 00143, and extract the square root of 14409616
- 3 Add together 55 5002, 3 17, 4 503 and 75 271, and find the value of 7365 of £3 6s 8d + 504 of £15 12s 6d + 2 102083 of £5.
- 4 Find by "Practice" the value of 2 tons 7 cwt 3 qrs 11 lbs at £21 12s 6d per cwt

5 A man can walk 600 miles in 35 days, resting 9 hours each day, how long will he take to walk 375 miles if he rests 10 hours each day, and walks $1\frac{1}{2}$ times as fast as before?

6 If the interest on money be one pie per rupee per month, what is the rate per cent per annum?

A man holds $15\frac{1}{4}$ shares of a bank, and receives £19 1s 3d per quarter. If the interest he receives be 5 per cent per annum, find the value of a share

1889

1 Multiply 0069347 by 74396

2 Divide 2100 005983 by 243 5846 correct to 5 places of decimals.

3 Find in any way the value of 1,317 cwt 3 qrs 21 lbs at £3 17s 10½d per cwt

4 Extract the square root of $1 + (0.634)^2$ to 6 places of decimals

5 Find in English money the value of Rs 100,000 at 1s 4½d per rupee

1890

1 Simplify $2\frac{2}{3}$ of $\frac{13\frac{1}{2} - 9\frac{3}{4}}{15\frac{1}{2} - 11\frac{7}{8}} - 3\frac{2}{3} + \frac{1\frac{1}{2}}{9\frac{5}{8} - 8\frac{1}{4}}$, and find by Practice the value of 3,049 articles at Rs 7 13a 7p each

2 Divide 27 03 by 0037, and reduce $75 - 102 - 27$ to a vulgar fraction

3 Find the cost of putting a fence round a square field, whose area is 13 225 acres at Rs 1 12a per yard

4 A piece of work can be done in 72 days by 17 men working together. If after 9 days of work, these are joined by 4 others, in how many days will the work be finished?

5 Find the price of $4\frac{1}{2}$ per cent Government Promissory Notes when an investment of Rs 59,422 8a produces a monthly income of Rs 213 12a

1891

1 Simplify the following expressions —

$$(a) \frac{2-2}{8-4} + \frac{4-1\frac{3}{4}}{\frac{1}{2}-1\frac{1}{4}}$$

$$(b) \frac{1}{4 - \frac{1}{2 - \frac{1}{1 - \frac{1}{2}}}}$$

2 Find the value of $24607 \times 06 - 375 \times 012 + 2163 - 103$

3 Find the value of 15 cwt 3 qrs 9 lbs at Rs 25 12a 7p per cwt

4 If a man walking at the rate of $3\frac{1}{2}$ miles an hour, walks to a place in 4 hours 30 minutes, how long will it take a man, walking at the rate of $3\frac{1}{4}$ miles an hour, to walk there and back?

5 A man invests a certain sum in $4\frac{1}{2}$ per cent Government Paper at 104. The price falling to 101, he sells out and loses Rs 600 by the transaction, exclusive of brokerage. Find the sum invested.

6 A gives B 10 yards' start and C 15 yards' start in a race of 100 yards, how much should B give C in 150 yards?

1892

1 Simplify $\frac{3\frac{5}{8} - 1\frac{1}{8} \text{ of } \frac{3\frac{1}{2}}{\frac{3}{8}}}{11\frac{2}{3} \text{ of } 1\frac{9}{4} \text{ of } \frac{2}{5}} - \frac{4\frac{1}{2} - 7\frac{5}{8} + 3\frac{3}{8}}{\frac{5}{6} \text{ of } 12}$

2 Find, to the nearest integer, the value of $\frac{39 \ 37 \times 760 \times 13 \ 596}{1 \ 293 \times 12}$

3 Find the square roots of 097344, of 009604, and of 996004.

4 Find the interest on 10 lakhs of rupees for 10 days at $4\frac{1}{2}$ per cent.

5 £3,000, which I held in the 4 per cents, was sold for me when they were at $82\frac{3}{4}$ by a broker whose commission is $\frac{1}{4}$ per cent, and the proceeds were re-invested by him in the four and a half per cent at $98\frac{1}{4}$. What amount of the latter stock did he purchase?

1893

1 Simplify —

(1) $1 + \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6} + \frac{6}{7}$

(2) $\frac{8\frac{1}{2} - 1\frac{1}{4}}{\frac{3}{4} + 1\frac{1}{2}} - \frac{1}{5\frac{3}{4} - 1\frac{1}{2}}$

2 Divide 184626 by 234. Express $45\frac{6}{7}$ and $65\frac{4}{5}$ as vulgar fractions reduced to their lowest terms, and their sum as a circulating decimal.

3 Find the cost of 73 cwt 3 qrs 14 lbs at £4 13s 6d per cwt.

4 Distinguish between true discount and banker's discount. Find the former in the case of a bill for Rs 3486 6a 8p due 16 months hence, the rate of interest being $5\frac{1}{2}$ per cent per annum.

5 A man invests Rs 163000, part in Govt 4 per cent stock at 108 and the remainder in Municipal 5 per cent Debenture stock at 109 $\frac{1}{2}$. Find how much he must invest in each that he may have an equal income from the two sources.

1894

1 In a compound metal containing tin and copper only, the proportion of tin to copper is 775 to 9225. Find to the nearest penny the value of 8 cwt 3 qrs of it. Tin costs £140, copper £80 per ton.

2 A rectangular court is 50 yards long and 30 yards broad. It has paths joining the middle points of the opposite sides of 6 feet in breadth and also paths of the same breadth running all round it. The remainder is covered with grass. If the cost of the pavement be 8d per sq ft and the turf 3s per sq. yd, find the cost of laying out the court.

3 Find the value of 2671875 of £3 in *s* *d* and decimal of a penny

4 Find the square root of $1-(.0678)^2$ to four places of decimals

5 At a cricket match a contractor provided luncheon for 24 and fixed the price to gain $12\frac{1}{2}$ per cent on his outlay Three persons were absent The remaining 21 paid the fixed price and the contractor lost 2 rupees What was the charge?

6 Find the simple interest on Rs 12345 for 134 days at $2\frac{1}{2}$ per cent

1895

1 Find the square root of $1+\frac{1}{2}(.0345)^2$ correctly to 4 places

2 Find the sum of money which put out at simple interest at $2\frac{1}{2}$ per cent will in 134 days exactly produce Rs 124 10s $11\frac{1}{2}\frac{1}{2}$ p (1 year=365 days)

3 If one pound sterling be worth 25-francs and 60 centimes, and also worth 6 thalers and 20 silver groschen, how many francs and centimes is one thaler worth? (NB—One thaler=30 silver groschen One franc=100 centimes)

4 Simplify $\frac{1\frac{1}{4}-1\frac{5}{8}+\frac{7}{9}}{1\frac{1}{4}+1\frac{5}{8}} \div \frac{9 \times 5}{14 \times 3} - \frac{11\frac{1}{2}}{15}$

5 I invest Rs 12805 in the 4 per cents at $98\frac{1}{2}$, and when they have risen to $102\frac{1}{2}$, I sell out and invest in the $4\frac{1}{2}$ per cents at $105\frac{3}{8}$, what is the change in my income? (Brokerage $\frac{1}{2}$ per cent on all transactions)

Or convert $1\frac{1}{8}\frac{1}{2}$ into a decimal fraction, pointing out accurately the recurring portion (if any)

1896

1 What greatest number and what least number can be subtracted from 23759143 that the remainders may be divisible by 24, 35, 91, 130 and 150?

2 (1) Simplify $\frac{5\frac{1}{2}}{6\frac{1}{2}}$ of $\frac{6\frac{1}{2}}{9\frac{1}{8}} - \frac{8}{9}(2\frac{1}{2} + 1\frac{1}{2})$ of $\frac{7s}{12s} \frac{6d}{6d}$

(2) Divide 0023465 by 03125

3 Extract the square root of $5\frac{1}{2}$ correct to 4 places of decimals

4 Find the simple interest on Rs 4235 12s $9\frac{3}{4}$ p for 3 years and 7 months at $3\frac{1}{2}$ per cent per annum

5 If by selling a horse for Rs 1100, I lose 18 per cent how much per cent should I have gained or lost, had it been sold for Rs 1320?

6 A man invested the same sum in two different stocks, $3\frac{1}{2}$ per cent Government Securities at 103 $\frac{1}{2}$ and 4 per cent Municipal Debentures at 105, his income from one was Rs 93 more than from the other what sum was invested in each stock?

1897

1 Reduce $\frac{2\frac{1}{2} - 1\frac{1}{2}}{2\frac{1}{2} + 1\frac{1}{2}} \times 15\frac{1}{2} - \frac{3\frac{1}{2} \times 3\frac{1}{2} \times 3\frac{1}{2} - 1}{3\frac{1}{2} \times 3\frac{1}{2} + 3\frac{1}{2} + 1}$ of 1 cwt 3 qrs 7 lbs to the decimal of $2\frac{1}{2}$ tons

(a) Find the vulgar fraction equivalent to the recurring decimal .133, without assuming any rule

2 What do you understand by an *aliquot* part of a quantity? Is an area equal to $15\frac{1}{8}$ sq yards an aliquot part of an acre?

Find by Practice the income tax on Rs 1250 10s 8d at the rate of 5 pies per rupee

3 What is meant by the *ratio* of one quantity to another?

What is a *proportion*?

320 people dine together 4 days a week, but on the remaining 3 days some are absent, the consumption of food is thus reduced, for the whole week, in the ratio of 109 to 112 Find the number of absentees

4 In what time will Rs 3546 amount to Rs 7683 at $3\frac{1}{2}$ per cent simple interest?

5 A person has stock in the $3\frac{1}{2}$ per cent Government Securities which yields Rs 2856 a year He sells out half of the stock at 109 $\frac{1}{2}$, and invests the proceeds in Howrah Mills shares at 153 What dividend ought the latter to pay that he may thereby increase his annual income by Rs 330?

6 Extract the square root of 3 14159 to 4 decimal places

1898

1 What is that least number, which, being divided by 48, 64, 72, 80, 120 and 140, leaves the remainders 38, 54, 62, 70, 110, and 130 respectively?

2 (a) Simplify

$$\frac{2\frac{2}{3}}{5\frac{1}{6}} \text{ of } \frac{7}{4} (\frac{7}{9} + \frac{1}{12}) - \frac{5\frac{7}{8}}{7\frac{1}{4}} \text{ of } \frac{2s}{3s} \frac{5d}{11d}$$

(b) What decimal of 2l 13s 4d is 0.625 of 2 6 of 1l 6s 8d?

3 Extract the square root of 54756, also of $(4.02)^3$ to 4 places of decimals

4 What sum will amount to Rs 300 in $3\frac{1}{2}$ years at $6\frac{1}{2}$ per cent. per annum simple interest?

5 A grocer buys 480 mds of sugar for Rs 6135 payable at the end of 3 months, and on the same day sells them at Rs 12 11a per maund ready money, what per cent does he gain or lose by the transaction, reckoning interest at 9 per cent per annum?

6 One-third of a certain capital is invested in the 3½ per cent Government Securities at 105, one-fourth in the 3 per cent Government Securities at 97½ and the remainder in the 4½ per cent Calcutta Municipal Debentures at 112½. If the total annual income is Rs 830, what is the capital?

1899

1 Find the greatest number which will divide 1028, 1629 and 2130, leaving the remainders 3, 4 and 5 respectively

2 (a) Simplify

$$\frac{\frac{2}{3} + \frac{7}{8}}{\frac{5}{8} + \frac{1}{10}} \text{ of } \frac{13^{\frac{3}{4}} - 51}{95 - 107} - \frac{2}{5} \left(\frac{1}{2} + \frac{1}{3} \right) \text{ of } \frac{3 \text{ tons } 3 \text{ cwt}}{4 \text{ tons } 3 \text{ cwt}}$$

(b) Prove that $234 = \frac{2}{5} \frac{2}{3} \frac{2}{6}$, without assuming the rule of converting a recurring decimal into a vulgar fraction

3 Find, by Practice, or otherwise, the value of 7 tons 2 cwt 2 qrs at Rs 3 2a per maund, assuming that 1 ton is equal to 27½ maunds

4 Extract the square root of 51076, and of 051076

5 A grocer mixed 20 maunds of one kind of rice at Rs 4 1 maund, with a certain quantity of a second kind of rice at Rs 3 8a a maund, and selling the mixed rice at Rs 3 12a a maund, gained Rs 10 on the whole. Find how many maunds of the second kind of rice he mixed, and the gain per cent on his outlay

6 Find the discount on Rs 1218 due 8½ months hence at 3 per cent per annum, simple interest

1900

1 What do you understand by the Greatest Common Measure and the Least Common Multiple of two or more whole numbers?

Nine bells begin to strike simultaneously, and strike at intervals of 1, 2, 3, 4, 5, 6, 7, 8, 9 seconds respectively. After what interval of time will they next strike simultaneously?

2 (a) Simplify $\frac{16\frac{1}{2} - 3\frac{1}{2} \text{ of } 2\frac{1}{2}}{22 \text{ of } 5\frac{1}{2} + 3\frac{1}{2}} \times \frac{21\frac{1}{2} \text{ of } 4\frac{1}{2} + \frac{2}{3} \text{ of } 13\frac{3}{8}}{5\frac{1}{2} - 4\frac{1}{2} \text{ of } \frac{5}{6}} - 1\frac{1}{2} - 1\frac{1}{2}$

(b) Reduce 0416 to its equivalent vulgar fraction in its lowest terms, and explain the reason for the process you employ

3 Find the value of $(125)^3 + 225 \times 125^2 + 375 \times (75)^2 + (75)^3$, without reducing the decimals to vulgar fractions

4 The length, the breadth, and the height of a room are 25 ft. 7 in., 20 ft 5 in. and 14 ft respectively. Its walls are papered at 3s 6d a sq yd, and its ceiling painted at 1s 2d a sq ft. Find the total cost.

5 The subscriptions to a certain memorial fund amounted to Rs 976 9a, and each person subscribed as many annas as there were subscribers altogether. Find the number of subscribers.

6 Explain clearly what you mean by saying that the $3\frac{1}{2}$ per cent Government Securities are at 101.

A person invests Rs 19,700 in the $3\frac{1}{2}$ per cent Government Securities at 98 $\frac{1}{2}$, and when they rise to 101 $\frac{1}{2}$, he sells out and invests the proceeds in the $4\frac{1}{2}$ per cent Calcutta Municipal Debentures at 114 $\frac{1}{8}$. Find the change in his income.

1901

1 (a) Simplify $\frac{306}{323} - \frac{204}{221}$ of $\frac{22\frac{5}{11}}{32\frac{9}{11}} - 583 \times 142857$, expressing your answer as a decimal.

(b) Reduce £3 15s 4d to the decimal of Rs 100 (£1 = Rs 15).

2 (a) What is meant by an *aliquot part* of a number?

Is $2\frac{1}{2}$ yds an aliquot part of a mile?

(b) Find by Practice, or otherwise, the value of 25 tons 15 cwt 3 qrs 17 $\frac{1}{2}$ lbs at £2 13s 4d per ton.

3 If the four penny loaf weighs 3 lbs 9 oz when wheat is at 9s 4d per bushel, what ought the six-penny loaf to weigh when wheat is at 11s 1d per bushel?

4 (a) Define *Interest*. What do you understand by the expression *Rate per cent per annum*?

(b) At what rate per cent per annum simple interest will £200 amount to £236 13s 4d in 4 years 7 months?

5 Extract the square root of 7466 4164.

6 A man invests one third of his capital in the $3\frac{1}{2}$ per cent Government Securities at 96 $\frac{1}{2}$, and the remaining two thirds in the $4\frac{1}{2}$ per cent Calcutta Municipal Debentures at 105 $\frac{1}{2}$. If the difference of the two annual incomes be Rs 1997, find his capital.

1902

1 (a) How can you ascertain whether a given vulgar fraction can be reduced to a terminating or a recurring decimal, without actually converting it into a decimal? What kind of decimal will the fraction $1\frac{1}{8}\frac{1}{16}$ produce?

(b) Simplify —

$$1 - \frac{2}{3 + \frac{4}{5 - \frac{6}{7 + \frac{1}{11}}}} - 2 \text{ of } \frac{2 \text{ cwt } 2 \text{ qrs } 21 \text{ lbs}}{10 \text{ cwt } 2 \text{ qrs } 11 \text{ lbs}}$$

and reduce the result to the decimal of 11

2 The area of a rectangular field whose breadth is 500 yds is 100 acres Find the cost of cultivating it at Rs 3 2s 8p per 100 sq yds and also the cost of fencing it round at Rs 2 8s per yard

3 If 12 men and 15 boys can do a piece of work in 30 days, working $7\frac{1}{2}$ hours a day, how many boys must assist 21 men to do a piece of work twice as great in 25 days, working 9 hours a day? (3 men are equivalent to 5 boys)

4 Extract the square roots of $5\frac{1}{4}$ and 76 195441

5 (a) Define *Discount*

(a) Find the discount on £700 due 3 years 4 months hence at 5 per cent per annum simple interest

6 Which is the better investment, the $3\frac{1}{2}$ per cent Government Securities at 95 $\frac{3}{4}$ or the 4 per cent Calcutta Municipal Debentures at 101 $\frac{1}{2}$? What will be the difference in the annual income by investing Rs 22127 in each of them?

1903

1 (a) Simplify $\frac{67 \times 67 \times 67 - 001}{67 \times 67 + 067 + 01} + \frac{57}{1 + \frac{1}{314}}$

(b) What decimal of a mile is 1 yard?

2 (a) What is meant by the aliquot part of a number? Is an acre an aliquot part of a square mile?

(b) Find by Practice, or otherwise, the price of 25 tons 12 cwt 3 qrs 17 $\frac{1}{2}$ lbs at £6 13s 4d per ton

3 Three taps A, B and C can fill a cistern in 5, 6 and $7\frac{1}{2}$ minutes respectively. They are all turned on at once, but after one minute, A is turned off. How much longer will B and C take to fill the cistern?

4 (a) Define the square root of $10\frac{3}{8}$, and of 4^2 to four places of decimals

5 A man buys wine at 5s a gallon, he mixes it with water, and by selling the mixture at 4s a gallon gains $12\frac{1}{2}$ per cent on his outlay. How much water did each gallon of the mixture contain?

6 (a) Define *Present worth*

(b) A tradesman marks his goods with two prices, one for ready-money and the other for 3 months' credit, allowing interest at $4\frac{1}{2}$ per cent per annum. If the credit price be marked at Rs 50 9a, what ought to be the cash price?

1904

1 Define the H.C.F. and the L.C.M. of two or more numbers

(a) Find the greatest number of six digits which is exactly divisible by 27, 45, 60, 72 and 96

2 Write down the local value of each of the figures in the number 010203

(a) Simplify $\frac{(01)^1 + (02)^1 + (03)^1}{(001 + 002 + 003)^2} = 02083 + \frac{£2 \ 3s}{£25 \ 16s}$ of $\frac{1}{2}$

3 A can do a piece of work in 25 days, B in 20 days, and C in 24 days. The three work together for 2 days, and then A and B leave, but C continues, and after $8\frac{1}{2}$ days is rejoined by A, who brings D along with him, and these three finish the remainder of the work in 3 days. In what time would D alone have done the whole work?

4 The area of a square cricket field is 9 ac 3 ro 8 16 po. Find the length of a side.

5 Define *Discount*

(a) The difference between the interest and the discount on a certain sum for 3 years 4 months at 5 per cent per annum is £16 13s 4d. Find the sum.

6 A person invests a certain sum in the $3\frac{1}{2}$ per cent Government Securities when they are at $97\frac{3}{8}$, had he waited till they had fallen to $97\frac{1}{8}$, he would have had Rs 400 more of Government Securities. How much money did he invest $\frac{1}{2}$ per cent being charged as brokerage in both cases?

1905

1 When is one number said to be a *measure* of another? What is a *Prime Number*?

A man bought two heaps of mangoes, one for Rs 10 5a and the other for Rs 18 0a 9p. If the price of each mango be the same, and not less than two and not more than three annas, find the total number of mangoes he bought.

2 (1) What is the meaning of $\frac{2}{3}$ and of $\frac{1}{2}$ of $\frac{2}{3}$?

(2) Simplify —

$$(5\frac{1}{2} - 1\frac{1}{2}) \text{ of } \left(\frac{5}{3\frac{1}{2}} + \frac{1}{2} \text{ of } \frac{4}{3} \right) - \frac{1}{2} \text{ of } \frac{3 \text{ tons } 3 \text{ cwt}}{9 \text{ cwt}}$$

3 Extract the square root of 19951 and of $\frac{1}{4}$ correct to three places of decimals

4 Find the cost of paving a pathway 6 ft wide, round and immediately outside a flower garden, 21 yds long and 10 yds broad, at 5 $\frac{1}{2}$ pice per sq yd

5 Find the price of 35 mds 13 $\frac{1}{2}$ srs of rice at Rs 3 2a per maund

If it is sold at the rate of Rs 3 3 $\frac{1}{2}$ a per maund, what is the profit per cent ?

6 I pay Rs 45000 to a Bank for a Bill of Exchange payable in London. The rate of exchange is 1s 4d for the rupee and the Bank charges me 2 per cent on the amount payable in England. How much will my agent in London receive ?

1906

1 (1) When is one number said to be a *multiple* of another ? How can you ascertain by inspection whether a given number is a multiple of 3 ?

(2) What is the greatest number consisting of five digits which can be added to 8321 so that the sum may be exactly divisible by 15, 20, 24, 27, 32 and 36 ?

2 (1) What is the meaning of $\frac{1}{2}$ of $\frac{1}{2}$? Give an illustration

(2) Simplify —

$$(1) 12 \times (2\frac{1}{2} - 1\frac{1}{2} - 1\frac{1}{2} - 1\frac{1}{2} - 1\frac{1}{2}) + 2\frac{1}{2} \div \frac{2\frac{1}{2}}{2\frac{1}{2}} \text{ of } \frac{11\frac{1}{2} 4d}{12\frac{1}{2} 3d}$$

$$(2) \frac{159 \times 159 - 41 \times 41}{159 - 41}$$

3 The cost of matting a room 16 ft broad and 12 ft high at 3a per sq yd is Rs 7 9a 4p. What will be the cost of papering its wall at the same rate, allowing for six doors, each 6 ft by 3 ft ?

4 Extract the square root of 027 and of $\frac{1}{2}$ correct to four places of decimals

5 A book sent from England costs me (including Re 1 2a postage) Rs 12 1a. But my bookseller allows me a discount of 2d in the shilling on the published price. What is the published price in English money, the rate of exchange being 1s 4d for the rupee ?

6 Define *Present Worth*

A man bought a horse for 30 guineas and sold him immediately for £36 1s payable at the end of 6 months. If interest be reckoned at 6 per cent per annum, find his gain per cent upon the transaction

1907

1 What do you understand by the G C M and the L C M of two or more integers? What is a prime number? Find the least number which is exactly divisible by 12, 34, 56 and 78

2 Simplify

$$(1) \frac{2 \times 2 \times 2 + 02 \times 02 \times 02}{6 \times 6 \times 6 + 06 \times 06 \times 06} - \frac{2\frac{1}{8} - 1}{2\frac{3}{4} + 1\frac{1}{8}}$$

$$(2) \frac{Rs\ 2\ 5a\ 6p}{Rs\ 3\ 12a} - \frac{1\ hr\ 16m\ 45\ sec}{2\ hr\ 7m\ 55\ sec}$$

3 Find the price of 8 mds 16 srs 2 chks of rice at Rs 5 9a per maund

4 How many paving stones, each of them 1 ft long and 9 in wide, will be required for paving a street 30 ft wide, surrounding the outside of a square grass plot, the area of the grass plot being 10 acres?

5 If 8 men or 15 women can earn Rs 120 in 30 days, how much can 21 men and 24 women earn in 45 days?

6 The debts of a bankrupt amount to £2134 10s 6d and his assets consist of property worth £916 5s 4d and an undiscounted Bill of £513 due 4 months hence, simple interest at 4 per cent. How much in the pound can he pay to his creditors?

1908

1 (1) When can a vulgar fraction be converted into a terminating decimal? What kind of decimal will the fraction $\frac{11}{138}$ produce?

(2) Simplify —

$$\frac{2\frac{1}{2} + \frac{3}{4} \text{ of } 2\frac{1}{2} - 1\frac{1}{4}}{36 + 15 \times 4 - 24 - 21} \text{ of } \frac{201 \text{ of } 1 \text{ md } 30\frac{1}{2} \text{ srs}}{175625 \text{ of } 2 \text{ mds } 20\frac{1}{2} \text{ srs}} \text{ of Rs } 2\ 8a$$

2 Find by Practice, or otherwise, the value of 5 acres 3 roods 7 poles $5\frac{1}{2}$ sq yds of land at £161 6s 8d per acre

Or

11 A reservoir is 25 ft 5 in long and 12 ft 10 in wide, how many gallons of water must be drawn off to make the surface sink one foot? (A cubic foot of water weighs 1000 ounces, and one gallon = 10lbs avoirdupois)

3 (1) The hands of a clock coincide after every 66 minutes of correct time. How much is the clock fast or slow in 24 hours?

(2) A race course is 440 yards long A and B run a race and A wins by 5 yards B and C run over the same course and B wins by 4 yards C and D run over it and D wins by 16 yards If A and D run over it, which would win, and by how much?

5 (1) What number multiplied by itself will produce $4\frac{28}{25}$?

(2) Extract the square root of $\frac{7}{8}$ correct to four places of decimals

Or

IV The discount on a certain sum due 2 years hence is Rs 638 8a, and the interest on the same sum for the same time is Rs 718 5a, find the sum and the rate per cent per annum

5 A trader allows a discount of 5 per cent to his customers What price should he mark on an article, the cost price of which is Rs 712 8a, so as to make a clear profit of $33\frac{1}{3}$ per cent on his outlay?

6 A person invests Rs 44100 in the $3\frac{1}{2}$ per cent Government Securities at 98, and when they rise to 98 $\frac{1}{2}$, he sells out and invests the proceeds in the 5 per cent Calcutta Municipal Debentures at 110 $\frac{1}{2}$ Find the alteration in his income

1909

1 Multiply 62031 by 46189, and divide the product by 7429

2 Simplify —

$$(1) \frac{10\frac{2}{3} - (5\frac{2}{3} + 4\frac{2}{3})}{10\frac{1}{3} - (2\frac{1}{3} - \frac{1}{3}) - 7} - \frac{1}{6 + \frac{1}{1 + \frac{1}{8}}}$$

$$(2) 2\ 1428571 - 07692307 \times 2\ \frac{2}{3}$$

3 Find, by Practice, the price of 28 bags of sugar, each weighing 3 cwt 2 qrs 8 lbs, at Rs 9 6a 4 $\frac{1}{2}$ p per cwt

Or

Extract the square root of 137769 395929

4 The area of a square garden is 10 acres On the inside of the garden and along four sides of it there is a gravel path 5 feet wide Find the cost of constructing the path at 1 anna 6 pies per square foot

Or

On what capital will the interest for 219 days at 4 per cent per annum amount to £14 2s 6d?

5 Among a certain number of children 91509 mangoes and also 83721 oranges may be equally divided How many are the children? Give all possible answers

O

What profit per cent is made by selling an article at a certain price, if by selling at two thirds of that price there would be a loss of 20 per cent

(Supplementary Examination)

1909

1 Multiply 18054 by 43565 and divide 722646078 by 56294

2 Simplify —

$$(a) \frac{10^8}{10^8} + \frac{1}{3^8} \text{ of } 1\frac{1}{4} - \frac{1}{11} \text{ of } 3\frac{2}{3} - 8\frac{1}{2}$$

$$(b) \frac{246 + 230}{3 + 127} + \frac{41}{19}$$

3 Find by Practice the cost of 1000 mds of sugar at Rs 8 13a 4p per maund

O

III Find the price of tiles, each 18 inches by 15 inches, that will be required to pave a square court, whose side is 30 yards, at Rs 25 per hundred

4 If 9 men do $\frac{2}{3}$ of a piece of work in 14 days working 10 hours a day, how many extra men must be employed to finish the work in 5 days more if all of them are now to work only 8 hours a day ?

O

IV In what time will the simple interest on £285 be £84 15s 9d at $3\frac{1}{2}$ per cent per annum ?

5 Extract the square root of 184389241

O

V Find the present worth of Rs 315 due 10 months hence at 6 per cent per annum

MATRICULATION EXAMINATION

1910

Compulsory Paper

1 Multiply 407566 by 800209, and divide 507233438305 by 670549

O

I Find the G C M of 253512 and 568512, and the L C M of 432, 720, 1152.

2 Reduce to its simplest form —

$$(1) \frac{4\frac{1}{2} \times 17\frac{1}{2} \times 2\frac{1}{2}}{5\frac{1}{2} \times 5\frac{1}{2} \times \left(\frac{1}{4\frac{1}{2}} + \frac{1}{6\frac{1}{2}}\right)},$$

$$(2) \frac{81 \times 005}{45}$$

Or

II A contractor engaged to finish six miles of railway in 200 days, but after employing 140 men for 60 days he found that only one and a half miles were completed. How many additional men must be engaged that the work may be finished within the given time?

3 (1) Find by Practice, or otherwise, the value of 458 things at Rs 8 5a 4p each

(2) In what time will a sum of money double itself at 6 per cent simple interest per annum?

Or

III The weight of a cubic inch of water is 253 17 grains and that of a cubic inch of air is 31 grains. Find to three places of decimals how many cubic inches of water weigh as much as one cubic foot of air

Additional Paper

1 Extract the square root of 6256586734489

Or

1 A cistern contains 243 $\frac{3}{4}$ cubic feet of water. Find the length of the side of a second cistern 4 ft 4 in deep, with a square base, which contains 4 times as much water as the first

2 (1) Calculate, correct to three places of decimals, the value of

$$1 + \frac{1}{12} + \frac{1}{123} + \frac{1}{1234} + \&c \text{ to infinity}$$

(2) A metre is defined to be the ten-millionth part of a quarter of the circumference of the earth, and is equal to 39 37079 inches. Find the circumference of the earth in miles

1911

Compulsory Paper

1 Multiply 87904563 by 7059089, and divide the product by 998875

Or,

A square grass plot whose side is 200 yds, is bordered on the outside by a path 10 ft wide. Find the cost of gravelling the path at Rs 2 8a per 100 square ft

2 (1) Simplify —

$$\frac{\frac{1}{2} + \frac{2}{3}}{\frac{5}{6} - \frac{7}{8} + \frac{9}{10}} - 142857 \text{ of } 10\frac{20}{103}$$

(2) What decimal of a rupee is a pie ?

Or,

What decimal of an hour is a second ?

3 (1) Find the value of 5 mds 25 sr 10 ch of milk at Rs 5 10a 8p per md

(2) What sum of money must be put out at $3\frac{3}{4}$ p c per annum, simple interest, in order to amount to £248 18s. 9d in $2\frac{1}{2}$ years ?

Or,

A contractor undertakes to execute a certain work in a given time, he employs 55 men, who work 9 hrs daily, when $\frac{2}{3}$ of the time has expired, he finds that only $\frac{2}{3}$ of the work is done, how many men must he now employ 11 hrs a day to fulfil his contract ?

Additional Paper

1 Find the square root of 220191808516,

Or,

$$2919\ 46783041$$

Or,

A general wishing to arrange his men, who were 335250 in number, into a solid square, found that there were 9 men over. How many men were there in the front ?

2 (1) Find a decimal that is within $\frac{1}{100000}$ of $\frac{355}{113}$

Or,

Find the value (correct to 5 places of decimals) of

$$1 - \frac{1}{3} + \frac{1}{8} - \frac{1}{4} + \frac{1}{5} -$$

(2) Assuming a metre to be $39\frac{3}{8}$ inches, find the nearest whole number of litres in one cubic foot

1912

Compulsory Paper

1 Multiply 814703 by 703692, and divide 246741768 by 75318

Or,

Reduce to its lowest terms $\frac{142593}{514199}$

- 2 (1) Reduce to the simplest vulgar fraction

$$\frac{2\ 46 - 2\ 30}{3 + 127} + \frac{43}{19}$$

- (2) Find the value of $\frac{3\ \text{cwt}\ 3\ \text{qrs}\ 14\ \text{lbs}}{2\ \text{cwt}\ 1\ \text{qr}\ 20\ \text{lbs}}$ of £7 18s 8d

Or,

(1) At what rate per cent simple interest will £440 6s 8d amount to £511 17s 9d in 5 years?

(2) Find the price of 12 maunds 8 seers 4 chattaks of ghee at Rs 36 4a per maund

3 If the wages of 45 women amount to £207 in 48 days, how many men must work 16 days to receive £76 13s 4d, the daily wages of a man being double those of a woman?

Or,

A rectangular courtyard 100 ft long by 80 ft wide has within it a gravel path 8 ft wide running round it Find the area of the path, and the cost of gravelling it at 5a 3p per sq yd

Additional Paper

- 1 Find the square root of 137769 395929

Or,

A rectangular court, three times as long as it is broad, is paved with 2028 stones, each $1\frac{1}{2}$ ft square Find the length of the court

2 If a metre be 32809 ft and the length of a line drawn on the earth from the North Pole to the Equator be 10,000,000 metres, find the circumference of the earth to the nearest mile

Or,

Find, correct to five places of decimals, the value of

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \frac{1}{2^6} + \frac{1}{7} + \frac{1}{2^7} + \frac{1}{9} + \frac{1}{2^9}$$

1913

Compulsory Paper

- 1 (1) Multiply 426042 by 90578

Or,

Divide 5208465 by 754

- (2) Find the G C M of 253512 and 568512

Or,

Find the L C M of 105, 135, and 210

2 (1) Simplify $\frac{1}{2} + \frac{2}{3} - \frac{3}{4} + \frac{4}{5} - \frac{5}{6} - \frac{1}{7}$

(2) Express in decimals the sum of 438×15 and $\frac{063}{28}$

Or,

(1) Find what decimal of a maund is a chattack

(2) Find the price of 432 pieces of cloth at Rs 5 7a 6p each

3 (1) If Rs 750 amount to Rs 873 12a in 5 years and 6 months, find the simple interest per cent per annum

(2) A can run 8 yd. in the same time that B can run 9. They start together, when B has run 252 yds, how far behind is A?

Additional Paper

1 Find the square root of 29 192409

Or,

Find the cost of fencing a square field of 10 acres at 6a 8p per yd

2 A room is 20 metres in length and 10 metres in breadth Find the number of square yards in the area of the floor, taking a metre as equal to 39 37 inches

Or,

Define a prime number, and state all the prime numbers between 70 and 90

1914

Compulsory Paper

1 Multiply $777\frac{378}{119}$ by 358, and express the result as a whole number and a proper fraction

2 Find the G C M of 7163 and 13091

Or,

Find the L C M of 48, 72, 80, 108, and 120

3 Simplify $\frac{3\frac{5}{8} + 7\frac{1}{2} + 9\frac{1}{4}}{11\frac{7}{8} - 5\frac{1}{8}} \div \frac{16\frac{3}{8}}{16\frac{3}{8} - 2\frac{1}{8}}$

Or,

Find the price of 273 maunds, 33 seers, 7 chattacks of ghee at Rs 53 8a per md

4 Add together 0.022 of £1, 0.946 of a shilling, and 3.48 pence, and subtract the sum from 0.26 of a guinea. Express the answer in pence and the decimal of a penny.

Or,

Find what sum will amount to Rs 723 or 10p in 6 years and 3 months at $4\frac{1}{2}$ per cent per annum, simple interest.

Additional Paper

1 Extract the square root of 7 correct to 3 places of decimals.

Or,

Show that 103 is a prime number.

2 Given one centimetre = 0.3937 inches, find in square metres the area of a floor whose length is 21 feet and breadth 10 feet 8 inches.

1915

Compulsory Paper

1 (1) Multiply 790463 by 95076.

Or,

Divide 277286112 by 35064.

(2) Find the G.C.M. of 253512 and 568512.

Or,

Find the L.C.M. of 125, 160, and 280.

2 (1) Simplify $2 - \frac{5}{3 + \frac{1}{2 - \frac{1}{1 + \frac{1}{3}}}}$

(2) Multiply 17.55 by 4004, and divide the product by 0.819. (The results are to be expressed in decimals.)

Or,

(1) Express as a recurring decimal $0.4 + \frac{15}{0.13}$

(2) Find the price of 729 slabs of marble at Rs 7 11as 3pils each.

3 (1) At what rate per cent per annum (simple interest) will a sum of money double itself in 10 years?

(2) Find the cost of papering the walls of a room 12 ft 6 in long, 7 ft 6 in wide, and 12 ft high, with half anna postage stamps measuring $\frac{1}{8}$ inch by $\frac{1}{4}$ inch.

Additional Paper

1 Find the square root of 170 485249

2 The palace of the King of Babylon contained a thousand rectangular courtyards, each 60 metres long and 54 metres broad. The courtyards were all paved with marble slabs, 18 inches long by 18 inches broad. Required the total number of slabs (Metre = 39 37 inches)

- Or,

Multiply 0.48785 by 0.85963 by a contracted method so as to obtain the result correct to five places of decimals

1916

Compulsory Paper

1 (1) Multiply 560789 by 987065

Or,

Divide 823479885 by 9897

(2) Find the G.C.M. of 36176 and 85085

Or,

Find the least whole number which is exactly divisible by 1, 2, 3, 4, 5, 6, 7, 8, and 9

2 (1) Simplify $\frac{3\frac{2}{3}}{2\frac{1}{4}} - \frac{2\frac{1}{2} - 7}{\frac{2}{3} \text{ of } \frac{4}{8}} + \frac{85}{204}$

(2) What decimal of a sovereign is a penny?

Or,

(1) Express $\frac{2051}{22800}$ as a terminating decimal fraction

(2) Find the cost of 153 articles at £1 2s 8d each

3 (1) If I have to pay 2 pies as interest on one rupee for one month, what is the rate per cent per annum?

(2) If 24 men can do a piece of work in 15 days, working $8\frac{1}{2}$ hours a day, how many men will be required to do another piece of work twice as great in 17 days, working 6 hours a day?

Additional Paper

1 Find the square root of 0.0041409225

2 Express the value of —

0.04375 kilogram + 0.3775 gram + 0.72 milligram as the decimal of a pound Avoirdupois

[1 gram = 15.432 grains, and one pound Avoirdupois = 7000 grains]

Or,

Divide 24494897 by 14142135 by a *contracted method*, correct to three decimal places

MADRAS MATRICULATION PAPERS

1885

1 Explain how the value of a fraction is not altered when its numerator and denominator are multiplied by the same number

Simplify $\left(\frac{1\frac{1}{2} \text{ of } 1\frac{1}{2} - \frac{1}{2} + \frac{2}{3} }{\frac{2}{3} \text{ of } 1\frac{1}{2} + 1\frac{1}{2}} \right) \times 41\frac{3}{8} - \frac{1}{8} \text{ of } \frac{1}{2}$

2 If the rupee is worth 15 6½d, express Rs 6-5-4 as a fraction of £1, and find the least number of rupees equal in value to an integral number of pounds

3 State the rule for converting recurring decimals into vulgar fractions, and find the value of 0.03 of 275 of £3-2 6+0.285714 of 13 of £7 5-10-0 5925 of £2-16-3

4 Find by any method the value of 5 cwt 2 qrs 21 lbs of goods at £3 7 6 per cwt

5 The carriage of 17½ cwt for 52 miles on a certain railway is 8s 4d, find what will be the cost of carrying 4½ cwt for 300 miles on a railway on which the rate per mile is 9 per cent lower

6 A landlord pays 1 per cent for collecting his rents and a tax of 7 pies in the rupee on what he receives after paying the collector. He has a clear rental of Rs 1,831 8 0. Find his gross rental

7 A grocer mixes four kinds of tea which cost him 5s, 4s, 3s, 2s per lb respectively, in the proportions of 2, 3, 4, 7 respectively. Find at what rate he must sell the mixture so as to gain 25 per cent on the whole

8 Define the terms *interest*, *discount*, and find in what time £533 6 8 will amount to £672 at 6½ per cent per annum, Simple Interest

9 A person invests £4800 in 4 per cent stock at 96, and after a year sells out at 92½, and invests the proceeds together with the interest for the year in stock at 96½. How much stock does he then purchase?

10 Find to four places of decimals the square root of $1\frac{37}{81}$; and calculate the cost of surrounding with a fence a square field of 22½ acres at 3d per yard

11 The population of a country increases at the rate of 7 per cent every 10 years. What was the population 20 years ago of a country whose present population is 4,007,150?

1886

1 State and explain the rule of the Multiplication of Vulgar Fractions

$$\text{Simplify } \frac{\frac{3}{4}(1\frac{3}{4} - \frac{3}{4} \text{ of } 1\frac{1}{2}) + \frac{1}{17} \times \frac{\frac{3}{4} + \frac{2}{3}}{\frac{1}{4} - \frac{2}{3}} - 20}{\frac{1}{2} \times 1\frac{1}{2} - 1\frac{1}{2} - \frac{1}{18}}$$

2 Express £66 14-5½ as the decimal of Rs 1,000, the rupee being worth 15 4½d

3 Distinguish between pure and mixed circulating decimals Find the value of 0.945 of £2-3 6½ + 0.37259 of £1-8-1½

4 Find by any method the rent of 156 ac 3 ro 24 po 11 sq yd at Rs 25-3 4 per acre

5 A clock which gains 3m 56s in 24 hours was set correctly at noon on the 1st of January, 1884 Find to the nearest minute the next date at which it indicated correct time

6 Twenty men are employed to make a tank 40 ft long, 20 ft broad, and 6 ft deep They work for 30 days and have just completed one-third of the work, when it was resolved to increase the length of the tank by 10 ft, the breadth by 4 ft and the depth by 2 ft How many additional men must be employed in order that the work may be completed in 30 days more?

7 The difference between the simple and compound interest on a sum of money for 3 years at 5 per cent is £7 12-6 Find the sum

8 The capital of a certain railway is £1,000,000 in 20,000 shares of £50 each, fully paid up The gross annual receipts are £105,000 of which 48 per cent is absorbed in working expenses, £4,600 goes to the reserve fund, and the remainder to pay dividend Find what annual income a person will obtain from the investment of £4,500 in the undertaking, the shares being at £62-10 0

9 Ice is manufactured for 6 pies a pound and sold for 9 pies a pound Two-thirds of the quantity made is kept for sale at the factory and the remainder is sent to branch shops If the average loss from melting of the former be 12½ per cent and that of the latter be 25 per cent, find the gain on every ton made

10 The average width and depth of a river at its mouth are 240 yards and 6 feet respectively, the average rate of flow is 3 miles per hour and the amount of sediment per cubic foot of water discharged is 1½ cubic inches Find the amount of sediment deposited annually, and the depth of the deposit, supposing it spread uniformly, (i.e., to the same depth throughout) over an area of 146 square miles

1888

$$2 \text{ Simplify } \frac{6\frac{3}{8} - 4\frac{1}{8}}{5\frac{1}{2} - 4\frac{3}{8}} - \frac{2\frac{2}{8} - 1\frac{1}{8} + 1\frac{1}{4} - \frac{1}{8} \text{ of } 3\frac{1}{4}}{\frac{1}{2} \times 3\frac{5}{8} - 5\frac{3}{8} - 3\frac{1}{8}} \times 13\frac{1}{2}$$

3 Find the value of $1\frac{3}{4}$ of 01236 of Rs 5-11 8, and taking the rupee as worth 1s 4 $\frac{1}{2}$ d, express the result as the decimal of one shilling

4 Find by any method the value of 9 tons 17 cwt 3 qrs 25 lbs of coffee at £72-18 4 per ton

5 When iron is at £3-7-6 a ton, the cost of laying a railway 10 mi 2 fur 20 po in length with rails weighing 270 lbs each is Rs 67,500 Find the cost of laying a railway 25 mi 220 yds long with rails of the same length weighing 500 lbs each, when iron is at £3-14 3 a ton

6 Find the present value of £482 6-10 $\frac{1}{2}$ due 3 years hence at 5 per cent Compound interest

7 When exchange is at the rate of 1s 4 $\frac{1}{2}$ d per rupee, a person in Madras orders from a bookseller in England a parcel of books, the published price of which is £5 The bookseller allows discount at the rate of 25 per cent on the published price, but includes in his bill a charge of 13s for packing, freight, &c When the books arrive in India, a further sum of Rs 2 8 has to be paid on account of landing charges and cost of delivery If the books can be obtained from a bookseller in Madras at the rate of 9 $\frac{1}{2}$ annas per shilling of the published price, find how much the person loses by ordering from England

8 A person holds forty Rs 500 shares in a concern which pays dividend at the rate of 6 per cent per annum When the shares are at Rs 675, he sells out and invests half the proceeds in 4 per cent stock at 90 With the other half he buys a house, for which he receives an annual rental of Rs 1,440 subject to a deduction of 3a 9p per rupee for repairs and taxes Find the alteration in his annual income

9 In a certain year a country produces 50,000,000 bushels of wheat Of this quantity 40 per cent is available for export at Rs 3-2 per bushel In the following year the acreage under wheat has increased 20 per cent., but the yield produce per acre is only seven-eighths of what it was in the previous year, while the quantity required in the country has increased 5 per cent If at the same time the export price has fallen to Rs 3 per bushel, find the increase in the value of the wheat available for export

10 The population of a country is 33,264,000 and there are 99 males to 101 females, 2 out of every 11 boys and one out of every 33 girls of school age are under instruction If the boys of school-age form one seventh of the male population and the girls of school-age form one seventh of the female population, find the total number of pupils under instruction

1889

$$2 \text{ Simplify } \frac{(\frac{1}{2} + \frac{1}{3}) - (\frac{1}{2} - \frac{1}{3})}{(\frac{1}{2} + \frac{1}{3}) - 1 - \frac{1}{3}} + \frac{1\frac{1}{2} + \frac{2}{3}}{1\frac{1}{2} - \frac{1}{3}} - \frac{\frac{1}{2} \text{ of } \frac{1}{3}}{\frac{1}{2} - \frac{1}{3}}$$

3 Multiply 41 36514 by 0019, expressing the result as a decimal, and find the value of 3472 of £1 4s - 03288 of £2 6s 3d

4 Find by any method the cost of 79 ca 17 m 5 v 25 pal of salt at Rs 21 10a 8p per candy

5 The cost of rice for a family of 2 adults and 3 children from Jan 1st, 1889, to Dec 11th, 1889, both days inclusive, during which time rice was selling at 15 4 sr per Re, was Rs 70 7a What will be the cost of rice for a family of 3 adults and 5 children from Dec 19th, 1889, to May 11th, 1890, both days inclusive, assuming that the price of the rice will be 14 7 sr per Re, and assuming also that the quantity required per day by each adult is the same in both cases, and that in both cases the quantity required by a child is $\frac{2}{3}$ of the quantity required by an adult?

6 On what sum due 1 yr 4 mo hence does the true discount amount to £100 18 9, Simple Interest being reckoned at $4\frac{1}{2}$ per cent per annum?

7 How much 3 per cent stock must a person sell when the selling price is 91 in order that by investing the proceeds in the $4\frac{1}{2}$ per cents at 113 $\frac{1}{2}$ he may derive from the investment an annual income of Rs 9817 8a, after paying income-tax at the rate of 5p per rupee?

8 A and B can do a piece of work in 10 days, B and C in 15 days, and C and A in 20 days They all work at it for 6 days, then A leaves and B and C go on for 4 days more If B then leaves, how long will C take to complete the work?

9 In a certain year the total amount received by a railway company for carriage of passengers was Rs 2,751,000 Of this sum 6 per cent was contributed by first class passengers, 5 per cent by second class, and the remainder by third class The fares were 18, 6, and 1 $\frac{1}{2}$ pies per mile for first, second and third class passengers respectively Assuming that the average distance travelled by each third class passenger was 36 miles and the average distance travelled by each passenger of the other classes was 160 miles, find the total number of passengers carried during the year

10 The length of a rectangular field is twice its breadth If the rent of the field at £3 7s 6d an acre is £151 17s 6d, find the cost of surrounding it with a fence at $4\frac{1}{2}$ d per yard

11 Extract the cube root of 9 to five decimal places

1890

1 Reduce 2149908480 sq in to acres, &c If this is the area of a rectangle, the length of which is 5 mi 7 fur 5 po 1 ft 6 in find its breadth

2 Simplify $\frac{1835}{2202} + \frac{5468}{12303} + \frac{147}{441} - 3\frac{1}{2}$ of $\frac{6}{5} \frac{25}{5}$ of $\frac{04}{1.285714}$

3 Find the value of 237 c 17 mds 6 v at Rs 4100 1a 4p per candy

4 300 coolies are set to build a tank-bund In 14 weeks they have done $\frac{7}{10}$ of the work, when rain stops the work for 4 weeks and washes away $\frac{2}{5}$ of what they have done At the end of that time the work is resumed with only 250 coolies In what time from the commencement will the work be finished ?

5 Find the amount of Rs 58,59,375 for 3 years at $4\frac{1}{2}$ per cent per annum, reckoning Compound Interest

6 Explain the difference between Discount and Interest If the discount on £2830 15s $7\frac{1}{2}d$ be equal to the simple interest on £2784 7s 6d for the same time, find the time, the rate of interest being 5 per cent per annum

7 A person invests £34,539 in the 3 per cents at 87 After receiving one year's dividend he sells out at 89 He then invests the whole in Railway stock paying 5 per cent at 115 What will the difference in his income be ?

8 A cistern 10 ft 6 in long by 7 ft 6 in wide by 3 ft 4 in high is lined inside with lead, 7 lbs of which cover a square foot Find the weight of the lead and its cost at 53s 4d per cwt

9 A cask contains 16 gals of spirits Two gallons are drawn off and the cask filled up with water Two gallons are again drawn off and the cask filled up as before This is done a third time Compare the quantities of spirits and water remaining in the cask

10 Find the square root of 379749833 583241

1891

2 Subtract 13 times Rs 17 6a 11p from 17 times Rs 13 6a 11p

3 Rs 330 3a 7p are to be divided among 193 persons, two of whom receive Rs 2 each, and ten Rs 3 each The others receive equal shares Find the value of each share

4 Find the value of $\frac{\frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}}{\frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6}} \times 3\frac{1}{2} - \frac{\frac{1}{2} + \frac{1}{3} + \frac{1}{4}}{\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{6}}$ and simplify (without reduction to vulgar fractions if you can)

$$2\ 03 + 1\ 34\dot{5} + 27\ 34 + 16\ 231\dot{7}$$

5 How long will it take to walk round a square field 14 acres a rood 24 poles in extent at the rate of 3 miles an hour ?

6 Find the cost of white-washing a room $22\frac{1}{2}$ ft by 12 ft and 11 ft high, at one anna per square yard, making allowance for four

windows each 4 ft \times 2 $\frac{1}{2}$ ft and two doors each 8 $\frac{1}{2}$ ft \times 4 ft Find also the cost of a carpet for the same room with 3 ft border all round, the carpet costing Rs 4 per square yard and the border Rs 6 per square yard

7 Find the compound interest on £3143 6s 8d for 3 years at 3 per cent per annum

8 A cistern can be filled by three pipes in 30, 40 and 60 minutes respectively, and emptied by an escape pipe in half an hour. The three taps are turned on at noon, but the escape pipe is at the same time accidentally left open and not closed for a quarter of an hour. At what time will the cistern be full?

9 I purchase 16 lbs of tea at 1s 7d per lb, 14 at 2s 2d and 17 at 1s 8d. Seven pounds of the mixture becoming spoiled have to be sold at a low price, but by selling half the remainder at 2s 4d per lb and the other half at 2s 7 $\frac{1}{2}$ d, I eventually make a profit of 25 per cent on the original outlay. At what price per pound was the spoiled tea sold?

10 A person invests a sum of money in the 4 per cents at 102. When they have risen to 104, he transfers Rs 6000 stock to another investment paying 5 per cent of which the shares are at 120. When the 4 per cents fall to par he transfers the remainder to the 5 per cents, which are still at the same price and now finds his income Rs 25 more per annum than it was at first. What was the sum originally invested?

1892

2 Simplify $\frac{\frac{5}{7} \text{ of } 1\frac{3}{4} - \frac{1}{2} \text{ of } \frac{5}{7}}{1 - \frac{1}{7} \times (\frac{1}{2} + \frac{1}{3})} \times \frac{1 + \frac{1}{7} - (1 - \frac{1}{7})}{(\frac{1}{2} + \frac{1}{3}) - \frac{1}{7} - \frac{1}{7}}$

3 Find the value of 0.416 of £33 7s 6d - 0.342 of £32 13s 1 $\frac{1}{2}$ d and express Rs 371 2a 6p as the decimal of a lakh of rupees

4 Find by any method the cost of making a road 37 mi 6 fur 31 po 3 yds long at Rs 1785 3a 4p per mile

5 Find the present value of £482 6s 10 $\frac{1}{2}$ d due three years hence at 5 per cent per annum, Compound Interest

6 Extract the square root of 13 697142031225 to six places of decimals

7 The annual rainfall of a district is 49.7 inches. Assuming that the fall is distributed uniformly over the district, and that a cubic foot of water weighs 62.5 lbs, find the weight in tons of the rain that falls throughout the year on a square mile

8 When exchange is 1s 2 $\frac{1}{2}$ d per rupee, a Madras bookseller sends to a London publisher a bill for £104 in payment of books ordered. Freight and landing charges amount to Rs 37 8a. The

publisher allows the bookseller discount at the rate of 35 per cent on the published price and the latter sells the books at the rate of $10\frac{1}{2}$ annās per shilling of the published price Find how much he gains on the transaction

9 In the year 1891, the cost of rice for a family of 2 adults and 4 children was Rs 86 7a 9p In that year rice is sold at 11 2 seers per rupee, and each child received two fifths of the amount given to an adult Assuming that in 1893, the price of rice will be 13 5 seers per rupee, what will be the cost of rice for the same family from January 5 to August 11 both days inclusive, if the allowance of each adult be increased by one-fourth and the allowance of each child be three sevenths of that of an adult?

10 The capital of a railway company amounts to Rs 18,90,00000, of which one-fourth is 5 per cent preference stock and one third $4\frac{1}{2}$ per cent preference stock In a certain year the receipts are Rs 18,150,000, and the working expenses amount to 55 per cent of the receipts Of the net receipts Rs 540000 are added to the reserve fund, and the remainder, after paying dividend on the preference stock, is divided among the ordinary shareholders What rate of interest will they receive?

11 In the ten years from 1871 to 1881 the population of a country increased at the rate of 9 5 per cent, and in ten years from 1881 to 1891, the rate of increase was 10 5 per cent If the population in 1891 was 31,023,759, find what it was in 1871

1894

2 Simplify $\frac{\frac{2}{3} - \frac{1}{5} \text{ of } \frac{7}{8} + \frac{8}{9}}{\frac{4}{5} - \frac{2}{7} - \frac{1}{5} + \frac{2}{3}} - \frac{1\frac{1}{2} - \frac{4}{9}}{1 - \frac{5}{8} - \frac{2}{3}}$

3 Find the value of 204752 of £2 2s 1d - 1734375 of £2 6s 8d

4 Find by any method the value of 59 ca⁶ 14 m. 7 v 27 pl of salt at Rs 26 10a 8p per candy

5 In a certain year the produce of a tea estate was sold in London at an average rate of 9½d per lb, and the amount realised was remitted at an average rate of exchange of 1s 2¼d per rupee In the following year the average price realised was only 8¾d per lb, but the quantity sold exceeded by 12½ per cent the quantity sold in the previous year, and the average rate of exchange at which remittances were made fell to 1s 1½d If in this year the total amount realised from sales in London was Rs 105000, find how much was realised in the previous year

6 A sum of money was invested for four years, interest payable annually The rate of interest was 5 per cent per annum for

the first two years and 4 per cent per annum for the last two, and the amount at the end of four years was £1,164 10s 3½d. What was the sum invested?

7 Ice is manufactured for 2½ pices per lb and sold at 6 pices per lb. Of the total quantity made one half is kept for sale at the factory, and the remainder sent to branch shops. The loss from melting is 12½ per cent in the case of the former and 25 per cent in the latter, and the agents at the branch shops receive commission at the rate of 15 per cent on the price of every pound sold by them. Find the profit on every ton of ice manufactured.

8 Two persons, *A* and *B*, set out together on a journey. They walked at the rate of 3 miles an hour, and after they had proceeded for three quarters of a mile, *B* returned, walking at the same rate, to the place of starting. Here he was detained three quarters of an hour. Setting out again he overtook *A* who had been walking all the time, at the end of 2½ hours from the second time of starting. At what rate did he walk?

9 A person sold 25 Bank of Madras shares and invested the proceeds in the Government 3½ per cents, when they were at 3½ premium. If his net annual income from the investment, after paying income tax at the rate of 5p in the rupee, be Rs 876 9a, find the price at which he sold each of his bank shares.

10 In the year 1891 the population of a country was 356,40,000 and there was 1025 females to every 1000 males. Of the total population 75 per cent could read and write, but of the females only 1 per cent could do so. Find what percentage of males could read and write.

11 Extract the square root of 81 13183159704101 to seven places of decimals.

BOMBAY MATRICULATION PAPERS

1885-86

1 Reduce $\frac{2\frac{1}{2}-\frac{5}{8}}{2\frac{1}{2}+\frac{5}{8}}$ of 2 gu + $\frac{7}{2}$ of $\frac{9-1}{14 \times 3}$ of 4 cl - $\frac{83 \text{ of } 1\frac{5}{7}-\frac{1}{7}}{1\frac{5}{7}}$

of £1 to the decimal of five half guineas, and prove that $\frac{6+5}{11+7}$ is greater than $\frac{1}{4}$ and less than $\frac{1}{5}$

2 A man contracts to perform a piece of work in 30 days and immediately employs 15 men on it, at the end of 24 days the work is only half done. How many boys should be given to assist them that the contract may be fulfilled, each boy working two-fifths as much as each man?

3 A person buys 80 tons of coal and after selling them again at 1s 6d per sack finds that he has gained £4, had he sold them for 1s 4d per sack he would have lost £6. Find the weight of each sack and the cost price per ton.

4 A field of 7 acres is sown with wheat, barley and maize, the areas of the crops being respectively $2\frac{1}{2}$, $3\frac{1}{2}$, $4\frac{1}{2}$. If the values of an acre of each be also respectively in the same ratios, and an acre of wheat be worth £7, what is the worth of all the crops in the field?

5 If the three per cents are at 92 $\frac{3}{4}$ and the four per cents at 123 $\frac{1}{4}$, in which should one invest? And how much is one investing when the difference in income is a shilling?

1886 87

1 Explain carefully the meaning of *prime number*, *factor*, *divisor*, *measure*, *multiple*.

Resolve 5005 into its prime factors

Add together as decimals 8 138, 14 65651, '20558963

2 The circumference of the fore-wheel of a carriage is 6 $\frac{7}{8}$ feet and that of the hind-wheel is 12 $\frac{1}{2}$ feet. How many feet must the carriage pass over before the wheels shall have made a complete number of revolutions?

3 A vessel is filled with a liquid, 3 parts of which are water and 5 parts syrup. How much of the mixture must be drawn off and replaced with water so that the mixture may be half water and half syrup?

4 (i) The surface of a cube is 308 16 square feet. Find the length of its edge.

(ii) Extract the cube root of 45 698 to four places of decimals.

5 If the price of gold be £3 10s 10½d an ounce and a cubic inch of gold weigh 10 ounces, what is the price of the gold that would be required to gild a dome whose surface is 5,000 square feet, the thickness of the gold gilding being 0002 of an inch?

6 A person invests in 4 per cent Government paper so as to receive 4 per cent clear when the income-tax is 5 pias in the rupee. What percentage will be received if the tax be increased to 7½ in the rupee?

1887 88

$$1 \text{ Simplify } \frac{142857 \times 076923}{010989} + \frac{275 \times 1125}{62}$$

2 If 9 lbs of rice cost as much as 4 lbs of sugar, and 14 lbs of sugar are worth as much as 1½ lbs of tea, and 2 lbs of tea worth 5 lbs of coffee, find the cost of 11 lbs of coffee, if 2½ lbs of rice cost 6½d

3 If Rs 165 14a 1½p be the discount of a debt of Rs 2820, simple interest being at the rate of 3½ per cent, how many months before due was the debt paid?

4 The price of gold is £3 17s 10½d per oz, a composition of gold and silver weighing 18 lbs is worth £637 7s, but if the proportions of gold and silver were interchanged it would be worth only £259 1s. Find the proportion of gold and silver in the composition and the price of silver per oz

5 By selling 4 dozen mangoes for 13 rupees, it was found that 10ths of the outlay was gained, what ought the retail price per mango to have been in order to have gained 60 per cent?

1888 89

1 One clerk has 24 428571 and a second clerk has 38½ sheets to engross they call in a third clerk and agree to divide the work equally among the three, and to pay the third clerk at the rate of 2430½ shilling per sheet, how much will he receive from each of them?

2 If the manufacturer makes a profit of 20 per cent, the wholesale dealer a profit of 25 per cent, and the shopkeeper a profit of 40 per cent, what was the cost of the manufacture of an article bought at a shop for 17s 6d?

3 If 15 men eat 28 shillings worth of bread in 14 days, when wheat is at 52 shillings per quarter, what must be the price of wheat per quarter that 18 shillings worth may provide bread for 13 men for 5 days?

4 Find the value of $\sqrt{(90252508017424)} - \sqrt{(347740371686161)}$

5 If the discount on £678 8s which is due at the end of a year and a half be £38 8s, what is the rate per cent of Simple Interest?

1889 90

1 Simplify $\frac{5\frac{1}{2} \text{ of } 2 \text{ of } 2571428 - 1 - (1 + 5)}{1 - \frac{1}{4} \text{ of } \left\{ 5 + \frac{1}{2} \text{ of } \frac{05}{142857 \text{ of } 1\frac{1}{20}} \right\}}$

2 A rectangular cistern, whose length is equal to its breadth is 5½ feet deep and contains 5 tons of water. If 1 cubic foot of water weighs 1000 ounces, find the dimensions of the cistern.

3 A, B and C can walk at the rate of 3, 4, and 5 miles an hour, they start from Pooná at 1, 2, 3 o'clock respectively, when B catches A, B sends him back with a message to C, when will C get the message?

4 If I borrow money at 3 per cent per annum, interest payable yearly, and lend it immediately at 5 per cent per annum, interest payable half yearly (receiving compound interest for the second half year), and gain thereby at the end of the year Rs 660, what was the sum of money which I borrowed?

5 A person buys tea at 6 annas per seer and also some at 4 annas per seer. In what proportions must he mix them so that by selling the mixture at 5½ annas per seer he may gain 20 per cent on each seer sold?

1891 92

1 Simplify (i) $\frac{1\frac{1}{2} \text{ of } 1\frac{1}{2}\pi + 6 \text{ of } \frac{1}{12}}{5\pi \text{ of } 1\frac{1}{2} - 1\frac{1}{2} \text{ of } \frac{1}{12}}$

(ii) $\frac{36428571 - (009923 + 0102 - 000123) \frac{145}{0056}}{\sqrt{345744 - 29663597}}$

2 Two passengers have together 5 cwt of luggage and are charged for the excess above the weight allowed 5s 2d and 9s 10d respectively, but if the luggage had all belonged to one of them he would have been charged 19s 2d. How much luggage is each passenger allowed to carry free of charge, and how much luggage had each passenger?

3 Two clocks A and B, whose rates are uniform, at noon yesterday indicated 11 hrs 55 min A M and on 2m P M respectively A indicated the correct time at 9 P M yesterday and B at 6 A M this morning. When did A and B last agree and what time did they then indicate?

4 A person borrows two equal sums of money at the same time at 5 per cent and 3½ per cent simple interest respectively, and finds that if he repays the former sum with interest on a certain date a year before the latter, he will have to pay in each case the same amount, viz Rs 736. Find the amounts borrowed.

1892 93

1 What decimal of a rupee is 964 pie? Find the value of Rs 97625

Simplify $\frac{\frac{1}{4} - \frac{1}{2} \text{ of } \frac{1}{2}}{\frac{1}{8} + \frac{1}{2} \text{ of } 3\frac{1}{4} - (\frac{7}{8} \text{ of } 2\frac{1}{4} - \frac{1}{2})} - \frac{\frac{1}{2} \text{ of } \frac{1}{2} + \frac{1}{2} \text{ of } 5}{9\frac{1}{2} - 1\frac{2}{3}}$

2 How long will two examiners, working 8 hours a day, take to look over the answers to this paper, if four examiners, working 5 hours a day, can do it in 8 days?

3 On a stream, *B* is intermediate to and equidistant from *A* and *C*, a boat can go from *A* to *B* and back in 5 hrs 15 min, from *A* to *C* in 7 hrs. How long would it take to go from *C* to *A*?

4 What income will a retired officer obtain in England, from one lakh of rupees, Indian Government $4\frac{1}{2}$ per cent bonds, when for drawing and remitting it, his agents in India charge him 3 per cent and exchange is at 1s 2½d for the rupee?

5 Three equal glasses are filled with a mixture of spirits and water, the proportion of spirits to water in each glass is as follows in the first glass as 2 3, in the second as 3 4, and in the third as 4 5. The contents of the three glasses are poured into a single vessel. What is the proportion of spirits to water in it?

1893 94

(Set in the Mofussil)

1 Divide each of the numbers 2,572,125 and 4,961,250 by 125, and express as a decimal the first quotient divided by the second

2 Find, by Practice, the value of 5 yds 22½ in at £2 1s 2d a yard

3 If the carriage of 2 cwt 1 qt 18 lbs of goods for 56 miles be £1 1s, what weight can be carried at the same rate, 200 miles for £4 3s 4d?

4 A man invests £3,000 in the 5 per cents. If after deducting an income tax of 8d in the pound, the man's clear income is £174, what is the price of the 5 per cents?

5 A cistern is filled by two taps *A* and *B* in 4 hours and 6 hours respectively, and is emptied by a waste pipe *C* in 3 hours. When the cistern is half full, *A* and *B* are closed, and *C* is opened, after one hour, *B* is turned on, and after half an hour more, *A* is turned on. In what time after *C* is first opened, does the cistern become full?

6 A person buys two kinds of tea, at 5s a lb and 6s a lb respectively, and after mixing them he sells the mixture at 6s 6d a lb, thereby gaining 17 per cent. In what proportion does he mix them?

(Set at Bombay)

1 Reduce to their simplest forms —

(i) $\frac{\frac{1}{2} + \frac{1}{3} - \frac{1}{6}}{\frac{1}{2} \text{ of } \frac{1}{3} \text{ of } \frac{1}{6}}$

(ii) $\frac{2}{3 + \frac{4}{5 - \frac{1}{2}}}$

2 Find, by Practice, the value of 9 cwt 3 qrs 24 lbs at £3 5s 8d per cwt

3 If 40 men, 60 women or 80 children can do a work in 6 months, in what time will 10 men, 10 women, and 10 children do $\frac{1}{4}$ of the work?4 A person invested £1,000 in the 3 per cents at $90\frac{1}{2}$, but the price rising to $91\frac{1}{2}$, he sold out and invested the proceeds in the 3½ per cents at $97\frac{1}{2}$, find the increase in his income

5 A cistern can be filled by two pipes, A and B, in 12 minutes and 14 minutes respectively, and can be emptied by a third C, in 8 minutes. If all the taps be turned on at the same moment, what part of the cistern will remain unfilled at the end of 7 minutes?

6 Two clocks point to 2 o'clock at the same instant on the afternoon of 25th April, one loses 7 seconds, and the other gains 8 seconds, in 24 hours, when will one be half-an hour before the other, and what time will each clock then shew?

1894-95

1 When the number representing the year is a multiple of four, it is a leap year consisting of 366 days, except when this number is a multiple of 100, in which case it is an ordinary year consisting of 365 days, but when the number is a multiple of 400, it is again a leap year, on this supposition calculate the number of days from the 1st January, 1495 to 31st December, 1894, both days inclusive

2 A school of boys and girls consists of 453 children, the number representing the boys is $\frac{5}{2}$ of the number of girls. How many boys were there?

3 Two thirds of a certain number of poor persons received 1s 6d each, and the rest 2s 6d each, the whole sum spent being £2 15s, how many poor persons were there?

4 If 3 men and 5 women do a piece of work in 8 days, which 2 men and 7 children can do in 12 days, find how long 13 men, 14 children and 15 women will take to do it

5 A sells a house to B for Rs 4860, thereby losing 19 per cent, B sells it out to C at a price which would have given A 17 per cent profit. Find B's gain

6 The compound interest on one rupee is one quarter of a rupee at the end of three years, find the rate per cent per annum, correct to two places of decimals, and calculate exactly the compound interest at the end of 9 years

PUNJAB ENTRANCE PAPERS

1835

1 Simplify $\frac{1-\frac{1}{2}-\frac{2}{3}}{\frac{1}{2}+\frac{1}{3}} + \frac{\frac{2}{3}+\frac{1}{6}}{\frac{1}{3}-\frac{1}{6}}$ of $\frac{1}{60} - \frac{3}{35}$, and find how many times 027 can be taken from 333

2 Convert $\frac{13}{20 \times 8}$ into a decimal, why is the result a terminating and not a recurring decimal? Subtract 03 from 03 and divide the result by 007

3 Find, by Practice, the value of 12 maunds 8 seers 4 chataks of ghee at Rs 72 8a per maund

4 A legacy of £1901 5s is to be distributed amongst a number of persons, in such a way that each shall receive as many shillings as there are persons, what will be the portion of each?

5 Find the Least Common Multiple of 35280 and 592704. What is the smallest number of square yards which can be measured, either by rods or square chains?

6 Four per cents are offered at Rs 98, five per cents at Rs 120 $\frac{20}{100}$, which is the better investment? How much is the investment when the difference of income is Rs 30?

1886

1 Simplify $\frac{44-283}{16+2629}$, and extract the square root of the result to three places of decimals

2 Reduce $\frac{5}{1}$ to a decimal fraction correct to four places
 $7 - \frac{1}{2 - \frac{1}{2}}$

Is there anything to suggest that the result will be a terminating or recurring decimal?

3 What fraction of £51,120 18s is 17 975 of £71 2s?

4 A clever housekeeper went out shopping and found that 2 cocoanuts were selling for the same price as 144 plums, she

bought half a dozen cocoanuts, exchanged one of them for 5 lemons, and a couple of lemons for 5 oranges, she then gave 3 oranges for 42 limes, and finally secured a couple of plums for 5 limes. Has she gained or lost in buying the plums?

5 Distinguish between Interest and Discount

Find the Interest and Discount of Rs 1,450 8a for 3 years at 4½ per cent per annum, Simple Interest

1887

1 (a) Write in figures—three billions, five millions, four hundred and nine thousand and sixty-two

(b) Write out the measures of length and surface, both English and Indian

(c) Express an acre as the decimal of a *bigha*, a cubit being equivalent to 18 inches

2 Owning $\frac{1}{7}$ of an estate, I sold $\frac{1}{11}$ of $\frac{2}{5}$ of my share for £400, what is the value of $\frac{1\frac{1}{2}}{4\frac{1}{2}}$ of $\frac{2}{5}$ of the estate at the same rate?

3 A merchant having 150 maunds of grain sold 50 maunds at Rs 9 1a 1½p per maund, and thereby gained 7½ per cent. At what rate should he sell the remainder 100 that he may gain 10 per cent on the whole?

4 A merchant in trade successively admits three partners at the end of 3 months, 5 months, and 6 months respectively from the opening of the business. The capitals embarked by them were Rs 400, Rs 450, Rs 480, and Rs 495 respectively. After 6 months more, the profit was found to be Rs 1,518. Divide this rateably between the partners.

5 What sum of money invested in the 4 per cents at par would realise the same income as Rs 10,200 invested in the 4½ per cents at 102½

6 Extract the square root of —

$$\begin{array}{r} 0025 \times 16 \\ 36-25 \end{array} \text{ of } \frac{426 \times 2625}{127-102}$$

1888

1 Simplify $\frac{1}{1-\frac{1}{12}} - \frac{1-\frac{1}{12}}{2-\frac{1}{3}} + \frac{1\frac{1}{2}}{4-1\frac{1}{2}} - \frac{6\frac{1}{2}-3}{6\frac{1}{2}} \times \left\{ \frac{1}{2} - \frac{\frac{1}{2}-\frac{1}{4}}{4\frac{1}{2}-3\frac{1}{4}} \right\}$

2 Express the difference between 378 of 13s 10½d and 378 of 26s 6d as the decimal of

$$426 \text{ of } \frac{33}{08} \text{ of } \frac{3}{735} \text{ of } \frac{147 \times 44}{111} \text{ of } \text{£} 1 \ 17s \ 6d$$

3 Four men working together all day, can finish a piece of work in 11 days, but one of them having other engagements can work only half time, another only quarter time. How long will it take the men to complete the work?

4 A merchant sells his goods worth Rs 500 directly for Rs 600, giving three months' credit. Find his profit per cent, interest being calculated at 12 per cent per annum.

5 Find the value of $\frac{12 + \sqrt{1009}}{1 - \sqrt{4}}$ correct to 3 places of decimals

1889

1 Express 800800800975 in words and give the local value of the digits. What decimal of Rs 75 is Rs 24 2a 6p?

What is the least number which, when divided by 22, by 88, by 132, and by 198, gives in each case a remainder 7?

2 Why is the fraction $\frac{3}{4}$ objectionable?

After walking $4\frac{1}{2}$ miles, a man has accomplished $\frac{2\frac{1}{2} - 1\frac{1}{2} \text{ of } 2\frac{1}{4} + 1\frac{1}{8}}{(2\frac{1}{2} - 1\frac{1}{2}) \text{ of } (2\frac{1}{4} + 1\frac{1}{8})}$

of $\frac{11 + \frac{1}{2}}{4 - 1\frac{1}{2}}$ of his journey, how far has he still to walk?

3 Add together $\frac{57}{152}$ and $\frac{0112}{74}$

Five bells which commence tolling together, toll at intervals of 12, 15, 175, 18, 21 seconds respectively, after what interval will they again toll together?

4 Define "present worth"

A farmer buys 57 sheep for Rs 120 payable at the end of 12 months and sells them directly at Rs 1 12a ready money, what does he lose by the transaction, supposing the interest of money to be 5 per cent?

Which is the better investment, the 3 per cents at $83\frac{1}{2}$ or the $3\frac{1}{2}$ per cents at 3 per cent discount?

5 Shew which is the greater, $\sqrt{2}$ or $\sqrt[3]{3}$?

1890

1 Simplify (a) $\frac{\frac{\frac{2}{8}}{1 - \frac{1}{28}} + \frac{1}{3} + \frac{1}{4}}{1 - \frac{1}{2} \left(\frac{-\frac{2}{8}}{1 - \frac{1}{28}} + \frac{1}{3} \right)}$ (b) $\frac{47 - (5 - 0303)}{0873 - (0083 + 07)}$

2 What part of $\frac{1}{8}$ of 5 cwt is $\frac{1}{100}$ of a ton?

Express $3\frac{7}{8}$ of 16s 6d as a decimal of '426 of £1 17s 6d

3 A man bequeathed $\frac{5}{8}$ of his property to one son, 30 per cent of the remainder to another and the surplus to his widow. The difference of his sons' legacies was £754. How much did the widow receive?

4 A ship with 1200 men on board had sufficient provisions to last 17 weeks. The survivors of a wreck having been taken aboard, the provisions were consumed in 15 days. How many men were taken aboard?

5 At what price must a person invest in the 4 per cent Government Promissory Note, so that after paying income tax at the rate of 5 pies in the rupee he may receive 41 per cent on his investment?

6 A and B travel together 120 miles by rail. A takes a return ticket for which he has to pay one fare and a half. Coming back they find that A has travelled cheaper than B by 40 2p for every 100 miles. Find the fare per mile.

1891

1 Simplify —

$$(1) \frac{1 + \frac{1}{2} + \frac{7}{1 - \frac{1}{2^4}}}{1 - 2 \text{ of } \left(\frac{1}{2} + \frac{7}{1 - \frac{1}{2}} \right)}$$

$$(2) \frac{3\sqrt{2} - 2\sqrt{3}}{3\sqrt{2} + 2\sqrt{3}} + \frac{\sqrt{12}}{\sqrt{3} - \sqrt{2}}$$

2 Express 2 75 or + 075 cwt as decimal of 2 25 of 27 of a ton

3 A sum of money invested at 5 per cent per annum, Simple interest, amounts in 6 years to Rs 1,326, in what time will it amount to Rs 1530?

4 What is Discount? Distinguish between True and Commercial Discount

The interest on a certain sum at 5 per cent per annum for a certain time is £50 and the discount at the same rate for the same time is £40. Find the sum and time

5 Nine gallons are drawn from a cask full of wine. It is then filled with water. Nine gallons of the mixture are drawn, and the cask is again filled with water. The quantity of wine now left in the cask is to that of the water in it as 16 : 9. How much does the cask hold?

1892

1 Find by how much the square root of

$$9 + \frac{1}{1 + \frac{1}{7 + \frac{1}{1}}}$$

$$3 + 16\sqrt{2}?$$

3 Four men working together all day, can finish a piece of work in 11 days, but one of them having other engagements can work only half time, another only quarter time. How long will it take the men to complete the work?

4 A merchant sells his goods worth Rs 500 directly for Rs 600, giving three months' credit. Find his profit per cent, interest being calculated at 12 per cent per annum.

5 Find the value of $\frac{12 + \sqrt{1009}}{1 - \sqrt{4}}$ correct to 3 places of decimals

1889

1 Express 80080080 0975 in words and give the local value of the digits. What decimal of Rs 75 is Rs 24 2a 6p?

What is the least number which, when divided by 22, by 88, by 132 and by 198, gives in each case a remainder 7?

2 Why is the fraction $\frac{1}{8}$ objectionable?

After walking $4\frac{1}{2}$ miles, a man has accomplished $\frac{2\frac{1}{2} - 1\frac{1}{2}}{(2\frac{1}{2} - 1\frac{1}{2})}$ of $2\frac{1}{2} + 1\frac{1}{2}$ of $(2\frac{1}{2} + 1\frac{1}{2})$ of his journey, how far has he still to walk?

3 Add together $\frac{57}{152}$ and $\frac{0112}{74}$

Five bells which commence tolling together, toll at intervals of 12, 15, 175, 18, 21 seconds respectively, after what interval will they again toll together?

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A farmer buys 57 sheep for Rs 120 payable at the end of 12 months and sells them directly at Re 1 12a ready money, what does he lose by the transaction, supposing the interest of money to be 5 per cent?

Which is the better investment, the 3 per cents at $83\frac{1}{2}$ or the $3\frac{1}{2}$ per cents at 3 per cent discount?

5 Shew which is the greater, $\sqrt{2}$ or $\frac{2}{3}$?

1890

1 Simplify (a) $\frac{\frac{\frac{2}{3}}{1 - \frac{2}{3}} + \frac{1}{3} + 1}{1 - \frac{1}{2} \left(\frac{\frac{2}{3}}{1 - \frac{2}{3}} + \frac{1}{3} \right)}$ (b) $\frac{47 - (5 - 0303)}{0873 - (0083 + 07)}$

2 What part of $\frac{1}{8}$ of 5 cwt is $\frac{1}{100}$ of a ton?

Express $37\frac{7}{8}$ of 16s 6d as a decimal of 42s of £1 17s 6d

3 A man bequeathed $\frac{1}{2}$ of his property to one son, 30 per cent of the remainder to another and the surplus to his widow. The difference of his sons' legacies was £754. How much did the widow receive?

4 A ship with 1200 men on board had sufficient provisions to last 17 weeks. The survivors of a wreck having been taken aboard, the provisions were consumed in 15 days. How many men were taken aboard?

5 At what price must a person invest in the 4 per cent Government Promissory Note, so that after paying income tax at the rate of 5 pies in the rupee, he may receive $4\frac{1}{2}$ per cent on his investment?

6 A and B travel together 120 miles by rail. A takes a return ticket for which he has to pay one fare and a half. Coming back they find that A has travelled cheaper than B by 40 p for every 100 miles. Find the fare per mile.

1891

1 Simplify —

$$(1) \frac{\frac{1}{2} + \frac{1}{3} + \frac{5}{1-2x}}{1 - 2 \text{ of } \left(\frac{1}{2} + \frac{5}{1-2x} \right)}$$

$$(2) \frac{3\sqrt{2} - 2\sqrt{3} + \sqrt{12}}{3\sqrt{2} + 2\sqrt{3} + \sqrt{3} - \sqrt{2}}$$

2 Express 275 oz + 075 cwt as decimal of 225 of $2\frac{1}{2}$ of a ton

3 A sum of money invested at 5 per cent per annum, Simple interest, amounts in 6 years to Rs 1,326, in what time will it amount to Rs 1530?

4 What is Discount? Distinguish between True and Commercial Discount

The interest on a certain sum at 5 per cent per annum for a certain time is £50 and the discount at the same rate for the same time is £40. Find the sum and time

5 Nine gallons are drawn from a cask full of wine. It is then filled with water. Nine gallons of the mixture are drawn, and the cask is again filled with water. The quantity of wine now left in the cask is to that of the water in it as 16 : 9. How much does the cask hold?

1892

1 Find by how much the square root of

$$9 + \frac{1}{1 + \frac{1}{7 + \frac{1}{1}}}$$

$$3 + 16\sqrt{2}$$

2 Find the value of $\left(\frac{0019}{3\ 16} \text{ of } \frac{4\ 4}{0005}\right) - \left(\frac{8\ 8}{7} \text{ of } \frac{4}{5\ 625}\right)$

3 A stream which flows at a uniform rate of 1 109 miles an hour, is 20 yards wide, the depth of a certain ferry being 6 ft how many gallons pass the ferry *in a minute*? (Each gallon contains about $277\frac{1}{4}$ cubic inches)

4 A person invests £14,970 in the purchase of the 3 per cents at 90 and the $3\frac{1}{4}$ per cents at 97 His total income being £500, how much of each stock did he buy?

5 A spirit merchant buys 80 gallons of whisky at 18s per gallon, and 180 gallons more at 15s per gallon and mixes them At what price must he sell the mixture to gain $8\frac{1}{2}$ per cent upon his outlay?

1893

2 Multiply 319 9657 by 0.4286

3 Find the value of $\frac{\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}$ correct to 5 places of decimals

4 Calculate the income tax on Rs 666 10s 8p at 5p per rupee

5 A local train which travels at the rate of 24 miles an hour, leaves Lahore at 20 min past 8 and reaches Amritsar at 5 min past 10 the same morning It stops at Mianmir for 10 min and at each of three other stations for 5 min Find the distance between Lahore and Amritsar

1894

1 Convert $\frac{1}{2}$ and $\frac{3}{4}$ into circulating decimals and point out the relation between the figures in their periods

2 The sides of a rectangle are as 3 4 and the area is 1452 square feet Find its length and breadth

3 Exchange Rs 7680 for English money at 1s $3\frac{1}{2}d$ per rupee

4 What is discount? How is it commonly calculated? If a sum of Rs 1000 becomes due three months hence, what is its present value as commonly calculated, and what as correctly calculated, interest being reckoned at 5 per cent?

5 Find the square root of 101 correct to 5 places of decimals

1895

1 Divide $\frac{48\frac{4}{7}}{1085\frac{7}{10}}$ by $\frac{7\frac{3}{11}}{174\frac{1}{17}}$, and reduce the quotient to a recurring decimal

2 The imperial gallon contains 277 27 cubic inches and a cubic foot of water at its maximum density weighs 62 42 lbs, find the weight of a pint of water correctly to two places of decimals

3 The capital of a firm consists of £713 3s, £964 17s, £2391 3s subscribed by three partners, divide £2231 among them in proportion to their several capitals

4 Find the square root of 5 correctly to seven places of decimals

5 The area of a rectangular field is $\frac{1}{2}$ of an acre, and its length is twice its breadth, determine the lengths of its sides approximately

1896

1 Make out a bill for the following articles supplied by Messrs Mool Chind & Co to Lal Gujar Mal —

10 lbs of tea at Rs 1 3a per lb, 6 seers of sugar at Rs 2 3a per bag of 5 seers, 4 tins of coffee at Rs 1 1a per tin, 8 silk handkerchiefs at Rs 3 8a per dozen, 3 mds 37 sq of Portland cement at 8 seers per rupee, a child's perambulator, price Rs 30. Subtract 10 per cent discount for cash

2 Reduce to its lowest terms $\frac{1}{10}$ of $4\frac{1}{3}$ of $\frac{\frac{1}{2} + \frac{1}{3} + \frac{1}{6}}{\frac{1}{4} + \frac{1}{5} + \frac{1}{6}}$

3 A cubic foot of copper weighs 560 lbs. It is rolled into a square bar 40 ft long. An exact cube is cut from the bar. What is its weight to four decimals of a pound?

4 The area of a country is 32,300,000 acres. It consists of three kinds of land, the areas of which are in proportion to the numbers 2, 3 and 7. How many acres are there of each kind of land?

5 If the 3 per cent stock is at 98 $\frac{1}{2}$, how much money must be invested in the stock to yield an annual income of Rs 120?

1897

1 Define a fraction, and prove that the value of a fraction is not altered by multiplying both its numerator and denominator by the same whole number. Deduce from this principle a rule for the addition of fractions.

2 The sum of £177 is to be divided among 15 men, 20 women and 30 children, in such a manner that a man and a child may together receive as much as two women, and all the women may together receive £60. What will they respectively receive?

3 Find the value of $\frac{\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}$ correct to 7 places of decimals

4 A garrison of 800 men has provisions sufficient for 10 weeks. How long would they last if the garrison were reduced to 560 men?

5 Find the L C M of $4\frac{1}{2}$, $5\frac{2}{3}$, $6\frac{1}{4}$ and $7\frac{1}{5}$

1898

1 State in words the value of the figures 200690125, and multiply 056931 by 1879 6958

2 On what day of the week will December 25 fall next year?

3 Find the cost of papering the walls of a room 22 ft long, 18 ft wide and 20 ft high, with rolls of paper 21 inches wide at Rs 2 10s per roll of 12 linear yards

4 Simplify $\frac{\frac{1}{17}(\frac{2}{18} \text{ of } 2\frac{2}{3} + \frac{1}{4} \text{ of } 1\frac{2}{3})}{\frac{1}{18} \times 1\frac{1}{8} \times 1\frac{1}{4} - \frac{1}{17}} - \frac{\frac{1}{2}}{\frac{2}{3} \text{ of } 3 - \frac{1}{4} \text{ of } 5\frac{1}{4}}$

5 A person holding £10,000 in the 3 per cent stock sells out at 93 $\frac{1}{2}$, and invests the proceeds in the 4 per cent stock at 101 $\frac{1}{2}$. Find the change in his income, allowing $\frac{1}{4}$ per cent commission in each transaction

1899

1 Express 7639 as a non recurring decimal

(a) Simplify $00621 + 10891 + 81 \cdot 0563 + 21 \cdot 02$ without reducing the terms to vulgar fractions

2 The length of a hall is three times the breadth. The cost of whitewashing the ceiling at 5 $\frac{1}{2}$ d per square yard is £4 12s 7 $\frac{1}{2}$ d and the cost of papering the walls at 1s 9d per square yard is £35. Find the height of the hall.

3 Shew that the difference between the interest and the true discount on a given sum at a given rate for a given time, is equal to the interest on the discount.

4 A man has £5 17s consisting of sovereigns, half crowns and shillings, in the proportion of 2, 3, 11. How many has he of each coin?

5 Which is the better investment, the 3 $\frac{1}{2}$ per cents at 102, or the 3 per cents at 97?

1900

1 Find the square root of 4001204 090601

2 Find the present worth of Rs 10000 due 8 years hence at 4 $\frac{1}{2}$ per cent

3 A rectangular court-yard the sides of which are as 5 11, costs Rs 144 6a for paving at 10a 6p per sq yd. Find the lengths of its sides.

4 Shew that *this* year the 23rd of March, and 23rd of November fall on the same day of the week.

5 Shew that Compound Interest reckoned quarterly at Rs 1 3s 7½p per cent is nearly equal to Interest reckoned yearly at 5 per cent

1901

1 Eight bells which toll at intervals of 1, 2, 3, 4, 5, 6, 7, 8 seconds respectively, begin tolling all simultaneously with the clock striking. How many hours must elapse before they all toll simultaneously again with the clock striking? (N.B.—The clock is supposed to strike at the hour only)

2 Find the true discount on a bill for £721 13s 8d paid 73 days before due, the rate of interest being 3½ per cent per annum

3 Divide each of the numbers 4061250 and 2572125 by 125 and express the ratio of the quotients correctly to three places of decimals

4 A man buys eggs at 1s 3d per dozen and sells them at 11s 8d per hundred. Find his gain per cent

5 There are four vessels of equal capacity, $\frac{1}{8}$ of the first is filled with spirit, $\frac{1}{4}$ of the second, $\frac{1}{2}$ of the third, and $\frac{3}{4}$ of the last. The first is then filled with water and from this mixture the second is filled up, again from the second mixture, the third is filled up, and in like manner the fourth from the third. What proportion of spirit to water is there in the fourth vessel?

1902

1 Define a prime number. Find the prime factors of 555,555.

2 A railway truck is 29 ft 4 in in length, how many such trucks will be required to fill up the entire length of the line between Lahore and Amritsar, a distance of 32 miles?

3 The difference between the simple and compound interest on a sum of money for 2 years at 5 per cent per annum is Rs 12. Find the sum.

4 If 3 fowls and 4 pigeons cost Rs 2 3s 6p, and 5 fowls and 2 pigeons cost Rs 2 12s, find what must be paid for 4 fowls and 3 pigeons.

5 A person sold 60 yards of cloth for Rs 28 2s gaining thereby the cost price of 9 yards. Find his gain per cent.

ALLAHABAD ENTRANCE PAPERS

1889

1 Define a fraction and show that $\frac{1}{2} = \frac{2}{4}$ By how much does the difference of $1\frac{1}{2}$ and $1\frac{1}{3}$ fall short of their sum? Express the defect as a decimal2 (a) Simplify $\frac{3\frac{1}{2} - 1\frac{1}{2} \text{ of } 1\frac{1}{2} - 1\frac{1}{2}}{(3\frac{1}{2} - 1\frac{1}{2}) \text{ of } (1\frac{1}{2} - 1\frac{1}{2})}$

(b) Subtract 03 from 03 and divide the result by 102

3 Find the square root of 001 to four places of decimals
What number has 1 for its square root?

4 What sum of money will amount to Rs 1381 4a in 15 months at 5 per cent per annum, Simple interest?

5 How long will it take to walk along the four sides of a square field which contains 16 acres 401 square yards, at 3 miles an hour?

6 A and B complete a piece of work in 8 days, B and C do the same in 12 days, and A, B and C finish it in 6 days. In how many days will A and C complete the work?

7 A who travels $3\frac{1}{2}$ miles an hour starts $2\frac{1}{2}$ hours before B who goes the same road at $4\frac{1}{2}$ miles an hour, where will he overtake A?

1890

1 Multiply 347695 by 2 0066, and divide the product by 01905

2 Simplify $1\frac{1}{2} + 3\frac{1}{2} - 5\frac{1}{2} + 2\frac{1}{2} - 1\frac{1}{2}$

3 Find by Practice or otherwise the value of 2345 mds 27 sl and 10 ch of wheat at Rs 3 10a 8p per maund

4 Extract the square root of $1 - (.00135)^2$ to 5 places of decimals

5 One cubic inch of water weighs 253 17 grains while one inch of air weighs 31 grain, find the number of inches of water (to three places of decimals) that would be equivalent to one cubic foot of air

6 On measuring a distance of 32 yds with a rod of a certain length, it was found that the rod was contained 41 times with $\frac{1}{2}$ an inch over. How many inches will there be over in measuring 44 yds with the same rod?

1891

1 Define "Notation," "Numeration", and prove that "three-times four" = "four times three"

2 Reduce to a single fraction $\frac{919\frac{2}{7}}{7954} \times \frac{4100}{442\frac{8}{9}} \times \frac{7}{11}$ of 07344

3 The wine in a pipe when full is worth £19 9s 9d How much has leaked away, if what is left is worth £9 16s 7 $\frac{8}{9}$ d ?

4 In discounting a bill, what do you mean by "the Banker's profit?" If the simple interest on £923 18s 1 $\frac{1}{2}$ d amounts to £17 9s 3 $\frac{1}{2}$ d exactly in 138 days, what is the rate of interest per cent per annum?

5 Extract the square root of 99,980,001, and of 60 $\frac{81}{100}$

1892

1 How is a fraction affected by adding the same number to the numerator and the denominator?

Prove that $\frac{3+4}{4+5}$ is greater than $\frac{1}{2}$ and less than $\frac{1}{3}$

2 (a) Divide $\frac{1}{3}[3+\frac{1}{3}\{3+\frac{1}{3}(3+1\frac{1}{2})\}]$ by 125

(b) Reduce $\frac{1}{7}\frac{4}{3}$ and $\frac{1}{2}\frac{5}{9}$ to their lowest terms and express their difference as a decimal

3 Forty men finish a piece of work in 40 days, if 5 men leave the work after every tenth day, in what time will the whole work be completed?

4 Find the difference between the Simple Interest and Discount of £330 in 4 years at 2 $\frac{1}{2}$ per cent per annum

5 Extract the square root of $\frac{100020001}{1000}$

1893

1 Two recurring decimals are added together, prove that the number of digits in the period of the result, cannot exceed the product of the numbers of the digits in the original periods

2 Find the value of $\frac{5}{4}$ of 3072 of 1 mile 5 fur 30 poles

3 Multiply Rs 2 1a by $\frac{1}{2} + \frac{6}{7} + \frac{1}{3}\frac{2}{5} + \frac{1}{3}\frac{1}{7}$

4 Find by Practice the cost of 10 cwt 3 qrs 23 lbs 8 oz at £1 5s 8d per cwt

5 A sum of money was divided amongst 5 people, 4 of them received respectively $\frac{1}{5}$, $\frac{1}{4}$, $\frac{1}{3}$ of the whole, while the 5th received £105 3s 6d What was the sum divided?

6 An oz of standard gold, $\frac{1}{2}$ of which is alloy is worth £3 17s 10 $\frac{1}{2}$ d, how many sovereigns would be coined from 36 lbs 8 oz of pure gold?

7 Find the square roots of 6246 057024 and 71 $\frac{1}{2}\frac{5}{8}$

1894

1 (a) A multiplication sum having been worked is partially rubbed out, the figures that remain are the entire multiplicand 999 and the last three digits 193 in the product. Restore the complete work.

(b) Simplify $\frac{1}{1\frac{1}{10}} \times \frac{1 + 0.025 \times 0.5}{1.0025 - 0.5} = \frac{45 \times 35}{8}$

2 (a) What decimal of Rs 100 must be added to $1\frac{41}{100}$ of Rs 5 10a 8p, that the sum may be 10 annas?

(b) Extract the square root of 25 6

3 Two trains start at the same time from Mirzapore and Delhi and proceed towards each other at the rates of 16 and 21 miles per hour respectively. When they meet it is found that one train has travelled 60 miles more than the other. Find the distance between the two stations.

4 Two years and six months ago I borrowed a sum which with simple interest at 6 per cent per annum now amounts to Rs 638 4 0. Find the sum.

1895

1 (a) Explain what is meant by the following terms —

Prime factors, common measure, common multiple, lowest common multiple

(b) A court-yard 452 feet long and 404 feet wide, is to be paved with square stones all of one size. What is the largest size which can be used?

2 (a) Simplify $\frac{5.75}{4.25}$ of $\frac{1}{2} - \frac{3}{4} + \frac{1}{6} \times \frac{3}{4} + \frac{1}{2}$

(b) Find the square root of 3 141 5926 to four places of decimals.

3 The *difference* between the interest for 4 months and the Discount on a certain sum due in 4 months at 4 per cent, is one rupee. What is the sum?

4 A merchant sells silk of two qualities which cost him Rs 5 5a 4p, and Rs 4 4a 4p per yard respectively. The selling price of the latter is two thirds that of the former, but the quantity sold is double and the merchant gains 25 per cent on the whole. Calculate the selling price per yard of each.

5 A policeman goes after a thief who has 100 yards' start, if the policeman run a mile in six minutes, and the thief a mile in ten minutes, how far will the thief have gone before he is overtaken?

1896

1 Simplify —

$$(a) 5 - 5 \times \frac{2 + 1\frac{1}{2}(2 + 1\frac{1}{2})}{1\frac{1}{2} + 2(2 + 1\frac{1}{2})} \quad (b) \frac{125 \times (175 \text{ of } 285714)}{00025}$$

2 (a) Express $\frac{2}{3}$ of 7s 6d + 1 25 of 5s - 545 of 9s 2d as a decimal of £10

(b) Extract the square root of 40000 400001

3 What is an aliquot part of a quantity ?

Find, by Practice, the time of building a wall 27 yards long, 1 yard thick and 6 ft high, of which one cubic yard is built in 3 hours 18 minutes and 45 seconds

4 How far shall I ride with a friend who leaves Allahabad at 9 A.M. and will drive to Karchana which is 10 miles from Allahabad in one hour, that I may by walking back at the rate of 4 miles an hour, reach home at 11-30 A.M. ?

5 A owes B Rs 1435 due at the end of 4 months, Rs 630 due at the end of 8 months, Rs 860 due at the end of a year B wants his money forthwith What ought A to pay him reckoning interest at $7\frac{1}{2}$ per cent ?

1897

1 What is the largest number which divides both 2397 and 2491 without remainder ? What is the smallest number which is divisible by both of these numbers ?

2 State and prove the rule for pointing in multiplication of decimals Why is the removal of the decimal point one place to the right equivalent to multiplication by 10 ? Illustrate your answer by comparing the numbers 23 015 and 230 15

Find the square root of 08027

3 A person lent another a sum of money for 72 days at 3 per cent per annum At the end of that time he received £293 12s 0½d What was the sum lent ?

4 The compound interest on a sum of money for 3 years at 5 per cent is £331 or 3d, what is the simple interest ?

5 If a rupee is worth one shilling and three pence half penny, and a shilling is worth 1 25 francs, what is the value in francs of 1,365 rupees ?

1898

1 Define *measure* of a number and find the G.C.M. of —

(i) Rs 2 4a and 10a (ii) $\frac{2}{3}$ and $\frac{5}{6}$

(a) Find the greatest number which will divide 13956 and 14565 and leave a remainder 7 in each case

2 Simplify —

$$(a) \frac{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{3}\right)^3}{\left(\frac{1}{3}\right)^2 + \left(\frac{1}{5}\right)^3} + \frac{125 \text{ of Rs } 5 \text{ } 10a \text{ } 8p}{037 \text{ of Rs } 7 \text{ } 12a}$$

$$(b) \frac{1}{2} - \frac{1}{3} - \frac{1}{8} \text{ of } \frac{1}{4} - \frac{1}{6} \times \frac{1}{8}$$

3 Extract the square root of $9 + \frac{1}{1 + \frac{1}{7 + \frac{1}{8}}}$ and calculate the

difference between this square root and $3 + \frac{1}{10}\sqrt{2}$ to three places of decimals

4 Find the cost in English money of travelling from Vienna to Trieste, a distance of 363 English miles, the average cost per German mile being 13 kreutzers (Given that 1 German mile = $4\frac{1}{2}$ English miles, £1 = 25 5 francs, 3 75 francs = 105 kreutzers)

5 What is the present value of a legacy of £149 1s 3d due 7 years hence, at $2\frac{3}{4}$ per cent simple interest?

1899

1 Simplify $\frac{\frac{1}{2} + \frac{1}{4} \text{ of } \frac{1}{4} + \frac{5}{8}}{\frac{1}{15} \text{ of } (1 + 5\frac{1}{2}) + \frac{1}{6} \text{ of } \frac{1}{25} \text{ of } (7 - 2\frac{3}{8}) - \frac{1}{3}}$ and express $\frac{2}{3}$ of Re 1 5a as the decimal of Re 1 4a

2 A number may be divided by 125 by multiplying it by 8, and then marking off the last three digits as decimals Explain the reason for this, and divide 5335 by 125

3 What is the meaning of an "*aliquot part*"?

Find by Practice the value of 24 tons 3 cwt 2 qrs 25 lbs at £17 11s 6d per ton

4 A piece of work can be done in 72 days by 17 men working together If after 9 days of work, these are joined by 4 others, in how many days will the work be finished?

5 Extract the square root of 5 and 5 each to 4 places of decimals, and shew that the square root of 4 is $\frac{2}{3}$

6 What is the difference between the interest on a bill of £138 13s 4d for 3 months, at 4 percent per annum, and the discount on the same for a quarter of a year, at the same rate?

7 (a) A speculator sells at a profit of 50 per cent, but his purchaser fails, and only pays 8a in the rupee How much per cent does the speculator gain or lose by his venture?

(b) A person investing in the 4 per cents, receives 5 per cent for his money What is the price of stock?

1900

1 State the rules for multiplication and division of decimal fractions

Assuming that the surface of a sphere is 3 1416 times the square of its diameter, and that the earth is a sphere whose diameter is 8000 miles, find what fraction of the whole surface of the earth is the area of India which is 1350000 square miles Express your result as a decimal fraction

2 What are circulating decimals ? Distinguish between pure and mixed circulating decimals

(a) Add together $\frac{1}{10}$, $\frac{1}{10}$, $\frac{1}{12}$, $\frac{1}{16}$ and express the sum as a mixed circulating decimals

(b) Reduce $0416 \times \frac{142857}{(\frac{5}{12} + \frac{1}{8}) \times 40}$ of Rs 8 5a to the fraction of 1a

3 (a) Find, by Practice, the price of 100 bags of Rosa sugar, each weighing 4 seers 2 powas and 3 chataks, at 6a 9p per seer

(b) Find the square root of 10 02 to three places of decimals

4 What sum of money will amount to Rs 3528 in two years at 5 per cent compound interest ? and what will it amount to in two more years ?

5 What monthly income will be derived from the investment of one lac of rupees in the $3\frac{1}{2}$ per cent Government of India paper at $100\frac{1}{8}$?

1901

1 (a) What is the greatest length which is contained a whole number of times exactly in both $25\frac{1}{2}$ feet and $21\frac{9}{10}$ feet ?

(b) Find the value of

$$\frac{49}{21} \text{ of } \frac{(3\frac{1}{2} - 2\frac{1}{2}) - \frac{1}{4} \text{ of } \frac{1}{4}}{2\frac{1}{2} + (\frac{1}{2} + \frac{1}{4})} \text{ of } £46$$

2 (a) Express the difference between 9428571 and 857142 as a vulgar fraction in its lowest terms

(b) Extract the square root of

$$\frac{0253 \times 365}{803} \text{ to five places of decimals}$$

3 In a two mile race A wins, B being 22 yards behind, and C, 106 yards behind B By how much would B, beat C in a three mile race ?

4 What sum at a compound interest will amount to Rs 650 at the end of the first year and to Rs 676 at the end of the second year ?

5 How much $3\frac{1}{2}$ per cent Government Securities at $95\frac{1}{4}$ must be sold out in order to purchase enough 5 per cent Calcutta Municipal Debentures at $119\frac{1}{4}$ to produce an annual income of Rs 665 ? (a brokerage of $\frac{1}{4}$ per cent being charged on each transaction)

1902

1 Find the G C M and L C M of 49 383 and 142569

2 Simplify $\frac{15}{075} \times \frac{3\frac{1}{2}}{1\frac{1}{2}} + \frac{1875}{21} \times \frac{35}{375} - 16$

3 Find by Practice the value of $246\frac{3}{4}$ maunds of sugar at Rs 13 5a 4p per maund

4 A and B have between them 132 horses, 25 of A's = 142857 of B's How many had each of them ?

5 Six men and five boys can do a piece of work in 7 days they work at it till they have completed $\frac{3}{4}$ of it, then two of the men leave and two more boys come How long will the work be in hand, if a boy does half as much work as a man ?

6 If I lend a friend Rs 1250 at 4 per cent Simple Interest and tell him to keep it until principal and interest amount to Rs 1666 10a 8p, how long will he have it ?

ANSWERS

Ex I (pp 78)

- 1 43, 79, 65, 84, 58, 97, 60, 87
- 2 449, 598, 704, 405, 235, 958, 725, 835
- 3 4000, 7804, 89063, 53223, 8046, 603240, 500505, 909009
- 4 341323 200075, 707070, 500000, 80008, 402700
- 5 9043602, 7859632, 3040020, 1403000, 5500676, 11000005, 1378267, 1010001
- 6 45387025, 92568985, 11565437, 40040005, 96096096
- 7 349004065, 100013001, 909009099, 842246484, 3452161, 494000000
- 8 99099099, 111650050, 640064600, 500703002, 609001208
- 9 2804252097 12036054079, 4000900005, 6304506506, 40280530259
- 10 400000010000 836573244006, 900900900009, 600060006006
- 11 9405004250, 413723009004, 5808068080
- 12 8000000207005, 3004702164722, 1000000300005
- 13 9900009009999, 100196400010009
- 14 654323004021050301, 47526870744103284
- 15 9000004000640000365 16 100000, 99999999, 6
- 17 88, 89, 90, 91, 92, 612, 613, 614, 615, 616, 617, 618, 619, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969
- 18 The correct writing is 5505505, hence find out the mistakes

Ex II (p 11)

1 Seventeen, twenty-four, thirty-five, forty-six, twenty-seven; forty-eight, fifty-nine, seventy-six, eighty-four, ninety-five, sixty-six, seventy-five, eighty-nine

2 Two hundred and seventeen, three hundred and nineteen, five hundred and eighty-three, six hundred and ninety-five, seven hundred and twenty-five, three hundred and eight, four hundred and six, eight hundred and forty-six, nine hundred and thirty-two, seven hundred and twenty-five

3 Three thousand, four hundred and six, five thousand, two hundred and sixty, four thousand, two hundred and thirty-six, three thousand, two hundred and ninety-eight, five thousand, six hundred

and seventy eight , two thousand, four hundred and five , nine thousand, two hundred and eighty six

4 Forty-three thousand, two hundred and one , eighty-seven thousand, and fifty-four , thirty four thousand and two , forty-nine thousand, eight hundred and three , fifty-eight thousand and thirty , seventy six thousand, five hundred and three

5 Nine hundred three thousand, seven hundred and fifty six , nine hundred three thousand, two hundred and eighty-four , eight hundred twenty seven thousand, one hundred and nine , three hundred nineteen thousand, four hundred and twenty , two hundred forty-three thousand and sixty-five , one hundred twenty three thousand, four hundred and fifty-six

6 Two million, seven hundred fourteen thousand, three hundred and twenty five , eight million, forty-seven thousand, three hundred and twenty-eight , four million, ten thousand and ten , eight million, four thousand, six hundred and forty , one million, two hundred thirty-four thousand and seven

7 Twelve million, eight hundred seventy thousand and forty-five , twenty million, eighty four thousand, two hundred and sixteen , seventy-nine million, thirty thousand, two hundred and eighty four , forty three million, two thousand and five

8 Three hundred twenty one million, four hundred eight thousand, six hundred and fifty three , four hundred eight million, seventy six thousand and thirty-two , three hundred fourteen million, one hundred fifty nine thousand, two hundred and sixty-five , one hundred twenty-three million, four hundred fifty-six thousand, seven hundred and eighty-nine

9 Five hundred seventy one million, two hundred sixty-eight thousand, four hundred and five , three thousand, one hundred seventy-nine million, forty thousand, six hundred and one , three hundred nineteen thousand, six hundred eighty million, two hundred nine thousand and seventy eight

10 One billion, two hundred thirty-four thousand, five hundred sixty seven million, six hundred fifty-four thousand, three hundred and twenty one , five billion, twenty thousand, forty million, three thousand and sixty , four billion, three hundred two thousand, five hundred million, seven hundred sixty four thousand and nine

11 Two hundred thousand, nine hundred million, six hundred thousand and two , forty-three billion, two hundred eighty-seven thousand million, six thousand, three hundred, and twenty one , sixty-four billion, two million, six hundred forty-six thousand and two

12 Three hundred nineteen thousand, eighty million, two hundred fifty nine thousand, four hundred and seventeen , two hundred thirty-six billion, forty five thousand, nine hundred seventy-eight million, two hundred thirteen thousand, four hundred and seventy-eight

13 One billion, three hundred twenty-seven thousand, eight hundred seventy-five million, four hundred thirty thousand and twenty-nine, five billion, four hundred thirty-two thousand, one hundred seventy-six million, nine hundred eighty-nine thousand and seven

14 97542, 24579

15 90, 5, 60, 4, 500, 70, 5, 8000, 200, 90, 7, 40000, 200, 70, 6, 3000, 200, 5, 400000, 70000, 8000, 200, 90, 6, 40000000, 300000, 2000, 600, 5, 50000000, 3000, 20, 9, 70000000, 300000, 6, 9000000000, 700000000, 80000000, 6000000, 2000, 30

16 Ninety-nine thousand, nine hundred and ninety-nine, one million

17 234, 243, 324, 342, 423, 432

Ex III (pp 12 13)

1 Nineteen thousand, two hundred and thirty-seven, sixty thousand and eighty-one, forty-nine thousand and twenty seven, one lac, sixty-seven thousand, two hundred and eight, two lacs, seven hundred and fifty-three, eight lacs, thirty thousand and five

2 Seventy lacs, ninety thousand, seven hundred and nine, eighty lacs, one thousand, and twenty-five, thirty-nine lacs, five thousand and eighty-six, two crores, forty lacs, fifty thousand and eight, forty lacs, one thousand, seven hundred and forty-five

3 Four crores, two lacs, seventeen thousand, eight hundred and fifteen, four hundred and three crores, twenty-four thousand, three hundred and forty, four hundred and seventy-eight crores, two lacs, thirty-thousand and sixteen, two crores, thirty-four lacs, fifty-six thousand

4 Twelve crores, thirty-four lacs, fifty-six thousand, seven hundred and eighty nine, six hundred and forty-five crores and three lacs, seventy six crores, two lacs, forty two thousand and nine hundred

5 Four hundred and fifty crores, two thousand, four hundred and thirty, eight hundred crores, seven lacs, and eighty five thousand, four hundred and two crores, five lacs, four thousand and eight

6 415208, 5604029, 84374209, 800005, 7500000, 3000708

7 21500004, 370704012, 1451900007, 50990405607

8 803001011, 42951400085, 754101409009

9 200, 1 thousand, 400

10 Forty crores, fifty lacs, seventy-five thousand, nine hundred and four

11 Sankha, trillion

12 The correct writing is 90504756, hence find out the mistakes

Ex IV (p 14)

1 7, 17, 21, 54, 34, 39

2 95, 48, 95, 214, 514, 419

3 1009, 1804, 1650, 1766, 1100000, 605

- 4 5555, 6550, 210440, 290540, 1000010, 2501100
 5 IX, XVI, XXXV, XLVI, LXVIII, LXXV, LXXXIX, XCIX, CV, CXLVIII
 6 XXXII, XXVIII, XLIX, LXIX, LXXVIII, XCV, CCXV, CCCXXVII, CDXXXIII, DXLIX
 7 DCCXLV, CMXXIII, DLXVII, MCCXXXIV, MDLXVII, MDCCCLIII, MCMXVII
 8 MCCXXXI, MCCLXII, MDCCCLXII, MDCCLXXVII, MCMXCIX, MMI, MDCCLXIX
 9 $\overline{XVCDXC VII}$, \overline{XXXV} , \overline{CCCL} , \overline{DCLIII} , \overline{MI} , $\overline{MMMMMLD}$.

Ex. V (pp 16 17)

7	11	8	13	9	16	10	23	11	9	12	10
13	23	14	30	15	27	16	20	17	33	18	21
19	25	20	16	21	19	22	29	23	50	24	21
25	23	26	37	27	36	28	40	29	128	30	57

Ex VI (pp 19 22)

1	(1) 102	(2) 208	(3) 200	(4) 115	(5) 213
	(6) 215	(7) 200	(8) 116	(9) 214	(10) 224
	(11) 276	(12) 222	(13) 503	(14) 2133	(15) 1697
	(16) 14624	(17) 1890	(18) 995	(19) 12345	(20) 2956
	(21) 2489	(22) 29125	(23) 25575	(24) 2239	(25) 780
	(26) 2227	(27) 2435	(28) 6553	(29) 7812	(30) 36092-
	(31) 28026	(32) 37667	(33) 16553	(34) 23724	(35) 17764
	(36) 26973	(37) 60714	(38) 336513	(39) 144563	(40) 400257-
	(41) 358064	(42) 390370	(43) 3140069	(44) 2329089	
	(45) 746506	(46) 23726503	(47) 30421482	(48) 25522084	
	(49) 28556362	(50) 20222215	(51) 3388360	(52) 4025738	
	(53) 37155818	(54) 260342508	(55) 171357572		
2	(1) 17866	(2) 172846	(3) 42612875		
	(4) 518890	(5) 19169327	(6) 12842644		
3	(1) 12891663	(2) 21086067	(3) 531904		
	(4) 2689134309	(5) 1788591628			
4.	162209	5 1694375	6 2230626	7 2294129927	
8	20566726566	9 13241749	10 15701653985		
11	1454	12 497	13 337	14 140	
15	1338	16 1464	17 9770	18 R42068	
19	50150009	20 17863411	21 R4387	22 3554	
23	6529	24 47423136	25 1847	26 3826	
27	382169	28 365	29 R232031	30 6116	

Ex VII (pp 24 25)

11	5	12	1	13	7	14	15	15	17-
16	11	17	5	18	1	19	11	20	10-

Ex VIII (pp 26 28)

- 1 (1) 17 (2) 34 (3) 16 (4) 16 (5) 9 (6) 154
 (7) 209 (8) 198 (9) 594 (10) 205 (11) 6239 (12) 2849
 (13) 1189 (14) 2886 (15) 4370 (16) 6092 (17) 4960
 (18) 18469 (19) 16909 (20) 16449 (21) 14759 (22) 668493
 (23) 327699 (24) 127589 (25) 229678 (26) 74819 (27) 286699
 (28) 18838478 (29) 246913578 (30) 5101262
 (31) 64446566 (32) 100909765 (33) 22591687
 (34) 555939946 (35) 4691357 (36) 272886756
 (37) 738776598 (38) 66765615 (39) 414866185
 (40) 100011 (41) 364179951
- 2 (1) 68999, 77036, 9999 (2) 99099, 766899, 173706
 (3) 60005393, 192484228 (4) 10942895, 67200757689
- 3 (1) 40101, 88890, 109089 (2) 1288874, 28890, 520986
 (3) 53318, 2378, 1188988 (4) 823611, 1213667908
- 4 4699, 2167090875 5 19279548
 6 809089 7 4091, 900000
- 8 1576542, 9542315, 967599, 2387655, 4959711, 9095493, 996535
 9 754321, 179400, 8480222, 6174909, 183227
- 10 304924818 11 619310439 12 748696147
 13 1 14 1688 15 1796 16 749 17 2327 18 417
 19 51 20 49 21 R891 22 12161 23 K897976
 24 8630098 25 76 26 A 23, B 47, C 35 27 R3087
 28 3361 29 650, 858 30 1900, 81 years

Ex IX (pp 29 30)

- 1 4 2 1609 3 92 4 396 5 1034
 6 3742 7 6140 8 15022 9 1273 10 682
 11 26, 19 12 6, 3, 57, 14, 426, 2

Ex X (pp 33 34)

- 4 96 5 88 6 126 7 54 8 81
 9 162 10 41, 60 11 108 12 10, 40, 36
 13 408 14 217 15 18 16 56, 18
 17 32 18 64 19 22 20 108

Ex XI (pp 35 36)

- 1 568, 4425, 11468, 18096, 24228, 404825, 396064, 231483, 109844
 2 (1) 11698, 17547, 23396, 29245, 35094, 40943, 46792, 52641, 64339
 (2) 115428, 192380, 269332, 346284, 423236, 500188, 538664, 577140, 731044
 (3) 7740984, 19352460, 11611476, 27093444, 15481968, 34834428, 23222952, 30963936, 42575412, 46445904, 58057380

- (4) 52070352, 45561558, 58579146, 71596734, 84614322, 97631910, 110649498, 123667086
 (5) 1975308642, 2962962963, 3950617284, 4938271605, 5925925926, 6913580247, 7901234568, 8888888889, 10864197531, 11851851852
- 3 (1) 388064, 416160, 541376, 833184
 (2) 346284, 10518588, 6050000, 8224776
 (3) 735675, 736944, 584212, 12019080
 (4) 64536612, 87585402, 1014848586
- 4 (1) 9809890, 98098900, 980989000, 9809890000
 (2) 2161530, 2882040, 5043570, 6484590, 7205100
 (3) 1827140, 18271400, 27407100, 456785000, 822213000
 (4) 94894200, 1186177500, 13443345000, 1028020500, 150249150000
 (5) 720774400, 1441548800, 12613552000, 162174240000
- 5 2274 6 74451 7 555555505 8 R811224
 9 214948 10 R1125 11 166 12 (i) 98 miles (ii) 1204 miles

Ex XII (pp 39 40)

- 1 57706, 77341, 42182, 79992, 281504, 308163, 1619723, 50516
 2 (1) 1287657, 1000055, 34381488, 1531335
 (2) 8539410, 11216556, 46634205, 48954719
 (3) 1013736849, 145651668, 311305816
 (4) 2518028865, 757030260, 37335129056
 (5) 1973316695, 2706262896, 4059394344, 7442222964
 (6) 24149786524, 296988105062, 5327809224181
 (7) 28631518784, 213248118864, 63840278567472
 (8) 2053737000, 59392800000, 622439160
 (9) 2851265148, 593928000000, 5994485049000
 (10) 2299320000, 51734700000, 24717690000, 1408333500000, 268445610000000
- 3 (1) 416948784 (2) 278178269193 (3) 19948130736
 (4) 78214076605 (5) 3405426851645 (6) 258656813296
 (7) 2090752670781 (8) 21862529907675
 (9) 22102928324900 (10) 34525538710
 (11) 3983561445637782 (12) 31260150931584
 (13) 604356745368450 (14) 742892741529300
 (15) 15241578750190521
- 4 (1) 433418175, 173367270, 337103025
 (2) 2600509050, 9101781675, 7313243616
 (3) 6535022130, 31260150931584, 4594091417461
 (4) 2809599487063727789412, 516796263529337751696165
 (5) 13453369324195728883956, 26803379349538380154320
- 5 0 6 R159152 7 34864128 8 125
 9 24941648 10 R42973403 11 10212310
 12 9630 13 2482 14 493314
 15 41610 16 2799992 17 57456
 18 6428506 19 92005008 miles 20 277182864

Ex XIII (p 41.)

- | | | | | |
|---|---------------|---------------|-----------------|-----------------|
| 1 | (1) 700 | (2) 3315 | (3) 6840 | (4) 36630 |
| | (5) 355752 | (6) 517248 | (7) 454720 | (8) 3797115 |
| | (9) 1441440 | (10) 68785605 | (11) 57737160 | |
| | (12) 58605120 | (13) 67886100 | (14) 3417187500 | |
| 2 | 62968425 | 3 595680000 | 4 62172 | 5 23328 |
| 6 | 99216 | 7 720 | 8 51948 | 9 4096 10 15360 |

Ex XIV (p 42)

- 1 (1) 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400, 441, 484, 529, 576, 625, 1521, 2116, 2916, 7396, 9801
- (2) 29584, 56169, 820836, 974169
- (3) 531441, 762129, 1087849, 30206016
- (4) 53904964, 88059456, 77369616, 1522756
- 2 (1) 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, 1331, 1728, 2197, 2744, 3375, 4096, 4913, 5832, 6859, 8000, 9261, 10648, 12167, 13824, 15625, 50653, 110592, 314432, 456533
- (2) 681472, 912673, 1860867, 94818816
- (3) 29218112, 672221376, 447697125, 997002999
- (4) 961504803, 156242452456, 963259373376, 1879080904
- 3 (1) 211309379856, 8653650625, 949005240561, 952857108736
- (2) 679740887296, 114478037712481, 4104198146048256, 9996000599960001
- 4 (1) 285 (2) 727 (3) 45 (4) 2025 (5) 96 (6) 910
- 5 (1) 55 (2) 150 (3) 672 (4) 88 (5) 466 (6) 18
- (7) 14767 (8) 141

Ex XV (pp 43 44)

- 4 6 times and 6 over, 13 times, 13 times and 6 over
- | | | | | | | | | | |
|----|----|----|--------------|----|----|----|-----|----|----|
| 5 | 4 | 6 | 4, 6, 12, 16 | 7 | 17 | 8 | 12 | 9 | 16 |
| 10 | 20 | 11 | 11 | 12 | 12 | 13 | 360 | 14 | 7 |
| 16 | 66 | 17 | 380 gain | 18 | 10 | 19 | 18 | 20 | 17 |

Ex XVI (p 46)

- 1 (1) 154, 77, 57, 6, 51, 3, 46, 2, 42, 38, 6
- (2) 227, 1, 170, 2, 113, 4, 85, 2, 75, 7, 62, 48, 10, 45, 7
- (3) 1685, 1203, 4, 1053, 1, 842, 5, 648, 1, 526, 9, 495, 10, 443, 8
- (4) 3438, 2292, 982, 2, 764, 625, 1, 573, 491, 2
- (5) 11766, 7059, 3, 3922, 5042, 4, 3529, 8, 2941, 6, 1961
- (6) 174, 1532, 69071, 16875
- (7) 10768, 4, 76582, 11979052, 1, 315836, 7
- (8) 795917072, 3, 112233444, 6, 823045, 3
- (9) 1974538, 2384163, 2645753
- (10) 2167022, 58097313, 54869684, 9, 51981806, 7, 49382716, 1
- 2 93 3 180 4. 9006, 9192 5 94
- 6 156 7 1728 8 Rs 20

Ex XVII (pp 49-50.)

- 1 (1) 4021 (2) 2050 8 (3) 1010 6 (4) 7002
 (5) 1849129 40 (6) 2176183 (7) 1367372 (8) 1736834 24
 (9) 1272250 6 (10) 84293 (11) 7629302 (12) 93845796
 (13) 593559 6 (14) 325698042 25 (15) 172956436 7
 (16) 45783975 (17) 987654321
- 2 (1) 3409371 (2) 11951629 79 (3) 356995601 29
 (4) 7071 (5) 57096 (6) 103944
 (7) 87997 214 (8) 7341069 (9) 137641 371
 (10) 967427210 61 (11) 87366 6076 (12) 190182
 (13) 19915 5559 (14) 65839 2 (15) 886797 1310
 (16) 3192 4966 (17) 1453 2286 (18) 43349 1140
 (19) 193421 811 (20) 16575 (21) 4938
 (22) 73086413 (23) 37956314 (24) 70080092 7322
 (25) 507001 4221 (26) 85802 2575858 (27) 9640 999821
 (28) 736 30167291 (29) 328 65220054 (30) 2796
 (31) 30305 (32) 987654321 (33) 574585614865
 (34) 800300500 (35) 2837154309
- 3 (1) 42439 5498, 26171 7874, 5822 6639, 4167 2898
 (2) 8122 9, 4061 9, 2707 19, 2030 29, 1624 29,
 1015 29, 902 49
 (3) 3426 4, 856 204, 571 4, 428 204, 380 604
 (4) 98168 426, 6544 6826, 24542 426, 165 159826
 (5) 12433128 54, 653266 334, 6566 27734, 42401 18834
 (6) 3216 6886, 229 47832, 3396 5094687
- 4 (1) 11 (2) 102 (3) 433 (4) 53 (5) 284074
 5 72158 6 17 7 13 8 75 9 21 10 420
 11 973 12 5669 13 1799 14 R11 15 478 16 201
 17 1888, 101001 18 66 19 15 3 20 186746 492

Ex XVIII (pp 51 52)

- 1 (1) 26 (2) 26 (3) 25 (4) 27 (5) 552 (6) -6
 (7) 815 (8) 303 (9) 69 (10) 130 (11) 778
- 2 (1) 1036 (2) 1008 (3) 1808 (4) 8969578 (5) 2067
- 3 382 4 1000 5 1675 6 3582 7 636
- 8 162 9 3370 10 1999999998 11 1098897
- 12 (1) 150 (2) 2034 (3) 83 (4) 172720 (5) 24
 (6) 554 (7) 21 (8) 24 (9) 93 (10) 1764.

Ex XIX (pp 57 58)

- 1 847091 2 38967 3 81757 4 7640 5 7124, 5516
 6 94 79 7 905, 521 8 Carriage Rs 511, Each horse Rs 173
 9 780, 420 10 105, 63 11 Ram 42, Gopal 35, Han 28
 12 50 oranges, 35 apples, 27 plums 13 A Rs 55, B Rs 65, C Rs 85
 14 A Rs 1244, B Rs 1460, C Rs 1796 15 3507 16 1887378
 17 9306 18 37246 19 175 20 5878 21 2107

- 22 31352 23 109, 278 24 403 25 141 26 (1) 1830
 (2) 5050 (3) 155 (4) 256 (5) 240 (6) 800 (7) 493 (8) 1430
 27 Rs 2704 28 960 miles 29 156 30 5193171
 31 99735, 10137 32 100191 33 9999700002
 34 9999, rem 999 35 10876799

Ex XX (p 60)

- 1 942636, 999457944, 992140936, 9920146
 2 (1) 4352 (2) 10924 (3) 3318939 (4) 3857
 3 (1) 60, 6, 10277 (2) 29889, 3318377 (3) 3349319,
 36438018 4 1370, 25780, 17363
 5 (1) 4355516, 7121448, 8552448, 5027211
 (2) 1252562304, 1613118976, 3769357248
 (3) 7490168136, 4334453248, 3160438848
 (4) 15735060, 461981520, 14850510675
 6 (1) 2558192, 2295864, 77967198, 448602324
 (2) 157739562, 303794712, 4288179204, 672604515
 7 (1) 49110419796, 144872064531, 63723226584
 (2) 996275287620, 397685408184, 5334673883463
 (3) 44818796323449, 603155680760244
 8 1269488804031, 25332654572848

Ex XXI (pp 63 64)

- 1 (1) 4396630, 21983150, 65949450, 109915750, 549578750
 (2) 959175, 2238075, 4795875, 7993125
 (3) 439556825, 2197784125, 6593352375,
 10988920625, 54944603125
 (4) 439545546, 4435414146, 44394100146, 443980960146
 (5) 556032, 5768832, 57885248, 57908416
 (6) 96081172, 1056892892, 11625821812, 873412792088
 2 (1) 1369, 2025, 2304, 3025, 4225, 5625, 4096, 5041, 6889,
 9216, 15625
 (2) 11664, 22201, 24336, 33489, 46225, 152881, 228484,
 207936, 274576
 3 $(59)^2 - (6)^2$, $(85)^2 - (11)^2$, $(105)^2 - (21)^2$, $(221)^2 - (24)^2$,
 $(397)^2 - (81)^2$
 4 (1) 150000, 400, 2140200, 2805000
 (2) 176384, 2892000, 7386000, 52640
 5 (1) 2 9669 (2) 500, 4124
 6 99995356 7 1006434 8 100028544, 99991331

Ex XXII (p 66)

- 1 (1) 45669, 6210 48, 9607
 (2) 122276 23, 78556 65, 99904 53
 (3) 79732 53, 181554 6, 450314 66

- (4) 2733534 7, 1195921 15, 1063041 7, 797280 79,
 154457 242 (5) 120696 136, 37663 30, 4565 512
 (6) 28716695. 68, 19339815 68, 10028052 152, 4099 133
 2 (1) 7592 4, 759 14, 75 464, 7 2964, 1518 14
 (2) 1750863 1, 583621 1, 350172 16, 250123 11,
 194540 16, 159169 21, 134681. 51, 116724 16
 (3) 722185 64, 515846 139, 401214 39, 328266 39
 (4) 1237807 1, 412602 126, 247561 251, 176829 501
 (5) 7859 3851, 11171 4593, 8787 1988

Ex XXIII (pp 68 69)

- 1 52 2 78 3 21 4 Rs 288 5 7 hours
 6 A Rs 224, B Rs 336, C Rs 448 7 A 410, B 902, C 1312
 8 Rs 265 9 33 10 Son Rs 45000, daughter Rs 15000
 11 Each man Rs 1050, each woman Rs 875, each child Rs 175
 12 1440 13 A Rs 422, B Rs 633, C Rs 844, D Rs 1055
 14 Rs 4 15 6

Miscellaneous Examples I (pp 71-76)

- 1 533884537 2 379708 3 63550 and 62128
 4 7222 and 8456 5 (i) 119 (ii) 2268 6 Rs 23006 7 38
 8 2378 9 172180 10 400 11 3679 12 191
 13 188521 males, 246090 females 14 $(4010)^2 - (2779)^2$
 15 22771 16 305 17 17350 18 723 19 1865
 20 49505296 21 480 22 2220 23 581506, 581550
 24 51, 6 25 House Rs 507200 26 13
 27 1st 8799, 2nd 3557, 3rd 5204 28 203 29 400
 30 52 years 31 Rs 2914 32 139 33 He gains Rs 25
 34 Eldest Rs 1230928, second Rs 706806, third Rs 489764
 35 180440 36 435 seers 37 C is 3 years older than D
 38 A has 80, B 69, and C 157 39 29 40 1429
 41 Rs 790 42 Rs 3327, Rs 3201, Rs 3070, Rs 2949
 43 99 hours 44 $(10208)^2 - (8983)^2$ 45 Quotient = 2442,
 rem = 102 46 Rs 8240 47 Rs 593810
 48 After 9 hours 49 C had Rs 1545 more than B
 50 4997 51 2568307 52 A Rs 25, B Rs 15
 53 A Rs 5, B Rs 10, C Rs 15 54 987 55 2664, 1590408
 56 38664 57

2	7	6
9	5	1
4	3	8

 58 286 59 3
 60 A 167, B 501, C 799 61 A Rs 500, B Rs 750,
 62 (1) 1440 (2) 300 C Rs 1000, D Rs 2750
 63 15 64 45 65 Rs 1250
 66 49110419796, 144872064531, 63723226584
 67 18441216 68 Rs 670 69 403 70 312
 71 139 72 1044 miles 73 224 miles 74 57606
 75 10 days 76 Rs 11 77 105 78 49
 79 A Rs 546, B Rs 569, C Rs 1160, D Rs 1700
 80 He gained Rs 4 a head, no of bullocks = 175

Ex XXIV (pp. 80 81)

- 1 (1) 272*a* , 304*a* , 672*a* , 720*a* , 1104*a* , 1344*a* , 1520*a*
 (2) 1392*a* , 1920*a* , 3920*a* , 7360*a* , 156*a* , 334*a*
 (3) 582*a* , 861*a* , 1403*a* , 1279*a* , 3755*a*
- 2 (1) 6528*p* , 10752*p* , 18624*p* , 28032*p* , 65664*p* , 95232*p*
 (2) 16188*p* , 14736*p* , 50988*p* , 154524*p* , 181512*p*
 (3) 2979*p* , 1511*p* , 1541*p* , 1857*p*
 (4) 81693*p* , 105647*p* , 240067*p* , 969769*p* , 87731*p* ,
 6073*p* , 65959*p*
- 3 (1) 3328*ps* , 9984*p* , 1216*ps* , 3648*p* , 7192*ps* , 21576*p* ,
 2350*ps* , 7050*p* , 1315*ps* , 3945*p*
 (2) 5609*ps* , 16827*p* , 11031*ps* , 33093*p* , 14438*ps* , 43314*p* ,
 30513*ps* , 91539*p* , 50051*ps* , 150153*p* , 875*ps* , 2625*p* ,
 13791*ps* , 41373*p*
- 4 (1) 6080*g* , 24320*c* , 10880*g* , 43520*c* , 17920*g* , 71680*c* ,
 24960*g* , 99840*c* , 33600*g* , 134400*c* , 27020*g* , 108080*c*
 (2) 32945*g* , 131780*c* , 7975*g* , 31900*c* , 129860*g* , 519440*c* ,
 24145*g* , 96580*c*
 (3) 15550*g* , 62200*c* , 17237*g* , 68948*c* , 3062696*g* , 12250784*c*
- 5 68950*c* , 87805*c* , 23570*c* , 7605831*c*
- 6 (1) 5600*ps* , 16600*p* , 5920*ps* , 17760*p* , 6760*ps* , 20280*p*
 (2) 850*ps* , 2550*p* , 24928*ps* , 74784*p* , 77280*ps* , 231840*p*
 (3) 547392*ps* , 1642176*p* , 40314*ps* , 120942*p* ,
 1087200*ps* , 3261600*p* , 26464*ps* , 79392*p*
- 7 (1) 1458 , 2916 , 5832 , 1850 , 3700 , 7400 , 2456 , 4912 , 9824 ,
 2854 , 5708 , 11416 , 8486 , 16972 , 33944 ,
 194806 , 389612 , 779224
 (2) 1717 , 3434 , 6868 , 19453 , 38906 , 77812 ,
 146492 , 292984 , 585968 , 115018 , 230036 , 460072
- 8 2412 , 9648 , 4242 , 16968 , 4801 , 19204 , 96181 , 384724
- 9 (1) 6400000 , 28501 , 48604 ; 40015 , 19729
 (2) 169674 , 99741 , 309206
- 10 (1) 6900*s* , 9960*s* , 15900*s* , 28040*s* , 181720*s* , 161840*s*
 (2) 1421*s* , 9818*s* , 15813*s* , 69137*s* , 128067*s*
- 11 (1) 15600*d* , 23520*d* , 37440*d* , 97200*d* , 443760*d* ,
 1210320*d* , 2216640*d*
 (2) 32340*d* , 47676*d* , 99972*d* , 126300*d* , 222324*d*
 (3) 560*d* , 9726*d* , 2733*d* , 89900*d*
 (4) 157362*d* , 17201*d* , 82481*d* , 305997*d*
- 12 (1) 4243*g* , 7391*g* , 13393*g* , 28364*g*
 (2) 97403*g* , 147042*g* , 576300*g* , 80563979*g*
- 13 (1) 372 , 744 , 450 900 , 334 , 668 , 468 , 936 , 212 , 424 , 418 , 836
 (2) 1958400 , 3916800 , 4131840 , 8263680 , 4192320 , 8384640 ,
 348912 , 697824 , 248472 , 496944 , 1020144 , 2040288
 (3) 38272 , 76544 , 2035 , 46070 , 187025 , 374050 , 422353 , 844706
 (4) 749049 , 1498098 , 1030047 , 2060094 , 4385659 , 8771318

- (5) 935760, 1871520, 576840, 1153680, 516480, 1032960, 313440, 626880, 70428, 140856, 55664, 111328, 48054, 96108, 4574304, 9148608, 778428 1556856, 61056, 122112, 936360, 1872720, 327520, 655040, 157728, 315456, 23796, 47592
- 14 (1) 7600, 5700, 3800, 10240, 7680, 5120, 646080, 484560, 523040, 148560, 111420, 74280, 726640, 544980, 363320 725760 544320, 362880
- (2) 936, 702, 468, 11588, 8691, 5794, 171644, 128733 85822, 341260, 255945, 170630, 261188, 195891, 130594
- 15 (1) 96615, 18033 (2) 5706, 3049, 2349, 28007
- (3) 7959, 167190 (4) 47855, 01000
- 16 (1) 72346, 90009, 696543 (2) 72346, 80563979, 696543
- 17 260 18 173 19 93 20 101

Ex XXV (pp 82 83)

- 1 (1) Rs 131 14a 5p, Rs 299 8a 5p, Rs 265 13a 3p, Rs 3538 0a 2p, Rs 197 8a 1p, Rs 2379 1a 6p
- (2) Rs 3814 8a 10p, Rs 3350 7a 10p, Rs 10324 11a 5p Rs 5039 7a 1p, Rs 5433 15a 8p
- 2 Rs 15436 10a 1ps, Rs 19290 1a 3ps, Rs 8551 14a 1ps, Rs 13983 11a, Rs 86528
- 3 (1) Rs 26, Rs 6, Rs 1185 14a 18g, Rs 637
- (2) Rs 32677 4a 2ps, Rs 16152 11a, Rs 842
- (3) Rs 5357 7a, Rs 9325 6a, Rs 739 15a 2ps, Rs 809 15a 3ps, Rs 902 11a
- 4 (1) Rs 824, Rs 473, Rs 316 4a
- (2) Rs 181, Rs 93, Rs 2876, Rs 19
- 5 (1) £288 1s, £253 19s, £375 19s 3d, £11 7s 9d, £374 11s 8d, £655 13s 6d
- (2) £153 3s 4d, £295 17s 11½d, £128 8s 6½d, £364 11s 8d, £83920 16s 2½d
- (3) £360 1s 7½d, £135 8s 7½d, £1041 13s 8¾d, £295 18s 3¾d, £433 1s 2¾d, £3802 8s 5d
- 6 (1) £449522 10s, £242778, £4807 6s, £110762 12s 6d, £178 12s 1d, £162 17s 3d
- (2) £252 5s 8d, £8255 12s, £3611 10s, £47 12s 4d, £2057 12s 2½d, £49649 5s
- (3) £31920 15s, £26106 3s, £4587 5s 6d, £3513 5s 6d, £42 0s 5d
- 7 £156 7s 4d 8 Rs 20 8a 6p 9 Rs 52 9a 10 Rs 1125

Ex XXVI (p 84)

- 1 60, 120, 100, 200, 92gs 4s, 184h gs 4s, 835, 1670, 512gs 8s, 1024h gs 8s, 10217gs 3s, 20434h gs 3s
- 2 1062, 2124, 2359, 4718, 1750, 3500, 2481, 4962, 20759, 41518
- 3 46145crs 2s, 356000, 147crs 2s 6d, 1250crs 2s
- 4 391, 3059, 4665, 2814.

- 5 2752 $\frac{1}{2}$ s 20s , 12771gs 19s , 5669gs 1s , 90gs 10s ,
616gs 4s 11d , 357gs 19s 0 $\frac{1}{2}$ d
6 154 $\frac{1}{2}$ gs 8s , 10349 $\frac{1}{2}$ gs 5s 6d , 100 , 6172 $\frac{1}{2}$ gs 4s 6d , 35
7 £2 11s 9d , £51 9s 5d , £182 11s 8d
8 Rs 327 4a 8p , Rs 966 12a , Rs 9038 12a 4p , Rs 283 8a ,
Rs 107 , Rs 21 4a
9 147840 10 Rs 8000 , Rs 6030 *Sic*

Ex XXVII (pp. 85-88)

- 1 (1) Rs 2 11a (2) Rs 2 6a 5p (3) Rs 2 5a 9p
(4) Rs 2 12a 6p (5) Rs 2 0a 8p (6) Rs 2 8a 6p
(7) Rs 29 5a 3p (8) Rs 57 0a 1p (9) Rs 313 2a 7p
(10) Rs 361 11a 1p (11) Rs 50 6a 5p (12) Rs 102 0a 1p
(13) Rs 206 7a 2p (14) Rs 308 7a 7p (15) Rs 1205 14a 2p $\frac{1}{2}$
(16) Rs 912 15a (17) Rs 2813 5a 4p (18) Rs 10256 9a 10p
(19) Rs 18632 2a 5p (20) Rs 25536 9a 8p
(21) Rs 28141 14a 11p (22) Rs 346001 0a 8p
2 (1) £2 10s 2d (2) £3 1s 4d (3) £3 18s 2 $\frac{1}{2}$ d
(4) £136 17s 7d (5) £150 8s 6 $\frac{1}{2}$ d (6) £200 17s 10 $\frac{1}{2}$ d
(7) £4908 1 4 $\frac{1}{2}$ d (8) £59667 3s 7 $\frac{1}{2}$ d (9) £10798 5s 9 $\frac{1}{2}$ d
(10) £6004 11s 7d
3 (1) Rs 44626 11a 11p (2) Rs 34717 4a 8p
(3) Rs 38349 10a 2p (4) £23506 14s 9 $\frac{1}{2}$ d
(5) £30465 17s 4 $\frac{1}{2}$ d (6) £24760 12s 6 $\frac{1}{2}$ d
4 £113 6s 5 £1667 0s 3d 6 Rs 368 3a 3p
7 Rs 9691 11a 6p 8 £311 11s 10 $\frac{1}{2}$ d

Ex XXVIII (pp 89-91)

- 1 (1) Rs 8 6a 11p (2) Rs 28 12a 11p (3) Rs 10 12a 5p
(4) Rs 25 14a 3p $\frac{1}{2}$ (5) Rs 43 10a 2p $\frac{1}{2}$ (6) Rs 125 15a 1p
(7) Rs 511 8a 10p (8) Rs 79 8a 1p
(9) Rs 2163 9a 8p (10) Rs 424 13a 10p
(11) Rs 2183 14a 9p (12) Rs 447 6a 10p
(13) Rs 518 13a 10p (14) Re 1 1a 1p
(15) Rs 485 14a 9p (16) Rs 898 5a 4p
(17) Rs 1364 4a 11p
2 (1) 6s 0 $\frac{1}{2}$ d (2) 11s 8 $\frac{1}{2}$ d (3) 4s 0 $\frac{1}{2}$ d
(4) 6s 7 $\frac{1}{2}$ d (5) 5s 7 $\frac{1}{2}$ d (6) £39 10s 7 $\frac{1}{2}$ d
(7) £58 7s 10 $\frac{1}{2}$ d (8) £788 3s 11 $\frac{1}{2}$ d (9) £49 19s 11 $\frac{1}{2}$ d
(10) £30 14s 9 $\frac{1}{2}$ d (11) £4 7s 8 $\frac{1}{2}$ d (12) £118 18s 5 $\frac{1}{2}$ d
(13) £467 4s 11 $\frac{1}{2}$ d (14) £179 16s 10 $\frac{1}{2}$ d (15) £446 14s 9 $\frac{1}{2}$ d
(16) £478 15s 7 $\frac{1}{2}$ d (17) £76 16s 9 $\frac{1}{2}$ d
3 (1) Rs 4727 15a 10p (2) Rs 2262 12a 10p
(3) Rs 1085 13a 10p (4) £1184 14s 10 $\frac{1}{2}$ d
(5) £1027 16s 6 $\frac{1}{2}$ d (6) Rs 32924 11a 11p
(7) Rs 79 15a 6p (8) £264 17s 6d
4 £197 16s 6 $\frac{1}{2}$ d 5 Rs 294 11a 1p $\frac{1}{2}$ 6 £3 1s 0 $\frac{1}{2}$ d
7 £5 9s 5d 8 Rs 590 2a 11p 9 Rs 57 2a 1p

10	<i>Rs</i> 56 10 <i>a</i> 6 <i>p</i>	11	<i>Rs</i> 365 6 <i>a</i>	12	£2 0 <i>s</i> 2½ <i>d</i>
13	<i>Rs</i> 250 12 <i>a</i> 11 <i>p</i>	14	<i>A</i> £26 15 <i>s</i> 7 <i>d</i> , <i>B</i> £43 9 <i>s</i> 10 <i>d</i> ,		
	<i>C</i> £37 6 <i>s</i> 3 <i>d</i>	15	<i>Rs</i> 16 13 <i>a</i> 4 <i>p</i>	16	£18 16 <i>s</i> 11½ <i>d</i>

Ex XXIX (pp 93-94)

- 1 (1) *Rs* 37 0*a* 8*p* , *Rs* 127 15*a* 6*p* , *Rs* 541 12*a*
 (2) *Rs* 206 14*a* 4*p* , *Rs* 475 0*a* 3*p* ; *Rs* 348 15*a* 6*p*
 (3) *Rs* 328 15*a* 4*p* , *Rs* 931 13*a* 7*p*
 (4) *Rs* 440 5*a* 6*p* , *Rs* 1191 12*a*
 (5) £159 8*s* 10*d* , £41 15*s* 9*d* , £874 9*s* 9½*d*
 (6) £341 18*s* 1½*d* , £2622 9*s* 11½*d*
 (7) £6046 10*s* 1½*d* , £943 7*s* 4½*d*
 (8) £995 19*s* 2½*d* , £1493 18*s* 9½*d* , £1991 18*s* 5*d* ,
 £2489 18*s* 0½*d* , £2987 17*s* 7½*d* , £3485 17*s* 2½*d* ,
 £3983 16*s* 10*d* , £4481 16*s* 5½*d* , £4979 16*s* 0½*d* ,
 £5477 15*s* 7½*d* , £5975 15*s* 3*d*
 (9) *Rs* 1333 5*a* 6*p* , *Rs* 2000 0*a* 3*p* , *Rs* 2666 11*a* ,
Rs 3333 5*a* 9*p* , *Rs* 4000 0*a* 6*p* , *Rs* 4666 11*a* 3*p* ,
Rs 5333 6*a* *Rs* 6000 0*a* 9*p* , *Rs* 6666 11*a* 6*p* ,
Rs 7333 6*a* 3*p* , *Rs* 8000 1*a*
 (10) £86 1*s* 5*d* , £92 13*s* 10*d* , £105 18*s* 8*d* ,
 £119 3*s* 6*d* , £132 8*s* 4*d*
 (11) *Rs* 1362 1*a* 5*p* , *Rs* 1466 13*a* 10*p* , *Rs* 1571 10*a* 3*p* ,
Rs 1781 3*a* 1*p* , *Rs* 1990 11*a* 11*p* , *Rs* 2095 8*a* 4*p*
 (12) £1297 19*s* 8½*d* , £1384 10*s* 4*d* , £1471 0*s* 11½*d* ,
 £1557 11*s* 7½*d* , £1644 2*s* 3½*d*
- 2 (1) *Rs* 4668 14*a* , *Rs* 6739 7*a* 6*p*
 (2) *Rs* 37559 5*a* 4*p* , *Rs* 18750 3*a*
 (3) *Rs* 72415 2*a* , *Rs* 10005 7*a* 6*p*
 (4) *Rs* 22168 11*a* , *Rs* 20182 8*a*
 (5) *Rs* 49492 12*a* 6*p* , *Rs* 3147603 7*a* 6*p*
 (6) £9495 12*s* , £2392 18*s* 10½*d* , £3676 13*s* 10½*d*
 (7) £7625 9*s* 8½*d* , £537 1*s* 3½*d* , £1818 11*s* 6*d*
 (8) *Rs* 59901 12*a* , *Rs* 275661 2*a* 8*p*
- 3 (1) *Rs* 1774 5*a* 8*p* , *Rs* 865 6*a* 8*p* , £309 9*s* 0½*d*
 (2) *Rs* 267 4*a* 10*p* , *Rs* 283 4*a* 2*p* , *Rs* 315 2*a* 10*p*
 (3) *Rs* 27527 6*a* 6*p* , *Rs* 23537 14*a* 10*p* , *Rs* 16356 13*a* 10*p*
 (4) £11653 12*s* 5½*d* , £12177 7*s* 7½*d* , £14796 3*s* 5½*d*
 (5) £63881 15*s* 8½*d* , £67116 6*s* 1½*d* , £105930 11*s* 1½*d*
 (6) *Rs* 7473 6*a* 6*p* , *Rs* 16662 5*a* 6*p*
 (7) *Rs* 10964 9*a* 8*p* , *Rs* 50451 6*a* 4*p*
 (8) *Rs* 372207 10*a* 6*p* , *Rs* 221737 8*a*
 (9) £4656820 11*s* 0½*d* , £4823315 15*s* 3*d* ,
 £13950280 8*s* 5½*d*
- 4 (1) £127903 1*s* 5*d* , £1993 7*s* 9*d*
 (2) *Rs* 61892 13*a* , *Rs* 86151 14*a*
 (3) £44154 18*s* 5*d* , £225382 12*s* 3½*d*

- (4) Rs 127023 13a 8p, Rs 299362 0a 8p
 (5) £4959308 18s, £56051090 3s
 5 (1) Rs 3 12a 2p (2) Rs 112 2a (3) Rs 56 8a
 (4) Rs 308 7a (5) £361 2s 6d (6) £278 6s 2d
 (7) Rs 3889 14a (8) £1046 17s 6d (9) Rs 5342 3a 3p
 (10) Rs 7223 8a 5p (11) £1462 17s 8½d (12) £1762 5s
 6 (1) Rs 3655 6a 4p (2) Rs 487 3a 8p (3) £16 2s 8d
 7 £1393 8s 10½d 8 Rs 2905 4a 4½p, Rs 219 11a 7½p
 9 £160 10s 10 He ought to pay Rs 49 6a
 11 £860 0s 7½d 12 £68423

Ex XXX (pp 97 98.)

- 1 (1) Rs 5 14a 10p, Rs 56 4a 4p, Rs 65 12a 8p
 (2) Rs 1720 8a 5p rem 1p, Rs 48 14a 10p, Rs 39 10a 8p
 (3) Rs 674 0a 2p, Rs 48 0a 8p, Rs 122 15a 5p
 (4) £13 7s 7½d, £9 15s 2½d, £87 14s 7¾d
 (5) £6 12s 9¾d, £9 16s 9¼d, £8 12s 11¼d
 P.6 (6) £3 16s 7¾d, £47 7s 10½d, £1 13s 4¼d
 (7) Rs 492 6a 6p rem 6p, Rs 164 2a 2p rem 6p,
 Rs 66 6a 11p rem 7a 6p
 (8) Rs 649 9a 10p rem 1a 4p, Rs 209 3a 2p rem 1a 4p,
 Rs 37 15a 7p rem Re 1 3a 7p
 (9) Rs 141 7a 7p rem 7p, Rs 37 6a 5p rem 9p,
 Rs 4 9a 1p rem Re 1 11a 8p
 (10) £9 0s 11½d, £7 2s 6d
 (11) 14s 7½d; £35 14s 11½d rem 2d
 (12) £8 15s 9¼d rem 10s 4¼d, £16 18s 10¾d rem 13s 8d
 2 (1) £119 17s 2½d, £79 18s 1½d, £59 18s 7d rem 2q,
 £47 18s 10¾d, £39 19s 0¾d, £34 4s 10¾d, rem 5q,
 £29 19s 3½d rem 2q, £26 12s 8½d, £23 19s 5¼d,
 £21 15s 10d rem 10q, £19 19s 6¼d rem 6q
 (2) Rs 544 6a, Rs 362 14a 8p, Rs 272 3a, Rs 217 12a,
 Rs 181 7a 4p, Rs 155 8a 6p, rem 6p,
 Rs 136 1a 6p, Rs 120 15a 6p, rem 6p, Rs 108 14a,
 Rs 98 15a 7p rem 7p, Rs 90 11a 8p
 (3) Rs 134 1a 8p, Rs 153 3a 2p, Rs 351 2a 6p
 (4) £47 18s 10¼d, £22 15s 0½d, £8 14s 7¼d
 3 (1) Rs 14 12a 8p, 12a 4p, Rs 39 1a 2p
 (2) Rs 44 14a 10p, Rs 24 2a 10p, Rs 40 0a 2p
 (3) £12 17s 8¾d rem 2¼d, £23 14s 6¼d rem 4d,
 £5 17s 9¾d
 (4) £3 9s 5¼d, £4 0s 0¼d, £3 18s 1¾d
 4 (1) Rs 262 8a 2p (2) £17 13s 8d (3) Rs 33 15a 4p
 (4) 14s 8¾d (5) £8 15s 2¾d
 (6) Re 1 2a 5p rem 8a 6p
 (7) £9 13s 2d rem 10s 7½d (8) Rs 11 7a 2d
 (9) Rs 8 10a 6p, 9a 8p, Rs 4 11a 10p
 (10) 8s 7¾d, 3s 3¾d, 2s 8½d

- 5 (1) Rs 87 0a 10p rem 6a , Rs 80 5a 11p rem 15a 8p ,
Rs 8 13a 1em Rs 4 10a 8p
(2) Rs 215 11a 8p rem 4a 10p , Rs 147 9a 4p rem 14a 2p ,
Rs 10 12a 2p rem Rs 32 5a 4p
(3) Rs 6 1a 1p rem Rs 14 6a 11p
(4) Rs 9 13a 3p rem Rs 14 5a 2p
(5) £2579 0s 0 $\frac{1}{2}$ d (6) £166 14s 5 $\frac{1}{2}$ d rem £1 3s 5d
(7) £191 10s 2 $\frac{1}{2}$ d (8) £1473 2s 9 $\frac{1}{2}$ d
- 6 (1) Rs 3 11a 8p (2) Rs 5 4a 5p (3) 15a 4p
(4) £12 10s 4 $\frac{1}{2}$ d (5) £3 4s 7 $\frac{1}{2}$ d (6) Rs 11 2a 2p
(7) Rs 5 3a 9p (8) £4 11s 9 $\frac{1}{2}$ d (9) £151 1s
(10) £2 3s 2 $\frac{1}{2}$ d 7 Rs 153 3a 2p 8 Rs 107 10a
- 9 5s 7 $\frac{1}{2}$ d 10 15a 10p
11 19s 8 $\frac{1}{2}$ d 12 Rs 13 4a 6p

Ex XXXI (p 99)

- 1 (1) 98, 41 (2) 69, 156 (3) 45, 178 (4) 184, 290
(5) 345, 648 (6) 347, 247
- 2 (1) 9 (2) 729 (3) 800 (4) 278 (5) 155
- 3 (1) 365 rem 2a (2) 22 rem Rs 1 1a 1p
(3) 399 rem 2s 2d (4) 74 rem 3s 7d
- 4 1142 5 13 6 121 7 921 8 480 9 96 10 4 gals

Ex XXXII (pp 102-105)

(Indian Bazar and Avondupois Weights)

- 1 (1) 52060 kan, 65075 to, 31720, 39650, 194108, 242635
(2) 119408, 149260, 64000, 80000 116000, 115000
- 2 (1) 78578 kan, 17153 (2) 538571, 564291
- 3 1610 mds 21 sr 14 ch, 11799 mds 3 sr 11 ch, 4695 mds
15 sr 15 ch 1 to 30476 mds 18 sr 3 po, 6750 mds
36 sr 2 ch, 1363 mds 11 sr 5 ch 3 to, 4890 mds 26 sr
- 4 2616, 110880 167895
- 5 873600, 65118176; 77414400 361576
- 6 8162 kan 18 mds 5 vis 1 sr 2 to, 62 kan 10 mds 8 sr 41 ta
1 m 4rat 1 dh, 13kan 13mds 10 tr, 5kan 14 mds 4vis 1sr
- 7 (1) 20895 oz, 5584 oz (2) 216160 oz, 10708 lbs
- 8 (1) 1441331, 2149817 1741872, 6912
(2) 31497, 4301, 534793
- 9 (1) 39 tons 2 cwt 2 qrs 14 lbs, 10 tons 11 cwt 26 lbs 11 oz,
2 tons 19 cwt 7 lbs 6 oz 3drs, 2 cwt 1 qr 3 lbs 9 oz 13drs
(2) 4 tons 1cwt 3qrs 7lbs 4 oz 12drs, 28 tons 3cwt 2qrs 107,
6 tons 12 cwt 1 qr 1lb 15 oz, 372831 tons 5 cwt 7 st
(3) 9 tons 13 cwt 17 lbs, 494 tons 19 cwt 1 qr 24 lbs 13 oz,
9 tons 7 cwt 3 qrs 26 lbs 14 oz 6 drs
1245 tons 1 cwt 1 qr 3 lbs 8 drs

- 10 (1) 451 mds 6 sr 10 ch (2) 573 mds 24 sr 10 ch 2 kan
 (3) 1177 mds 36 sr 2 ch 3kan (4) 694 tons 5 cwt 27 lbs 7 oz
 (5) 120 cwt 2 qrs 6 lbs 2 oz (6) 150 tons 13 cwt 1 qr 26 lbs
- 11 (1) 133 mds 22 sr 13 ch (2) 544 mds 27 sr 11 ch 1 kan
 (3) 214 mds 24 sr 11 ch 2kan (4) 14 cwt 18 lbs
 (5) 33 tons 12 cwt 2 qrs 25 lbs (6) 53 cwt 1 qr 23 lbs 14 oz
- 12 (1) 2652 mds 18 sr , 3978 mds 27 sr , 7957 mds 14 sr
 (2) 32482 mds 9 sr 8 ch , 28421 mds 38 sr 5 ch ,
 81205 mds 23 sr 12 ch
 (3) 2645 tons 13 cwt 16 lbs , 2866 tons 2 cwt 2 qrs 8 lbs
 (4) 3734 tons 3 cwt 1 qr 4 lbs , 44809 tons 19 cwt 1 qr 20lbs
 (5) 144 tons 13 cwt 3 qrs 26 lbs 14 oz 10 drs ,
 233 tons 2 cwt 2 qrs 4 lbs 7 oz 9 drs ,
 377 tons 16 cwt 2 qrs 3 lbs 6 oz 3 drs ,
 1069 tons 3 cwt 1 qr 13 lbs 12 oz 1 dr
- 13 (1) 4 mds 2 ch 2 kan rem 58 kan , 3 mds 2 ch rem 4 ch
 (2) 10 mds 39 sr 7 ch 3 kan. rem 44 kan ,
 3 mds 26 sr 7 ch 3 kan rem 268 kan
 (3) 18 mds 28 sr 9 ch rem 46 kan , 31 sr 3 ch rem 714 kan
 (4) 5 tons 8 cwt 2 qrs 18 lbs 9 oz ,
 2 tons 17 cwt 2 qrs 3 lbs 3 oz 13 drs rem 3 drs ,
 1 cwt 3 qrs 23 lbs 1 oz 1 dr rem 476 drs
 (5) 4 cwt 1 qr 2 lbs 13 oz 3 drs rem 80 drs , 2 qr 23 lbs
 12 oz 13 drs rem 800 drs (6) 28 , 27 , 17
- 14 £1. 5s 7½d 15 2s 10¾d 16 150

(Indian Jeweller's and Troy Weights)

- 1 9939 , 57956 , 62534
- 2 146 to 11 m 3 ra 2 dh , 556 to 8a 2 ra , 97 to 7 m 3 ra ,
 338 to 1a 4 ra
- 3 (1) 74294 , 91697 , 52272 (2) 92160 , 4750 , 79480
- 4 2 lbs 4 oz 6 dwts 16 grs , 25 lbs 4 oz 16 dwts 10 grs ,
 1670lbs 5oz 3dwts , 561lbs 7oz , 151lbs 7oz 16dwts 17grs
- 5 (1) 383 to 8 m 3 ra (2) 402 to 6a 1 ra 3 dh
 (3) 140 lbs 9 oz 3 dwts 14 grs
- 6 (1) 202 to 7 m. 5 ra 3 dh (2) 198 to 12a 3 ra 3dh
 (3) 66 lbs 10 oz 16 dwts 23 grs (4) 57lbs 8oz 15 dwts 20grs
 (5) 1 lb 11 dwts 19 grs (6) 187 to 12a 3 ra 2 dh
- 7 (1) 8325 to 11 m 2 ra , 9251 to 4 ra
 (2) 27805 to 15a , 30123 to 1a 3 ra 2 dh
 (3) 1448 lbs 9 oz 1 dwt 8 grs , 2173 lbs 1 oz 12 dwts
 (4) 2159 lbs 10 oz 7 dwts 10 grs , 10882 lbs 4 oz 14 dwts 7grs ,
 21349 lbs 5 oz 2 dwts 13 grs
- 8 (1) 8 to 6 m 2 ra 2 dh rem 115 dh ,
 7 to 9 m 6 ra 1 dh rem 139 dh
 (2) 5 to 14a 5 ra 2 dh rem 170 dh ,
 1 to 15a 3 ra 3 dh rem 342 dh

- (3) 4 lbs 7 oz 10 dwts 23 grs rem 7 grs ,
 1 lb 2 oz 11 dwts 1 gr rem 360 grs
 (4) 9 oz 15 dwts 16 grs rem 96 grs ,
 4 oz 17 dwts 20 grs rem 96 grs
 (5) 365 9 £3 17s 10½d 10 80

(Native Physicians' and Apothecaries' Weights)

- 1 8275 , 10478 , 2862
 2 15 to 2 m 4 ra , 41 to 2 m , 1332 to 6 m 6 ra 3 dh , 30 to 6 ra
 3 17599 , 11800 , 104932
 4 46lbs 10 oz 5 drs 1 scr 3 grs , 93 lbs 8 oz 2 drs 2 scr ,
 16 lbs 2 scr , 9 lbs 2 ½ , 3 lbs 5 ½ 19 grs
 5 (1) 304 to 4 m 1 dh (2) 5 lbs 9 oz 1 dr 1 scr 3 grs
 (3) 143 lbs 9 ½ 5 ½ 1 gr 6 (1) 66 to 6 m 6 ra 2 dh
 (2) 15 lbs 10 oz 7 drs 12 grs (3) 25 lbs 8 ½ 4 ½ 1 ½
 7 (1) 4337 to 3 m 5 ra 4698 to 7 m 1 ra 1 dh
 (2) 729 lbs 11 oz 4 drs 2 scr 8 grs , 2919 lbs 10 oz 3 drs 12 grs
 (3) 374 lbs 5 oz 2 drs 1 scr 2 grs , 501 lbs 7 oz 2 drs 1 scr
 14 grs , 17662 lbs 3 oz 7 drs 1 scr
 8 (1) 7 m 3 ra 1 dh rem 10 dh , 6 m 5 ra rem 46 dh
 (2) 1 lb 1 scr 18 grs rem 112 grs ,
 10 oz 7 drs 2 scr 11 grs rem 100 grs
 (3) 5 lbs 10 oz 4 drs 1 scr 9 grs rem 26 grs ,
 5 lbs 1 oz 1 dr 2 scr 16 grs rem 12 grs , 3 oz 1 dr 2 scr 18 grs
 (4) 252 , 28 9 140 pills + 80 grs

Ex XXXIII (pp 106-108)

- 1 8 , 245 , 528 , 6 2 (1) 237600 373140 , 660600 , 94860
 (2) 132401045 6679602762½ , 1155000
 3 24 lbs 3 oz 13 dwts 8 grs , 6 drs 1 scr 4 grs , 12 dwts 12 grs ,
 5 mds 10 sr 4 36 , 216 mds 3 vis 19 pal 1 to 117 grs ,
 3760460 mds 10 sr 10 ch
 5 19 mds 27 sr 8 ch , 14 mds 35 sr 98 mds 17 sr 8 ch ,
 107 mds 25 sr , 6 mds 22 sr 8 ch
 6 5 cwt 16 lbs , 11 cwt 2 qrs 26 lbs , 6 cwt 3 qrs , 7 cwt 2 qrs
 24 lbs , 1 ton 19 cwt 24 lbs , 16 cwt 8 lbs
 7 936lbs , 394lbs 11oz 11 dwts 16 grs , 703lbs 5 oz 18 dwts 13grs
 8 585 Mad mds , 648 Bom mds and 9 lb, over , 4 tons 12 cwt ,
 1 ton 11 cwt 1 qr 9 442 10 50 lbs 9 oz 16 dwts 3 grs
 11 6600 12 274 mds 35 sr 13 144
 14 8773 ka 7 mds 24 sr , 5848 ka 18 mds 16 sr ,
 12761 ka 5 mds 24 sr , 13950 ka 5 mds 20 sr
 15 117 tons 18 cwt 1 qr 16 lbs 16 35 sr , 14 mds 4 sr 6 ch ,
 12 lbs 8 oz 17 700lbs , 48lbs , 6 drs 2scr
 18 54 tolas , 384 lbs Troy 19 25 20 801 tons 8 cwt
 3 qrs 21lb, 21 154 22 3 mds 2 ch rem 4 ch
 23 13797 ka 8 mds 24 1943 25 2 mds

- 26 7895 ka 6 mds 32 sr , 3838 ka 10 sr , 82243 ka 2 mds
 20 sr 26061 ka 2 mds 10 sr 27 (1) 3 sr 27 ta
 (2) 12 mds 15 sr 4 prl 28 84 29 213
 30 114 grs 31 187 32 1290 cwt 9 lbs 13 oz
 33 845 34 8 lbs 9 oz , 5 dwts 35 Feathers , 1240 grs

Ex XXXIV (pp 110-111)

- 1 124032, 2976768 , 215120, 5162880 , 1616, 38784 ,
 2063, 49512 , 83503, 2004072
 2. 713 gay 1 ht 2 gi 2 an , 5912 gay 1 ht 5 gi ,
 8757 gay 1 ht 2 gi 1 an , 439 gay 1 ht 3 gi
 3 (1) 28624 , 324003 , 18465 , 760320
 (2) 185184 , 2573426 (3) 2000000 , 1635033
 (4) 1600555 , 205862
 4 13600 yds , 15620 vds , 29005 ft , 5031 poles
 5 (1) 32 mi 4 fur 33 po 1 vd 1 ft 6 in , 349 mi 7 fur 18 po 1 ft ,
 4 mi 1 fur 36 po 1 vd 1 ft 7 in , 57 mi 6 fur 5 yds ,
 10 mi 1 fur 56 vds
 (2) 17 mi 110 yds 131 mi 2 fur 31 vds , 6 mi 1 fur 86 yds 4 in ,
 6 mi 6 fur 150 vds , 31 mi 4 fur 115 yds 1 ft 8 in
 6 11 lea 1 mi 6 fur 110 yds , 2399 mi 2 fur 5 po 4 yds 1 ft 8 in
 7 (1) 333 (2) 4400 (3) 716 (4) 726½ (5) 1000 (6) 550
 (7) 260000 (8) 40000 (9) 75 (10) 24½
 8 (1) 80 vds 1 qr 3 nl 1½ in (2) 7098 nl 18 ka
 (3) 699 drd 2 haths 12 ang 2 brv (4) 3096 gay 6 tasu
 (5) 56 mi 1440 vds (6) 33 kros 3520 haths (7) 383999
 9 (1) 32 yds 4 in (2) 43 po 1 ft 9 in (3) 195 mi 1 fur 23 po 4½ yds
 (4) 46 mi 6 fur 2 po 5 vds 8 in (5) 167 yds 1 na. (6) 142ells 1 na
 10 (1) 12 mi 2 fur 29 po (2) 1 fur 18 po 5 yds
 (3) 2 mi 7 fur 2 po 2½ vds (4) 9 yds 2 qrs 1 nl
 11 (1) 12 lea 1 mi 4 fur 16 vd 8 in 1705 yds
 (2) 585 lea 1 mi 6 fur 10 po , 1352 lea 1 mi 2 fur 25 po
 (3) 1446 mi 1 fur 7 po 3 vds 10 in ,
 2143 mi 3 fur 7 po 3 yds 2 ft 4 in
 (4) 81 d in 3 hr 4 gi 102 dan 1 ha 3 gi 245 dan 2 ha 4 gi
 12 (1) 69 lea 2 fur 36 po , 44 yds 2 nl 2 in rem ¼ in
 (2) 4 mi 7 fur 36 po 7 in rem 1 ft 4 in
 34 po 4 yds 1 ft 11 in rem 12 po 4 yds 2 ft 9 in
 (3) 61 mi 6 fur 111 vds 1 ft 3 in rem 9 in ,
 5 mi 5 fur 72 yds 1 ft 7 in rem 2 yds 1 ft 6 in
 (4) 91 dan 1 gay 1 ha 4 gi , 55 dan. 4 gi , 27 drd 1 gay 2 gi ,
 22 drd 1 gi 1 ha 7 gi (5) 58
 13 34 yds 3 qrs 4 in 14 11 mi 3 fur 14 po
 15 29 kros 155 dan 1 gay 1 ha 5 gir 16 61

Ex XXXV (pp 113-115)

- 1 33080 , 291020 , 164895 , 867520 , 2725717 , 191040

- 2 1117 bi 11k 12ch, 1bi 12k 12ch 6gan, 13orbi 13ch 15sq cub -
73obi 12k. 3ch, 392bi 17k 8ch
- 3 198000, 1093940, 690800, 3388255
- 4 34936012800, 62585395200, 107491250688, 29376720
- 5 38160, 61875, 30604 6 65343, 235500, 16912
- 7 (1) 30 bi 15 bisv 2 bisvansi 13 kachvan
(2) 86282 bi 3 pands 16 ka
(3) 5 caw 5 grounds 533 $\frac{1}{2}$ q ft 98 sq in
(4) 8400 cha 3 ruk 1 bi 2 pands 14 ka
(5) 97 $\frac{1}{2}$ q mi 164 caw 4 man
(6) 6069 ghu 1 bi 1 kanal 11 marla 2 sar
- 8 (1) 23184, 4827, 37584, 71240, 5399316
(2) 107183736, 9531756 5595452, 355433005
(3) 242868780, 12702096 12043468800, 26109864, 263450880.
- 9 (1) 103 ac 1 ro 33 po, 2 ac 3 ro 12 po 5 sq yds,
3 ac 3 ro 25 po 3 sq yds 108 sq in, 518 ac 2 ro 13 po,
185 ac 2 po 26 sq yds 4 sq ft 72 sq in, 62 ac 32 po 5sq ft
(2) 19 ac 2 ro 29 po 2 sq yds 5 sq ft 81 sq in,
15 ac 10 po 15 sq yds 1 sq ft 138 sq in,
72 ac 2 ro 17 po 23 sq yds 88 sq in,
11480 ac 2 ro 11 po 28 sq yds 6 sq ft 116 sq in,
56 ac 2 ro 25 po 37 sq yds 5 sq ft 73 sq in
- 10 (1) 1400000 sq links (2) 2 sq mi 523 ac
(3) 1 sq mi 434 ac 1840 sq yds (4) 268468992 $\frac{1}{2}$ q in
(5) 351 sq chs and 5 sq po (6) 1035760 bi
- 11 1800 ac, 7744 bi, 3200 ac, 43560 bi, 2400 ac, 279389 bi
- 12 332800, 9600, 80195, 1452000, 29160000
- 13 (1) 136 bi 13 k 11 ch (2) 134 sq yds 6 sq ft 53 sq in
(3) 148 ac 4 po (4) 30 ac 2 ro 14 po 0 sq yds
(5) 98 ac 2 ro 18 po 23 sq yds
(6) 86 ac 1 ro 32 po $2\frac{1}{2}$ sq yds 4 sq ft 109 $\frac{1}{2}$ q in
- 14 (1) 48 bi 15 k 12 ch (2) 66 ac 1 ro 29 po
(3) 5 ac 2 ro 10 po $28\frac{1}{2}$ sq yds
- 15 (1) 5915 bi 16 k 10 ch, 19371 bi 5 k 8 ch
(2) 1255 ac 3 ro 32 po 4 ac 5 po 4 sq yds 5 sq ft 36 sq in,
6 ac 2 ro 35 po 7 sq yds 5 sq ft 108 sq in
(3) 302 ac 3 ro 39 po 16 sq yds 3 sq ft 130 sq in,
2840 ac 2 ro 15 po 21 sq yds 7 sq ft 21 sq in
- 16 (1) 1 bi 2 k 13 ch rem 7 ch, 15 bi 1k 14 ch rem 4 k 3 ch
(2) 1 bi 3 k rem 12 ch, 1 ac 13 po rem 2 ro 28 po
3) 94 ac 3 ro 38 po 17 $\frac{1}{2}$ q yds 7 sq ft 25 sq in,
3 ac 1 ro 36 po 1 sq yd 3 sq ft 72 sq in rem 9 sq in
(4) 23, 17 17 297 18 653 $\frac{1}{2}$ q mi and 439 ac
- Ex XXXVI (pp 115 116)
- 1 2742, 5630, 9376, 9216
- 2 1131984, 841082, 599616, 1074088
- 3 4 cub yds 7 cub ft 1280 cub in, 2 cub yds 26 cub ft 57 cub in,
3 cub yds 1 cub ft 25 cub in, 18 cub yds 13 cub ft 33 cub in

- 4 566386 chouk 4 cub yds 6 cub cubits ,
 67324 chouk 2 cub yds 4 cub cubits ,
 13470 chouk 2 cub yds 1 cub cubit
- 5 932014080 cub angulis 6 (1) 464 chouka 7 cub vds
 (2) 189 c yds 22 c ft 431 c in (3) 2627 c yds 3 c ft 27 c in
- 7 (1) 9 c yds 18 c ft 1534 c in (2) 90 c yds 1 c ft 727 c in
 (3) 247 c yds 25 c ft 1470 c in
- 8 (1) 24 c yds 5 c ft 832 c in , 50 c yds 16 c ft 640 c in
 (2) 15435 c yds 11 c ft 88 cub in
- 9 (1) 5 c yds 21 c ft 1639 c in rem 20 c in
 (2) 354 c yds 13 c ft 1025 c in rem 1 c in ,
 13 c yds 6 c ft 822 c in rem 246 c in
 (3) 26 times and 11 c ft 700 c in over 10 7

Ex XXXVII (pp 117-119)

- 1 1603 , 890 , 3655680 , 80520 , 5735200 , 29560
- 2 6 mds 2 ch , 1573 mds 1 do 1 pa 1 rek ,
 76190 mds 5 do 1 rek 15 ch , 1140 kah 3 bis 8 arh 6kat ,
 7711 mds 7 do , 43347 do 1 pa
- 3 576000 , 47923200 , 52190 , 2166 mudas 12 ph 13 paylis
 1 sr 1 tipar. , 1465 ka 7 ph 2 pavlis 2 sr 28 tanks
- 4 65600 , 40152 , 9270 , 10 garces 4 markals 5 padis 4 ollaks ,
 385 ph 4 markals , 58 ph 6 markals , 15431 markals 2 pad
- 5 188 , 1158 , 3518 , 41528
- 6 6480 , 9240 43832 , 91380 3936
- 7 9354 qrs 7 bus , 10007 lasts 1 qr 1 co 2 bus ,
 1606 co 2 bus 1 pot , 246688 qrs 1 str
- 8 561 lds 1 bus 1 pk , 22 lds 7 bus 1 pk 2 qts 1 pt ,
 278 lds 1 qr 2 bus 3 pks 3 qts , 6250 lds ,
 38 lds 2 pks 1 gal 2 pts , 13 lds 3 qrs 2 pks 1 gal
- 9 3136 , 20160 , 8616 , 28032 , 225516
- 10 1032 1400 , 284463 , 44284 11 10256 , 459705 , 24040
- 12 55 pipes 110 gals 1 pt , 992 tuns 16 gals 659 gals 3 qts 1 pt ,
 1960 gals 2 qts 1 pt 1 gill 1816 qrs 6bus 1pk 1gal 1qt 1pt
- 13 48 tuns 1 butt 1 hhd 1 fir 8 gals 2 qts 1 pt
 6 pipes 5 gals 1 pt 1 gill , 83 butts 2 kil ,
 2852 lasts 1 ld 4 qrs 2 bus 2 pks 1 gal 3 qts 1 pt ,
 23211 hhds 8 gals
- 14 11 C 30 19 fl oz 23m , 760 C 60 5 fl oz 1 fl dr 32m
- 15 1 cwt 1qr 3 lbs 12 oz 16 16 kan
- 17 21000 lbs , 26 tons 15 cwt 2 qrs 24 lbs
- 18 (1) 623 mds 1 rek (2) 187 gals 1 qt 1 pt 3 gills
 (3) 21 lds 3 qrs 1 pk 1 gal (4) 191 gals 1 qt 1 pt
 (5) 178 lds 3 qrs 2 bus (6) 8 C 6 O 7 fl oz
- 19 (1) 30 gals 2 qts 1 pt 3 gills (2) 99 gals 1 pt
 (3) 1 tun 2 hhds 30 gals 4 pts (4) 3 lds 3 qrs 5bus 2pks 1gal
 (5) 17 bus 1 pk 1 gal (6) 3 C 4 O, 14 fl oz 3 fl dr 35 m
- 20 (1) 856 qrs 3 bus 1 pk , 1760 qrs 3 bus 1 gal
 (2) 1006 gals 3 qts 1 pt , 6600 gals 2 qts 1 pt

- 21 (1) 732 gals 3 qts 1 pt 3 gills rem 1 pt 3 gills ,
78 gals 1 pt 1 gill rem 2 gals 1 qt
(2) 1477 lbs 7 bus 3 pks rem 3 gals ,
110 lbs 1 qr 1 bus 2 pks rem 21 gals
(3) 67 qrs 1 bus 3 pks 1 gal , 79 , 12 fl oz 3 fl dr 2 m
- 22 128 23 172 days and 4 pts over 24 283 gals 2 qts 25 96

Ex XXXVIII (pp 123-124)

- 1 (1) 1188000 (2) 20738400 (3) 798660000
(4) 16395480000 (5) 450542400 (6) 9720000
(7) 1179360000 (6) 155520000 (9) 137700000
(10) 221071189860
- 2 (1) 16815600 , 23137 3283200 (2) 107362800 , 6637437
(3) 190310 , 10969200 , 94867200
- 3 7921587 , 170120 , 2030400
- 4 (1) 156 da 4 pr 1 dan 12 pals ,
4 d1 33 d1 14 pals 5 kip 32 anupals
(2) 313 pr 3 dan 20 pals 15 kip , 345 dan 31 pals 15 kip 8 anu
(3) 52 da 18 pals 14 kip 3 anu , 1686 uru 11 ma 10 da 5 dan
- 5 (1) 428 wks 4 da 15 hrs , 4 wks 3 da 7 hrs 45 min 59 sec ,
26 days 18 hrs 47 min
(2) 34 hrs 17 min 36 sec , 40 d1 13 min 14 sec ,
1 yr 37 days 11 hrs 34 min
- 6 2 yrs 101 da 20 hrs 25 min ,
41 yrs 97 da 16 hrs 35 min 10 sec , 22 yrs 216 d1
20 hrs 50 min 10 sec , 13 yrs 49 da 22 hrs 37 min
- 7 (1) 131 sap 6 da (2) 212 d1 38 pals 9 kip 33 anu
(3) 229 din 24 d1 41 pals 46 kip 7 anu
(4) 152 hrs 20 min 45 sec (5) 285 d1 13 hrs 47 min 52 sec
(6) 224 wks 2 da 2 hrs 10 min
- 8 (1) 108 sap 5 da 6 pr (2) 29 din 54 dan 49 pals 44 kip
(3) 9 da 7 hrs 28 min 55 sec (4) 19 d1 17 hrs 27 min 39 sec
(5) 2 wks 3 d1 22 hrs (6) 3 yrs 220 d1 16 hrs 51 min 46 sec
- 9 (1) 524 din 48 dan 53 pals 30 kip , 992 din 50 d1 35 pals
(2) 418 ba 11 ma 23 d1 55 dan 35 pals 51 kip 16 anu
652 ba 10 ma 18 d1 15 pals 24 kip 4 anu
(3) 3371 days 3 hrs 45 min , 6435 days 20 hrs 15 min
(4) 158 wks 6 da 1 hr 8 min 24 sec ,
1394 wks 3 da 7 hrs 20 min 24 sec
- 10 (5) 2491 yrs 247 da 2 hrs 16 min 48 sec
(1) 21 din 5 pr 5 dan 30 pals
(2) 23 ba 7 ma 19 d1 37 dan 45 pals
(3) 2 da 13 hrs 6 min 37 sec rem 47 sec
(4) 97 wks 4 da 9 hrs 44 min 10 sec rem 6 sec ,
15 wks 4 da 19 hrs 59 min 36 sec
- 11 (1) 212 days (2) 366 days (3) 162 days (4) 659 days
- 12 78894000 kip 13 (1) 20927½ sec (2) 65472½ sec
- 14 262 15 Wednesday

Ex XXXIX (p 125)

- 1 (1) 619705 (2) 991826 (3) 519158 (4) 203665
 2 (1) 57° 17' 45" (2) 2 rt ang 60° 23' 28" (3) 2 rt ang 19° 31' 16"
 (4) 7 rt ang 8° 21' (5) 14 rt ang 43° 25'
 3 951° 28' 6" 4 127° 42' 51" 5 1016° 35' 12", 692° 39'
 6 1° 33' 20" rem 12", 6° 34' 43" rem 21' 7 26880
 8 86829755 9 5701 kahans 7 pans 6 ga 1 unit, 15 doz
 10 752 kahans 11 pans 15 ga, 3176 kahans 2 pans 15 ga

Ex XL (pp 125 127)

- 1 £297 10s 2 154, 370843 3 2723, 778
 4 3100 5 £461 16s 6d, £860 or 7½d
 6 £219 13s 9d 7 £4 4s 3½d 8 11 mi 3 fur 14 po
 9 9 yrs 131 days 18 hrs 12 min 54 sec
 10 1767 din 6 pr 2 dan 8 pals 45 bip 15 anupals
 11 48 kros 1582 dan 1 hath 1 big 1 mush 3 ang,
 2340 bi 8 k 3 ch 11 gan
 12 18783 times, and 18 in over 13 1705 yds 14 6d
 15 2s 6d 16 41 lbs 6 oz 11 dwts 17 7 da 13 hrs 30 sec
 18 £23 6s 8d 19 Rs 8 20 7 21 144062
 22 564394385 grs 23 46 ac 3 ro 27 po 13 sq yds 8 sq ft
 65 sq in 24 2 tons 4 cwt 2 qrs 16 lbs
 25 1 lb of sugar 26 £3 17s 10½d 27 57
 28 £4 17s 10d 29 91 mi 3 fur 15 po 5 yds 1 ft 3 in
 30 25 yrs 200 da 9 hrs 31 118 cwt 3 qrs 23 lbs 12 oz, 25½
 32 108, 275 33 (i) 21120 (ii) 53712 34 272 35 245
 36 2161, 11040 37 Friday, Monday 38 Friday, Wednesday

Ex XLI (p 129)

- 1 £49 4s 2 £31 17s 6d 3 £1 4s 7½d 4 £1 5s 7½d
 5 Rs 196 8a 6 Rs 21. 6a 9p 7 Rs 54 10a 8 Rs 930 4a
 9 72 days 10 100 miles 11 160 12 Rs 69 14a
 13 Rs 315 14 75 ac 2 ro 10 po 15 25 men
 16 Rs 11 13a 3p 17 22 mds 15 sr 18 6 cwt 3 lbs
 19 £18 16s 8d 20 168 days

Ex XLII (pp 130 131.)

- 1 3 miles 960 yds 2 202 miles 3 2880 4 4 yds 1 ft
 5 4 mi 5 fur 10 po 2 yds 2 ft 2 in 6 6400 7 7 ft 9 in
 8 4675 9 10240 10 36 mi 2 fur 18 po 1 yd
 11 4 yds 1 ft 4 in 12 5 yds 2 ft 5 in 13 4 yds 1 ft
 14 2 ft 9 in 15 6 fur 2 po 2 yds 1 ft

Ex XLIII (p 133)

- 1 Rs 36 5a 2p rem 6p 2 (1) Rs 76 6a (2) Rs 63 10a 4p
 3 1½d 4 Rs 18 7a 6p and 1a over 5 54s 6½d

- 6 Rs 3339 10a 11p 7. 12359 ft 8 14s 7½d
9 6a 8p 10 Rs 3740 2a

Ex. XLIV (pp 134-135)

- 1 Rs 125 2 Rs 40 3 Rs 171 4a gam 4 £86 13s 11½d
5 Rs 467 8a 6 Rs 2 13a 7 Rs 104 10a 8 3a 8p
9 96 books 10 226 gals 11 Rs 595, Rs 37
12 12 chairs 13 £10 10s 8d 14 £3 11s 15 10 gals

Ex. XLV (p 136)

- 1 3550 2 1775 3 115 4 Receives Rs 136 14a
5 Rs 100000 6 128 lbs 7 720 yds 8 1771
9 72 lbs 10 £2. 3s 4½d 11 Re 1 7a 4p 12 185

Ex XLVI (pp 137-138)

- 1 20 2 25 3 240 4 418 5 19 6 585
7 45 8 Rs 64, 128 half Rs, 320 gr Rs, 512 two anna pieces
9 15 mds 34 sr 6 ch 10 88

Ex XLVII (pp 139 140)

- 1 Re 1 6a 8p 2 5½d 3 Re 1 7a 8p 4 5 lbs
5 3a 4p 6 2s 10½d 7 13 8 5d 9 4½d 10 14 gals

Ex XLVIII (pp 140 141)

- 1 Rs 28748 7a 2 Rs 9 12a 3 Rs 5520 10a
4 Rs 1252 8s 5 £253 16s 3d 6 £100 3s 1½d
7 £256 8 Rs 8750 9 2s 6d 10 Rs 431 5a 2p

Ex. XLIX (p 142)

- 1 A Rs 3 10a 4p, B Rs 6 15a 8p, C Rs 13 15a 4p
2 Rs 48 13a 8p, Rs 24 6a 10p 3 Rs 47 5a 2p, Rs 54 1a 4p
4 £14 7s 6d, £5 15s 5 A Rs 10448 9s 8p,
B Rs 8661 13a 8p, C Rs 5404 8a 8p
6 A Rs 278 14a, B Rs 836 10a, C Rs 1394 6a
7 £73 11s 10½d, each of the other two £57 16s 10½d
8 Rs 17 8s 9 A gets Rs 60, B Rs 180, C Rs 450
10 Rs 1,2 11a 4p, Rs 129 12a 10p, Rs 91 10a,
Rs 61 1a 4p, Rs 38 2s 10p 11 £2 17s 8½d, £3 16s 5½d
12 Rs 16053 15a 4p, Rs 23343 0a 8p, Rs 29030 3a 4p

Ex L (pp 143 144)

- 1 Each man receives 13s 10½d, each woman 4s 7½d
2 Rs 2 1a 3 A man Rs 1426 4a a woman Rs 427 14a,
a boy Rs 142 10a 4 A man Rs 660 3a, a woman
Rs 330 1a 6p, a boy Rs 110 0s 6p 5 6s 4½d
6 Rs 29 12a 7 Rs 5 8 70 men 9 Rs 25 10a 4p
10 A goat Rs 4 4a 7p, a lamb Rs 2 5a 5p, a calf Rs.6 0a 1p

Miscellaneous Examples II (pp 146 152)

- 1 Rs 11 4a 2 1065 3 587 4 2s 3d, £11250
 5 Rs 50 7 A Rs 12 4a B Rs 10 6a, C Rs 15 2a
 8 Rs 32 8a 9 Rs 331 4a 2p 10 Rs 1074 12a 10p
 11 Re 1 7a 4p 12 247 sr 8 ch tea, 1897 sr 8ch sugar,
 2530 sr rice, 690 sr coffee 13 Rs 4 6a
 14 Rs 49573 5a 4p 15 54 gals 16 Rs 61 12a 17 589 days
 18 Rs 230 12a 19 Rs 2 7a 20 A 10 ac, B 1 ro,
 C 1 p 9½ sq yds 21 Thursday 22 42 gals
 23 1600 yds 24 £2993 25 Rs 52 4a
 26 The first gains 1a 4p more than the 2nd 27 Rs 3 1a 1p
 28 80 h lf Rs, 240 q1 Rs 29 A shall pay B Rs 72 8a
 30 23013 days 31 Rs 1250 32 Piano costs Rs 372 9a 4p,
 table Rs 174 12a 8p, carpet Rs 85 6a 33 100 lbs,
 Re 1 5a 4p 34 A Rs 37 14a, B Rs 25 4a
 35 Rs 25, 50 half Rs, 100 four-anna bits, 200 two anna bits
 36 C shall have to pay A Rs 35 4a, and to B Rs 26 2a
 37 Rs 1025 12a 6p 38 A Rs 2630, B Rs 1252 8a,
 C Rs 1007 8a 39 50 tolas 40 2a 41 Tuesday
 42 5 md 8 sr, Rs 96 8a 43 25 days 44 Loss Rs 7 8a 8p
 45 1st Rs 3785 5a 4p 2nd Rs 2534 1a 4p, 3rd Rs 3937 5a 4p
 46 A man 10a, a woman 6a, a boy 4a
 47 14 d uble pice 36 single pice 48 7th Feb 1833, Thursday
 49 First £400, 2nd £800, 3rd £1600 50 A gainer by Rs 85 2a
 51 A boy Rs 2, a woman Rs 4 2a, a man Rs 6 6a
 52 Monday, 16th March, 1863 53 Rs 953, Rs 422 12a 6p,
 Rs 105 0a 6p 54 Rs 59 55 Re 1 12a 56 20
 57 20, 40 58 1 mi 7 fur 22 po 5 yds 59 3s 7½d
 60 £494812 10s 61 941070 sec 62 91 63 45 miles
 64 1 mi 901 yds 2 ft 65 26 days 20 min, 40000 66 10 sr
 67 Rs 48243 68 107 tons 2 cwt 3 qrs 12 lbs
 69 1 sr 2 ch 2 to 70 First Rs 1920 2nd Rs 3810,
 3rd Rs 5760, 4th Rs 7680 71 671 lbs 4 oz 72 Rs 140 2a
 73 10 ten-seer weights, 30 five-seer weights, &c
 74 10 sov, 20 cr, 50 half-cr, 80s, 120 six pences
 75 2nd 16 mds 5 sr, 3rd 48 mds 15 sr

Ex LI (pp 156-157)

- 2 (1) $2^6 5$, $2^2 5 23$, $2 3 7 11$, $3^2 5 7$, $2^2 3^2 17$, $5 11 13$,
 $2^2 3^2 47$, $3^3 5 7$, $3 5 7^2$
 (2) $2^2 3^1 11$, $7 11 17$, $3^2 7 29$, 11^3 , $2^4 7 13$, $3^3 5 11$,
 $3 5^2 7^2$, $2^2 3 5 7 11$
 (3) $2 3 5^3 7$, $2^2 3 5 7 131$, $2^3 3^3 7 11$, $7^2 31^2$,
 $7 13 19 31$, $3 5^2 7 13^2$, $3^2 5^2 7^2$
 (4) $2 5^1 11^2 17$, $7^2 13^2 23$, $17^2 29 31$, $11^2 13 17 19$,
 $7^2 17^2 293$, $2 5 7^4 13^2$
 (5) $2^2 3 5^4 7 11 13$, $13^2 17^3 89$, $2^8 \times 3^4 \times 7^2 \times 11$,
 $2.3^6 5 7^3 11 13$, $2^2 3^6 5^3 7^3$

- 3 (1) 31, 53, 167 are primes, 2×43 , $2^5 \times 3$, $2^2 \times 3 \times 11$,
 $5^2 \times 11$, $2^5 \times 3 \times 5$, $2^3 \times 107$, $3^2 \times 97$
 (2) 397, 461, 727, 953, 971, 997 are primes,
 17^2 , 23×29 , 23×37
 (3) 1009, 3389 are primes, 37×41 , $7 \times 13 \times 19$,
 $2^5 \times 11 \times 13$, 41×61 , 53×89
- 4 (1) 3, 5, 11, 2, 3, 4, 6, 8, 9, 12, 2, 3, 4, 6, 9, 12, 5,
 3, 9, 2, 3, 4, 6, 8, 9, 12, 13, 2, 4, 7, 2, 4, 8, 3, 5
 (2) 2, 4, 7, 2, 4, 7, 2, 3, 4, 6, 7, 12, 3, 7,
 2, 3, 4, 6, 11, 12, 13
 (3) 5, 13, none, 3, 9, 2, 3, 4, 6, 8, 12,
 3, 7, 11, 13, 2, 3, 5, 6, 9, 10
 (4) 2, 4, 8, 11, 2, 3, 6, 11, 3, 7, 9, 2, 4, 13
- 5 (1) 18 (2) 9 (3) 16 (4) 4 (5) 8 (6) 30
- 6 4, 8, 11, 16, 22, 44, 88, 7, 11, 14, 22, 28, 44, 77, 154,
 8, 9, 11, 12, 18, 22, 24, 33, 36, 44, 66, 72, 88, 99, 132, 198, 264, 396

Ex LIII (p 158)

1	9	2 32	3 24	4 28	5 9	6 37	7 41
8	25	9 46	10 14	11 22	12 19	13 8	14 124
15	9	16 81	17 442	18 2	19 4	20 2	

Ex LIV (pp 159 160)

1.	6.	2 38	3 6	4 2	5 28	6 39	7 27
8	17	9 12	10 39	11 53	12 131	13 113	14 173
15	147	16 55	17 221		18 1536	19 25	20 105
21	142857		22 142	23 3	24 1246	25 37	26 571
27	2476099		28 1031	29 2003	30 11	31 57	32 8221
33	31		34 17	35 23	36 21	37 163	38 6862
39	3432		40 84				

Ex LV (pp 160 161.)

- 1 (1) Yes (2) Yes (3) Yes (4) Yes (5) No (6) Yes
 (7) Yes (8) Yes
 2 (1) 37 (2) 1287 (3) 3432 (4) 257 (5) 37

Ex LVI (pp 162 163)

- 1 6p 2 8 3 6912 4 40 grs 5 58, 870, 174, 754,
 290, 638, 406, 522 6 57 min
 7 11 8 5, 1140, 15, 380, 20, 285, 95, 60
 9 24060 and 21672, 13, 3, 1 10 24720 and 4155
 11 20 pairs 12 2 pairs 13 19 14 9 15 4d
 16 Rs 7 a head, A 97, B 843, C 962 17 9971, 10140
 18 28 19 23 21 9920, 10044 22 999984, 100149, 753.

Ex LVII (pp 163 164)

- 2 (1) 144 (2) 240 (3) 168 (4) 1680

(5) 2100	(6) 810	(7) 540540	(8) 1225449
(9) 43344	(10) 2286900	(11) 11025	(12) 25200
(13) 4204200	(14) 236876164	(15) 21600	(16) 27720

Ex LVIII (p 164)

1 5491	2. 3396	3 3556	4 7980	5 734877
6 67868155	7 4040400	8 11754483	9 14664	
10 39648	11 159137	12 2880	13 4029051	
14. 61688187	15 7258671	16 119268375		

Ex LIX (p 165)

1 240	2 80	3 525	4 180	5 120	6 68	7 144
8 12600	9 810	10 1680.	11 840	12 11088	13 1260	
14. 7560	15 7560	16 352800	17 6300	18 10800		
19 285120	20 95040	21 180180	22 9828			
23 120929760	24 5040	25 9009	26 98280			
27 6630	28 602910	29 138600	30 27720 , 720720			

Ex LX (pp 166 168.)

1. 77	2 £10 10s	3 6 min	4 982832	5 144 lbs	Avoir
6 7 hours , 210, 84, 60, 42, 35, 30				7 42 min.	
8 13 min past 1 P M	9 £1	10 2108	11 875	12 842	
13 960 pages , 25 chap	14 60 min	15 9883	16 1197		
17 6193	18 2521	19 4199 , 2431	20 288 , 432 , 576		
21 Rs 105	22 48 hours.	23 840 hrs	24 218 yds 2 ft 3 in		
25 22 ft	26 2 mds 10 sr	27 740 miles			
28 (1) 10117800	(11) 94500				

Ex LXI (p 171)

1 $\frac{11}{12}$, $\frac{101}{100}$	2 $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{9}{10}$, $16\frac{4}{5}$, $200\frac{1}{11}$, $94\frac{1}{7}$
3	Two-thirds , three-fourths , eight-ninths , fifteen-seventeenths , eighty seven one hundredths , three, and four sevenths , eight and eleven-sixteenths , twenty four, and twenty-seven forty-fourths , one hundred twenty-five, and ninety-one one thousandths
4 (1)	$\frac{2}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{7}{8}$ $\frac{48}{37}$, $\frac{72}{37}$, $\frac{120}{37}$, $\frac{168}{37}$, $\frac{216}{37}$, $\frac{264}{37}$, $\frac{288}{37}$, $\frac{312}{37}$, $\frac{432}{37}$
(2)	$\frac{540}{28}$, $\frac{1020}{28}$, $\frac{1200}{28}$, $\frac{1425}{23}$, $\frac{1680}{23}$, $\frac{2255}{23}$, $\frac{3132}{109}$, $\frac{5916}{109}$, $\frac{6960}{109}$, $\frac{8265}{109}$, $\frac{9744}{109}$, $\frac{13650}{109}$
5 (1)	$\frac{1}{12}$, $\frac{5}{21}$, $\frac{5}{36}$, $\frac{1}{40}$, $\frac{8}{55}$, $\frac{7}{72}$, $\frac{1}{84}$, $\frac{5}{91}$, $\frac{1}{128}$, $\frac{2}{35}$, $\frac{3}{35}$, $\frac{5}{36}$, $\frac{5}{91}$, $\frac{1}{112}$, $\frac{1}{128}$, $\frac{1}{188}$, $\frac{1}{189}$, $\frac{1}{252}$
(2)	$\frac{10}{1332}$, $\frac{21}{2512}$, $\frac{24}{2880}$, $\frac{31}{3512}$, $\frac{19}{4144}$, $\frac{10}{5600}$, $\frac{26}{5724}$, $\frac{26}{10812}$, $\frac{26}{12720}$, $\frac{26}{16108}$, $\frac{26}{17808}$, $\frac{26}{24768}$

Ex LXII (p 172.)

1 $\frac{38}{18}$, $\frac{65}{18}$, $\frac{81}{18}$, $\frac{104}{18}$, $\frac{105}{17}$, $\frac{234}{17}$, $\frac{260}{18}$, $\frac{325}{18}$
--

- 2 $\frac{338}{13}, \frac{2025}{25}, \frac{4375}{75}$ 3 $\frac{120}{120}, \frac{120}{120}, \frac{120}{120}, \frac{120}{120}, \frac{120}{120}, \frac{120}{120}$
 4 $\frac{56}{118}, \frac{756}{1050}, \frac{756}{112}, \frac{756}{980}$ 5 (i) Each = $\frac{1}{6}$ (ii) Each = $\frac{5040}{11820}$
 6 $\frac{1}{2}, \frac{1}{40}, \frac{1}{75}, \frac{1}{11}, \frac{1}{30}$

Ex LXIII (pp 173 174)

- 2 (1) $\frac{1336}{11}, \frac{902}{11}, \frac{540}{11}, \frac{2025}{11}, \frac{2038}{11}, \frac{1535}{11}, \frac{359}{11}$
 (2) $\frac{4140}{12}, \frac{38576}{10}, \frac{10000}{10}, \frac{1773}{10}, \frac{4137}{10}, \frac{1899}{10}, \frac{2805}{10}$
 (3) $\frac{89716}{128}, \frac{1407}{880}, \frac{6057}{408}, \frac{00389}{2400}, \frac{684}{880}, \frac{25}{880}, \frac{97526098}{8879}$
 4 (1) $\frac{3313}{13}, \frac{1274}{13}, \frac{13027}{13}, \frac{32217}{13}, \frac{82359}{13}, \frac{2217}{13}, \frac{1627}{13}, 25$
 (2) $\frac{13117}{13}, \frac{26175}{13}, \frac{14250}{13}, \frac{91452}{13}, \frac{339102}{13}, \frac{28488}{13}, \frac{136144}{13}$
 (3) $\frac{70851}{13}, \frac{830501}{13}, \frac{116477}{13}, \frac{403327}{13}, \frac{5030111}{13}, \frac{10010}{100001}$
 5 $\frac{27}{13}, \frac{378}{13}, \frac{314}{13}, \frac{6874}{100}, \frac{3082}{13}, \frac{4618}{13}, \frac{341152}{13}, \frac{471159}{13}$
 6 (a) $\frac{1000}{100}, \frac{6220}{100}, \frac{2385}{210}, \frac{2608}{240}$ (b) $\frac{10150}{20}, \frac{12660}{120}, \frac{7155}{720}, \frac{7524}{720}$
 7 (1) $\frac{53}{13}, 12, \frac{443}{13}, 178, 76, \frac{281}{13}$
 (2) $\frac{49}{13}, \frac{12}{13}, \frac{120}{13}, \frac{103}{13}, \frac{1}{13}, \frac{11}{13}$

Ex LXIV (p 175)

- 1 $\frac{3}{8}, \frac{5}{8}, \frac{1}{8}, \frac{85}{8}, \frac{5}{8}, \frac{50}{8}$ 2 $\frac{176}{13}, \frac{40}{7}, 25, \frac{15}{13}, \frac{26}{13}$
 3 $\frac{21}{13}, \frac{1}{7}, \frac{5}{13}, 4, \frac{100}{13}, \frac{83}{13}$ 5 $\frac{17}{8}$ 6 $\frac{231}{13}$ 7 $\frac{65660}{27}$
 8 $\frac{910}{13}$ 9 9 10 $\frac{6}{13}$ 11 $\frac{64672}{13}$ 12 $\frac{1}{13}$
 13 120 14 6 15 6 16 $\frac{6}{13}$

Ex LXV (p 176)

- 2 (1) $\frac{11}{11}, \frac{20}{11}, \frac{13}{11}, \frac{1}{11}, \frac{12}{11}, \frac{17}{11}, \frac{2}{11}, \frac{41}{11}$
 (2) $\frac{119}{11}, \frac{127}{11}, \frac{100}{11}, \frac{15}{11}, \frac{43}{11}, \frac{51}{11}, \frac{980}{11}, \frac{50}{11}$
 (3) $\frac{17}{11}, \frac{17}{11}, \frac{107}{11}, \frac{1}{11}, \frac{8}{11}, \frac{57}{11}, \frac{96}{11}$
 4) $\frac{11}{11}, \frac{1}{7}, \frac{46}{11}, \frac{2}{7}, \frac{405}{11}, \frac{72}{11}, \frac{8}{11}$ (5) $\frac{50}{11}, \frac{117}{11}, \frac{5}{7}, \frac{7}{11}, \frac{11}{11}, \frac{7}{11}$
 (6) $\frac{478}{11}, \frac{2311}{11}, \frac{1480}{11}, \frac{174}{11}, \frac{53}{11}, \frac{24681}{11}$
 3 (1) $\frac{10}{13}, \frac{2}{13}, \frac{21}{13}, \frac{7}{13}, \frac{28}{13}$ (2) $\frac{27}{13}, \frac{1}{13}, \frac{1}{13}, \frac{1}{13}$

Ex LXVI (p. 178)

- 1 (1) $\frac{10}{13}, \frac{13}{13}$ (2) $\frac{8}{13}, \frac{14}{13}$ (3) $\frac{13}{13}, \frac{16}{13}$ (4) $\frac{35}{13}, \frac{56}{13}, \frac{56}{13}$
 (5) $\frac{174}{13}, \frac{20}{13}, \frac{241}{13}$ (6) $\frac{105}{13}, \frac{100}{13}, \frac{134}{13}, \frac{170}{13}$
 (7) $\frac{1105}{13}, \frac{1288}{13}, \frac{1485}{13}, \frac{1540}{13}, \frac{1575}{13}$ (8) $\frac{140}{13}, \frac{231}{13}, \frac{230}{13}$
 9) $\frac{105}{13}, \frac{1265}{13}, \frac{1575}{13}$ (10) $\frac{4312}{13}, \frac{395}{13}, \frac{6720}{13}, \frac{16184}{13}$
 (11) $\frac{15}{13}, \frac{14}{13}, \frac{12}{13}$
 2 (1) $\frac{1}{13}, \frac{1}{13}, \frac{1}{13}$ (2) $\frac{16}{13}, \frac{18}{13}, \frac{21}{13}$ (3) $\frac{87}{13}, \frac{86}{13}$
 (4) $\frac{80}{13}, \frac{1008}{13}, \frac{1500}{13}$ (5) $\frac{210}{13}, \frac{110}{13}, \frac{101}{13}$ (6) $\frac{75}{13}, \frac{70}{13}, \frac{48}{13}, \frac{81}{13}$
 (7) $\frac{404}{13}, \frac{441}{13}, \frac{443}{13}, \frac{482}{13}$ (8) $\frac{34}{13}, \frac{27}{13}, \frac{30}{13}, \frac{32}{13}$

- Ex LXVII (pp 179 180)

- Ex LXVIII (pp 181 182)

- Ex LXIX (p 184)

- 2 (1) $\frac{1}{10}, \frac{2}{15}, \frac{3}{20}, \frac{4}{25}, \frac{5}{30}, \frac{6}{35}, \frac{7}{40}$ (2) $\frac{1}{12}, \frac{1}{18}, \frac{1}{24}, \frac{1}{30}, \frac{1}{36}, \frac{1}{42}, \frac{1}{48}$
 (3) $6\frac{1}{11}, 6\frac{2}{11}, 6\frac{3}{11}, 6\frac{4}{11}, 6\frac{5}{11}, 6\frac{6}{11}, 6\frac{7}{11}$ (4) $39\frac{1}{11}, 39\frac{2}{11}, 39\frac{3}{11}, 39\frac{4}{11}, 39\frac{5}{11}, 39\frac{6}{11}, 39\frac{7}{11}$
 3 (1) $\frac{1}{2}$ (2) $\frac{1}{3}$ (3) $\frac{1}{4}$ (4) $\frac{1}{5}$ (5) $\frac{1}{6}$ (6) $\frac{1}{7}$ (7) $\frac{1}{8}$
 (8) $\frac{1}{9}$ (9) $\frac{1}{10}$ (10) $\frac{1}{11}$ (11) $\frac{1}{12}$ (12) $\frac{1}{13}$
 (13) $\frac{1}{14}$ (14) $\frac{1}{15}$ (15) $\frac{1}{16}$

Ex LXX. (p. 185)

- 2 (1) $23\frac{1}{2}$, $37\frac{1}{2}$, $38\frac{1}{2}$ (2) $2\frac{1}{2}$, $8\frac{1}{2}$, $11\frac{1}{2}$, $11\frac{1}{2}$
 (3) $11\frac{1}{2}$, $46\frac{1}{2}$, $61\frac{1}{2}$ (4) $41\frac{1}{2}$, $56\frac{1}{2}$, $79\frac{1}{2}$, 498
 (5) $188\frac{1}{2}$, $263\frac{1}{2}$, $376\frac{1}{2}$ (6) $26\frac{1}{2}$, $80\frac{1}{2}$, $86\frac{1}{2}$, $240\frac{1}{2}$
 (7) $1912\frac{1}{2}$, $37389\frac{1}{2}$, $111\frac{1}{2}$, $58727\frac{1}{2}$, $168726\frac{1}{2}$
- 3 (1) $599\frac{1}{2}$, $799\frac{1}{2}$, $1498\frac{1}{2}$, $1798\frac{1}{2}$, $2497\frac{1}{2}$ (2) $98994\frac{1}{2}$, $549969\frac{1}{2}$
 (3) $12499\frac{1}{2}$, $24999\frac{1}{2}$, $37499\frac{1}{2}$, $49999\frac{1}{2}$, $74999\frac{1}{2}$,
 $99999\frac{1}{2}$, $124999\frac{1}{2}$ (4) $3223\frac{1}{2}$, $32399\frac{1}{2}$, $998999\frac{1}{2}$

Ex LXXI (pp 186 187)

- 2 (1) $3\frac{1}{2}$, $7\frac{1}{2}$, $7\frac{1}{2}$, $7\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$ (2) $4\frac{1}{2}$, $7\frac{1}{2}$, $11\frac{1}{2}$, $1\frac{1}{2}$, $1\frac{1}{2}$
 (3) $18\frac{1}{2}$, $86\frac{1}{2}$, $91\frac{1}{2}$, $61\frac{1}{2}$, 17 (4) $1\frac{1}{2}$, $414\frac{1}{2}$, $3\frac{1}{2}$
 (5) 26 , $1267\frac{1}{2}$, $5\frac{1}{2}$
- 3 (1) $3\frac{1}{2}$, $4\frac{1}{2}$, $4\frac{1}{2}$, $8\frac{1}{2}$, 2 (2) $1\frac{1}{2}$, $4\frac{1}{2}$, 2 , $1\frac{1}{2}$ (3) $242\frac{1}{2}$, $498\frac{1}{2}$
 (4) $70\frac{1}{2}$ (5) $5\frac{1}{2}$
- 4 (1) $7\frac{1}{2}$ (2) 3 (3) $1\frac{1}{2}$ (4) $1\frac{1}{2}$ (5) 1 (6) $1\frac{1}{2}$
- 5 (1) $66\frac{1}{2}$, $33\frac{1}{2}$, $3\frac{1}{2}$ (2) $1\frac{1}{2}$, $3\frac{1}{2}$, $4\frac{1}{2}$
 (3) $22\frac{1}{2}$, $31\frac{1}{2}$, $14\frac{1}{2}$ (4) $5\frac{1}{2}$ (5) $7\frac{1}{2}$ (6) 2

Ex LXXII (p 189)

- 2 (1) $6\frac{1}{2}$, $15\frac{1}{2}$, $20\frac{1}{2}$, $20\frac{1}{2}$ (2) $1\frac{1}{2}$, $2\frac{1}{2}$, $10\frac{1}{2}$
 (3) $1\frac{1}{2}$, $3\frac{1}{2}$, $1\frac{1}{2}$ (4) $1\frac{1}{2}$, $1\frac{1}{2}$, $1\frac{1}{2}$
- 4 (1) $1\frac{1}{2}$, $2\frac{1}{2}$, $2\frac{1}{2}$, $1\frac{1}{2}$ (2) $1\frac{1}{2}$, $1\frac{1}{2}$, $1\frac{1}{2}$, $4\frac{1}{2}$
 (3) $2\frac{1}{2}$, $1\frac{1}{2}$, $4\frac{1}{2}$, $1\frac{1}{2}$ (4) $3\frac{1}{2}$, $1\frac{1}{2}$, $1\frac{1}{2}$, $3\frac{1}{2}$

Ex LXXIII (p 189.)

- 1 (1) $1\frac{1}{2}$, $19\frac{1}{2}$, $3\frac{1}{2}$, $1\frac{1}{2}$, $1\frac{1}{2}$, $1\frac{1}{2}$, 156
 (2) $1\frac{1}{2}$, $7\frac{1}{2}$, $2\frac{1}{2}$, $4\frac{1}{2}$, $1\frac{1}{2}$ (3) $14\frac{1}{2}$, $8\frac{1}{2}$, $19\frac{1}{2}$ (4) $3\frac{1}{2}$
- 2 (1) $2\frac{1}{2}$ (2) $1\frac{1}{2}$ (3) $7\frac{1}{2}$ (4) $9\frac{1}{2}$ (5) $7\frac{1}{2}$ (6) $1\frac{1}{2}$
 (7) $7\frac{1}{2}$ (8) $1\frac{1}{2}$ (9) 1 (10) 2 (11) $1\frac{1}{2}$ (12) $1\frac{1}{2}$

Ex LXXIV (pp 191-192)

- 1 (1) $1\frac{1}{2}$, $4\frac{1}{2}$, $3\frac{1}{2}$, $3\frac{1}{2}$, $1\frac{1}{2}$, $1\frac{1}{2}$, $1\frac{1}{2}$, $7\frac{1}{2}$, $7\frac{1}{2}$
 (2) $2\frac{1}{2}$, $1\frac{1}{2}$, $1\frac{1}{2}$, $1\frac{1}{2}$, $1\frac{1}{2}$ (3) $3\frac{1}{2}$, $7\frac{1}{2}$, 2 , $2\frac{1}{2}$
 (4) 3 , $1\frac{1}{2}$, 1 (5) $2\frac{1}{2}$, $2\frac{1}{2}$, $1\frac{1}{2}$
- 2 (1) $1\frac{1}{2}$, $2\frac{1}{2}$, $4\frac{1}{2}$, $1\frac{1}{2}$ (11) $\frac{3\frac{1}{2}}{5}$, $\frac{10}{11\frac{1}{2}}$ 3 The first is the least

and the second is the greatest

- 4 (1) 15 (2) $10\frac{1}{2}$ (3) $1\frac{1}{2}$ (4) $5\frac{1}{2}$
 5 (1) $4\frac{1}{2}$ (2) $1\frac{1}{2}$ (3) 7

- 6 (1) $5\frac{2}{3}$ (2) $10\frac{1}{2}$ (3) $3\frac{7}{8}$ (4) $1\frac{7}{8}$ (5) $\frac{23}{416}$ (6) $7\frac{1}{2}$
 (7) 26 (8) 55 (9) $3\frac{1}{3}$

Ex LXXV (pp 192 194)

- 1 (1) $72\frac{1}{2}$ (2) $16\frac{1}{2}$ (3) $25\frac{1}{2}$ (4) $194\frac{2}{3}$ (5) $70\frac{8}{10}$
 (6) $182\frac{1}{2}$, $92\frac{1}{2}$ (7) $378\frac{1}{2}$, $31\frac{1}{2}$ (8) $2108\frac{1}{2}$
 2 (1) $2\frac{3}{8}$ (2) $\frac{1}{3}$ (3) $\frac{2}{3}$ (4) $2\frac{1}{2}$ (5) $3\frac{1}{2}$ (6) $4\frac{1}{2}$
 (7) $14\frac{1}{2}$ (8) $3\frac{1}{2}$ (9) $1\frac{1}{2}$ (10) $1\frac{1}{2}$
 (11) $2\frac{1}{2}$ (12) $1\frac{1}{2}$ (13) $4\frac{1}{2}$ (14) $4\frac{1}{2}$
 (15) $6\frac{1}{2}$ (16) 2 (17) $3\frac{1}{2}$ (18) $2\frac{1}{2}$ (19) $1\frac{1}{2}$ (20) $1\frac{1}{2}$
 3 (1) $16\frac{1}{2}$ (2) $10\frac{1}{2}$ (3) $19\frac{1}{2}$ (4) $15\frac{1}{2}$ (5) $3\frac{1}{2}$
 (6) $1\frac{1}{2}$ (7) $5\frac{1}{2}$ (8) $\frac{1}{2}$ (9) $\frac{1}{2}$ (10) 5

Ex LXXVI (pp 195 197)

- 1 $1\frac{1}{2}$ 2 $1\frac{1}{2}$ 3 $4\frac{1}{2}$ 4 $7\frac{1}{2}$ 5 $1\frac{1}{2}$ 6 2
 7 $7\frac{1}{2}$ 8 9 9 2 10 $1\frac{1}{2}$ 11 $\frac{1}{2}$ 12 1 13 10
 14 $\frac{1}{2}$ 15 1 16 $\frac{1}{2}$ 17 $1\frac{1}{2}$ 18 3 19 $\frac{1}{2}$
 20 1 21 $1\frac{1}{2}$ 22 $\frac{1}{2}$ 23 2 24 1 25 $2\frac{1}{2}$
 26 $1\frac{1}{2}$ 27 $1\frac{1}{2}$ 28 $1\frac{1}{2}$ 29 $1\frac{1}{2}$ 30 $7\frac{1}{2}$ 31 $4\frac{1}{2}$
 32 $1\frac{1}{2}$ 33 902 34 $1\frac{1}{2}$ 35 $4\frac{1}{2}$ 36 $20\frac{1}{2}$
 37 $36\frac{1}{2}$ 38 $1\frac{1}{2}$ 39 $2\frac{1}{2}$ 40 $1\frac{1}{2}$ 41 11 42 $1\frac{1}{2}$
 43 $\frac{1}{2}$ 44 3 45 $1\frac{1}{2}$ 46 1 47 2 48 $\frac{1}{2}$
 49 $\frac{1}{2}$ 50 $3\frac{1}{2}$ 51 2, $2\frac{1}{2}$ 52 $\frac{1}{2}$ 53 $5\frac{1}{2}$
 54 $\frac{1}{2}$ 55 $1\frac{1}{2}$, $2\frac{1}{2}$ 56 $1\frac{1}{2}$

Ex LXXVII (pp 198 199)

- 1 (1) $1\frac{1}{2}$, $13\frac{1}{2}$ (2) $3\frac{1}{2}$, $5\frac{1}{2}$ (3) $1\frac{1}{2}$, 1 (4) $3\frac{1}{2}$, $25\frac{1}{2}$
 (5) $3\frac{1}{2}$, 24 (6) $1\frac{1}{2}$, 60 (7) $1\frac{1}{2}$, $18\frac{1}{2}$ (8) $3\frac{1}{2}$, 720
 (9) $1\frac{1}{2}$, $180\frac{1}{2}$ (10) $1\frac{1}{2}$, $70\frac{1}{2}$ (11) $1\frac{1}{2}$, 1890 (12) $3\frac{1}{2}$, 5060
 2 $3\frac{1}{2}$ in 3 211 4 $33\frac{1}{2}$ 5 10 times 6 6 min
 7. $4042\frac{1}{2}$ ft 8 720

Ex LXXVIII (pp. 200 201.)

- 1 $1\frac{1}{2}$, $1\frac{1}{2}$ 2 $2\frac{1}{2}$ 3 1 5 $3\frac{1}{2}$, $8\frac{1}{2}$ 6 $1\frac{1}{2}$
 7 60, $2\frac{1}{2}$ 8 $1\frac{1}{2}$ 9 $492\frac{1}{2}$ 10 $13\frac{1}{2}$ 11 2500
 12 $2\frac{1}{2}$ 13 $1\frac{1}{2}$ 14 $1\frac{1}{2}$ 15 1, $\frac{1}{2}$ 16 $\frac{1}{2}$
 17 $3\frac{1}{2}$ 18 $1\frac{1}{2}$ 19 $1\frac{1}{2}$ 20 $5\frac{1}{2}$ 21 $109\frac{1}{2}$
 22 $1\frac{1}{2}$ 23 $\frac{1}{2}$ 24 $1\frac{1}{2}$ 25 $20\frac{1}{2}$ 26 99 27 $5\frac{1}{2}$
 28 $1\frac{1}{2}$, $1\frac{1}{2}$ 29 $2\frac{1}{2}$ 30 B receives $1\frac{1}{2}$, C receives $1\frac{1}{2}$,
 B receives $1\frac{1}{2}$ of A's money after loss, and C receives $1\frac{1}{2}$
 of A's money after loss 31 $3\frac{1}{2}$ 32 $1\frac{1}{2}$, $2\frac{1}{2}$

Ex. LXXIX. (pp 202 203.)

- 1 (1) Rs 28 15a. 9 $\frac{3}{8}$ p (2) Rs 87 15a 8 $\frac{5}{8}$ p (3) £32 12s 9 $\frac{8}{16}$ d
 (4) £70 10s 11 $\frac{3}{4}$ d (5) 1 lb 2 oz 14 dwts 4 $\frac{1}{2}$ grs
 (6) 19 cwt 1 qr 17 lbs 7 $\frac{7}{10}$ oz (7) 2 fur 8 po 5 yds 1 $\frac{1}{2}$ in
- 2 (1) Rs 5 1a 5 $\frac{1}{2}$ p (2) Rs 10 15a 2 $\frac{3}{8}$ p (3) £13 13s 8 $\frac{1}{8}$ d
 (4) £98 19s 1 $\frac{3}{8}$ d (5) 6 cwt 1 qr 18 $\frac{3}{8}$ lbs (6) 9 cwt 10 $\frac{1}{8}$ lbs
 (7) 3 fur 10 po 2 yds 9 $\frac{1}{8}$ in (8) 7 hrs 54 min 46 $\frac{1}{4}$ sec
- 3 (1) Rs 74 0a 9 $\frac{1}{4}$ p, Rs 101 13a 0 $\frac{1}{2}$ p, Rs 416 8a 3 $\frac{3}{4}$ p,
 Rs 1286 9a 1 $\frac{3}{4}$ p
 (2) £302 15s 0 $\frac{1}{8}$ d, £1135 6s 6d, £6660 11s 5 $\frac{3}{8}$ d,
 £7266 1s 7 $\frac{1}{8}$ d
 (3) 8 tons 19 cwt 3 qrs 2 lbs 0 $\frac{1}{4}$ oz, 17 tons 19 cwt 2 qrs
 4 lbs 1 $\frac{1}{2}$ oz, 71 tons 18 cwt 16 lbs 4 $\frac{1}{2}$ oz
 (4) 408 mds 10 sr 2 $\frac{1}{2}$ ch, 1088 mds 27 sr 1 $\frac{1}{3}$ ch,
 1587 mds 12 sr 2 $\frac{1}{8}$ ch
 (5) 21 mi 6 fur 7 po 2 yds 3 $\frac{1}{2}$ in, 47 mi 7 fur 8 po 1 yd 10 $\frac{1}{2}$ in
- 4 (1) Rs 20 9a 1 $\frac{1}{2}$ p, Rs 17 10a 1 $\frac{3}{8}$ p, Rs 9 7a 10 $\frac{1}{16}$ p,
 Rs 4 4a 1 $\frac{1}{8}$ p
 (2) £38 7s 7 $\frac{1}{8}$ d, £18 16s 6 $\frac{2}{8}$ d,
 £11 17s 7 $\frac{1}{8}$ d, £6 17s 7 $\frac{1}{8}$ d
 (3) 112 lbs 13 oz 4 $\frac{1}{2}$ drs, 52 lbs 10 oz 7 $\frac{1}{2}$ drs,
 11 lbs 12 oz 9 $\frac{1}{2}$ drs
 (4) 2 mds 29 sr 7 $\frac{1}{8}$ ch, 1 md 12 sr 7 $\frac{1}{8}$ ch
 (5) 2 $\frac{4}{1000}$, 7 $\frac{20}{1000}$ (6) 41, 10 $\frac{22}{1000}$

Ex LXXX (pp 204 205)

- 1 3 $\frac{1}{2}$ a, 4 $\frac{1}{4}$ a, 13 $\frac{1}{2}$ a, 4 $\frac{1}{5}$ a, 1 $\frac{1}{2}$ a, 15g 2 34 $\frac{1}{2}$ d, 106 $\frac{3}{4}$ d, 11 $\frac{1}{2}$ d, 194 $\frac{1}{4}$ d
 3 $\frac{1}{8}$, $\frac{3}{16}$, $\frac{1}{6}$ 4 $\frac{1}{16}$, $\frac{1}{8}$, $\frac{1}{4}$ 5 $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{2}$
 6 $\frac{1}{16}$, $\frac{1}{8}$ 7 $\frac{1}{16}$, $\frac{1}{8}$ 8 $\frac{1}{16}$, $\frac{1}{8}$, 210 ch
 9 $\frac{1}{16}$, $\frac{1}{8}$ 10 147 $\frac{1}{2}$ po., 2660 $\frac{1}{2}$ yds 11 $\frac{1}{16}$, $\frac{1}{8}$, $\frac{1}{4}$
 12 $\frac{1}{16}$ 13 $\frac{1}{4}$ ac, 6091 $\frac{1}{2}$ ds, $\frac{1}{4}$ 14 $\frac{1}{16}$

Ex LXXXI (p 207.)

- 1 (1) 13a 4p, 9a 4p, Rs 16 14a, Rs 6 9a 7 $\frac{1}{2}$ p,
 Rs 6 12a 9 $\frac{1}{2}$ p
 (2) Rs 364, Re 1 5a 4p, Rs 4 10a 6 $\frac{3}{8}$ p, 3 $\frac{1}{2}$ p
 (3) 12s, 6 $\frac{1}{2}$ d $\frac{3}{4}$ q, 5s 10d, 16s 8d, 1s 4d, 10s 7 $\frac{1}{2}$ d
 (4) £3 5s, 2s 8d, 9s 4 $\frac{1}{2}$ d, £3 0s 8d, £3 12s 6d
 (5) £2 6s 8d, 10s 3 $\frac{1}{2}$ d $\frac{1}{4}$ q, 8s, £4 17s 6d
 (6) 2 qrs 24 lbs, 10 lbs, 6 oz, 2 qrs 26 lbs 4 oz,
 21 cwt 1 qr 9 lbs 5 oz 5 $\frac{1}{2}$ drs

- (7) 4cwt 2qrs 8lbs, 16s 8d, 1s 6½d ½g, 4 fur 26po 3yds 2ft
 (8) 6 or 17 dwts 3½ grs, 9 oz 2½ drs, 6 oz 6 drs 2 scr 11½ grs, 2 lbs 5 oz 13 dwts 8 rs
 (9) 1 yd 11t 10½ in, 1 po 1 yd 1 ft 5½ in, 2 ro 16 p, 37 po 24 sq yds 6 sq ft 108 sq in
 (10) 3 qrs, 1 da 22 hrs 40 min 3 da 17 hrs 36 min
 (11) 1 pk, 1 pk 1 gal 1 gal 1 qt 1½ pts, 3 lbs 7 us 3 pks 1½ pts
 (12) 148 c yds 12 ft 1036½ in, 27 mds 10 sr 13 ch 13 to, 145 tons 16 cwt 2 qrs 18 lbs 10 oz 10½ drs
 (13) 5 hrs 36 min; 7 hrs 12 min, 2 pipes 115 gals 2 qts
 (14) £7, 109 da 13 hrs 20 min, 4s 8d
 (15) 3 cwt 1qt 6 lbs, 3 da 11 hrs 13 min, £4 14s 11d
 2 (1) £4 1s 8d, £2 18s 4d (2, 19s 6½d (3) £1 2s
 (4) 12 cwt 2qrs 14lbs 10 oz 10½ drs, 2qrs 17 lbs 10 oz 11½ drs
 (5) 4 da 23 hrs 31 min 31½ sec (6) 2 mi 6 fur 22 po 3 in, 17 mds 27 sr 5½ ch (7) Rs 5 14a (8) £50

Ex. LXXXII (pp 208 209.)

- 1 (1) £1 12s 10½d 3g, £4 15s 11½d 1, £39 11s 11½d
 (2) Rs 186 10a 6p, Rs 951 7a 11½p, Rs 2687 9a 3½p, Rs 7898 2a 6½p
 (3) £82 18s 0½d ½g, £98 19s 11½d, £402 17s 5½d ½g
 (4) 25 tons 3 cwt 3 qrs 7 lbs 8½ oz, 10½ tons 6 cwt 2 qrs 10 lbs 14½ oz, 242 tons 9 lbs 4½ oz, 1100 tons 9 cwt 1 qr 5 lbs 6½ oz
 (5) 10 da 10 hrs 36 min 5½ sec, 119 ac 1 ro 1 po 21½ sq yds
 2 (1) Rs 1766 12a 11p, Rs 401 15a 1½p, £1 8s 6½d
 (2) £42 7s 3½d 3g, £46 4s 0½d 3g, £3 12s 4½d ½g
 (3) Rs 65 Rs 26 7a 10½p, Rs 17 11a 7½p, Rs 8 5a 4p
 (4) 3 cwt 3 qrs 18 lbs 12½ oz, 20 ac 23½ po
 (5) 1 da 19 hrs 29 min 46½ sec, 7 c yds 11 c ft 1210½ c in
 (6) 2 fur 124 yds 2 ft, 4 mds 29 s
 3 (1) Rs 3 4a 6p, Rs 1 12a 6p, Rs 26 4a, 8a
 (2) Rs 25 5a 4p, Rs 133 4a, Rs 1575 5a 4p
 (3) £4 1s 8½d, 3s, 4s 4) £12 3s, £8 14s 6½d, Rs 40 8a
 (5) 11 mds 11 sr 5 ch, 9 cwt 2 qrs
 (6) 4 ft 6½ in, 3 sq ft 96 sq in 1 c ft 512 c in
 (7) 102 mi 4 fur 11 po 3½ dds, £40 4s 2d
 (8) 26 cwt 24 lbs 4 oz, 36 po
 (9) 32 da 9 hrs 49 min, Rs 79 9a 3½p
 (10) 210 18½ po, £22 16s
 4 (1) Rs 56 4a (2) Rs 85 5a 11½p. (3) £5 12s 6d
 (4) £6 10s 3½d (5) £50 (6) £5 12s 8½d
 (7) Rs 62 11a 9½p (8) 6 mds 18 sr
 (9) 7 yrs 169 da 34 min (10) 1 po 34 yds 1 ft 5½½ in

Ex. LXXXIII. (pp. 210 211)

- 1 2, 15, 25, 40, 28, 10, 1490
 2 240, 480, 192, 192, 480, 480

- 3 £144, £342, £378, £540
 4 Rs 3s Rs 8s, Rs 15½s, Rs 81½s
 5 2s 6d, 2s 2d, 1s 8d 6 3 2s 6d, 2s
 8 4½s mds, 2½s bu, 2½s 9 4s 10 12s 6d, 5s
 11 2s, 1s 6d 12 1s 6d, 1s

Ex LXXXIV (pp 212 215)

- 1 (1) 1½s 2s (2) 10s (3) 1s, 1s 2 (4) 2s (5) 1s, 2s 6d
 (6) 2s 6d (7) 10s (8) 2s 6d
 2 (1) 10s 1s (2) 1½s (3) 10s, 1s 6d (4) 1s
 (5) 2s 6d, 1s 6d (6) 1s (7) 2s (8) 2s, 2s
 (9) 1s (10) 10s
 3 2s 4 2s 6d 5 2s 6d 6 1s 6d 7 1s 6d, 2s
 8 1s 6d 9 2s 6d, 1s 6d 10 2s 6d, 1s 6d 11 1s, 2s 6d
 12 2s 6d 13 (1) 1s (2) 1s (3) 1s (4) 2s 6d
 14 1s 6d, 1s 15 1s 16 (1) 1s 6d (2) 2s, 2s (3) 1s 6d
 (4) 2s 6d, 1s (5) 1s (6) 1s (7) 2s 6d
 17 In order of value the fractions will stand thus —
 (1) 1s of 1 cr, £10, 1s of a gu (2) 1s of a gu, £11, 1s of 3s 9½d
 (3) 1s of Rs 10 = 1s of Rs 10 8s, 1s of Rs 7 13s
 (4) 1s of a md, 1s of 3 sr 6 ch, 1s of 14 sr
 (5) 1s of 5 days, 1s of 20 hrs, 1s of 59 min
 18 2s 6d 19 2s 6d 20 1s 6d 21 1s 6d 22 1s 6d 23 1s 6d
 24 2s 6d 25 Rs 1 5s 5½p 26 1 fur 27 Rs 10, 2s 6d
 28 7 cwt 11 lbs.

Ex LXXXV (pp 215-216)

- 1 3 days 10 hrs 25 min 3 sec 2 25 mds 30 sr.
 3 £3 13s 8½d 4 5 po 10 in 5 31 years 203 da 18 hrs
 6 242½ days 7 344½s 8 2440½s.
 9 79 wks 1 da 22 hrs 49 min 48 sec 10 12½, 1½
 76½, 5½

Ex LXXXVI (pp 218 220)

- 1 Rs 32 6s 2 55s 3 £1 2s 6d 4 Rs 2 12s
 5 £44 17s 8½d 6 £4 16s 7 £200
 8 111835½ metres 9 6s 3½p 10 Rs 225 11 2
 12 1s 6d, Rs 31250 13 9 men 14 7½ days 15 41½ days
 16 15½ days 17 7 miles. 18 60 cwt 19 2lbs 3 oz
 20 4 wks 2 da 21 85 days 22 12800 23 Rs 4 8s
 24 75 25 100 days 26 Rs 43 8s 27 10 yds 11 in
 28 Rs 46 10s 8p

Ex LXXXVII (pp. 221 222)

1	12a 8p	2	£379 11s 3½d	3	6a 3½p	4	Rs 2376
5	10a 9½p	6	12a, A Rs 3867	7	B Rs 3052	8a	C Rs 2200
7	Rs 3000	8	£722 13s 4d	9	£412	10	3s 4d
11	Rs 2790 10a	12	Debts = £25230, assets = £15558 10s				
13	Rs 37350	14	£8000	15	Debts = Rs 13600, assets = Rs 9350	16	£6666 13s 4d

Ex. LXXXVIII. (pp 223 224.)

1	Rs 1752 12a	2	£631 10s	3	Rs 360	4	£598
5	£451 10s	6	Rs 12000	7	£1040	8	6p
9	£108	10	Rs 11480	11	£70	12	Rs 960
13	£3642	14	£280				

Ex LXXXIX (pp. 226 227)

1	4200	2	Rs 2160	3	120 gals	4	210	5	Rs 16 13a 2p
6	Rs 22500	7	3 ro 24 po	1	ac 2 ro 30 po	8	£7 8s		
9	£1 5s	10	38 yds 1 ft 6 in	11	A Rs 22 14a 8p ,				
			B Rs 20 13a 4p	12	Eldest Rs 12000 , younger Rs 4500 ,				
			wife Rs 2700	13	Rs 16 4a	14	Rs 15000 , Rs 4500 each		
15	960	16	£4000	17	A Rs 114 4a 6½p , B Rs 128 9a 1½p ,				
			C Rs 157 2a 3¾p	18	Rs 3333 5a 4p , 30				

Ex XC. (pp 229 230)

1	6 hrs 40 min	2	24 hrs	3	3 hrs 38½ min	4	12 min
5	3½ hrs	6	7	7	12 min past 5	8	58 min 40 sec, 120
9	5½ hrs	10	30 min	11	1½ min, 237½ gals	12	4 min 42 sec

Ex. XCI. (pp 233 235.)

1	61 $\frac{2}{3}$ days	2	4 days	3	The whole	4	36 days
5	15 days	6	3 $\frac{1}{3}$ days	7	10 $\frac{2}{3}$ days	8	A 8 days,
	B 24 days, C 4 $\frac{1}{2}$ days	9	18 $\frac{1}{2}$ days	10	$\frac{4}{3}$		
11	120 days	12	A 45 $\frac{1}{2}$ days	B 10 $\frac{10}{13}$ days	13	1 $\frac{1}{2}$ hours	
14	3 $\frac{1}{2}$ hours	15	12 $\frac{1}{6}$ days, A 4 $\frac{1}{2}$ ac, B 5 $\frac{1}{2}$ ac	16	A 41 $\frac{8}{13}$ da,		
	B 8 $\frac{1}{7}$ da, C 3 $\frac{1}{7}$ da	17	16 $\frac{1}{14}$ days	18	18 days		
19	8 days	20	4 $\frac{1}{5}$ days	21	A 12 da, B 24 da		
22	14 $\frac{2}{3}$ days	23	A 4 days, B 12 days	24	26 days		
25	41 $\frac{2}{3}$ days	26	9 $\frac{1}{2}$ days	27	11 $\frac{1}{2}$ weeks	28	23 $\frac{1}{2}$ hrs

Ex. XCII (p 237)

1	56	2	112	3	90	4	15	5	172	6	3456, 2304
7	120, 20, 1½, 1½	8	Rs 150	9	Rs 24	10	Rs 2400				
11	36 gals	12	60000 men								

Ex XCIII (pp 239 240.)

1	A Rs 1696	15a	10p, B Rs 2196	1a	8p, C Rs 2595	6a	4p
2	A Rs 15 8a, B Rs 22 4a, C Rs 37 12a						

- 3 *A Rs 600, B Rs 480, C Rs 320*
 4 *A Rs 58 12a 2p, B Rs 117 8a 4p, C Rs 176 4a 6p*
 5 *A Rs 1200, B Rs 400, C Rs 200*
 6 *A Rs 3762, B Rs 2280, C Rs 6498*
 7 *A Rs 768, B Rs 672, C Rs 560*
 8 *Man, Rs 7, wife, Rs 3 8a, child, Re 1 2s 8p*
 9 *A £50, B £100, C £300, D £1200*
 10 *A Rs 2782¹⁰/₁₀₀, B Rs 3130²⁰/₁₀₀, C Rs 2347¹¹/₁₀₀*
 11 *A Rs 6048, B Rs 5760, C Rs 5670, D Rs 5600*
 12 *A Rs 8505, B Rs 7276 8a, C Rs 9172*

Ex. XCIV (pp. 241-242.)

- 1 14 min 24 sec 2 6 hrs 3 36¹/₂ days 4 7¹/₂ hours
 5 72 hours 6 258 h s 7 258 hrs
 8 (i) 120 hrs (ii) 120 hrs

Ex XCV (pp 243-244)

- 1 5¹/₇ ds 2 8a 3 200 4 10 gulden 5 2601
 6 1296 7 82212 8 Rs 1066 10a 8p
 9 £11760 10 1136¹/₂ gals 11 £802 10s 0¹/₄d 4q
 12 2s 0¹/₄d nearly

Ex XCVI (pp 246-248)

- 1 ³/₂₀ 2 ²/₁₀ 3 16 4 21 5 Rs 400 6 Rs 3000
 7 429 8 ¹/₁₀ 9 3 mds 10 r 10 3¹/₂ 11 *A Rs 2 13a, B Rs 3 12a, C 15a*
 12 *A Rs 10, B Rs 20, C Rs 10*
 13 Rs 9984 14 Rs 53 2a 15 Rs 32
 16 *A 131⁴/₅ days, B 40⁸/₅ days, C 67²/₅ days, D 93¹/₅ days*
 17 9s 6p, 12a 6p, 15a 6p, Re 1 2a 6p, Re 1 5a 6p
 18 Rs 5 19 *A Rs 2 8a, B Re 1 8a, C 8a* 20 20³/₅ in

Miscellaneous Examples III (pp 248-253)

- 1 2⁴/₁₀₀₀, 3⁴⁰⁰/₁₀₀₀ 2 12¹/₅ 3 ²/₁₂ 4 *A Rs 6666 10a 8p, B Rs 5000, C Rs 4000, D Rs 3333 5a 4p*
 5 *A Rs 1344, B Rs 1053* 6 40¹⁰/₁₁ days
 7 80 apple, 60 pear 8 cherry, 40 filbert, 12 walnut trees
 9 Rs 9 2a 8p 10 Rs 2102 8a 11 2¹/₁₀ days
 12 Rs 4 9a 6p 13 20¹/₅ days 14 ³/₁₀₀ 15 ³/₁₀₀
 16 6⁸/₁₀₀ days 17 600 18 (a) 258²⁴/₁₀₀ (b) 1³/₂
 19 37610528 20 54 days 21 £22 8s 22 Yes, ¹/₂₈₀
 23 71³/₄ hours 24 Rs 5040 25 107⁸/₁₀₀ days
 26 300 gals 27 7¹/₁₀ 28 *D's share = Rs 30847*
 29 ⁹/₁₄ and ¹/₁₄, 2¹/₂
 30 14 each 31 50 yds 32 ²/₇ 33 Rs 4200
 34 Rs 6111 10a 8p 35 Rs 484 36 10s 10d to be added
 37 *A Rs 200, B Rs 100, C Rs 700, D Rs 3200* 38 13a 4p.
 39 *A's money 18s, B's 6s 8d, C's 12s* 40 ¹/₂
 41 *Rs 7 7a 7p* 42 8a 6p to be subtracted 43 10 weeks

- | | | | | | | | | |
|----|---------------------|--------------------|----------------------|--------------------|----------------------|----------------------|-------------------|---------------------|
| 44 | $R_3 132$ | 45 | $10\frac{1}{2}$ days | 46 | 3To^n days | 47 | £80 | |
| 48 | 16720 tons | 49 | $\frac{1}{4}$ | 50 | $16\frac{1}{2}$ day- | | | |
| 51 | $A R_5 6$ 1a | 1 st p. | $B R_5 6$ 6a | 10^2 p | 52 | 50 days | 53 | $2\frac{1}{2}$ days |
| 54 | $5\frac{1}{2}$ dlys | 55 | 4 hrs | $7\frac{1}{2}$ min | 56 | 4 men | 57 | 30 men |
| 58 | 169, 260 | 59 | 17 | 60 | 3024 | 61 | 12 A M , 7-30 P M | |
| 62 | 12 dlys | 63 | 4 men | 64 | 1st class | $3\frac{1}{2}$ mds , | | |
- 2nd 1st mds , 3rd 1st mds , 4th 2 mds

1 (1) 4, 18, 006 (2) 37, 154 (3) 24,079, 000001
(4) 579 (5) 0006, 00007, 0000005 (6) 315 00005

2 (1) Five tenths, twenty five hundredths, three hundred and twenty six thousandths, one-tenth, one-hundredth, one ten thousandth, five and thirty seven hundredths, twenty five ten thousandths
(2) Forty three thousand two hundred and sixty eight hundred thousandths, three thousand four hundred and five millionths, four hundred and fifty six ten millionths, ninety eight and seven millions six hundred fifty four thousandths, three hundred and twenty one ten-millionths, one hundred thousand and one millionths

3 (1) $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, $\frac{1}{10000}$, $\frac{1}{100000}$, $\frac{1}{1000000}$, $\frac{1}{10000000}$, $\frac{1}{100000000}$
(2) $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, $\frac{1}{10000}$, $\frac{1}{100000}$, $\frac{1}{1000000}$, $\frac{1}{10000000}$, $\frac{1}{100000000}$

4 (1) $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, $\frac{1}{64}$, $\frac{1}{128}$, $\frac{1}{256}$
(2) $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, $\frac{1}{64}$, $\frac{1}{128}$, $\frac{1}{256}$
(3) $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, $\frac{1}{64}$, $\frac{1}{128}$, $\frac{1}{256}$

5 (1) $5\frac{1}{2}$, $6\frac{1}{2}$, $4\frac{1}{2}$, $7\frac{1}{2}$, $9\frac{1}{2}$, $307\frac{1}{2}$, $8\frac{1}{2}$
(2) $376\frac{1}{2}$, $970\frac{1}{2}$, $732\frac{1}{2}$, $23\frac{1}{2}$

6 (1) 3, 7, 13, 47, 13, 407, 801, 559, 057
(2) 0017, 00965, 0303, 000009, 0000057, 4676
(3) 213, 0213, 000213, 0300507, 00076359, 20403005
(4) 16, 75, 031, 0004, 0275, 000079, 0001001, 00000079, 00261, 000000000328

1 (1) 8, 80, 800, 80000, 8000000
(2) 53, 53, 5300, 530000 (3) 130 014, 1300 14, 13001 4,
130014, 13001400 (4) 800 3065, 80030 56, 80030560
2 (1) 071, 0071, 000071, 00000071
(2) 07358, 007358, 00007358, 0000007358
(3) 00007, 000007 00000007, 0000000007
(4) 001, 00000001, 00057426

1 (1) 756 2407 (2) 4009 360626 (3) 538 6422021
(4) 433 416458 (5) 442 10265 (6) 4852 77877

- 2 (7) 634 5529699 (8) 100 (9) 61 58125 (10) 100 1111111
 (1) 1589 6738 (2) 1152 445834 (3) 3033 9704833
 (4) R1783 378 (5) £878 368 (6) 2623 952038 cwt
 (7) 2170 0457 yds (8) 6 171256

Ex C (p 259)

- 1 (1) 18971, 11 203, 52 5564 (2) 117864, 806423, 09914
 (3) 8 00001, 03035039, 51635 (4) 09, 0001, 623967
 2 (1) 27 849, 7040 322, 31 580901 (2) 1 0473, 3 021975, 0324
 (3) 4 911002, 1431, 7 09484
 3 (1) 786 214, 11 206157, 00099
 (2) R6 417, R2 23224, R3 340526
 (3) £ 12084, £54 3249, £ 20994
 (4) 6 399936 mds, 22 5124 tons, 1 43856 ft
 4 9 413 5 86 7358 6 (1) 2 6686, 2 102
 (2) 644 447 (3) 5 45414 (4) 527 02949
 7 2218487, 999544, 1 345679, 457 575, 1 101, 99999
 8 3 1415927 9 54321

Ex. CI (pp. 260 261)

- 1 (1) 40926, 04032, 9, 7777 (2) 100 17484, 21 983472,
 8 2940365 (3) 230 625593 4155 84, 00003738028
 (4) 123123, 0019610652875, 0000008
 (5) 0000003217, 6 7254023544, 5 548075
 (6) 00128, 000000015625, 0000017084592
 2 (1) 26325, 0001410816, 00177089, 6 240312
 (2) 120 89115, 0011238573, 147168
 (3) 20 796875, 026649, 133 46322
 (4) 108 243216, 9 12850715 (5) 0672, 001024
 (6) 000000001, 9 56709
 3 (1) 21 09301541972 (2) 210 795537 (3) 12 66806,
 23 68676, 001 (4) 678 593664, 14726336, 9 141474375
 4 0177775, 001

Ex CII (pp 263-264)

- 1 (1) 156 7, 31 34, 6 268, 1 2536, 12536
 (2) 128 947, 59 514, 9 919, 4 578, 1 526, 763
 (3) 00001108, 00000554, 00000277, 0000006925
 (4) 5 12, 427, 2 417 (5) 70, 7900, 3250, 22500
 (6) 24172, 2 421875, 0000746
 (7) 10, 10 01, 1545, 110000, 10
 (8) 281 315, 039 (9) 1 092, 1092, 1092, 0001092
 (10) 15 67, 313 4, 6268, 1 2536, 12 536, 125 36, '000012536
 (11) 007853, 0032, 602400 (12) 300
 2 (1) 3 42465, 6 26829, 17404 27807, 00261

- (2) 00650, 3 21685, 00025, 1 31579
 (3) 6 03604, 91 33632, 36542894 03280
- 3 (1) 12 096, 5 184, 4 032, 576, 288, 19 2, 151 2
 (2) 85, 5 1, 0075, 00425, 2 5, 01
- 4 (1) 00134, 13405, 1 34057, 00521, 5 21333, 14 89523
 (2) 76293021842 10526, 2167022 5625
- 5 (1) 30685, 147, 0007 (2) 32236 38, 2296
 (3) 20500, 1 569995 (4) 00011, 15 845686

Ex. CIII (p 265)

- 1 (1) 5, 25, 75, 125, 625, 1875, 16, 3 1171875, 008125,
 072, '7578125
 (2) '006640625, 03808; 4 0064, 3 16, 4 3125, 6 0005859375;
 7 425
 (3) 1 4, 053125, 1 687841796875, 1 0693359375, 7 00048
 3 00224, 4 0010624
- 2 (1) 12 75, 76 234375, 8 75, 2 59375
 (2) 1 25, 0176, 326 4, 1128, 6, 1 37
 (3) 83 03355, 007080078125 (4) 89 395, 0029
- 3 '333333, 285714, 272727, 384615, 269230, 309523,
 2 916666, 8 642857, 24 352941, 7 326086
- 4 (1) $\frac{5}{8}, \frac{3}{8}, \frac{3}{8}$ (2) $\frac{1}{2}, \frac{1}{8}, \frac{5}{8}$ (3) $\frac{5}{8}, \frac{3}{8}, \frac{5}{8}$ (4) $\frac{1}{8}, \frac{5}{8}, \frac{4}{8}, \frac{3}{8}$
- 5 (1) 120 (2) 9 (3) 85 1 (4) 10 125 (5) 0000125
 (6) 6400 (7) 3 (8) 562 1, 7 174970942857142'

Ex. CIV. (p. 266)

- 1 (1) 36 (2) 2223 (3) 1 115 (4) 0094. (5) 004 (6) 003
 (7) 16 (8) 0008
- 2 (1) 1260 (2) 12 6 (3) 1466 4. (4) 5319 6 (5) 14360 58
 (6) 8433621'6646

Ex. CV. (p. 267.)

- 1 $\frac{1}{8}, \frac{2}{8}, \frac{11}{8}, \frac{11}{8}, \frac{1}{8}, \frac{9}{8}, \frac{18}{8}, \frac{18}{8}$ 2 2, 4, 5, 8, 10, 16, 20

Ex. CVI. (pp. 268 269)

- 1 2, 27, 13, 135, 2 345, 1 6; 4 8, 1 90, 714285,
 7 461538, 126984, 9 309
- 2 307692, 12195, 1509433962264; 3 614583,
 4 803571428, 7 1893, 5 2871 3 3567
- 3 0126, 5 783; 2 733108, 1 0850694, 1 29573170,
 75240384615, 3'4556097, 1 00198
- 4 2'457002, 3 780003, 1 761143, 56349, 72509,
 4990774, , 907967032

Ex CVII (pp 269 270)

- 1 3 252525 , 4 77777 , 290 02222 , 36666 , 21 141414 ,
0352352 , 70655 , 046363 , 3 4534534
- 2 5777777 , 2 343434 , 0645645
- 3 1232323232323 , 1234234234234 , 123412341234
- 4 846153 , 0714285 , 2941176470588235 , 105263157894736842 ,
3478260869565217391304 , 0344827586206896551724137931 ,
193548387096774 , 1886792452830
- 5 (1) 3 07666666 , 9 24524524 , 20303030
(2) 888888 , 878787 , 876876
(3) 4141414141414 , 0352352352352 , 6 1013101310131
(4) 550777777777777 , 046363636363636 ,
1 413413413413413 , 706570657065706
(5) 78548548 , 59595959 , 14 57777777 , 00454545
(6) 970121212121212 , 4 403333333333333 ,
10 8492137137137137 , 2186518651865186

Ex CVIII (p 272)

- 1 (1) $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$
(2) $\frac{1}{10}$, $\frac{1}{11}$, $\frac{1}{12}$, $\frac{1}{13}$, $\frac{1}{14}$, $\frac{1}{15}$, $\frac{1}{16}$, $\frac{1}{17}$, $\frac{1}{18}$
(3) $\frac{1}{19}$, $\frac{1}{20}$, $\frac{1}{21}$, $\frac{1}{22}$, $\frac{1}{23}$, $\frac{1}{24}$, $\frac{1}{25}$, $\frac{1}{26}$
(4) $\frac{1}{27}$, $\frac{1}{28}$, $\frac{1}{29}$, $\frac{1}{30}$, $\frac{1}{31}$, $\frac{1}{32}$
(5) $\frac{1}{33}$, $\frac{1}{34}$, $\frac{1}{35}$, $\frac{1}{36}$, $\frac{1}{37}$, $\frac{1}{38}$, $\frac{1}{39}$, $\frac{1}{40}$
- 2 1 , 4 37 , 458 , 26 , 15 9 , 3 79 , 60 , 01
- 3 10 and 21 , 233333

Ex CIX (p 278)

- 1 (1) 4 94332695 , 8502588315 , 2 738437
(2) 45 7678297 , 115 18275 (3) 44 5012779837
(4) 41 2553124691 (5) 40 1413813787
- 2 (1) 13 3848250 , 16 9602603 (2) 611 4024344
(3) 275 0304909 (4) 47 4114553

Ex. CX. (p. 274)

- 1 (1) 4 7517 (2) 07220 (3) 11872508. (4) 4 350896
(5) 216 29543 (6) 8 6198 (7) 3 661923286
(8) 7131493505. (9) 6 114237

- 2 (1) 032712 (2) 58 628447 (3) 33 022222
 (4) 133 874328 (5) 44 494309 (6) 1 817687
 3 95436 , 9210 , 74 3579629 4. (1) 4 98381933
 (2) 157 5779684 (3) 186 8538187 (4) 23 30798.

Ex CXI (p 275)

- 1 (1) 3 3878 , 4 1407 , 13 927 , 6 382798
 (2) 38 0375 , 2715 1213693 31 8236520733
 (3) 284 , 0862 , 51014730 , 36 1753
 (4) 13 36 , 731 164390 , 71 8803178235
 2 (1) 1 185 , 67 45 580 90 , 120
 (2) 3 63 , 63 , 46178 68,621 (3) 5694 , 2 , 14 7 , 323863
 (4) 10284 , 5 , 69 95100308641975

Ex CXII. (p 276)

- 1 (1) 06 , 047619 , 7 417 , 824157490 , 0026604 , 0153846
 (2) 432244318 , 5 376548977 , 4 818163910389
 (3) 490578703 , 10 881077 , 6 269360 , 1 527
 (4) 00200212 , 4783 , 280 58,613016947250280
 2 (1) 1 42065 , 3 27 , 8 18 , 47 3 , 2 527
 (2) 28 65789 , 24 2 0714285 , 4459
 (3) 68921875 , 02674540 , 5 , 92 5
 (4) 21 6 7047619 8 18746

Ex CXIII (p 277)

- | | | | | | | | | | |
|----|-------|----|------------|----|----------|----|------------|----|----------|
| 1 | 7 137 | 9 | 1 6875 | 3 | 1 851 | 4 | 10 | 5 | 13 13493 |
| 6 | 2715 | 7 | 1 2825 | 8 | 283 5 | 9 | 1461 91304 | | |
| 10 | 8863 | 11 | 0245700 | 12 | 11'11 | 13 | 007 | | |
| 14 | 20 | 15 | 25 | 16 | 5 62682 | 17 | 3125 | 18 | 84575 |
| 19 | 7 | 20 | 1 22974537 | 21 | 1 216803 | 22 | 14 765625 | | |
| 23 | 5 | 24 | 11344 6 | 25 | 3 783 | 26 | 125 | | |
| 27 | 4 | 28 | 41 6 | 29 | 26 415 | 30 | 55 1625 | | |

Ex CXIV (p 279)

- 1 57d 9d , 851 472d , 1 5d , 67 5d
 2 1 59 , 8599 , 1 129 , 4529 , 3689
 3 961 2p , 530 96256p 744 6p , 585 406p
 4 62 26ps , 26 2ps , 43 6ps , 40 6ps , 21 7945ps
 5 1408 3328 oz , 7655 2 yds , 47491 92 sec , 219321 sec

- 6 3570 grs , 6840 min
 7 17847 016 sq yds , 38 pt , 1455 376 ch , 4356 yds
 8 £ 035 , £ 1675 , £ 0265625 , £ 00042375
 9 0004396 da , 086945 ac , 1260 po
 10 021309375 ton , 9114583 oz , 18½ ton
 11 5952380 gu , 1190476 half-gui
 12 29965875 mi , 7 158857142 oz Av , 00055803571428 cwt

Ex CXV (pp 280 281)

- 1 (1) Rs 5 1a 168p , Rs 80 1a 24p , 32p. , Rs 227 11a 763p
 (2) 9s , 10s 1½d , £14 4s 3d. , 11s 4½d , 11 616d ,
 15s 6½d nearly
 (3) 6s 9½d , 7½d 52q , 9s 8d nearly , £5 12s. 3d , 8s 9d
 (4) 7s 10½d , £1 1s 6¼d 2q , 15s 3¼d , 5½d.
 (5) 1qr 14lbs , 2qrs. 3na , 13ac 1ro 14po , 15hrs 43min 12sec
 (6) Rs 5 12a 94p , 17 cwt 1 qr 20 lbs 8 oz 8⅞ drs ,
 2 days 12 hrs 55 min. 21 sec , £47 5s 7½d
 (7) 2½ in , 12 lbs , 15 po
 (8) 4 ch , 1s 3¼d , 4 ac 3 ro 16 po
 2. (1) 4375s , 0625s , 8 9375s , 1 27083s , 34 875s
 (2) £ 628125 , £ 790625 , £ 853125 , £ 0072916 , £ 2 790625
 (3) 46875 , 515625 , 84765625 , 53 85416
 (4) 9027 , 09975 (5) 572916 , 1'43625 , 61
 (6) 628125 , 00628125 , 628 125 (7) 882899305 , 7265
 (8) 455921875 , 4 34375 , 89533203125
 (9) 538461 , 7882571428 , 089285714
 (10) 00625 , 4125 , 00390625
 (11) 005563 , 30016741 , 021614583
 (12) 807244318 , 9471590 , 509837962

Ex CXVI. (pp. 282 283)

- 1 (1) Rs 3. 1a 4p , Rs 15 13a 4p. , Rs 38 12a 6p
 (2) 6s 2d , 4s 10½d 68q , 11s 5½d , £4 13s 9d
 (3) 1½d , 10s 1½d 3r 6d
 (4) Rs 246 4a 199999488p , 12a 6p , 4 bi 8 kat 2 ch
 (5) 2 sr 4 ch , 19 mds 30 sr 648ch , 1 fur 17 po 3 yds 10 8 in.
 (6) 9s 10 16d ; 3 tons 11 cwt 2 qrs. , 61 mi 377 yds 2 ft 6 96 in
 (7) £1 9s 7½d , 1d 8594q , 7s 7½d

- (8) £9 4s 0½d 65625g, £1640
 (9) £23 10s 10d, £19 2s 9½d
 (10) Rs 1175 1a 945p, Rs 51612 13a 7 56p., Rs 17 11a 4p
 (11) 3 sq ft 67 5 sq in, 2 tons 17 cwt 1 qr 27 lbs 7 oz 4 drs
 (12) 25 days 21 hrs 3 0028 min, 12 lbs 2 oz 1 152 drs.
- 2 (1) Rs 1 5a 4p, Rs 6 5a, Rs 1 2a 9 3p
 (2) 7s 6d, £1 9s 1 478d, Rs 31 10r 8p
 (3) £1 14s 4d 8296g, Rs 67 8a, 2 448p
 (4) 1 cwt 2 qrs 6 lbs 6 oz, 52 mi 2 fur 25 po 3 yds
 (5) 20 lbs, 15s 4d, £1 14s 7½d, 24 bi 6½k, 16 qrs 4 bus
 (6) 7a 2 4p, 13a 5 616p, Rs 189 15a 4p, Rs 2 7a 1p
- 3 £4 13s 8½d ½g, Rs 12 4 1 oz 17 dwts 22 grs
- 5 (1) Rs 22 14a 6 4p (2) Rs 6 1a 2p (3) 10s 11d
 (4) 6s 6d, 16s 11½d (5) Rs 21 12a 1 8p (6) 5 mds 9sr 4ch
 (7) 19s 1½d (8) £1 18s 7d (9) £15 14s 10½d
 (10) 1 ton 17 cwt. 2 qrs 4lbs

Ex CXVII (pp 284 285)

- 1 (1) 0525 (2) 4729 (3) 675 (4) 0572916 (5) 375
 (6) 875 (7) 44472 (8) 1'32531 (9) 015873
 (10) 4878 (11) 875 (12) 0714285 (13) 14 40
 (14) 10885416 (15) 01037 (16) 0625 (17) 95
 (18) 34375 (19) 71 703 (20) 04(875. (21) 36
 (22) 003 (23) 1 86 (24) 7 2 8293 3 59 52
4. 6875 5 6305 6 171296 7. 0816 8 2083
- 9 3 28 10 45. 11 0625 12. 5

Ex CXVIII (pp 289 291)

- 1 (1) 3010300 (2) 11704 013860 (3) 5 74 (4) 392754.
 (5) 46287 739 (6) 95424 (7) 985 128337 (8) 1 1038
 (9) 229 92815 (10) 60911 2 (1) 14 26 (2) 1495
 (3) 2115 1921 (4) 285119 0879907 (5) 30 799 (6) 13050
 (7) 318310 (8) 7157788 (9) 7 037 (10) 26544
- 3 (1) 25 (2) 16 (3) 2 (4) 1'410'861.
 (5) 202733 (6) 009706 (7) 3 141593 (8) 46
 (9) 489898 (10) 583 4 (1) 324048, rem 6 ; 8340153, rem 5
 (2) 521695, rem 96, 72773, rem 34
 (3) 7211, rem 772 581, rem 425
 (4) 59, rem 1667, 739, rem 2423
 (5) 23, rem 6939, 7205, rem 40081

Miscellaneous Examples IV. (pp. 292-295)

- 1 961 6832321 2 0002938 3 360 4 The latter
 5 3 6 02525 7 23957 8 Rs 7206 6a 4 8p
 9 Rs 285120 10 44 11 861 12 48 13 5
 14 40 15 £6 1s 6d 16 A Rs 360, B Rs 270, C Rs 240
 17 54p 18 Rs 2571 15a 19 168 20 5 077881
 21 A's property exceeds B's by Rs 49000 22 2530 6
 23 075 24 225625 25 00256256, 256 256, 0256256, 63
 26 1 414 27 03 28 00297 29 Rs 17 1a 8p
 30 6p 31 $\frac{1}{16}$ 32 A 28 $\frac{3}{4}$ days, B 35 $\frac{1}{8}$ days,
 C 38 $\frac{1}{16}$ days 33 32 34 13 $\frac{3}{8}$ = 13 48 $\frac{3}{8}$
 35 Rs 88200 36 200 gals 37 120 runs 38 (1) $\frac{1}{8}$ (2) 708
 (3) 4 4 39 126 sec 40 6 75 41 20 hrs 30 min
 42 69 e,gs 43 Rs 247 8a 44 35584345, rem 1s 0035 in
 45 022916 46 Rs 2428 12a, Rs 1238 10a 7 2p,
 Rs 1190 1a 4 8p 47 A's inc = $\frac{1}{4}$ of B's inc, Rs 4200
 48 Rs 15 49 Rs 137 8a 50 £419 19s 3d

Ex. CXIX. (p 299)

- 1 Rs 541 11a 6p 2 Rs 542 4a 10p 3 Rs 965 5a 4p
 4 Rs 427 5 Rs 1975 6 £54 4s 7 $\frac{1}{2}$ d 7 £144 10s 7 $\frac{1}{2}$ d
 8 £138 14s 4d 9 £5429 0s 4 $\frac{1}{2}$ d 10 Rs 6839 9a 4p
 11 Rs 3750 12 Rs 12241, 7a 3 $\frac{1}{2}$ p 13 Rs 9065 4a
 14 £6893 19s 8 $\frac{1}{2}$ d 15 £1156 0s 7 $\frac{1}{2}$ d 16 £730 10s 10 $\frac{1}{2}$ d
 17 £7620 18s 0 $\frac{1}{2}$ d 18 £13903 10s 6d 19 £2357 16s 10d
 20 Rs 109320 7a 6 $\frac{1}{2}$ p 21 Rs 1689 3a 7 $\frac{3}{8}$ p 22 £35978 18s 1 $\frac{1}{8}$ d
 23 £20504 0s 1 $\frac{1}{2}$ d 24 Rs 315597 3a 2p 25 Rs 16532 8a 8 $\frac{1}{2}$ p
 26 Rs 202 13a 10p 27 Rs 3379 5a 2p 28 Rs 50324 3a. 6p
 29 £5392 1s 9 $\frac{1}{2}$ d $\frac{1}{4}$ q 30 £1873 18s 3 $\frac{1}{2}$ d $\frac{1}{8}$ q
 31 £625 19s 0 $\frac{1}{2}$ d $\frac{1}{4}$ q 32 Rs 12506
 33 £2683 0s 9 $\frac{1}{2}$ d $\frac{1}{4}$ q 34 Rs 7 2a 10 6p
 35 £491 7s 11 $\frac{1}{2}$ d $\frac{1}{4}$ q 36 £470 2s 0 $\frac{1}{8}$ d
 37 Rs 102781 6a 9 $\frac{1}{8}$ p 38 Rs 593 11a 1 $\frac{1}{8}$ p
 39 Rs 5708 1a 7 $\frac{1}{8}$ p 40 Rs 4706 10a 9p
 41 Rs 17739 4a 1 6p 42 £831 3s 2 $\frac{1}{2}$ d $\frac{1}{4}$ q
 43 £12126 7s 11 $\frac{1}{2}$ d $\frac{3}{4}$ q 44 Rs 208336 5a
 45 Rs 124479 2a 8p 46 £24617 0s 1 $\frac{1}{2}$ d $\frac{1}{8}$ q

Ex. CXX. (pp 301 302)

- 1 Rs 198 2a 1 $\frac{1}{2}$ p 2 Rs 55 3a 10p 3 Rs 506 4a 0 $\frac{1}{2}$ p
 4 Rs 31619 1a 6 4p 5 Rs 9404 3s 0 10625p 6 £18 14s 11d
 7 £316 17s 3 $\frac{1}{2}$ d 8 £13 4s 4 $\frac{1}{8}$ d 9 £140 18s 5 $\frac{1}{8}$ d
 10 £89 6s 1 $\frac{1}{8}$ d 11 £1052 13s 11 $\frac{1}{2}$ d $\frac{1}{4}$ q
 12 £496 1s 9 $\frac{3}{8}$ d 13 Rs 221 0a 7 $\frac{1}{2}$ p
 14 £2 3s 0 $\frac{1}{2}$ d $\frac{3}{8}$ q 15 Rs 71369 12a 6 $\frac{3}{8}$ p

16 Rs 61 14a 0 $\frac{1}{2}$ p
 18 Rs 1058 11a 6q
 20 £2495 4s 3 $\frac{1}{2}$ d $\frac{1}{2}$ q.
 22 £46 4s 10 $\frac{1}{2}$ d
 24 £2395 7s 7 $\frac{1}{2}$ d $\frac{1}{2}$ q
 26 £8095 4s 2 $\frac{1}{2}$ d
 28 Rs 568 5a 10 $\frac{1}{2}$ p
 30 £897 6s 6d
 32 Rs 430 10a
 34 £44 10s 5 $\frac{1}{2}$ d $\frac{1}{2}$ q

17. Rs 1014 5a 075p
 19 Rs 13881 5a 2 $\frac{1}{2}$ p
 21 £105 4s 7 $\frac{1}{2}$ d $\frac{1}{2}$ q
 23 £275 2s 0d $\frac{1}{2}$ q
 25 £23264 13s 3 $\frac{1}{2}$ d
 27 £16 18s 609d
 29 Rs 1548 1a 3765p
 31 Rs 185 9a 0 $\frac{1}{2}$ p
 33 £33 8s 3 $\frac{1}{2}$ d $\frac{1}{2}$ q
 35 Rs 23 11a 10 $\frac{1}{2}$ p

Ex CXXI (pp. 303 304)

1 Rs 2277 13a 6p	2 Rs 12973 6a 6p	3 £44 3s 0 $\frac{1}{8}$ d
4 £11870 3s 4 $\frac{1}{2}$ d 97q	5 £836 16s 4 $\frac{1}{2}$ d	6 Rs 7 11a 5 $\frac{1}{2}$ p
7 Rs 26882 0a 6p	8 £1144 0s 11 $\frac{3}{8}$ d	9 £117 0s 3d
10 53703lbs 18 dwts 6 grs	11 182 miles 3 fur 143 $\frac{1}{2}$ fyds	
12 Rs 448 7a	13 181 mds 34 sr 4 ch	
14 Rs 11237 9a 4p	15 Rs 1 07 1a	
16 Rs 6089 9a 5 $\frac{1}{2}$ p	17 Rs 12659 6a 81p	
18 £428 9s 1 $\frac{1}{8}$ d	19 Rs 470	20 23, lb, Tr 4 oz 19 $\frac{1}{16}$ grs

Ex CXXII (p. 306.)

1 Rs 23 0a 6p	2 Rs 9 4a 6 $\frac{1}{2}$ p	3 Rs 22 10a 2p
4 £8 19s 11 $\frac{1}{2}$ d	5 Rs 293 4a 4p	6 Rs 197 6a 5 $\frac{1}{2}$ p
7 Rs 850 5a 8p	8 Rs 274 7a 8p	9 Rs 40 0a 11p
10 Rs 41 7a 4p		

Ex CXXIII (p 309)

1 (1) 29791	(2) 168210432	(3) 855625	(4) 31255875
(5) 38385223	(6) 218401	(7) 100875809	(8) 14885593
(9) 124618464	(10) 3559996	(11) 53841087	
2 (1) 12321	(2) 388129, 38812900		
3 (1) 7, 14, 17, 19, 18, 16, 11, 20, 12			
(2) 25, 23, 30, 36, 132, 252, 315			
(3) 825, 6930, 3528, 1575, 7600			
4 7, 77, 91, 385, 6, 14, 3157, 30030			5 900

Ex. CXXIV (p 312)

1 (1) 26, 38, 127, 145, 517, 999, 267
(2) 45, 812, 234, 907, 9878, 3163
(3) 5746, 6772, 2403, 7925, 2005
(4) 80700, 7008, 309000, 62573
(5) 90888, 9688669, 887145333
2 (1) 47, 58, 329, 478, 1369, 359
(2) 08, 073, 0231, 095, 6049

(3) 34 12, 370 09, 490 07, 15 367
 (4) 203 975, 0708069; 0007008

3 19 4 144 5 210 6 124

Ex CXXV (pp 313-314)

- 1 (1) 1 4142, 1 7320, 2 2360, 2 4494, 2 6457, 2 8284, 3 4641;
 3 6055, 4 2426, 4 4721, 5 6568, 6 1644
 (2) 6 6332, 7 1414, 8 4852, 8 9442, 9 7467, 25 2586, 28 2134,
 28 3019, 31 6227
 (3) 75 5843, 19 0525, 187 4033, 94 0053, 906 9983,
 8513 0157, 86 2090
- 2 (1) 3162, 4472, 5477, 6324, 7071, 7745, 8366, 8944,
 9486, 1 0954, 1 2649
 (2) 2236, 2 2583, 2 2135, 4, 1264, 1, 7141, 2258, 2 0074.
 (3) 1417, 4743, 1449, 5773, 2 1343, 5 9033, 17 9368, 4 3
 (4) 1 7724, 13 2382, 3 5449, 5 4233, 88 8516
 (5) 1 5367, 2 2390, 2 0074, 7 0003, 2659, 7 5878, 1410
- 3 (1) 0415692193, 3 1288975694, 2613426869, 5888123640,
 6 6546224536, 1 2529964086, 8 6802649729
 (2) 9219544457, 2645751311, 1 7320508075, 9 8944428848,
 0059430631, 5 2700094876, 1910497317

Ex. CXXVI. (p 315)

- 1 (1) $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$
 (2) $2\frac{1}{2}$, $3\frac{1}{2}$, $18\frac{1}{2}$, $5\frac{1}{2}$, $5\frac{1}{2}$, $6\frac{1}{2}$, $23\frac{1}{2}$
 (3) $55\frac{1}{2}$, $268\frac{1}{2}$, $175\frac{1}{2}$, $122\frac{1}{2}$, $393\frac{1}{2}$
- 2 (1) 7905, 3 0822, 2 5298, 32, 2625, 8 7619, 20 4939, 0029
 (2) 7745, 4 6612, 2 0586, 9607, 9701, 4 6150, 3717, 5 2164
 (3) 6060, 5678, 3118, 2373, 16 9595, 0574, 19 1646, 27 5111
- 3 (1) 3, 16, 4 $\frac{1}{2}$, 1 83, 5 3, 016, 68 83
 (2) 036, 26, 7 $\frac{1}{2}$, 06, 2 $\frac{1}{2}$, 13, 2

Ex CXXVII. (pp. 317 318.)

- 1 (1) 11, 25, 36, 13, 57, 49 (2) 89, 97 321, 247, 473
 (3) 956, 5836, 8888 (4) 2 6, 5 1, 7 9, 40 1, 2 65
 (5) 197, 957, 101, 0299 (6) $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$
 (7) 10 1120, 1 8081, 1 7099, 4286, 1 5135
 (8) 7539, 8505, 8939, 1 9813, 2 6487, 2 6536
 (9) 12599210, 14422495, 23513346, 28844991, 1 29802461
- 2 (1) 53, 425, 5 $\frac{1}{2}$, 425
 (2) 15 6, 3, 11 6, 9283; 6463, 6876, 4 3411
- 3 37 ft
 4 6 16; 1 232

Ex CXXVIII (p. 319)

- 1 18, 35, 45 6 2 12, 17, 51 3 11, 347032, 49374,
1 65970 4 4, 8, 12 5 2½, 21, 799

Miscellaneous Examples V (pp 319-322)

- 1 53² 11² 7² 13², 5 2 8p 3 100000 4 15700
5 Rs 603 13a 9p 6 3759 7 The second, 07083
8 2 9 Rs 51738 3a 3p 10. £6 16s 5d 11 1
13 Wife Rs 14508, eldest son Rs 12090, youngest Rs 12896,
daughter Rs 9672 14 £213 12s
15 63 ac, A 90 days, B 63 days, C 70 days
16 (1) 6 6140 (2) 2752057 17 2333283½ francs
18 1055 19 Rs 5888 20 35 measures
21 (1) 5 99027 (2) 0 (3) 77 22 Rs 4 23 Rs 5 11a
24 6½, 0505 25 105570 and 950370 26 1 5795
27 3a 624p 28 80 29 1512497 30 2 571428
31. £2 15s 2½d 32 The first is greatest; second is least
33 128½ years 34 1½ 35 645, 151, 567
36 00307692 37 1 qr 13 lbs to be subtracted
38 9975 and 9925, 10000 and 10015 39 7575½ hrs 40 1
41 Rs 42 14a 0s 10p 42 1½, 7½, 8½ 43 420 rix-dollars
44 11½ = 11 8208 45 8 175, 816, 27, 75, 135 1940625
46 300 miles 47 Rs 258 9a 7½p, Rs 86 12a 8p
48. £55 6s 10½d 49 0030422 50 5½ 51 1½
52 1½d, 160 oz 53 1 000049, 266 6 54 67 ft, 125

Ex CXXIX. (pp. 326-329)

- 1 (1) 47 sq yds 2 ft (2) 52 sq yds (3) 25 sq yds 8 ft 48 in
(4) 35 sq yds 7 ft 32 in (5) 11 sq yds 3 ft 129 in
(6) 42 sq yds 1 ft 50 in (7) 24 sq yds 4 ft 54 in
(8) 36 sq yds 2 ft 115½ in
2 (1) 683 sq yds 2 ft 25 in (2) 10 sq yds 5 ft 90 in
(3) 43 sq yds 5 ft 128 in (4) 16 sq yds 6 ft 27 in
3 (1) 2 yds 1 ft 5 in (2) 11 ft 11 in (3) 2 ft 9 in
(4) 13 ft 1 in (5) 7 yds 2 ft
4 55 sq yds 7 ft 73 in 5 96 sq ft 93 in
6 7 ft 10 in, 15 yds 1 ft 8 in, 15 ft 5 in 7 2 ft 6¾ in
8. 2 ac 4 po 9 5760 sods 10 84 11 29400
12 72½ 13 1707 14 142 sq yds 90 in 15 759
16 85 sq ft 56 in 17 3 chs 50 lks off the length 18 16½ yds
19 131 ft 20 6160 acres 21. (1) Rs 7468 12a
(2) Rs 678 5a 4½p (3) Rs 68 10a 8½p (4) £451 14s 4½d
22 (1) 52½ yds (2) 142 yds 1 ft 11½ in (3) 26 yds 4 in
23 (1) Rs 114 13a 4p (2) Rs 228 9a 6p (3) Rs 130
(4) £21 24 48 ft 25 14 ft 3 in 26 20 ft

27	<i>Rs</i> 2 12 <i>a</i>	28	2 ft	29	<i>Rs</i> 135	30	15 ft 6 in
31	1406½ sq ft	32	195 sq yds	33	14 yds 1 ft	34	1310
35	41 yds, 20½ yds			36	<i>Rs</i> 1050		

Ex CXXX (pp. 331-333)

1	(1) 875 sq ft	126 sq in	(2) 605½ sq ft	(3) 798 sq ft	
	(4) 79 sq ft	132 sq in			
2	(1) 250 yds	(2) 248 yds	211 ft	(3) 648 ft	(4) 115 yds
3	(1) Rs 28 6a			(2) £13 19s	10½d
	(3) Rs 1 9a			(4) £3 10s	
4	(1) Rs 76 2a 4p	(2) Rs 61 15a 8p	(3) £9 9s 818d		
5	Rs 47 1a 4p	6 Rs 294 14a 2108p	7 £8 15s		
8	£5 10s	9 113400	10 Rs 12 6a 4p		
11	37 sq yds 5 ft	12 8 ft 10½ in	13 11 ft 6½ in		
14	10 ft	15 16 ft	16 Height 10 ft		
	breadth 12 ft	17 Rs 37 8a	18 77824		
19	13 ft 6 in	20 12 ft 6 in	21 Rs 125		
22	20½ ft, Rs 42 11a 4p		23 15 ft		

Ex CXXXI (pp 335-336)

1	6 ac 3 ro 30 po	22½ sq yds	2	1913 sq yds	3 ft
3	<i>Rs</i> 125 8 <i>a</i> 4 <i>p</i>	4 60750	5	<i>Rs</i> 312 12 <i>a</i>	6 <i>Rs</i> 10464
7	300 stone	8 £117 15 <i>s</i>	9	£176 17 <i>s</i> 9½ <i>d</i>	
10	352 or 360	11 34½ sq ft	12	<i>Rs</i> 1965	
13	55 sq ft 80 in	, <i>Rs</i> 9 4 <i>a</i> 1½ <i>p</i>	14	8 <i>a</i>	15 <i>Rs</i> 274 4 <i>a</i> 5½ <i>p</i>
16	378 11 ft sq cub	, 75 mds 25 sr 8 ch	, <i>Rs</i> 378	3 <i>a</i>	

Ex CXXXII (pp 339-342)

1	(1) 4 cub yds	8 ft 648 in	(2) 38 cub yds	22 ft 592 in
	(3) 59 cub yds	19 ft 764 in	(4) 260 cub yds	4 ft 1053 in
	(5) 69 cub yds	16 ft 1026 in		
3	<i>Rs</i> 246 15 <i>a</i> 3 <i>p</i>	4 2 yds	5 1½ in	
6	123 ft 11½ in	7 1 ft 6 in	8 5044	9 2 ft 6 in
10	11 ft 6 in	11 507 ft	12 29 cub yds	17 ft
13	1 ton 3 cwt 1 qr 0½ lb		14 2304	
15	£4 1 <i>s</i> 10½ <i>d</i> ½ <i>q</i>	16 28 tons 9 cwt 1 qr 11 lbs	14½ oz	
17	3 ft 6 in	18 16 sr 11½ ch		
19	115 cub ft 30 in	20 1950	21 101½ tons	
22	32 ft 23 114	24 4 3½ yds	25 <i>Rs</i> 276 5 <i>a</i> 3 <i>p</i>	, 31440
26	34 yds <i>Rs</i> 3984 6 <i>a</i>		27 Depth 1½ ft, breadth 3 ft	
	length 6 ft, <i>Rs</i> 55 2 <i>a</i>		28 <i>Rs</i> 2 6 <i>a</i> 11½ <i>p</i>	
29	2 tons 1 cwt 2 qrs	14½ lbs	30 <i>Rs</i> 389 9 <i>a</i> 4 <i>p</i>	
31	2 ft 3 in	32 0000459 in	33 ½ mile per min	
34	1 11 miles	35 22 min 24 sec	36 <i>Rs</i> 41 2 <i>a</i> 4 <i>p</i>	
37	30 cub ft 1404 in	, 33 mds 28 sr 8 ch	38 135 times.	
39	11 mds 26½ sr	, <i>Rs</i> 64 2 <i>a</i> 8 <i>p</i>	40 277½ cub in	
41	4 sq in	42 8 yds, 3 hrs		

Ex CXXXIII (p 344)

- 1 (1) 5 yds 2 ft $3\frac{1}{2}$ in , 8 yds 1 ft $11\frac{5}{8}$ in , 48 yds 2 ft $6\frac{1}{2}$ in
 (2) 2 sq yds $138\frac{2}{3}$ in , 24 sq yds $72\frac{2}{3}$ in , 30 sq yds 4 ft $58\frac{1}{8}$ in
 (3) 20 cub ft 1377 in , 11 cub ft 675 in , 3 cub yds 2 ft $934\frac{1}{2}$ in
 (4) 8 yds 1 ft $3\frac{1}{2}$ in , 25 sq yds 6 ft 135 in 86 cub yds 19 ft $800\frac{2}{3}$ in
- 2 (1) 13 ft 5' 3 , 59 ft 6' 8" , 22 ft 6' 8" 29 ft 4' 9"
 (2) 50 sq ft 5' 8" , 47 sq ft 5' 3" , 300 sq ft 2' 4" 3"
 (3) 4 cub ft 7' 6" 8" , 42 cub ft 2' 3" 10" , 39 cub ft 7' 9" 4" 6" ,
 18 cub ft 11' 6' 8"

Ex CXXXIV (pp 345-346)

- 1 (1) 182 sq ft 66 in (2) 200 sq ft 58 in (3) 212 sq ft 7 in
 (4) 27 sq ft 1' 8" 8" (5) 51 sq ft 10' 4" 0" 4
 (6) 336 sq ft 9' 6" 8' 8" (7) 154 sq yds 6 ft 7' 6" 4" 6"
 (8) 2203 sq yds 2 ft 8' 2" 2' 9" (9) 216 sq ft 6' 0" 10" 6"
 (10) 147 sq yds 6 ft 2' 11" 6' 10" (11) 4' 3" 9" 9" 8"
 (12) 2 sq ft 4" 7
- 2 (1) 170 cub ft (2) 59 cub ft 2' 8' 4" (3) 167 cub ft 6'
 (4) 180 cub ft 9' 2' (5) 132 cub yds 4 ft 10' 4" 9" 6"
- 3 (1) 5 sq yds 8 ft $82\frac{2}{3}$ in 2' 1696 sq yds 6 ft $38\frac{1}{2}$ in
- 4 (1) 42 cub ft 1512 in (2) 56 cub yds 22 ft $1196\frac{1}{2}$ in
 (3) 44 cub yds 16 ft $298\frac{1}{2}$ in 5 18 ft 9 in

Ex CXXXV (pp 346-347)

- 1 (1) 6 bi (2) 4 bi (3) 1 bi 5 kat 9 ch 12 ga
 (4) 32 bi 17 kat 6 ch 8 ga (5) 36 bi 6 kat 1 ch 12 ga
 (6) 136 bi 14 kat 12 ch $16\frac{1}{4}$ ga (7) 71 bi 18 kat 12 ch 16 ga
 (8) 55 bi 2 kat 6 ch 8 ga (9) 136 bi 10 kat 15 ch 2 $\frac{1}{2}$ ga
 (10) 109 bi 6 kat 10 ch $19\frac{3}{4}$ ga (11) 15 bi 8 kat 12 ch
 (12) 28 bi 1 kat 1 ch 12 ga
- 2 (1) 8064 c cubits (2) 8736 c cubits (3) 163680 c cubits
 (4) 26970 c cubits (5) 2592 c yds (6) 2900 c yds

Ex. CXXXVI (pp 349-350)

- 1 Rs 7 8a 2 150 3 11268 4 Rs 182 0a 8p 5 660
 6 75 7 15 cwt 8 4 days 9 30 10 Rs 491 4a
 11 30 12 100 13 Rs 1 8a 14 3 lbs 11 oz 7 $\frac{1}{2}$ drs
 15 Rs 505 16 9 months 17 6 18 2 days

Ex CXXXVII. (pp. 351-352)

- 1 Horse Rs 230 , cow Rs 160 2 Tea Rs 1 5a 4p , coffee 12a
 3 Turkey 15s , fowl 3s 4d 4 $1\frac{1}{2}$ days 5 $33\frac{1}{2}$ days
 6 3 days 7 2 boys 8 Man 7 $\frac{1}{2}$ hrs , boy 18 hrs , together $5\frac{1}{2}$ hrs
 9 35 days 10 Horse Rs 24 , cow Rs 12
 11 Man 5a 3p , woman 3a 5p 12 12

Ex CXXXVIII (p. 353)

1	1263	2	839	3	903	4	8398	5	1078
6	119	7	892430	8	5390	9	1662	10	5
11	9999988	12	10004390						

Ex CXXXIX (pp 354 355)

1	39 yds	2	25 yds	3	$39\frac{1}{2}$ yds	4	4 (4 $\frac{1}{2}$)	5	$31\frac{2}{3}$ yds
6	$106\frac{2}{3}$ yds	7	9 points	8	4min 47sec	9	A 3 min 37 $\frac{1}{2}$ sec		
	B 3 min 48 sec		C 4 min	10	350 yds	11	10 points		
12	$5\frac{1}{2}$ min	13	C	14	5 points	15	C by $\frac{1}{2}$ yd	16	$5\frac{1}{4}$ min
17	C, 40	18	A 7 min 9 $\frac{1}{2}$ sec				B 9 min 15 $\frac{1}{4}$ sec		

Ex CXL (pp. 357 358)

[The numbers denote minutes past the hour]

1	(1) (i) $5\frac{5}{11}$	(ii) $21\frac{9}{11}$, $54\frac{1}{11}$	(iii) $38\frac{2}{11}$
	(2) (i) $10\frac{10}{11}$	(ii) $27\frac{7}{11}$, 3 o'clock	(iii) $43\frac{7}{11}$
	(3) (i) $16\frac{4}{11}$	(ii) 3 o'clock, $32\frac{1}{11}$	(iii) $49\frac{1}{11}$
	(4) (i) $21\frac{9}{11}$	(ii) $5\frac{5}{11}$, $38\frac{2}{11}$	(iii) $54\frac{1}{11}$
	(5) (i) $27\frac{7}{11}$	(ii) $10\frac{10}{11}$, $43\frac{7}{11}$	(iii) 6 o'clock
	(6) (i) $32\frac{1}{11}$	(ii) $16\frac{4}{11}$, $49\frac{1}{11}$	(iii) 6 o'clock
	(7) (i) $38\frac{2}{11}$	(ii) $21\frac{9}{11}$, $54\frac{1}{11}$	(iii) $5\frac{5}{11}$
	(8) (i) $43\frac{7}{11}$	(ii) $27\frac{7}{11}$, 9 o'clock	(iii) $10\frac{10}{11}$
	(9) (i) $49\frac{1}{11}$	(ii) 9 o'clock, $32\frac{1}{11}$	(iii) $16\frac{4}{11}$
	(10) (i) $54\frac{1}{11}$	(ii) $5\frac{5}{11}$, $38\frac{2}{11}$	(iii) $21\frac{9}{11}$
	(11) (i) 12 o'clock	(ii) $10\frac{10}{11}$, $43\frac{7}{11}$	(iii) $27\frac{7}{11}$
	(12) (i) 12 o'clock	(ii) $16\frac{4}{11}$, $49\frac{1}{11}$	(iii) $32\frac{1}{11}$
2	(1) (i) $5\frac{5}{11}$, $27\frac{7}{11}$	(ii) $43\frac{7}{11}$, $54\frac{1}{11}$	(2) (i) $10\frac{10}{11}$, $32\frac{1}{11}$
	(ii) $49\frac{1}{11}$, 5 o'clock	(3) (i) $21\frac{9}{11}$, $43\frac{7}{11}$	(ii) $5\frac{5}{11}$, 7 o'clock
	(4) (i) $27\frac{7}{11}$, $49\frac{1}{11}$	(ii) 7 o'clock, $10\frac{10}{11}$	
3	3 times	4	3 times
7	$36\frac{1}{2}$ min past 4	8	$24\frac{1}{2}$ min past 6
10	$10\frac{1}{4}$ min gain	11	$\frac{1}{3}$ min div put back
12	(i) $4-38\frac{2}{3}$, or $4-37\frac{1}{3}$ P M	(ii) $4-54\frac{1}{4}$ P M	(iii) $4-20\frac{1}{2}$ P M
13	(i) $4-20\frac{2}{3}$ P M	(ii) $3-49\frac{2}{3}$ P M	(iii) $1-59\frac{1}{2}$ P M
14	9 hrs 18 min $22\frac{1}{2}$ sec P M	4 min $27\frac{1}{2}$ sec	

Ex. CXLI (pp 360 362)

1	887 $\frac{1}{2}$ days	2	10 hrs 40 min 36 $\frac{1}{8}$ sec	3	48 min. 7 $\frac{1}{2}$ sec
4	60 days, 12 hrs 14 min	5	11 hrs 44 min	6	51 $\frac{1}{2}$ min before 4
7	5 hrs 10 min 20 $\frac{2}{3}$ sec	8	11 o'clock P M	9	5 hrs 10 min
10	35 min 27 $\frac{1}{2}$ sec past noon	11	5 hrs 48 min	12	26 sec loss
13	4 hrs 32 min	14	18 $\frac{1}{2}$ min to 12	15	On Tuesday evening,
	when one clock marks 9 hrs 11 min and the other 8 hrs				
	54 min 30 sec	16	11 $\frac{1}{2}$ min	17	13 $\frac{2}{3}$ min
18	7 o'clock	19	1 hr 28 $\frac{1}{4}$ min		

- 19 At 3 P M on Dec 3
 21 $40\frac{2}{3}\frac{1}{4}$ min past 8 P M
 23 $8\frac{1}{4}$ sec
 26 1008 days
 28 3 hrs 24 min
 30 1 day 16 hrs from the time the second was put right
- 20 1 min $52\frac{1}{2}$ sec to 9 P M
 22 Loses $29\frac{1}{4}\frac{1}{4}$ sec
 25 At 11 P M
 27 12 days before, 528 days
 29 May 21, at 6 P M

Ex OXLII (p 363)

[The numbers denote degrees]

- 1 (i) (i) $4\frac{1}{2}$ (ii) $3\frac{1}{2}$ (2) (i) 20 (ii) 16 (3) (i) $47\frac{1}{2}$
 (ii) $38\frac{1}{2}$ (4) (i) $71\frac{1}{2}$ (ii) $56\frac{1}{2}$ (5) (i) $-33\frac{1}{2}$ (ii) $-26\frac{1}{2}$
 2 (i) (i) 68 (ii) 16 (2) (i) 113 (ii) 36 (3) (i) $136\frac{1}{2}$
 (ii) $46\frac{1}{2}$ (4) (i) $197\frac{1}{2}$ (ii) $73\frac{1}{2}$ (5) (i) 14 (ii) -8
 3 (i) (i) $110\frac{1}{2}$ (ii) $43\frac{1}{2}$ (2) (i) 167 (ii) 75 (3) (i) 203
 (ii) 95 (4) (i) $9\frac{1}{2}$ (ii) $-12\frac{1}{2}$ (5) (i) $-24\frac{1}{2}$ (ii) $-31\frac{1}{2}$

Ex CXLIII (pp 369-375)

- 1 (i) 45 min (ii) 4 hrs
 3 At 1 A M, 17 and 14 miles
 5 At 1 P M, 210 miles
 7 At 12-35 P M, $89\frac{1}{4}$ miles
 9 $3\frac{1}{2}$ miles per hour
 11 A 7 miles
 14 B 6 miles 352 yds
 16 1 mi, 40 yds, B 9 yds, 1 hr 15 min
 18 90 miles
 20 35 mi, 25 mi
 22 $60\frac{1}{2}$ seconds, 530 yds
 23 3 min
 25 19 mi 35 mi
 26 $4\frac{1}{2}$ mi
 28 $\frac{1}{10}$ mile
 29 $31\frac{1}{2}$ miles per hour
 31 $2\frac{1}{2}$ hours
 32 3 and $1\frac{1}{2}$ miles per hour
 34 300 35 600
 36 56 yds
 37 $28\frac{1}{2}$ miles
 38 746 $\frac{1}{2}$ yds
 39 $9\frac{3}{4}$ miles
 40 (i) $55\frac{1}{2}$ sec (ii) $7\frac{1}{4}$ sec
 41 15 hrs 35 min
 42 9 hrs 48 min
 43 $463\frac{1}{4}$ hrs
 44 16 min $32\frac{1}{4}$ sec
 45 9 sec
 46 $22\frac{1}{2}$ miles per hour
 47 $7\frac{1}{2}$ sec
 48 1 min. $7\frac{1}{2}$ sec
 49 50 and 30 miles per hour
 50 297 ft, 231 ft
 51 3 hrs 1 min 30 sec
 52 15 mi 180 yds, 14 yds and 1 yd per sec
 53 22 mi 280 yds, 20 ft and 60 ft per sec
 54 $17\frac{1}{2}$ miles
 55 $4\frac{1}{2}$ miles per hour
 56 15 miles
 57 2 miles
 58 165 miles
 59 (i) 1 sec (ii) 4 sec
 60 $13\frac{1}{2}$ miles
 61 $2\frac{1}{2}$ ft
 62 2740 yds, $13\frac{1}{2}$ miles
 63 25 and 30 miles per hour
 64 $79\frac{1}{2}$ miles

Miscellaneous Examples VI (pp 378-385)

- 1 66150140
 2 Rs 628 oa. $10\frac{1}{4}$ p
 3 $87\frac{1}{2}$ yds
 4 £365
 5 Rs 182 4a
 6 25 sec

- 7, 8100, Rs 50 12a 9½¢ 8 ४०४४ in
 9 £286492500 10 Rs 2164 6a 2½¢ 11 121 98
 12 7½¢ ft 13 396 or 406 14 64800
 15 18 16 875 17 He walked up 6½ miles
 18 9 hrs 37½ min 19 1109894 20 1009805
 21 Re 1 2a 22 45 miles 23 Rs 3 24 31116
 25 3 times or 4 time if we reckon the starting point
 26 75 days 27 240 men 28 942 29 1683 and 2431
 30 60 pieces, 5 6304 in 31 Rs 18 10a 8¢ 32 Rs 420
 33 1½ cwt 34 Rs 963 35 3 hrs 30 min
 36 1 mile 37 000004 in 38 2s 4d per stone
 39 36 and 24 miles per hour 40 60
 41 20 miles per hour 42 A Rs 12, B Rs 9
 43 3½ min to 6 A M, 6 o'clock A M 44 171
 45 25' 26" 6 to be subtracted 46 9½ seers
 47 9½ days 48 53 min 30.33 sec past 1
 49 9a 3½¢ 50 567 51 792 yd- or 1572 yds
 52 A 12 days, B 15 days, C 20 days 53 27½ yds
 54 6 min 55 2½ to his 57 At 4-25 P M 58 2½ miles
 59 80 and 75 60 38, 40 and 45 61 Rs 458
 62 A Rs 2400, B Rs 900, C Rs 240, D Rs 60 63 45 and 35
 64 23 years 65 14 beggars 66 44 day-
 67 45 men 68 5 yards 69 2½ miles
 70 564 miles 71 It will be more advantageous to
 employ the boys 72 Rs 4 6a 73 4½ miles per hour
 74 It loses 1½ min per day 75 30 days
 76 After 8 days 77 At 23½ min past 8, 7½ miles
 78 48 centres, 31 outers 79 611½ hrs 80 10 ft, Rs 296
 81 Rs 446 10a, 1¢ ft 82 124 yds

Ex CXLIV (pp 387-388)

- 1 (1) $\frac{3}{8}, \frac{1}{2}, \frac{3}{4}, \frac{1}{4}, \frac{1}{2}, \frac{5}{8}$ (2) $\frac{1}{2}, \frac{1}{4}, \frac{1}{2}, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$
 (3) $\frac{3}{8}, \frac{3}{8}, \frac{3}{8}, \frac{1}{4}, \frac{1}{4}, \frac{2}{4}$ (4) $\frac{2}{3}, \frac{1}{3}, \frac{1}{4}$ (5) $\frac{1}{17}, \frac{2}{17}, \frac{1}{17}$
 (6) $\frac{2}{3}, \frac{1}{3}, \frac{1}{3}$
 2 (1) 7 11, 17 23, 34 27 (2) 17 18, Equal, 1½ 1½
 (3) 17 18, 2 ½, 2 ½
 3 (1) 8 9, 2½ 3½ (2) 75 96, 4 7
 (3) 3 gal 3 qts 42 pts
 4 The first and second are increased and the third is diminished
 5 The first is increased and the second and third are diminished
 6 (1) 4 35, 1 7 (2) 45 364, 100 483 (3) 9 50, 10 21
 7 10½ 8 52 9 2 ro 13 po 10 25 39
 91 81 12 16 24 30 35

Ex CXLV (pp 391-392)

- 1 (1) 36 (2) 509 $\frac{1}{4}$ (3) 57 (4) 22 $\frac{1}{2}$ (5) 22 (6) 5 15
 (7) 26 208 (8) Rs 116 10a 8p (9) Rs 405
 (10) 11 tons 15 cwt 2qrs (11) 40 tons 10 cwt 12 17s 0 $\frac{1}{2}$ d
 2 63 31 $\frac{1}{2}$ 3 50 , 74 $\frac{3}{4}$
 4 The 3rd term is $\frac{1}{16}$, the second term is 0000406
 5 (1) 8 (2) 25 (3) 171 (4) $\frac{2}{3}$ (5) 6 (6) 01
 6 (1) 72 (2) 2 $\frac{5}{8}$ (3) 9 (4) 13 (5) 8 $\frac{1}{4}$ (6) 05
 7 (1) 6400 (2) $\frac{1}{4}$ 8 70 105 126 144
 9 1 3 10 2 15 11 1 $\frac{1}{2}$ gals 12 217 865
 13 9 8 14 19 17 15 739 257

Ex CXLVI. (pp. 397-401.)

- 1 Rs 1059 6a 2 60 days 3 £189 3s 11 $\frac{1}{2}$ d
 4 25 days 5 Rs 56 4a 6 8lbs 3 oz
 7 162 mds 8 Rs 10 11a 4p 9 Rs 263 4a
 10 £61 9s 1d 11 £27 16s 3 $\frac{1}{2}$ d 12 26 hours
 13 Rs 516 4a 14 £1590 0s 2 $\frac{1}{2}$ d 15 25 mds 29 sr
 16 Rs 657 8a 17 £118 13s 6d 18 2 tons 3 cwt 3 qrs
 19 Rs 17184 15a 2 $\frac{1}{2}$ p 20 Rs 101 5a 8p 21 365 days
 22 15 days 23 75 ft 6 in 24 6 yards
 25 £126 12s 10d 26 Rs 15 8a 27 8 months
 28 Rs 4 12a 29 2a 9 $\frac{1}{2}$ p 30 Rs 1084
 31 Rs 8707 8a 32 £23 3s 3 $\frac{1}{2}$ d 33 54756
 34 £257 10s 35 9s 7 $\frac{1}{2}$ p 36 65 hours
 37 Rs 10500 38 Rs 7 14a 6p 39 Rs 11452 13a
 40 Rs 3650 41 Rs 14000, 2 $\frac{1}{2}$ p 42 20 gals
 43 £1386 44 £2185 16s 3d
 45 He will lose Rs 10 6a 8p 46 74 $\frac{1}{2}$ days
 47 Rs 236 15a 8p , 16 yds 48 Rs 50
 49 £61 12s 50 £3 7s 10d 51 12a 7 2p
 52 9a 53 25 $\frac{2}{3}$ days 54 8min 45 $\frac{1}{2}$ sec past 8
 55 47 $\frac{1}{4}$ days 56 89 ft 2 $\frac{1}{4}$ in 57 28
 58 12800 59 March 8, Sunday at 4 P M
 60 £1 10s 61 Rs 17 14a 62 Rs 535 12a 6p
 63 10 days , 12 $\frac{1}{2}$ days 64 9 min
 65 101 days 66 124 $\frac{1}{4}$ days 67 6 months
 68 2 $\frac{3}{4}$ ch 69 150 leaps 70 8 ch 2 to
 71 368 men 72 Brandy 18s , rum 14s gin 10s 6d
 73 Rs 68 4a 11 $\frac{1}{2}$ 74 4 miles
 75 21 days 76 Rs 87 7a 8 $\frac{2}{3}$ p 77 5 $\frac{1}{2}$ days
 78 48 men 79 £1306 10s 80 202 $\frac{1}{2}$ miles

Ex. CXLVII (pp. 404-409)

- 1 6 days 2 Rs 282 14a 3 1878 men 4 5 months
 5 63 tons 6 174 miles 7 974 mds 38 $\frac{1}{2}$ sr 8 1s 6d
 9 £4 0s 10d 10 7 men 11 2250 men 12 9 days

13	<i>Rs</i> 564 2 <i>a</i> 8 <i>p</i>	14	26 lbs 0½ oz	15	300 men	16	540 guns
17	48 days	18	300 men	19	156 men	20	17 cwt 2 qrs 9⅞ lbs
21	19 36 days	22	9 <i>s</i> 0½ <i>d</i> ½ <i>q</i>	23	25½ days	24	67⅛ days
25	24 times	26	9 days	27	32 days	28	16¼ hours
29	6⅜ oz	30	14 women	31	18 men	32	12⅞ hours
33	542 bi 11 kat 12 ch	34	<i>Rs</i> 392 4 <i>a</i> 8 <i>p</i>	35	45 men		
36	<i>Rs</i> 168 12 <i>a</i>	37	7½ hours	38	125 men	39	27 days
40	180 men	41	2½ ft	42	15 men	43	100 men
44	1515½ bricks	45	1250 men	46	6 48 miles	47	4 dozen
48	222½ days	49	<i>Rs</i> 330 12 <i>a</i>	50	<i>Rs</i> 5096	51	84 men
52	4 adults	53	<i>Rs</i> 39 1 <i>a</i>	54	39⅞ weeks	55	16½ ft

Ex. CXLVIII (pp 412-414)

1	4 $\frac{3}{4}d$	2	19 $\frac{1}{2}$ days	3	$Rs\ 4200$	4	10 days
5	14 men	6	$\pounds\ 3\ 5s\ 6\frac{1}{2}d\ \frac{1}{2}q$	7	3 lbs 11 oz	7 $\frac{1}{2}$	drs
8	174 miles	9	178 mds 20 sr	10	$Rs\ 204\ 12a$	11	240 men
12	11 months	13	13 4s	14	229 yds 1 ft	15	7 hours
16	1 $\frac{1}{8}$ hours	17	20 cannon	18	180 men	19	5 2 . 4 $\frac{1}{8}$ ac
20	1s 7 $\frac{3}{4}d\ \frac{1}{2}q$	21	40000 men	22	6 men	23	131 $\frac{1}{4}$ days
24	120 days	25	29 $\frac{1}{2}$ days				

Ex CXLIX (pp 415-417)

1	(1) $\frac{1}{8}$	(2) $\frac{3}{8}$	(3) $\frac{3}{100}$	(4) $\frac{8}{10}$	(5) $1\frac{1}{10}$	(6) $\frac{1}{10}$
2	(1) 21 6	(2) $Rs\ 18$	(3) 58 mds 20 sr	(4) $Rs\ 109\ 6a\ 4\ 8p$		
	(5) 187 bi 17 kat 8 ch	(6) 171 yds	(7) $Rs\ 450\ 6a\ 3p$			
	(8) 2 hrs 12 min 36 sec					
3	(1) 28 $\frac{1}{2}$	(2) 49 $\frac{1}{2}$	(3) 8985375	(4) 26 $\frac{5}{11}$	(5) 2 530.	
	(6) 9 5	(7) 103	(8) 11 $\frac{1}{3}$	(9) 5 $\frac{1}{11}$		
4	(1) 37 $\frac{1}{2}$	(2) 16 $\frac{3}{4}$	(3) 460	(4) 125	51 366 $\frac{2}{3}$	(6) 590
5	20 p c	6 25 p c	7 43797	8 75844	9 38896200	14 600
10	58000	11 33 $\frac{1}{3}$ p c	12 $Rs\ 2607$	13 91 2 p c		
15	640	16 3608	17 120, 156, 174, 150			
18	66 $\frac{2}{3}$ gold, 33 $\frac{1}{3}$ silver	19 $Rs\ 3379\ 11a\ 3\frac{3}{4}p$	20 $Rs\ 2500$			
21	$Rs\ 15000$	22 $Rs\ 1500$	23 $\frac{1}{2}$ p c gain	24 $Rs\ 18000$	25 200	

Ex CL. (pp 418-419)

1	(1) 27	(2) 738 $\frac{5}{7}1428$	(3) 13 $\frac{1}{2}$	(4) 7 699794	
	(5) 26 9625	(6) 10 154875			
2	18 552 yrs	3 Rs 36 5a 2 $\frac{1}{2}$ p	4 10 st 10 $\frac{1}{2}$ lbs , 9 st 6 $\frac{1}{2}$ lbs		
5	1541	6 25311 84	7 8 667 years		
8	60 $\frac{1}{2}$ years	9 60	10 14 63 yrs	11 13 st 21lbs.	
12	51 $\frac{1}{2}$	13 Rs 30	14 78	15 13 yrs	16 21 $\frac{1}{4}$

Ex CLI. (pp 420-424.)

1	12 per cent	2	Rs 656 8a	3	Rs 1 10a 10p
4	Rs 275	5	7s	6	Rs 1 9a 10 1/2p
				7	Rs 850

- | | | | | | |
|----|------------------------|----|--------------------------|----|---------------------|
| 8 | 4s $1\frac{1}{2}d$ | 9 | Rs 9 9a 4p | 10 | Rs 9 2a 8p. |
| 11 | 40 per cent | 12 | Gain, $7\frac{1}{2}$ p c | 13 | $42\frac{1}{2}$ p c |
| 14 | Rs 18 4a | 15 | $62\frac{1}{2}$ p c | 16 | $96\frac{1}{4}$ p c |
| 17 | Rs 958 5a 4p | 18 | Rs 270 | 19 | Rs 7 5a 4p, 60 p c. |
| 20 | Rs 1 10a 2p | 21 | 5 p c | 22 | Loss, 65 p c |
| 23 | $30\frac{1}{4}$ p c | 24 | 32 | 25 | $3\frac{1}{2}$ p c |
| 26 | Rs 206 4a, 25 per cent | 27 | Rs 2500, 30 p c | 28 | 400 |
| 29 | 8 cwt | 30 | 16 per cent | 31 | 10a 32 £35 |
| 32 | 1 cwt 73 lbs 10 oz | 33 | 6 scores | 34 | Rs 3 2a |
| 35 | Rs 5 13a 4p | 36 | Rs 4 10a $6\frac{1}{2}p$ | 37 | 10s 5d |
| 38 | Rs 750 | 39 | Rs 7 8a | 40 | 3 cwt 2 qrs 23 lbs |
| 41 | Rs 100 | 42 | Rs 100 | 43 | Rs 250 |
| 44 | Rs 400 | 45 | £48 | 46 | 12½ per cent |

Ex CLII (pp 426-427)

- 1 (1) 224, 336, 448 (2) Rs 2500, Rs 3750; Rs 8750, Rs 10000
 (3) Rs 33000, Rs 22000 Rs 16500, Rs 13200
 (4) £320, £370, £384 (5) 180 bl 12 k 6 ch, 236 bl 3 k 14 ch, 277 bl 17 k 8 ch
 (6) Rs 50, Rs 12 8a, Rs 5, Rs 2 8a, Rs 1 10a 8p
 (7) 264 lbs, 56 lb., 40 lbs (8) £121 5s 6d, £179 11s, £292 12s
- 2 Saltpetre 336 sr, sulphur 44 8 sr, charcoal 67 2 sr
 3 3 mds 12 sr 4 270 lbs, 198 lbs (nitre)
 5 N 851 $\frac{1}{2}$ sr, S 106½ sr, C 162½ sr
 6 Men Rs 12 8a, women Rs 7 8a, boys Rs 6
 7 A Rs 4242 B Rs 7070 C Rs 7777
 8 C £630, B £600, A £400
 9 A Rs 1312 8a, B Rs 3500, C Rs 7875, D Rs 2100
 10 Rs 433 5a 4p 11 30, 48, 72
 12 A Rs 90, B Rs 120, C Rs 180
 13 Men 5400, women 675, boys 150, girls 45 14 £83 17s $10\frac{1}{2}d$
 15 A 200 mds, B 300 mds, C 525 mds, D 1155 mds
 16 56 half crs, 40 florins, 24 shillings
 17 12 half crs, 24 pence, 36 sovs, 96 shillings
 18 50 sovs, 60 half sovs, 200 florins
 19 A Rs 1920, B Rs 1440, C Rs 1350, D Rs 4800
 20 78 rupees, 132 half-rupees, 168 quarter rupees

Ex CLIII. (pp 429-430.)

- 1 Rs 150, Rs 200, Rs 250 2 Rs 22500, Rs 15000, Rs 7500
 3 A Rs 1500, B Rs 1312 8a, C Rs 1187 8a
 4 The share of each is Rs 1190 5 A £540, B £900
 6 A Rs 2240, B Rs 2360 7 A Rs 14337, B Rs 14458 8a,
 C Rs 11340 8 A Rs 160, B Rs 150, C Rs 400
 9 A £168, B £280 10 A Rs 35000, B Rs 49000, C Rs 70000,
 D Rs 84000 11 A Rs 1790 10a 8p, B Rs 2574 1a 4p

12	$A \text{ £}8 \text{ } 11s \text{ } 6d$, $B \text{ £}13 \text{ } 9s \text{ } 6d$	
13	$\text{£}486 \text{ } 13s \text{ } 4d$, $\text{£}730$, $\text{£}2433 \text{ } 6s \text{ } 8d$
14	$\text{Rs } 1600$	15	$23\frac{1}{4}$ months
17	$10\frac{1}{2}$ months	18	$45\frac{1}{2}$ days
		16	$22\frac{1}{4}$ days

Ex. CLIV. (pp. 431-432.)

1	$5\frac{1}{4}$ months	2	9 months	3	$7\frac{1}{2}$ months
4	28th May	5	$92\frac{1}{4}$ days		
6	7 months	7	80 months	8	24th April

Ex CLV. (pp 434-436)

1	Rs 7 3a 12½p	2	Re 1 2a 9½p , Re 1 1a 5½p
3	7 4	4	3 3 3 1
6	2 13	7	3 mds, 3 mds, 5 mds
9	28 lbs , 56 lbs , 56 lbs , 28 lbs , or 42 lbs of each	8	2 7
10	3 lbs of 1st , 1 lb of 2nd , 12 lbs of 3rd		
11	9 gals , 18 gals 27 gals , 9 gals	12	4½ lbs
13	3 lbs , 3 lbs , 3 lbs , 2 lbs		
14	50 lbs of brass , 200 lbs of pewter	15	185 lbs 10 oz.
16	115 oz , 85 oz	17	7 oz gold , 2 oz silver , 2 oz copper
18	20 7, 5s 1½d	19	40 apples , 20 pears
20	Gold £3 17s 10½d , silver 5s 1½d		

Ex CLVI (pp. 437-438)

1	32 oxen	2	$9\frac{1}{4}$ days	3	70 oxen	4	32 oxen.
5	190 oxen	6	3 ac	7	40 oxen	8	25 oxen
9	4 taps	10	7 sr 13 ch	11	9 min	12	21 days

Miscellaneous Examples VII (pp 441-446.)

1	5a 2p	2	9 per cent	3	Re 1 3a pei 100
4	22 women	5	A Rs 375 , B Rs 500 , C Rs 333 5a 4p.		
6	33½ p c	7	1500000	8	Loss 7 p c
9	Rich 15, poor 85	10	18s	11	1000 and 800
12	131 lbs	13	28¼ p c	14	625 15 £248
16	11½ carats	17	150 at Rs 4 , 250 at Rs 3		
18	Tea, 2s 6d , sugar, 6d	19	Tea, Re 1 14a ,		
	coffee, 13a 4p	20	38½¾ per cent	21	2 1
22	3 7	23	200 cu ft	24	£96 11s 8d
25	Rs 3 4a	26	£89 8s 9d	27	A 120 , B 75 , C 125
28	Rs 16500	29	200	30	A Rs 30 , B Rs 20
31	Rs 84 , A Rs 44 , B Rs 40	32	Men, Rs 72 14a 2½p ,		
	women, Rs 85 5 4p , boys, Rs 81 12a 5½p				
33	2 1	34	Rs 30	35	9½¾ p c gain , 1200
36	51½ p c gain	37	Rs 375	38	3 2 39 42, 36 and
	30 years	40	70 oz	41	300
	C Rs 1182	42	A Rs 850 , B Rs 846 ,		
43	Copper, 12a , brass, 9a , 5 6	44	390 , 150		
45	40 horses , 25 oxen	46	15 carats	47	24

- 48 35, 15 49 Mangoes, 12, plantains, 24
 50 12, 28 and 44 years 51 Rs 3 4a, Rs 6 2a
 52 3 mds 30 sr; 4 mds 35 sr 53 Rs 60
 54 $\frac{5}{10}$ p c increase 55 16 $\frac{1}{4}$ days 56 200
 57 Rs 200, A Rs 20, B Rs 40, C Rs 60 D Rs 80
 58 Rs 125 A Rs 50, B Rs 41 10a 8p, C Rs 33 5a 4p
 59 21 gals from the 1st, 8 gals from the 2nd 60 44 91

Ex CLVII (pp 447-448)

- 1 (1) Rs 10 14a (2) Rs 21 8a (3) Rs 130 12a 6p
 (4) Rs 10 $\frac{1}{2}$ 1a (5) Rs 1209 6a (6) Rs 481 4a
 (7) £32 3s 1 $\frac{1}{2}$ d (8) £103 11s 10 $\frac{1}{2}$ d
 2 £10 3 Rs 10 2a 4 Rs 60 5 Rs 50

Ex CLVIII (pp 449-450.)

- 1 (1) Rs 272 (2) Rs 49 3a 96p (3) Rs 88 1a 96p
 (4) Rs 169 13a 3 8p (5) Rs 28 14a 96p (6) £118 3s 3d
 (7) £159 17s 3d (8) £24 6s 2 4d (9) £140 14s 4 5d
 (10) £2 3s 3 $\frac{1}{2}$ d
 2 (1) Rs 91 14a (2) Rs 1444 14a 4 95p (3) Rs 647 3a
 (4) Rs 890 10a (5) Rs 117 1a 5 28p (6) Rs 474 3a 0 675p
 (7) £221 (8) £102 9s 9 $\frac{1}{2}$ d 2q (9) £11 17s 6d
 (10) £75 10s 3d cq (11) £1356 10s 6 $\frac{3}{4}$ d 4475q.
 (12) £273 12s
 3 (1) Rs 22 6a 10p (2) Rs 107 15a 4p
 (3) £36 16s 7 $\frac{1}{2}$ d (4) £73 15s 1 $\frac{1}{2}$ d (5) Rs 9792 12a 3p
 (6) £81 10s 3 $\frac{1}{2}$ d (7) Rs 39 2a 2p (8) Rs 1714 14a 8p
 (9) £3 15s 3 $\frac{1}{2}$ d (10) £2 19s 8 $\frac{1}{2}$ d
 4 (1) Rs 5916 2a 8p (2) Rs 3873 12a 96p
 (3) £959 13s 8 $\frac{1}{2}$ d $\frac{1}{8}$ 6p (4) Rs 2701 3a 9 $\frac{3}{8}$ p
 (5) Rs 18702 14a 4p

Ex CLIX. (pp 452-454)

- 1 Rs 4688 2 Rs 1205 3 Rs 3250 4 3 $\frac{1}{2}$ p c 5 2 $\frac{1}{2}$ p c.
 6 5 $\frac{1}{2}$ p c 7 Rs 12163 2a 6 $\frac{1}{8}$ p 8 £678 3s 11 $\frac{1}{2}$ d
 9 Rs 8489 4a 9p nearly 10 4 $\frac{1}{2}$ yrs 11 16 yrs 12 20 yrs
 13 4 $\frac{1}{2}$ p c 14 6 p c 15 6 yrs 8 mo 16 4 $\frac{1}{2}$ p c nearly.
 17 16 yrs 18 2 $\frac{1}{2}$ p c 19 Rs 1140 10a 20 Rs 43312 8a
 21 15 yrs 22 12 $\frac{1}{2}$ p c 23 6 $\frac{1}{2}$ yrs 24 £640
 25 Rs 6878 9a 26 5 $\frac{1}{2}$ p c 27 Rs 20855 10a
 28 Rs 3333 5a 4p 29 £10950 30 2 $\frac{1}{2}$ p c
 31 6 $\frac{1}{2}$ p c 32 3 and 5 yrs 33 Rs 40
 34 Rs 156 4a 35 Rs 1600, Rs 2400

Ex CLX (pp 457-458)

- 1 (1) Rs 406 0a 10p (2) Rs 262 11a 9 $\frac{3}{4}$ p
 (3) Rs 1495 11a 6p (4) Rs 3454 4a 2b

- (5) Rs 171 2a 4p (6) £934 3s 5½d
 (7) £1237 19s 2½d (8) £553 os 4½d nearly
 (9) Rs 1553 11a 8p
 2 (1) Rs 459 1a (2) Rs 297 6a 8p (3) Rs 1285 10a 10p
 (4) £33 13s 10½d 20384q
 3 (1) Rs 204 oa 77184768p (2) Rs 446 5a 4884168p
 (3) Rs 961 13a 8p
 4 (1) Rs 8104 10a 3p (2) Rs 3193 1a 8p
 (3) Rs 7155 15a 11p (4) £5140 10s
 (5) Rs 19422 6a 467712p (6) £2714 9s 9472d
 5 Rs 147 4a 4p nearly 6 £18 14s 8½d nearly
 7 874278 8 Rs 15540 6a 98p nearly

Ex. CLXI. (pp 460-461)

- 1 Rs 40000 2 £333333 6s 8d 3 £133 6s 8d
 4 Rs 42500 5 2 per cent 6 5 per cent
 7 2½ years 8 2½ years 9 3 years
 10 Rs 14080 10a 11 £800 12 £430 14s 6096d
 13 10 p c , Rs 2000 14 £3497 5s 4½d 15 Rs 125428 8a 2p

Ex. CLXII (pp. 462-463.)

- 1 (1) Rs 8500 (2) Rs 1200 (3) Rs 7212 8a
 (4) Rs 3150 (5) £45 11s 0½d 32½q (6) £350
 (7) £367 1s 6½d (8) Rs 10445 15a 2½p (9) £237 10s
 (10) £8000
 2 (1) Rs 1200 2a (2) Rs 350 (3) £5 5s (4) £15 3s 9d
 (5) £70 17s 6d (6) Rs 482 14a 8p (7) £12 16s 8d
 (8) £10 10s 10d (9) Rs 622 11a 10½p
 (10) £296 13s 4½d

Ex. CLXIII. (p. 464.)

- 1 (1) 7 p c (2) 6 p c (3) 4 p c (4) 2½ p c
 (5) 4 p c (6) 5 p c
 2 (1) 8 months (2) 3 yrs (3) 3½ yrs (4) 4 mo
 (5) 3½ yrs (6) 1½ yrs
 3 3 per cent 4 6s 0½d ½q 5 Nothing

Ex. CLXIV (pp 466-468)

- 1 Rs 8134 8a 2 £808 1s 4d 3 £231 15s
 4 6½ yrs 5 2½ per cent 6 7½ yrs
 7 £6¼ p c 8 Rs 7768 14a 2½p 9 Rs 180
 10 Rs 250 11 Rs 5746 8a , 6¼ p c
 12 Rs 337 14a 13 £1598 6s 8d , 1½ per cent
 14 80 83 , Rs 320 15 (i) Rs 45 13a 4p (ii) Rs 13 1a 6½p
 16 Rs 322 8a 5½p 17 17s 4½d 18 10 months

- 19 £130 20 Rs 9 9a 10²/₃p 21 £2500
 22 Disc. 16²/₃ p c, 1nt 20 p c 23 16 copies
 24 8¹/₂ p c 25 Rs 1200 26 18¹/₂ p c
 27 Bs by Rs 33 5a 4p 28 Rs 7014 5a 6p nearly
 29 Rs 2115 1a 4p nearly, 4 66 mo 30 Rs 6078 nearly

Ex CLXV. (pp 470-471)

- 1 Rs 318 10a 6p 2 Rs 11685 5a 4p 3 Rs 8 14a 1725p
 4 2s 6625d 5 Rs 9 6 Rs 121 8a 0534 p
 7 Rs 255 8. Rs 336 1a 9²/₃p 9 Rs 1585 8a 9²/₃p
 10 £547 2s 2d 2²/₃q, 7s 9¹/₂d 2²/₃q 11 Rs 51238 14a 1²/₃p
 12 Rs 57 0a 5¹/₂p Re 1 0a 5¹/₂p 13 £24 5s 11¹/₂d
 14 £6020 15 £61 15s 16 Rs 617624 13a 0¹/₂p
 17 31st May 18 11¹/₂ p c

Ex CLXVI (pp 473-474)

1. Rs 65 12a 4p 2 Rs 3507 7a 0²/₃p 3 9s 1¹/₂d
 4. Rs 128 0a 4p 5 £4488 15s 6 Rs 969 4a 9 6p
 7. £768 2s 6d 8. Rs 16170 9 Rs 5105 6s
 10 Rs 693333 5a 4p 11 4¹/₂ per cent
 12. Rs 23149 5a 4¹/₂p Rs 766666 2a 13p 13 Rs 38 10a 11p
 14. £486 3s 10¹/₂d 3¹/₈ £7q 15 Rs 68806 7a 5¹/₂p

Ex CLXVII (pp 479-480.)

1. Rs 18772 10a 9²/₃p 2 17511 fr 4¹/₂ c
 3 2648 fr 6971 c 6294 fr 6 61875 c 4. 11474 dol 38¹/₂ c
 5. 124 2675 fr or 124¹/₂ fr nearly
 6 30d per rouble, gain £20 16s 8d 7 25 fr 73 c
 8. 33s 4d 9 (i) 9¹/₂d in £1 (ii) 11¹/₂ fr in 300 fr
 10 £10 14s 3¹/₂d 11 996 87 fr 12. £1011 12s 67d
 13. £2000 14. £250 15 1 dol = 4 22429 s = 4s 2¹/₂d nearly
 16 25 001 fr or 25 fr 2¹/₂ nearly
 17 Gain £5 9s 0¹/₂d nearly 18 20 marcs 42¹/₂ pf

Ex CLXVIII (pp 481-482)

- 1 3²/₂ per cent 2 28¹/₂ years 3 Rs 1600 4. 4 p c 5 5 p c
 6 25 96 years 7 £2 7s 4¹/₂d 8 25¹/₂ years

Ex CLXIX. (pp. 487-488.)

- 1 (1) Rs 3960 (2) £5418 2s 9d (3) £108 5s 11¹/₂d 18¹/₂q
 (4) Rs 37660 2a 2¹/₂p (5) £588 5s (6) Rs 29808 10a 3¹/₂p
 (7) Rs 9175 10a (8) £1226 1s 10¹/₂d
 2. (1) Rs 14100 2) Rs 8065 14a 2¹/₂p (1) Rs 8051 14a 9²/₃p
 (4) £15573 8s 10¹/₂d (5) Rs 70850 1a 3¹/₂p
 3. (1) Rs 9000 (2) £1000 (3) £2955 1s 1¹/₂d 5¹/₂q
 (4) Rs 49788 6a 9²/₃p (5) Rs 24200 14a 7¹/₂p

- (6) £1015 1s 0 $\frac{1}{2}$ d $\frac{1}{11}$ g (7) Rs 8670 7a 1 $\frac{5}{8}$ pb
 (8) £566 13s 4d
 4 £10039 2s 2 $\frac{3}{4}$ d $\frac{5}{11}$ g 5 Rs 238 14a 6 £50 16s 8d
 7 Rs 22699 2a 2 $\frac{1}{2}$ pb 8 112 $\frac{1}{2}$ 9 Rs 6250
 10 Rs 7500 11 £175 12 93 $\frac{3}{8}$
 13 Rs 624 11a 10 $\frac{5}{11}$ pb

Ex CLXX. (pp. 489-491)

- 1 (1) £165 (2) Rs 3155 4a (3) £108 3s 3d (4) Rs 2878 12a
 2 (1) Rs 2400 (2) £159 12s (3) Rs 1368 8a
 (4) Rs 1093 13a 2 $\frac{1}{4}$ pb (5) Rs 843 9a 5 $\frac{1}{2}$ pb
 (6) Rs 9311 5a 6 $\frac{3}{8}$ pb
 3 (1) Rs 17000 (2) £1785 (3) Rs 44072 1a 4pb (4) £5805
 4 Rs 52 5a 4 $\frac{1}{2}$ pb 5 £83 6s 8d 6 £122 3s 4d
 7 91 $\frac{1}{2}$ 8 Rs 54309 10a 10 $\frac{3}{4}$ pb
 9 £15 3s 0 $\frac{1}{4}$ d increase 10 Rs 20 gain 11 £26 13s 4d
 12 £34 10s gain 13 £5 12s 6d loss 14 90
 15 Rs 47250 16 Rs 455 5a 8 $\frac{1}{2}$ pb 17 85 $\frac{1}{2}$
 18 94 $\frac{1}{2}$ 19 Rs 20000 20 90 $\frac{1}{3}$
 21 Rs 25 gain 22 £5300 23 Rs 9080 8a 5 $\frac{1}{8}$ pb
 24 £52 10s, £58 6s 8d more

Ex CLXXI (pp. 492-496)

- 1 (1) 4 $\frac{8}{11}$ p c (2) 3 $\frac{1}{2}$ p c (3) 3 $\frac{1}{2}$ p c (4) 5 p c
 2 (1) 80 (2) 87 $\frac{1}{2}$ (3) 77 $\frac{1}{2}$
 3 95 $\frac{7}{8}$ 4 85 $\frac{3}{4}$ 5 83 $\frac{1}{2}$ 6 4 p c 7 3 $\frac{1}{2}$ p c
 8 Rs 13800 9 83 $\frac{1}{2}$ 10 Rs 125000 11 Rs 18240
 12 £5631 5s, £90 6s 3d gain 14 117 $\frac{1}{2}$ 15 3 p c, £582 10s
 13 Rs 138548 12a 17 Rs 96 5a 8pb 18 106 $\frac{1}{16}$
 16 £3000 17 Rs 13409 8a 4 $\frac{1}{2}$ pb 20 94 $\frac{1}{2}$
 19 Rs 17414 15a 5 $\frac{3}{8}$ pb, Rs 13409 8a 4 $\frac{1}{2}$ pb 22 Rs 14150 15a 1 $\frac{1}{32}$ pb 23 Rs 49700
 21 £9880 25 Rs 25200
 24 Rs 19 13a 4pb less 27 Rs 16000, Rs 24000
 26 Rs 27000, Rs 185 more 29 109 $\frac{1}{11}$ 30 Rs 27000
 28 Rs 645000 32 £257 5s 5d 33 6d in the £
 31 £32 5s more 35 85 $\frac{1}{2}$ 36 Rs 4800, Rs 5200
 34 Rs 192000 38 Rs 16000000 39 15 years
 37 Rs 4000, Rs 12000 41 Rs 41066 10a 8pb, Rs 45600
 40 Rs 6750, Rs 5250 43 £400, £1600
 42 £1000 45 £242914 19s 7 $\frac{3}{4}$ d
 44 £3000

Ex. CLXXII (pp. 498-499.)

- 1 197 ft 2 45 09 ft 3 38 ft 9 in, 54 ft 9 6 in nearly
 4 3365 yds 5 14 02 ft 6 153 miles 7 273 642102 miles
 8 520 195 ft. 9 12 ft 10 19' 11 35 ft 12 1045 yds.

- 13 (1) 1 (2) 1 (3) 25 (4) 100. (5) 5725
 (6) $1\frac{25}{8}$ (7) $41\frac{5}{8}$ (8) $3\frac{1}{8}$
 14 $3\frac{1}{16}$ $15\frac{5}{8}$ ft $7\frac{1}{2}$ in

Ex CLXXIII (p 502)

- 1 (1) 136602 (2) 667423 (3) 50401 (4) 71070
 (5) 82185 (6) 43417 (7) 482825 (8) 405143
 (9) 261803 (10) 1 (11) 14 (12) 138742
 (13) 157313 (14) 26795 (15) 0
 2 (1) $\frac{2}{3}$ (2) $\frac{4}{19}$ (3) $\frac{4}{15}$ 3 $\frac{2}{81}$ 4. 294151

Ex. CLXXIV. (pp. 504 508.)

- 1 (1) 17030003 kilom, 17030003 centim
 (2) 50000037 kilom, 50000037 centim
 2 (1) 5330003000002 sq m, 5330003000002 sq millim
 (2) 725 sq m, 7250000 sq millim
 3 (1) 116075 gr, 116075 kilog (2) 53045 gr, 53045 kilog
 4 7630005 ares 5 530000465 decil
 6 (1) 2414 fr, 2414 c (2) 4800575 fr, 4800575 c
 (3) 508 fr, 508 c (4) 55536 fr, 55536 c
 7 127 fields 8 12 9 180 fr 74 c, $\frac{1}{2}$ 16s $10\frac{1}{2}$ d nearly
 10 A will have 24 hectar 80 ares, B 22 hectar 4 ares, C 33 hectar
 16 ares 11 $7\frac{1}{2}$ days 12 30 miles nearly, $1\frac{3}{4}$ d nearly
 13 297 fr 50 c 14 (1) 6 c. (2) Rather more than 1s $2\frac{1}{2}$ d
 15 2 fr 91 c 16 1600 fr 17 6 p c 18 $\frac{1}{7}$
 19 19781 met 20 158 of each of two first coins and 79 of each
 of three last coins 21 (1) 34 fr 7 c (2) 2 fr 37 c
 (3) 29 fr 87 c, or £1 3s 5d nearly 22 £110 1s 3d nearly
 23 A's share=24 hectar 75 ares, B's share=13 hectar 50 ares,
 C's share=4725 hectar 24 15 fr 3268 c 25 8 years
 26 £19 7s 10d nearly 27 3419 fr 58 c
 28 (1) 500 fr $26\frac{1}{2}$ c (2) 446 fr $6\frac{1}{2}$ c 29. 1785 fr 50 c
 30 1 hr 25 min 20 sec 31 1st 3 fr 25 c, 2nd, 4 fr 63 c
 32 The 5 per cents, £21 4s 11d nearly 33 £1 19s 215d
 34 27153 fr $78\frac{1}{2}$ c 35 (1) 7s $1\frac{1}{2}$ d (2) 8 fr 94 c
 36 82 fr 50 c 37 259 fr $70\frac{5}{8}$ c 38 6 ares, 6 deciar
 2 centiar nearly, 79179 sq yds 39 12059 fr
 40 2 decim 4 centim 41 9 kilog 126 grs 42 £864 8.
 43 15 fr 14 c nearly 44 1 m 6 decim
 45 No of boxes=14, no of oranges=588
 46 7 Nap 15 fr 85 c nearly

Ex CLXXV (p 509)

- 1 (1) 59170 m, £976 3fl 5c 8m (2) 18065 m., 96239 m
 (3) 14435 m, £254 5fl 2c 5m

- 2 (1) 1 c $8\frac{1}{2}$ m , 4 c $1\frac{1}{2}$ m , 2c 2 916m , 2fl 5c , 3fl 1 c $6\frac{1}{2}$ m ,
1 fl 5 208 $\frac{1}{2}$ m , 1 fl 5 c $3\frac{1}{2}$ m
(2) 2 fl 1 c $2\frac{1}{2}$ m , 7 fl 8 c $4\frac{1}{2}$ m , £12 6 fl 4 c 8 958 $\frac{1}{2}$ m ,
£4 4 fl 2 c 83 m
(3) 7 fl 8 c $3\frac{1}{2}$ m , 6fl 4 c 8 958 $\frac{1}{2}$ m , £5 4 fl 6 c 9 7916 m ,
£6 8 fl 9 c 7 916 m
- 3 (1) 18s $1\frac{1}{2}$ d , £1 9s 9d , £1. 15s 8d , 18s $1\frac{1}{2}$ d 0864g
(2) £3 12s $5\frac{1}{2}$ d , £7 16s $5\frac{1}{2}$ d , £29 17s 6 24d , £25 18s 10 $\frac{1}{2}$ d
- 4 £990 6 fl 7 c 3 m 5 £243 2 fl 6 c 4 m
6 £863 2 fl 8 c 5 m 7 £14 3 fl 2 c 5 m , £6 2 fl 7c 4 m
8 £174 2 fl 3 c 7 m 9 £196 6 fl 5 c , £11012 4 fl
10 £3051 7 fl 2 c 1 m 11 £76 8 fl 3 m 12 7 fl 8 c 2 2359 m
13 £2 9 fl 9 c 9 m , £46 3 fl 6 c 14 789
15 0732 , 001464

Miscellaneous Examples VIII (pp 512-519)

- 1 Rs 2140 2 11th March 3 11 $\frac{1}{2}$ p c 4 £273 8s 9d
5 £2376 5s 6 £2577 12s 1d 7 10a 8p 8 £81 $\frac{1}{2}$ $\frac{1}{8}$
9 155 147 10 £390 2s $5\frac{1}{2}$ d 11 224 seers
12 £160 14s $3\frac{1}{2}$ d 13 4 p c loss 14 First Re 1 8a per lb ,
second Re 1 12a per lb 15 £250
16 £192307 $\frac{1}{8}$, £57692 $\frac{1}{4}$ 17 £2200 18 Rs 3535
19 £8 15s $2\frac{1}{2}$ d 20 933 $\frac{1}{2}$ lbs 21 £5977 22 46 $\frac{2}{3}$ sr
23 £2450 , £2205 24 6 $\frac{1}{2}$ $\frac{1}{2}$ s 25 600 sq ft 26 Rs 264
27 The 3 per cents , 19s $7\frac{1}{2}$ d gain 28 £1500 29 Rs 39440 ,
Rs 1560 30 Rs 6776 31 £10 16s decrease
32 3 $\frac{1}{2}$ p c 33 £1200 34 Rs 280 35 '96 19
36 25 for 11d 37 5 9 38 60 ft 39 17s $2\frac{1}{2}$ d
40 First £10 13s 4d , second £11 13s 4d
41 Rs 72900000 42 32 25 p c 43 A 28 yrs , B 32 yrs
44 4 $\frac{1}{2}$ p c
45 A 32 yrs , B 28 yrs 46 Circuitously, by 35 985 milrees
47 5 51 p c nearly 48 4 p c 49 $\frac{1}{2}$ 50 12 yds from B
51 9 $\frac{1}{2}$ d 52 22 yrs ago , 18 yrs hence
53 Rs 21106 8a , Rs 18232 8a , Rs 15000
54 0102045 07 , 25 17 francs 55 25 fr 53 $\frac{1}{2}$ c , 25 fr 14 $\frac{1}{2}$ c
56 £1687 14s 8d 57 Rs 5110 58 Rs 2600 , Rs 3580
59 £4092 60 £10 61 (1) 23515302409
(2) 10192 62 He loses £240 63 6 $\frac{1}{2}$ p c
64 £2275 65 (1) Rs 137 8a (11) Rs 39 4a 6 $\frac{1}{2}$ p
66 Rs 12960 , Rs 11220 67 A £150 , B £144 ,
C £90 , D £60 68 Rs 6500 69 2s 8d
70 Rs 210 , Rs 105 , Rs 72 , Rs 42 71 18 $\frac{1}{2}$ p c
72 17 $\frac{1}{2}$ p c 73 £14700 74 9 $\frac{1}{11}$ per cent
75 A receives 6s 8 0325d more than B 76 Nothing
77 Gain = 623 $\frac{1}{2}$ francs , 37 $\frac{1}{2}$ p c 78 Gain 74 $\frac{1}{2}$ p c
79 £600 80 12 p c 81 9 5 ft nearly

Miscellaneous Exercises (pp. 520-557)

- 1 857628 2 62968425 3 37 4 £1 3s 2d
 5 12 lbs 6 £2 14s 1½d 7 477 times 8 A Rs 48,
 B Rs 72, C Rs 112 9 Rs 500 10 795
 11 33, £3048751 15s 12 Sum = 17624, product = 76945744
 13 612 14 5445928167g 15 £73 16s 2½d
 16 2415 times 17 76 times 18 29 19 Rs 7 13a
 20 256 weeks 21 Five millions two hundred and ninety-nine
 thousand eight hundred and thirty
 22 167073002551134812 23 2378 24 477648483
 25 £33438069 12s 3½d 26 Yes, 6p 27 £392 19s 3½d
 28 25 hens 29 262080 min, 797 30 £57 12s 6d
 31 One hundred millions, seven hundred and ninety six thousand,
 nine hundred and fifty-seven 32 17
 33 1861 34 95 35 511 ac 2 ro 8 po 5 sq ft 129 sq in
 36 4 gals 37 3½s 38 7 horses, 7 pigs, 21 cows, 105 sheep
 39 Rs 116666 10a 8p 40 Rs 15 41 668674698½s
 42 1 ac 10 po 4 yds 2 ft 43 £11 4s 6½d 44 57 years
 45 (i) A Rs 22222½, B Rs 33333½, C Rs 44444½
 (ii) D Rs 461531½, E Rs 30769½, F Rs 230761½
 46 119, 17 47 £52 15s 6d 48 1½ 49 £10 8s
 50 Rs 170 4a 51 2, 3, 7, 11, 13, 37, 71716
 52 Quot = 17430, Rem = 12 53 90945547 miles 2 fur 8 po
 1 yd 2 ft 3 in 54 2½ 55 1533½, 71½
 56 £11 3s 10½d 57 £233 17s 10d, 5½d
 58 A Rs 22840, B Rs 11420, C Rs 3806 10a 8p, D Rs 7613 5a 4p
 59 13½ days 60 Rs 103 1a 3p 61 Rs 774 0a 6½p
 62 £4 0s 9d 63 12 mds 64 9 weeks
 65 88½, Rs 13500 66 54 67 7½d
 68 Rs 333 5a 4p 69 A £317 9s 2½d, B £267 17s 1½d;
 C £253 19s 4½d, D £160 14s 3½d
 70 £2440, £73 4s 72 2967 73 1 74 756 cub ft
 75 55 sq ft 80 sq in, 18s 6½d 76 A's Rs 586½, B's Rs 391½;
 C's Rs 352½ 77 £20 78 4½ ft 79 £8000, £7500
 80 £1239 13s 4½d 81 15099500 82 1568
 83 112345569322391104 84 03212 85 Rs 767 13a 5½p
 86 90 days 87 17 ft 4 in 88 £104 89 44½ p c
 90 £4687 10s 91 763 92 794997 93 03493
 94 £26 13s 4d 95 8½ 96 1050 97 £1682
 98 7 99 83, 591 100 12½ p c
 101 14 days 7 hrs 11 min 17 sec, 2674 days 9 min 59 sec
 102 3998936616 103 4s 7½d 104 2721551 105 1 4
 106 33½ lbs 107 4½s 108 30 seers 109 439 824 yds
 110 Gain Rs 770, 1½ p c 111 24754 112 425
 113 20 hrs 16 min 114 3 tons 4 cwt 3 qrs 4 lbs 13 oz.
 115 307½ days 116 25 miles 117 Rs 9180 118 £1600
 119 Rs 1666 10a 8p monthly 120 72 men, 288 women

121	45	122	126.	123	4806	124	108 $\frac{0}{8}$ grs , 00221142857
125	<i>Rs</i> 14586			126	11 ft 6 $\frac{1}{2}$ in		
127	A 3 $\frac{1}{2}$ s , B 4 $\frac{1}{2}$ s , C 5 $\frac{1}{2}$ s , D 7 $\frac{1}{2}$ s	128	<i>Rs</i> 9920	15a	6 $\frac{3}{4}$ h		
129	\pounds 818 8s	130	\pounds 250 , 4 p c	131	73	132	2 $\frac{1}{2}$ h
133	<i>Rs</i> 288 12a	134	2 068501	135	150		
136	<i>Rs</i> 52000	137	5 p c	138	<i>Rs</i> 25000		
139	<i>Rs</i> 72	140	32	141	3612924 , 357250824		
142	$\frac{8}{9}$, 11s 5d	143	600600 , 6 006 , $\frac{8}{9}$, 8 8	146	3 $\frac{1}{2}$ h hours		
144	24	145	<i>Rs</i> 1500	149	$\frac{3}{4}$ gal	150	\pounds 1350
147	5 per cent	148	3 $\frac{1}{8}$ p c	153	\pounds 7 9s 7 $\frac{1}{2}$ d		
151	14a	152	6 times	156	14 $\frac{1}{2}$ days		
154	\pounds 1 13s 6 $\frac{1}{2}$ d	155	11 oz	159	<i>Rs</i> 1026		
157	12 lacs , <i>Rs</i> 5	158	<i>Rs</i> 3 2a	162	Eldest <i>Rs</i> 5184 ;		
160	29040	161	<i>Rs</i> 120	163	(i) $\frac{1}{2}$ (ii) 0513		
164	124 rings	165	7 days 10 hrs 52 min 30 sec	168	<i>Rs</i> 2319	169	624
166	20°	167	6a	172	320		
170	Loss <i>Rs</i> 1000	171	9 days	175	The clock ought to have		
173	10 half-crowns , 25 shillings , 50 six-pences	176	\pounds 31 5s.	179	Loss 80 p c		
174	A \pounds 1650 , B \pounds 1540	181	0	182	4 $\frac{1}{2}$ meas		
	been set at 5 h 30 $\frac{1}{2}$ min P M	183	A <i>Rs</i> 450 , B <i>Rs</i> 900 , C <i>Rs</i> 2250	184	20	185	16s 5 $\frac{1}{2}$ d
177	10 yds 2 ft 5 $\frac{1}{2}$ in	186	36 $\frac{1}{2}$ days	187	12s	188	72398369 $\frac{1}{1000}$ gals
180	11 2	189	391 $\frac{1}{2}$ rev , diamrs 2 ft 4 $\frac{1}{2}$ in , 4 ft 3 $\frac{1}{2}$ in	190	2 years		
183	A <i>Rs</i> 450 , B <i>Rs</i> 900 , C <i>Rs</i> 2250	191	2 $\frac{1}{2}$ f	192.	(i) $\frac{0}{8}$ (ii) 0		
186	36 $\frac{1}{2}$ days	193	32 seers	194	1 , 0447	195	1s 11 $\frac{1}{2}$ d
189	391 $\frac{1}{2}$ rev , diamrs 2 ft 4 $\frac{1}{2}$ in , 4 ft 3 $\frac{1}{2}$ in	196	8 $\frac{1}{2}$ days	197	176 ac 540 sq yds	198	<i>Rs</i> 2
191	2 $\frac{1}{2}$ f	199	\pounds 1 200 1s 10 $\frac{1}{2}$ d	201	13025 22 , an abstract number		
193	32 seers	202	1 $\frac{1}{8}$ 203 50 $\frac{1}{8}$ days	204	1513 $\frac{1}{2}$	205	$\frac{3}{4}$ in
196	8 $\frac{1}{2}$ days	206	20546 3	207	\pounds 4600	208	4 years
199	\pounds 1 200 1s 10 $\frac{1}{2}$ d	209	Author <i>Rs</i> 5322 , Publisher <i>Rs</i> 1008 , Bookseller <i>Rs</i> 4920	211	528093440	212	1 25 lbs
202	1 $\frac{1}{8}$ 203 50 $\frac{1}{8}$ days	210	9600 yds	213	A \pounds 212 2s , B \pounds 353 10s , C \pounds 388 17s		
206	20546 3	214	12 $\frac{1}{2}$ days	215	A , B and C each <i>Rs</i> 615 , D <i>Rs</i> 410		
209	Author <i>Rs</i> 5322 , Publisher <i>Rs</i> 1008 , Bookseller <i>Rs</i> 4920	216	23 boys	217	4 cwt	218	\pounds 1 0s 10d
210	9600 yds	219	\pounds 200	220	\pounds 106 8s 5 $\frac{1}{2}$ d gain		
213	A \pounds 212 2s , B \pounds 353 10s , C \pounds 388 17s	221	B is $\frac{1}{2}$ of a mile in advance of A	222	$\frac{1}{8}$		
214	12 $\frac{1}{2}$ days	223	33 $\frac{1}{2}$ years.	224	336 yds	225	\pounds 80 7s. 1d , 183 $\frac{1}{2}$ days
216	23 boys	226	163 $\frac{1}{2}$ days	227	29th March 1860 , 7 hrs 12 min		
219	\pounds 200	228	69 3109 miles	229	26 $\frac{1}{10}$ p c	230	Gain \pounds 24
221	B is $\frac{1}{2}$ of a mile in advance of A	231	\pounds 10 10s , $\frac{1}{2}$ s.	232	20 yds	233	1s 10 $\frac{1}{2}$ d
223	33 $\frac{1}{2}$ years.	234	33 $\frac{1}{2}$ per cent	235	3 029 in	236	15 , 1000 cub in
226	163 $\frac{1}{2}$ days	237	15 404 ft	238	<i>Rs</i> 75 7a 6 48p	239	<i>Rs</i> 621 $\frac{1}{11}$ each
228	69 3109 miles	240	31 $\frac{1}{2}$ miles	241	25 m 6 fur 6 po 2 ft 4 in , 42524 hhd		
231	\pounds 10 10s , $\frac{1}{2}$ s.		57 gals 3 qts 1 pt 2 gills ; 146929459 sq in	242	12 $\frac{1}{2}$ ft		
234	33 $\frac{1}{2}$ per cent	243	16 days	244	<i>Rs</i> 7992	245	86 7 $\frac{1}{2}$, \pounds 179 4s

246. 11 days. 247. Rs.1032. 248. £303. 15s.
 249. 11461 yds. 250. 35. 251. 550 : 215 : 20356 ; 2
 252. Tea Rs.1 :- Coffee 12s. 4d. ; Sugar 22 4d. per lb.
 253. 21 times. 254. A 162 : B 108 : C 72 runs. 255. $\frac{5}{7}$ m⁷e.
 256. Less by $\frac{7}{11}$ per cent. 257. £1000 : £7000. 258. 13s. 4s.
 259. £3. 17s. 260. 1000 : 27 2½d. 261. 1280730
 262. 189 tons 10 cwt. 1 cwt. 10 lbs. : 6 lbs. 6½d. oz.
 263. 3 : 2. 264. Loss $\frac{21}{100}$ p c. 265. $\frac{1}{2}$
 266. A 15 days : B 10 days : C 12 days. 267. Rs.1061. 7a.
 268. £172 5s. 269. 6 years. 270. The quantities are in order
 of magnitude. 271. 8½. 272. £75 7s 9½d.
 273. £6 7s. 10½d. 274. A 6 days ; B 18 days ; C 9 days.
 275. 12 passes. 276. 10 ft. 277. £5561. 13s 4d
 278. 1½ miles ; 1 hour. 279. Rs. 50 : Rs. 75 : Rs. 125
 280. Gain £17 3s. 5d. 281. (5009)² - (1012)² = (5015)².
 282. 5 (½) 283. 23 6s. 284. Rs. 7. 8a.
 285. 18½ p. c. 285. (i) 779 : 3 : 856.
 287. 87 anna 45½ yds 288. 75 ac. 289. £7 15s 7½d.
 290. 15 ft 7 in 291. Dr. 561 ; Q. 102 913
 292. 1071 and 1517 292. 294 276 tons. 295. 1115718
 296. 22½ days 297. £1500 298. 1 p c.
 299. £70. 300. 2 miles per hour. 301. 12
 302. 256 ; £5 7s 6d. 303. 150000 yds. 304. 687½ rev
 305. 3½ hours. 306. The steamer ; 16 hours
 307. 32 days. 307. 7½ min. or 57 ½ min past 6
 310. £2387 311. Dr. 779. Rem 270. 312. 11s 7½d
 313. 46½ cub ft. : 10½ cub ft. 314. 2075. 315. 21½ p c.
 316. A and B each takes 7½ days. C 157½ days 317. £200
 318. Rs. 24½ 319. 3s 9½d ; £1782. 320. 5 min , 10 hrs
 50 min 15 sec. 321. 7 ac. 2 ro 28 po 22 sq yds 5 sq ft
 113½ in 322. 2s 7½d. 323. Zinc 1512 tons,
 lead 1161 tons ; tin 327 ton
 324. Land 21½ ac ; iron 110 tons 6½ cwt. Int. = Rs 3090 13a 1½p
 325. 30 per min. 326. £212 5s 8½d. 29 327. £16
 328. Rs 180 329. 16580 330. 3 yrs. 8 mo 24 days
 331. 93324 332. 200. 333. 2880 334. 75
 335. 8 days, 12½ days 336. (i) 15660 (ii) 23434 (iii) 12702
 337. The 3½ per cents. ; 28½½ 338. £1 1s
 339. Rs 344. 7a 5 088p 340. (i) £1260 (ii) £31010
 341. Horse Rs 350 ; Cow Rs 120 342. 240 sq ft 343. Re 1 12a 4p
 344. 1 ft. 5 in 345. Man. Rs 8 5a 4p , woman Rs 2 11a 4p
 346. 104 boys ; 46 girls 347. 18 days 348. 234375 per cent.
 349. £30000 350. 12 per cent. 351. 780, 468, 520 acres
 352. £718 15s 353. A runs by ½ mile. 354. £15400
 355. 63 years 356. 60 p c. 357. £92
 358. £8 5s 4½d ; 7875 359. 52½ per cent. 360. Yes ; side of
 cube = 159 ft. side of square = 1944 in 361. 15 + 16 + 17 + 18

- 362 16 i 363 8 9 364 $8\frac{1}{2}$, $2\frac{1}{4}$
 365 0061 366 (i) $\frac{3}{4}$ d, 20 p c 367 9 gals
 368 (i) 2 hrs (ii) $1\frac{1}{4}$ hrs 369 $5\frac{1}{8}$ grs 370 £14440
 371 82, 820, 1053 372 $2\frac{1}{2}$ 373 90 miles
 374 Rs 8250 each 375 Rs 10210 4a $0\frac{1}{2}$ p 376 54 times
 377 B walks a mile in $13\frac{1}{4}$ min, he loses by $11\frac{1}{4}$ min, and by $\frac{1}{8}$ of a mile 378 45 days from the commencement
 379 One of the latter = two of the former 380 $\frac{1}{2}$
 381 999663, 100203 382 12 383 1015
 384 9 hours 385 90 quarters 386 6
 387 3 ft $11\frac{1}{8}$ in 388 Rs 112 6, 389 $7\frac{1}{2}$ miles
 390 $13\frac{1}{4}$ min or $16\frac{1}{4}$ min past 3 391 13221, 12231, 15201, 10251, 14211, 11241, 15291, 16281, 17271, 18261, 19251 392 1057
 393 (i) $6\frac{1}{8}$ (ii) 9 394 $1\frac{1}{2}$ ft 395 15 masters, 345 boys
 396 11 2 397 £6000 398 25
 400 Length 27 ft, breadth 18 ft, height 12 ft
 401 $2^4 3^3 5^2 7 11$, 118580 402 Rs 4800
 403 8 miles per hour 404 Rs 162 8a 405 £15 15s
 406 660 ft, 495 ft, area = $7\frac{1}{2}$ ac 407 35 days
 408 One orange 6p, one peach 3p, one mango 12p or 1a
 409 136801 cub yds 410 £94 $\frac{1}{2}$ 411 Multiplier 4253
 412 45 times 413 3 414 Rs 27 15a $0\frac{1}{4}$ p
 415 25 p c loss 416 30 min $41\frac{1}{2}$ sec 417 $8\frac{1}{2}$ days
 418 12 ft $7\frac{1}{2}$ in 419 $3\frac{1}{2}$ p c, £275 10s 420 6 mo
 421 48 days 422 Brus Kid 9 8 423 11s 3d, 5
 424 A ought to pay Rs 34 2a 8p, B Rs 44 2a 8p, C Rs 50
 Rent per acre per annum = Rs 55 9a $9\frac{1}{2}$ p
 425 A's Rs 1295, C's Rs 3885 426 £6 15s
 427 A $10\frac{1}{8}$ miles, B $10\frac{1}{8}$ miles 428 18 ft 429 $8\frac{1}{2}$ in
 430 Thrice 431 45 sq miles 432 230 stones 433 $3\frac{1}{2}$ days
 434 20 yds, 60 coolies 435 Dist 150 miles, rate 25 miles per hr
 436 $16\frac{1}{2}$ hours 437 50 438 Rs 3 $9\frac{1}{8}$ a 439 Rs 3,
 Rs 2 8a, Rs 2 440 17s 6d 441 28284 mile
 442 8 yds 443 Rs $38\frac{1}{2}$ 444 £4800
 445 1 8 446 $9\frac{1}{8}$ d 447 33 24 in 448 $49\frac{1}{2}$ p c
 449 (i) $2\frac{1}{4}$ min (ii) $2\frac{1}{8}$ min 450 Rs 489 10a 8p
 451 13212 452 640 and 350 453 6 454 25
 455 52 days 456 Rs 3312 + Rs 219 457 Rs 7678 2a, 10a 2 85p
 458 2 seconds 459 500 cub ft 460 $25\frac{1}{2}$ miles per hour

Problems (pp. 558-582)

- 1 £275 2 $\frac{1}{13}$ of a mile 3 Rs 400 4 1166 $\frac{1}{2}$, 1169, 1000, 1002
 5 397 yds per min 6 4a 8p, 8p 7 £2394 8 £1000
 9 Rs 9 7a 3p 10 $36\frac{1}{2}$ miles per hour, 37 min past 8 A M
 11 Rs 4942 $\frac{1}{4}$ 12 $23\frac{1}{2}$ months, nearly 13 £2 per cent.
 14 Rs 2933 5a 4p 15 18 miles per hour 16 The first
 17 360 gals, gain 1 gal. per hour 18 £125000 19 10 per cent.

- 20 £463 1s 21 $6\frac{2}{3}$ min 22 $3\frac{1}{2}$ min , 10 miles an hour.
 23 3 231 in 24 $1437\frac{1}{2}$ bricks 25 £1000
 26 £322, £627 4s, £2060 16s 27 3016 28 $34\frac{1}{2}$ miles
 29 8s 4d 30 $998\frac{2}{3}$ oz 31 $8\frac{1}{2}$ min 32 $16\frac{2}{3}$ miles
 33 1 o'clock 34 6 hr 30 min P M 35 £2000
 36 50000000 quarteils 38 £5 17s 39 Re 1 8a , Re 1 , 12a
 40 Rs 80 41 51 men 42 Gain £2 16s
 43 7496192000 cub yds 44 65 per cent 45 12 weeks
 46 $10\frac{1}{6}$ hrs 47 291600 48 Rs 6000 49 55 5 yds nearly
 50 Rs 850 , 26 p c 51 7 men 52 Rs 95 53 Rs 144
 55 8960 in 56 A's cap Rs 450000 , B's cap Rs 300000 ,
 A's profit Rs 19500 , B's Rs 13000 57 112 days
 58 A $16\frac{1}{2}$ rounds , B 17 rounds , 405 hrs 59 5 gals
 60 Express 2 hrs 13 min , Ordinary 3 hrs 10 min
 61 1300 , no in stalls 300 , boxes 500 , pit 400 gallery 100
 62 A 4s $2\frac{1}{2}d$, B 6s $7\frac{1}{2}d$ C 1s 9d 63 26 men
 64 £38 65 Rs $549\frac{1}{10}$ 66 18942 pipes
 67 $4\frac{1}{2}$ min spaces 68 $16\frac{2}{3}$ min 69 123 2 grs
 70 10 30 P M yesterday , 10 30 P M , 45 days later
 71 133 72 12 hrs 73 B , $3\frac{1}{2}$ 74 £19 14s 3d
 75 3375 dollars 76 3s 4d and 5s 4d 77 £121
 78 $4\frac{1}{2}$ days 79 Equal 80 9 miles 1035 yds , 9 miles
 660 yds 81 15 min 15 sec past 11 82 £26 7s $7\frac{3}{4}d$
 83 $2\frac{1}{11}$ sec 84. (i) 207 yds (ii) 1207 yds 85 Rad of inner
 circles = 31 0813 ft , rad of 2nd circle = 46 622 . ft
 86 253 377 87 $\frac{25}{28}$ 88 4 mds 8 sr 89 1000 ft 90 8s
 91 135 days 92 121 yds , $6\frac{1}{2}$ sec
 93 15 men , 18 women , 27 children 94 14s 6d and 15s 6d per gal
 95 8 horses 96 $2\frac{1}{2}$ miles 97 352 ft slower , 264 ft faster
 98 2 men 99 £1334 11s $9\frac{1}{2}d$ 100 £1782 101 12 days
 102 10 ft 103 $7\frac{1}{2}$ seers 104 15 lbs 10 oz
 105 no of fruits, 288 , no of two anna pieces, 287 106 230 min
 the boys bringing pint and gallon coming up 1 min too late
 107 $11\frac{3}{8}$ sec 108 14 076 min , $1\frac{5}{8}$ of the cistern
 109 Rs 12600 110 $\frac{1}{2}d$ 111 Rs 8 , Rs 16 , Rs 24
 112 £72 6s 8d 113 Rs 8 , Rs 16 114. $10\frac{1}{2}$ pies
 115 £556 10s 116 21 ft 117 First class, Rs 5 , 2nd class
 Rs 3 5a 4p , rate per mile = 1a 4p 118 10 miles
 119 Rs 400 120 Rs 387 8a 121 12 min before 9 o'clock ,
 12 hours 24 min 122 Rs 106666 10a 8p 123 Rs 18984 6a
 124 A Rs 162 , B Rs 324 , C Rs 405 , D Rs 486 , E Rs 648
 125 2 ft 126 $1\frac{1}{2}$ yds 127 22 miles 128 £5565 129 $10\frac{1}{2}$ in
 130 Rs 5376 , Rs 3584 131 $7\frac{1}{2}$ months 132 $9\frac{1}{2}$ miles per hour
 133 £1 11s 3d 134 14 cwt 3 qrs 13 lbs , 9 cwt 3 qrs 18 lbs ;
 8 cwt 3 qrs 19 lbs 135 6 min 6 08 sec
 136 6 miles per hour 137 £9309 $\frac{1}{11}$, £271 $\frac{1}{11}$, £101 $\frac{1}{11}$
 138 £264 6s 8d 139 Gold $46\frac{7}{8}$ lbs , silver 64 lbs , copper $6\frac{3}{8}$ lbs

- 140 Width = 13 ft 2 in , length = 271 ft 5 in , height = $14\frac{177753}{111108}$ ft.
 141 (i) 2hrs 37 $\frac{1}{2}$ min (ii) 2hrs 11 $\frac{1}{4}$ min , 3hrs 3 $\frac{1}{2}$ min 142 37800
 143 Rs 1000 144 14 years 145 For ploughing the field
 with oxen, £4 7s 9d , for ploughing with horses, £3 18s 9 $\frac{1}{2}$ d.
 146 Rs 127 8a 147 £60 , 7 $\frac{1}{8}$ p c 148 A Rs 10500 ,
 B Rs 7000 , C Rs 7350 149 6 miles 150 15s 6d
 151 15 oxen 152 £150 15s 153 £97826 $\frac{2}{3}$, £4591 $\frac{1}{8}$
 154 7 $\frac{1}{2}$ hours 155 62 hrs 33 min 45 sec , 48 min 45 sec
 156 25 min 18 $\frac{1}{4}$ sec 157 3 hrs 25 min
 158 They first cross at $\frac{3}{4}$ mi from starting-post, A going towards
 it ; they next cross at $\frac{5}{8}$ mi from starting post, A going from
 it , they next cross at $\frac{1}{2}$ mi from starting-post, A going
 towards it , and so on 159 1 $\frac{1}{2}$ miles , 2 hrs 160 2a
 161 £147 162 First 11 $\frac{1}{2}$ p c , 2nd 8 $\frac{1}{4}$ p c , 3rd 8 $\frac{1}{4}$ p c .
 163 10 times , 1 $\frac{1}{4}$ hrs or $\frac{3}{2}$ hr 164 Rs 525 165 10 in
 166 5 $\frac{1}{8}$ miles from Dover , 6 o'clock 167 4 miles 73 $\frac{1}{2}$ yds
 168 (i) A's work B's work 3 2 , A's rate B's rate 2 1
 (ii) A's work = 0 , A does nothing
 169 £1 10s 11 $\frac{1}{2}$ d 170 13 20 171 19 miles
 172 £1640 12s 6d 173 266 $\frac{2}{3}$ per cent 174 1114 4826 oz
 175 Rs 2170 176 13 times 177 A 90 min , B 72 min , C 60 min.
 178 At the summit 179 £2000 180 £7 11s 3d
 181 20 miles an hour 182 94 $\frac{1}{2}$ miles from Arconum
 183 200 years 184 First day 1 $\frac{1}{8}$ d , second day $\frac{1}{16}$ d
 185 75 days 186 1 mile , 6 5 187 (i) £5 5s (ii) 30 $\frac{1}{2}$ days.
 188 Rs 4000 189 Length = 1 ft 11 in , breadth = 1 ft 5 in ;
 depth = 1 ft 1 in , 2236 cases 190 90 $\frac{1}{8}$ days
 191 A Rs 60 , B Rs 70 , C Rs 84 , D Rs 105 , E Rs 101
 192 A and C each travels 8 miles per hour
 193 24 men, 20 women and 15 boys 194 1 hr 21 min 18 $\frac{1}{2}$ sec.
 195 4 $\frac{1}{2}$ years 196 (i) 48 hrs (ii) 54 hrs 197 1 $\frac{1}{8}$
 198 Rs 180 199 27 $\frac{2}{3}$ days 200 Indian, Rs 4500 201 12 cub ft.
 202 20 and 30 miles per hour 203 30 $\frac{1}{2}$ and 17 $\frac{1}{2}$ miles per hour

EXAMINATION PAPERS

CALCUTTA ENTRANCE PAPERS

- 85 1 $\frac{1}{2}$, $\frac{87}{10}$ 2 12 , 2 , 30472 3 3461538 , £1 10s
 4 £513 6s 6 $\frac{1}{2}$ d , 3 1225 , 2828 5 18 , 8 $\frac{1}{8}$ per cent
 6 The first investment is better , £1342 10s 31 $\frac{6}{7}$ per cent
 86 1 $\frac{87}{10}$ 2 $\frac{308}{1000}$ 3 $\frac{1}{8}$, $\frac{210}{100}$ 4 5 , 001136
 5 £36 17s 6d 6 R28659 6a 7 R12-12-9 $\frac{1}{2}$, gain R133 5-4
 87 1 (a) $\frac{1}{8}$, (b) 350 2 0203125 3 (a) £17-12-2 $\frac{1}{2}$, (b) R2000
 4 10 5 R510 6 13 31 , 471
 88 1 $\frac{1}{12}$ 2 11200 , 37 96 3 138 4497 $\frac{1}{2}$, £20 16s 9 $\frac{1}{10}$ d
 4 £1034 14s 4 $\frac{1}{8}$ d 5 15 $\frac{1}{8}$ days 6 6 $\frac{1}{2}$, £100

- 89 1 51 59139412 2 862126 3 £5247 2s 6 $\frac{1}{2}$ d
4 1000127 5 £6705 14s 7d
- 90 1 3, R23931 7a 7p 2 7305 40s, 1 $\frac{1}{2}$ h 3 R1771
4 60 days 5 R104 4a
- 91 1 (a) $\frac{1}{2}$ h, (b) $\frac{1}{4}$ 2 2 202642 3 R408 3a 41 $\frac{1}{2}$ p
4 91 $\frac{1}{2}$ hrs 5 R19992 6 8 $\frac{1}{2}$ yds
- 92 1 1 $\frac{1}{2}$ h 2 26219 3 312, 098, 998
4 R1232 14a 0 $\frac{2}{3}$ h 5 £2500
- 93 1 (1) 51 $\frac{1}{2}$ (2) 3 2 0789, 2 $\frac{1}{2}$ h, 2 $\frac{1}{2}$ h, 1 3 £345 7s 3 $\frac{1}{2}$ d
4 R238 3a 2 $\frac{1}{2}$ p 5 R90000, R73000
- 94 1 £37 or 8 $\frac{1}{2}$ d 2 £491 8s 3 16s 0 375013d
4 9998 5 R6 each 6 R124 10 11 $\frac{1}{2}$ h
- 95 1 100001 2 R12345 3 3 fr 84 centimes
4 5 $\frac{1}{2}$ 5 R47, increase, 6832876712
- 96 1 23704543, 8143 2 (i) $\frac{1}{2}$ (ii) 075088 3 2 2677 4
4 R531 3a 10 064p 5 Loss, 16 p c 6 R21735
- 97 1 0725 (a) $\frac{1}{2}$ 2 Yes, $\frac{1}{10}$ part, Rs 32 9a 1 $\frac{1}{2}$ p
3 20 4 33 $\frac{1}{2}$ years 5 Rs 6 6 17724
- 98 1 20150 2 (a) $\frac{1}{2}$ (b) 083 3 234, 8 06007
4 Rs 250 5 1 $\frac{1}{2}$ per cent gain 6 Rs 23400
- 99 1 25 2 (a) 1 $\frac{1}{2}$ 3 Rs 606 11a 9 $\frac{1}{2}$ p
4 226, 226 5 60 mds, 31 $\frac{1}{2}$ p c 6 Rs 18
- 1900 1 42 min 2 (a) 6 (b) $\frac{1}{2}$ 3 8
4 £55 10s 3 $\frac{1}{2}$ d 5 125 6 £100 gain
- 1901 1 (a) 1 416 (b) 565 2 (a) Yes, $\frac{1}{10}$ (b) £68 15s 9d
3 4 $\frac{1}{2}$ lbs 4 (b) 4 p c 5 86 42 6 Rs 122169
- 1902 1 (a) Terminating decimal (b) $\frac{1}{2}$, 036
2 Rs 15326 10a 8p, Rs 7340 3 35 boys
4 2 $\frac{1}{2}$, 8 729 5 (b) £100 6 4 p c, Rs 60
- 1903 1 (a) 1 (b) 000568i 2 (a) Yes, $\frac{1}{10}$ (b) £170 19s 4 $\frac{1}{2}$ d
3 1 min 40 sec 4 (b) 3 $\frac{1}{2}$, 1 7728 5 $\frac{1}{2}$ gal 6 (b) Rs 50
- 1904 1 (a) 997920 2 1 $\frac{1}{10}$ or 0, 1 $\frac{1}{10}$, 1 $\frac{1}{10}$ or 0, 1 $\frac{1}{10}$, 1 $\frac{1}{10}$ or 0,
1 $\frac{1}{10}$ (a) $\frac{1}{2}$ 3 22 $\frac{1}{2}$ days 4 396 poles 5 £700
6 Rs 151710
- 1905 1 165 or 154, 176, 187, 2 (2) $\frac{1}{2}$ 3 4 4666, 7905
4 Rs 4 1a 5 Rs 110 7a 2 $\frac{1}{2}$ p, 3 per cent 6. £3000
- 1906 1 (2) 99679 2 (2) (1) 1 (2) 2 3 Rs 17 2a
4 16, 7745 or 7746 nearly 5 17s 6d 6 11 $\frac{1}{2}$ p c
- 1907 1 37128 2 (1) $\frac{1}{2}$ (2) $\frac{1}{2}$ 3 Rs 46 11a 10 $\frac{1}{2}$ p
4 110400 5 Rs 760 8a 6 13s 3 $\frac{1}{2}$ h
- 1908 1 (2) Re 1 8a 2 £934 18s 2d 3 (1) 11 $\frac{1}{2}$ min;
(2) 7 $\frac{1}{2}$ h yds 4 (1) 2 $\frac{1}{2}$ h (2) 5345
5 Rs 1000 6 Rs 425
- 1909 1 2865149859, 385671 2 (1) 7 (2) 65
3 Rs 939 13a 6p III 371 173 4 Rs 1228 2a
IV £588 10s 10d 5 1947, 3, 11, 59, 33, 177, 649
V 20 p c

Supplementary Examination

- 1 786522510, 12837 2 (a) $\frac{4}{1}$ (b) 10 58 $\frac{3}{4}$ 3 Rs 8833 5a 4p
 III Rs 1080 4 12 IV 8 $\frac{1}{2}$ yrs 5 13579 V Rs 300

MATRICULATION EXAMINATION

1910

Compulsory Paper

- 1 326137981294, 756445 I 504, 17280 2 (1) 15 $\frac{7}{8}$
 (2) 009 II 40 men 3 (1) Rs 3816 10a 8p
 (2) 16 $\frac{2}{3}$ yrs III 2 115 cubic inches

Additional Paper

- 1 2501317 I 15 ft 2 (1) 1718 (2) 24855 2967

1911

Compulsory Paper

- 1 620526133723107, Quot 621225011, rem 860482 (Alt) Rs 610
 2 (1) 1 (2) 0052083 of a rupee (Alt) 00027 of an hr
 3 (1) Rs 31 15a 5p (2) £227 12s (Alt) 180 men

Additional Paper

- 1 469246 (Alt) 54 0321 (Alt) 579
 2 (1) 3 14159 (Alt) 78539 (2) 28

1912

Compulsory

- 1 573299983476, 3276 (Alt) 11 $\frac{3}{8}$
 2 (1) $\frac{1}{2}$, (2) £12 13s 2d (Alt) (1) 3 $\frac{1}{4}$ p c,
 (2) Rs 442 7a 7 $\frac{1}{2}$ p
 3 25 men (Alt) 2624 sq ft, Rs 95 10a 8p

Additional

- 1 371 173 (Alt) 117 ft 2 24855 (Alt) 54925

1913

Compulsory

- 1 (1) 38590032276, (Alt) 6907, Rem 587
 (2) 504, (Alt) 1890
 2 (1) $\frac{1}{8}$ (2) 2907 (Alt) (1) 0015625, (2) Rs 2362 8a
 3 (1) 3 p c (2) 28 yds

Additional

- 1 5403, (Alt) Rs 366 10a 8p
 2 239 197, (Alt) 71, 73, 79, 83, 89

1914

Compulsory

- 1 278523 $\frac{2}{3}$ 2 247, (Alt) 2160
 3 1 $\frac{63223}{67520}$, (Alt) Rs 14650 3a 6 $\frac{1}{2}$ p
 4 45 408d (Alt) Rs 564 5a 4p
Additional

- 1 2 645 2 20 $\frac{12560620}{15499969}$

1915

Compulsory Paper

- 1 (1) 75154060188 (Alt) 7908 (2) 501 (Alt) 28000
 2 (1) 2 (2) 70 2702 and 85 8 (Alt) (1) 11 938461 $\frac{1}{2}$
 (2) Rs 5615 9a 3p
 3 (1) 10 p c (2) Rs 3072

Additional Paper

- 1 13 057 2 15499969 slabs (Alt) 41937

1916

Compulsory Paper

- 1 (1) 553535194285 (Alt) 83205 (2) 119 (Alt) 2520
 2 (1) 1 (2) 00416 (Alt) (1) 1035546875 (2) £173 8s.
 3 (1) 12 $\frac{1}{2}$ p c (2) 60 men

Additional Paper

- 1 06435 2 09728381300571428 (Alt) 1 732

MADRAS MATRICULATION PAPERS.

- 85 1 1 2 $\frac{9}{12}$, Rs 64 3 £1-6 0 $\frac{1}{2}$ 4 £19-3-10 $\frac{1}{2}$
 5 11s 10 $\frac{1}{2}$ d 6 R1920 7 3s 9d 8 4 yrs
 9 £5000 10 9196, £16 10s 11 3500000
 86 1 1 2 9705 3 £2-11-7 $\frac{1}{2}$ 4 Rs 3955-3-11
 5 1 13 P M 2nd July 6 80 men 7 £1000
 8 £180 9 R17 8a 10 520344000 cub ft, 1 $\frac{1}{2}$ in
 88 2 $\frac{1}{2}$ 3 1a 4p, 114583 4 £721-15-6 $\frac{1}{8}$
 5 R335000 6 £416 13s 4d 7 R7 2a
 8 Increase, R502 8a 9 R500000 10 500400

- 89 2 $\frac{241}{81}$ 3 08273029, 6s 9 $\frac{1}{2}$ d 4 R1730-13-6 $\frac{1}{2}$ 5 R48 2a
 6 £1694-13 9 7 R280000 8 10 days
 9 7500274 10 £39 3 9 11 2 08008
 90 1 342 ac 2 ro 39 po 2 sq ft. 36 sq in, 160 yds 2 1 5
 3 R975358 9 2 $\frac{1}{2}$ 4 30 wks 5 R6744273 6 4 mo
 7 Increase, £397 8 12 cwt 1 qr 19 lbs 4 oz, £33 2s 6d
 9 343 169 10 19487 171
 91 2 R1-11-8 3 R1-10 2 $\frac{1}{2}$ 4 9, 46 947 177
 5 12 min 6 R6 6-4, R158 7 £291-9-5 $\frac{1}{2}$ nearly
 8 20 min (afternoon) 9 10d 10 R9180
 92 2 $\frac{2}{3}$ 3 5s 3d, 0037115625 4 R67567-9-7 $\frac{1}{2}$
 5 £416 13-4 6 3700965 7 3221625 tons
 8 R355-13 4 9 R55 8-4 10 3 $\frac{3}{8}$ p c 11 25640000
 94 2 $\frac{1}{2}$ 3 5s 2 $\frac{1}{2}$ d 4 R1593 4a 5 6p
 5 R93333 5a 4p 6 £976 11s 3d 7 R23 12a 4p
 8 4 $\frac{1}{2}$ mi per hour 9 R1062 10 14 1625 p c.
 11 9 0073210

BOMBAY MATRICULATION PAPERS

- 85 86 1 4857142 2 113 boys 3 1 $\frac{1}{2}$ cwt, £1 1s 6d
 4. £72 6s 8d 5 In the latter, £457 5s 1 $\frac{1}{2}$ d
 86 87 1 5,7,11,13, 22 99998912084530386257958 2 192 $\frac{1}{2}$ ft
 3 $\frac{1}{2}$ 4 (1) 7 ft 2 in (11) 3 5752 5 £5103 6 31 $\frac{1}{8}$
 87 88 1 6 2 19s 3d 3 20 months 4 20 7, 5s 1 $\frac{1}{2}$ d 5 5a 4p
 88 89 1 10d, 4s 3 $\frac{1}{2}$ d 2 8s 4d 3 108s
 4 94298 11 5 4 p c
 89 90 1 2 2 5 $\frac{1}{2}$ ft x 5 $\frac{1}{2}$ ft x 5 $\frac{1}{2}$ ft 3 5-15'
 4. R32000 5 3 13
 91-92 1. (i) $\frac{1}{8}$ (ii) 8 $\frac{3}{4}$ 2 100 lbs, 2 cwt, 3 cwt
 3 10 30 P M yesterday, 10 h 30' 50' 4. R640
 92 93 1 0050208 $\frac{3}{4}$, 15a 7 44p, $\frac{1}{7}$ 2 10 3 3 $\frac{1}{2}$ hrs
 4 £259 3s 5 $\frac{1}{2}$ d 5 401 544
 93-94 1 20577, 39690, 51844293272864701436130007
 (Mof) 2 £11 11s 6 $\frac{1}{2}$ d 3 2 cwt 2 qrs 20 lbs 4 £83 6s 8d
 5 12 $\frac{1}{2}$ hrs 6 4 lbs of the infr to 5 lbs of the supr quality
 93 94 1 (i) 24, (ii) 11 $\frac{1}{8}$ 2 £32 14-3 $\frac{1}{2}$ 3 31 $\frac{1}{8}$ mo
 (Bom) 4 £3 2 0 $\frac{3}{4}$ 5 $\frac{1}{4}$ 6 2 P M, 23 Aug, 1-46', 2-16'
 94-95 1 146097 days (actual 11 days less) 2 156 3 30
 4 1 $\frac{1}{8}$ days 5 R2160 6 7 72, 15a 3p

PUNJAB ENTRANCE PAPERS

- 85 1 1, 123 times 2 08125, 0003, 53896i
3 R884 15a 3b 4 59 15s
5 2963520, 2420 sq yds 6 The latter, R49000
- 86 1 375, 612 2 7895 3 16 4. She loses
5 Rs 195 13a 0 96b, Rs 172 8a 5 12-7
- 87 1 (c) 3 025 2 133 6s 8d 3 R9 6a 21p a md
4 Rs 480, Rs 405, Rs 336, Rs 297 5 Rs 11250 6 04
- 88 1 12 2 0625 3 16 days 4 16 108 p c 5 36 284
- 89 1 322083, 799 2 2 1/2 miles 3 37651, 2 min 6 sec
4 R14 8a. 62p, the latter 5 3/3 is the greater
- 90 1 (a) 1, (b) 03 2 1/3, 390625 3 1274
4 8320 men 5 R863 1/4 6 2p
- 91 1 (i) 1-1/2 (ii) 11 2 00622 3 10 years
4 200, 5 yrs 5 45 gals
- 92 1 1/11, 1/11 2 5 90625 3 218972 16 gals
4 10166 2/3, 6000 5 17s 3d
- 93 2 13 713729902 3 41421 4 R17-5 9 1/2 5 32 mi
- 94 1 571428, 42847i 2 44 ft, 33 ft 3 511
4 R987-10 5 1/2, R987 8s 5 10 04987
- 95 1 1 1/2, 1 0714285 2 1 25 lbs 3 1391, 1529, 1311.
4 2 2360680 5 76 2 yds, 38 1 yds
- 96 1 Rs 63 102 2 1/2 3 2 2136 lbs
4 11400000 ac, 17100000 ac, 3800000 ac 5 Rs 3930
- 97 2 Men 63, women 60, children 54 3 2 4142136
4 14 1/2 weeks 5 540
- 98 1 2 thousand million, 9 hundred thousand, 9 ten thousand,
1 hundred, 2 tens, 5 units 107 0129615898 2 Monday
3 Rs 66 10a 8p 4 1 5 Rs 66 10a 8p gain
- 99 1 764 (a) 102 1916349780924038 2 6 yds
4 4 sov, 6 h cr, 22 sh 5 The 3 1/2 per cents
- 1900 1 2000 301 2 Rs 7352 15 1/2 a 3 10 yds by 22 yds
- 1901 1 7 hrs 2 5 or 4d 3 32490, 20577, 1 579 .
4 12 per cent 5 79 49
- 1902 1 3 5 7 11 13 37 2 5760 3 Rs 4800
4 Rs 2 7a 9p 5 15 per cent

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- 89 1 1 384615 2 (a) 31 1/2 (b) 0003, 0029644268
3 0316, 01 4 R1300 5 12 min 40 1/9 sec
6 8 days 7 39 1/2 miles from the starting place

- 90 1 697684787, 36 62387 $\frac{3}{4}$ 2 $\frac{1}{4}$ 3 R8600 13a 10 $\frac{1}{2}$
 4 99999 5 2 115 cub in 6 11 $\frac{8}{11}$ in
- 91 2 $\frac{1}{8}$ 3 62 $\frac{1}{8}$ gals 4 5 5 9999, 7 $\frac{1}{8}$
- 92 2 (a) 12 (b) $\frac{428}{883}$, $\frac{7}{4}$, 846714 3 56 $\frac{1}{2}$ days
 4 £3 5 10001
- 93 2 2 fur 12 po 1 yd 3 R3 1a 6p 4 £14 1s 3 $\frac{1}{2}$ d
 5 £350 11s 8d 6 1869 7 79 032, 8 $\frac{1}{8}$
- 94 1 (a) 999 \times 807 = 806193 (b) -1 2 (a) 0009, (b) 5 0596
 3 444 mi 4 R555
- 95 1 (b) 4 feet square 2 (a) 1 $\frac{511}{8180}$, 17724 3 Rs 5700
 4 Rs 7 6a 11 $\frac{1}{4}$ p and Rs 4 15a 3 $\frac{1}{4}$ p 5 150 yds
- 96 1 (a) 1 $\frac{1}{10}$ (b) 25 2 (a) 0203125 (b) 200 001
 3 178 hrs 52 min 30 sec 4 7 $\frac{1}{2}$ miles 5 Rs 2800
- 97 1 47, 127041 2 28 $\frac{3}{4}$ 3 £291 17s 6d nearly
 4 £315 5 2203 90625 francs
- 98 1 (i) 2a (ii) 1 $\frac{1}{2}$ (a) 29 2 (a) 33 $\frac{1}{182}$ (b) -4 $\frac{1}{2}$
 3 3 $\frac{1}{2}$, 001 4 £1 9s 4 $\frac{1}{8}$ d 5 £125
- 99 1 $\frac{1}{4}$, 63 2 42 68 3 £425 1s 5 $\frac{253}{120}$ d 4 60 days
 5 2 2360, 7071 6 3 $\frac{148}{108}$ d 7 (a) 25 per cent loss (b) 80
- 1900 1 0067143 2 (a) 2995670 (b) $\frac{1}{10}$ 3 (a) Rs 197 12a 0 $\frac{1}{2}$ p
 (b) 3 165 4 Rs 3200, Rs 3889 62 5 Rs 289 $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{8}$
- 1901 1 (a) 1 $\frac{1}{10}$ ft (b) £8 1s 2 (a) 3 $\frac{3}{8}$ (b) 03391
 3 160 yds 4 Rs 625 5 Rs 16800
- 1902 1 G C M = 000279, L C M = 25234713
 2 44 3 Rs 3285 4 A 48, B 84
 5 7 $\frac{1}{8}$ days 6 8 $\frac{1}{8}$ years
-

APPENDIX

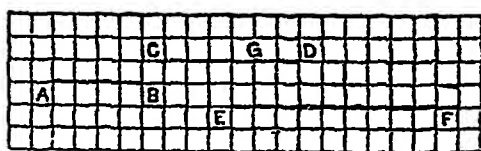
GRAPHS

1 In this section a few arithmetical sums will be solved graphically

Examples worked out

Ex. 1 Find the G C M of 4, 6 and 10 graphically

Let AB represent four units of length, CD six units and EF ten units of length

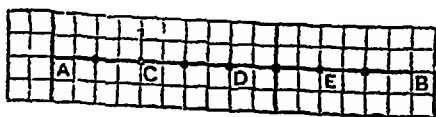


First, place AB along CD and let CG be equal to AB. Then it is evident that GD represents two units of length. Place GD along AB successively. Since AB denotes four units of length, therefore AB contains GD exactly twice. Similarly place GD along EF and it is evident that EF contains GD an exact number of times. Hence GD which represents two units of length is the G C M of AB, CD and EF, that is, the G C M of 4, 6 and 10 is 2.

(NB—From the solution it is clear how to divide an arithmetical sum graphically)

Ex. 2 Shew graphically that $\frac{2}{3} = \frac{4}{6}$

Let AB represent the unit of length. Divide AB into four equal parts. Then from the fig it is evident that $AE = \frac{2}{3}$ of AB. Again



let each of AC, CD, DE and EB be divided into two equal parts, then AB contains eight of these parts. But AE contains 6 of these parts

$$\therefore AE = \frac{6}{8} \text{ of } AB \quad \text{But } AE = \frac{2}{3} \text{ of } AB$$

$$\therefore \frac{2}{3} = \frac{6}{8}$$

Ex 3 Shew graphically that $\frac{2}{5} + \frac{1}{3} = \frac{6+5}{15} = 1\frac{1}{3}$ and $\frac{2}{5} - \frac{1}{3} = 1\frac{1}{3}$

Let AB represent the unit of length. Divide it into five equal parts and also into three equal parts



Therefore $AY = \frac{2}{5}$ of AB and $AX = \frac{1}{3}$ of AB. Now divide AB into fifteen equal parts and take YZ and YZ' equal to AX on the opposite sides of Y.

$$\therefore AZ = AY + YZ = AY + AX,$$

$$\therefore 11\text{-fifteenths} = 6\text{-fifteenths} + 5\text{-fifteenths}; \text{ i.e. } 1\frac{1}{3} = 1\frac{0}{3} + \frac{5}{3},$$

$$\therefore \frac{2}{5} + \frac{1}{3} = 1\frac{1}{3}$$

Similarly,

$$AZ' = AY - YZ',$$

$$\therefore 1\text{-fifteenth} = 6\text{-fifteenths} - 5\text{-fifteenths},$$

$$\therefore \frac{2}{5} - \frac{1}{3} = \frac{1}{15}$$

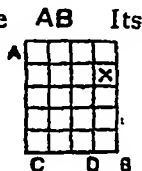
Ex 4 Shew graphically that $\frac{3}{4} - 5 = \frac{3}{20}$

Suppose the unit is represented by the rectangle AB. Its length contains five equal parts and breadth four equal parts

Then the rect is divided into 20 equal parts as shewn in the fig

Now $\frac{3}{4}$ is represented by the column AC, $\therefore \frac{3}{4}$ of the unit is represented by AD, $\therefore \frac{3}{4} - 5$ is represented by AX and it contains three of such equal parts,

$$\therefore \frac{3}{4} - 5 = \frac{3}{20}$$



(NB — Similarly multiplication of fractions can be graphically solved)

Examples I.

1 Add together 3, 5, 7 and 9, 4, 8, 12 and 20, 125, 140 and 175

2 Represent graphically the result of —

$$15 - 7, 25 - 10, 149 - 100, 1225 - 525$$

3 Multiply graphically — $3 \times 5, 7 \times 9, 10 \times 15; 2 \times 5 \times 3$

4 Divide graphically —

$$15 \text{ by } 5, 33 \text{ by } 9, 289 \text{ by } 17; 510 \text{ by } 115$$

5 Find graphically the G C M of — 3, 12, and 21, 12, 20 and 24, 35, 49 and 77, 125, 275 and 325

6 Establish the truth of the following graphically —

$$\frac{1}{2} = \frac{1}{2}, \frac{1}{4} = \frac{1}{4}, \frac{1}{6} = \frac{1}{6}, \frac{1}{11} = \frac{1}{11}, \frac{1}{13} = \frac{1}{13}$$

7 Simplify the following graphically —

$$(1) \frac{1}{2} + \frac{1}{8}, \frac{1}{2} + \frac{1}{4}, \frac{1}{11} + \frac{1}{13}, \frac{1}{17} + \frac{1}{19}, \frac{1}{7} + \frac{1}{3}$$

$$(2) \frac{5}{6} - \frac{1}{2}, \frac{3}{4} - \frac{1}{8}, \frac{5}{6} - \frac{1}{6}$$

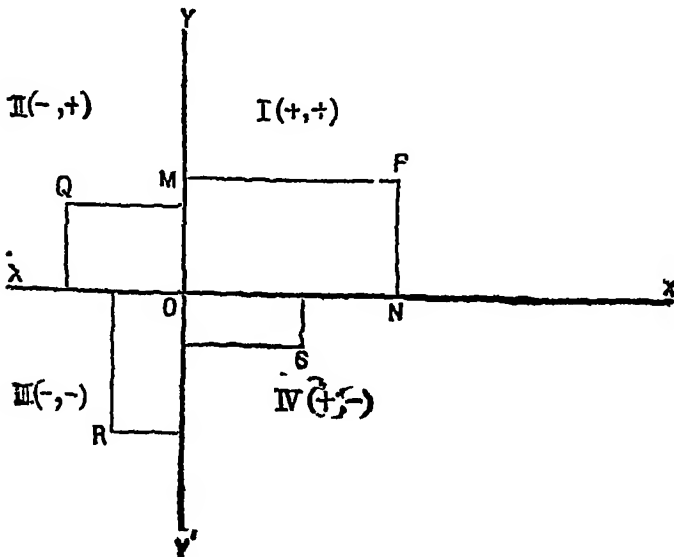
$$(3) \frac{2}{3} \times \frac{1}{2}, \frac{1}{2} \times \frac{1}{3}, \frac{2}{3} \times \frac{1}{3}, \frac{1}{3} \times \frac{1}{2}$$

$$(4) \frac{2}{3} - 5, \frac{1}{6} \div 3, \frac{1}{11} - 4, \frac{1}{2} - 3$$

2 APPLICATION OF GRAPHS.

Axes of Co ordinates

Let the horizontal line XOX' be cut at right angles by the vertical line YOY' at the point O , thus dividing the plane in which they are into four equal spaces called *Quadrants*, viz XOY , YOX' , $X'OY'$ and $Y'OX$ respectively



Take any point P in their plane and draw PM and PN parallel to OY and OX respectively

Let PM or $ON = x$

and PN or $OM = y$

then for P x and y are fixed

The numbers x and y are called the *co ordinates* of the point P, the lines XOX' and YOY' are called the *axes of co-ordinates* and generally the *axes*, they are the lines of reference and are respectively called the *axes of X and Y*

The point O is called the *origin*, x is called the *abscissa* and y the *ordinate* of the point P. It is common in graphs to denote a point by its co ordinates. Hence in this case the point P is described as the point (x, y)

The values of x are measured from O along the axis of x , the values are *positive* when drawn to the *right* of O along OX and *negative* when drawn to the *left* of O along OX'. The values of y are *positive* when drawn *above* OX' and *negative* when drawn *below* OX'.

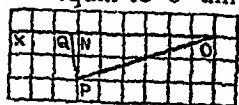
Thus a point $(5, 7)$ is represented by first taking ON equal to 5 units along the axis of x to the right of O and then by drawing a perp at N above OX and measuring in it PN equal to 7 units of length. The pt required is the pt P.

(For a fuller description see graphs of Algebraical Functions)

Examples worked out

Ex 1 A pedestrian walks westward 6 miles and then turns south and walks 2 miles, how far is he from the starting pt ?

Suppose O be the starting point. Measure ON equal to 6 units of length along OX'. Draw NP perp to and below the line OX', and cut off from it NP equal to 2 units of length. Then evidently P is the halting place. Join OP and with centre O and radius OP describe a circle cutting OX' at Q. Then $OP = OQ$. But $OQ = 6.3$ approximately. Therefore he is 6.3 miles away from the starting place.



Ex 2 Given that 1 kilogramme = 2.2 lbs. Draw a graph which will enable you to read off any number of lbs in kilogramme (up to 50 lbs), and read off the value of 25 kilogrammes in lbs and of 32.5 lbs in kilogrammes.

Measure lbs along OX and kilogrammes along OY

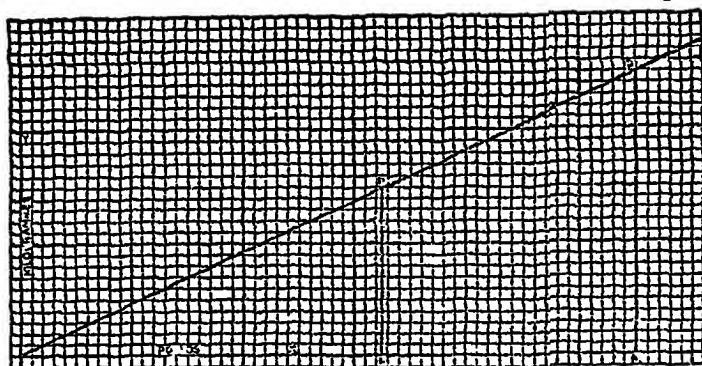
If x lbs = y kilogrammes

$$\text{then } \frac{x}{2.2} = y$$

$$\therefore x, y = \frac{5}{11} x, y$$

therefore the graph is a st line passing through the origin.

Therefore OP represents the graph



Since corresponding to the ordinate 25 we have the abscissa 55, therefore 25 kilogrammes are equivalent to 55 lbs

Similarly, corresponding to the abscissa 32.5 we have the ordinate 14.7 approximately, therefore 32.5 lbs are equivalent to 14.7 kilogrammes.

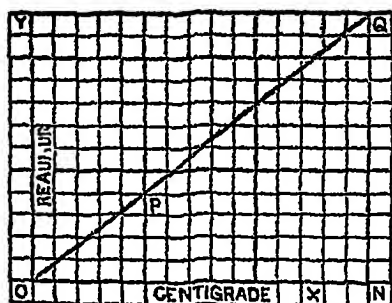
Ex 3 In a Reaumur thermometer the freezing pt stands at 0° and the boiling pt at 80° , in a Centigrade the freezing point at 0° and the boiling at 100° . Draw a chart to convert Centigrade degrees into Reaumur degrees and vice-versa and read off 15°C in R

If τ degrees in Centigrade be equivalent to j' degrees in Reaumur, then

$$\frac{\tau}{100} = \frac{j'}{80}$$

$$\therefore j' = \frac{80}{100} \tau$$

$$\therefore j' = \frac{4}{5} \tau$$



Therefore the graph is a st line passing through the origin

Measure the Centigrade degrees along the axis of τ and the Reaumur degrees along the axis of j' . Since corresponding

to the abscissa 15, we have the ordinate 12, therefore $15C = 12^{\circ}R$

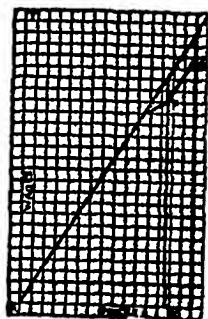
Ex 4 In a certain business it was arranged that the rate of wages of the labourers would rise or fall with the rise or fall of the price of the merchandise. The following table shows corresponding prices and wages

Price	Rs 5	Rs 12 8a	Rs 15	Rs 17 8a	Rs 20
Wages	Rs 8	20	24	26	30

Determine graphically whether the wages are proportional to the prices, if not, take the first pair of prices and wages as the standard, and find graphically what wages would be proportional to the remaining prices

Measure prices along the axis of x and the wages along the axis of y and let us begin from the pt which represents (Rs 5 and Rs 8)

From the fig it is evident that the last two wages are not proportional. The wages Rs 28 and Rs 32 are proportional to the prices

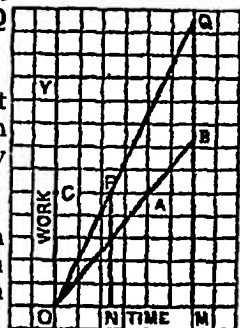


Ex 5 A can do a piece of work in 4 days, B can do it in 6 days, how many days will they take to finish it jointly?

Measure time along the axis of x and let OC represent the whole amount of work. Then OA is the graph of the work done by A. From B, the pt where it meets the vertical line take BQ equal to OC. Join OQ, then OQ is the graph of the work done by them jointly.

From the fig since $PN = OC$ = the amount of work, therefore ON represents the time in which the work is done and ON is evidently 2 4 hrs

Ex 6 A rectangle ABCD contains an area of 80 sq ft. If the length is increased in the ratio of 3 : 2 and the breadth increased in the ratio of 5 : 4. What is the new area?



A 10x10 grid with the following labels:

- Row 1: Y
- Row 2: D
- Row 3: C
- Row 4: (empty)
- Row 5: (empty)
- Row 6: (empty)
- Row 7: (empty)
- Row 8: (empty)
- Row 9: A
- Row 10: B
- Row 11: X

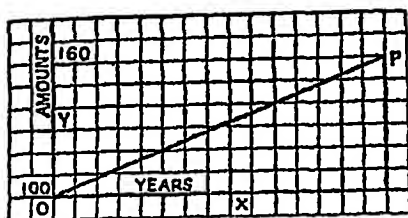
Ex 7 A starts walking at the rate of 3 miles per hour, 30 min after B starts from the same place at the rate of 4 miles per hour. Find graphically when and where B overtakes A.

\therefore OP represents the graph of A's motion B starts 30 min after, therefore M denotes B's starting pt As before let MQ represent the graph of B's motion Then the intersection of MQ and OP viz N will give the place and the time when B overtakes A From the fig it is clear that B will overtake A 2 hrs after A's starting and at 6 miles from the place where A started

Years	3	6	9	12	15
Amount at S I	112	124	136	148	160

The amounts of Simple Interest for 3, 6, 9, 12 and 15 years are Rs 12, Rs 24, Rs 36, Rs 48, Rs 60

Therefore representing the years by the axis of x and the amounts by the axis of y , we get the following graph



Examples II

1 Draw by straight lines a sketch of a section of a mountain from the following data —

Horizontal distance	1000	2000	2800	3000	3700		
Corresponding height	820	2090	2540	3440	2500		

2 The readings of a centigrade thermometer in degrees and the corresponding readings on a Fahrenheit thermometer in degrees are given in the following table —

C	5	10	15	20	30	50	80
F	41	50	59	68	86	122	176

Illustrate graphically the connection between the two scales
Express 140°F in Centigrade

Ans 60°C .

3 If a man first walks 8 miles east and then 5 miles north, how far is he from the starting point?

Ans 9.4 miles nearly

4 Represent graphically the increase of Velocity of a train starting from rest

Distance travelled	50	200	800	1800	3200	5000	feet
Velocity	5	10	20	30	40	50	miles per hr

5 Show graphically the amounts at Simple Interest and Compound Interest of £1 at 5 p c for different periods

No of years	0	5	10	15	20	25
Amt at S I	1	1 25	1 50	1 75	2 00	2 25
Amt at C I	1	1 28	1 63	2 68	2 65	3 39

6 A man bicycles from A to B at 10 miles an hour, and returns from B to A at 15 miles an hour. If he takes 5 hours to go there and back, find the distance from A to B and also his average speed per hour.
Ans 30 miles, 12 miles

7 A starts from Calcutta to walk to Burdwan a distance of 68 miles at 3 miles an hour, two hours later B starts from Burdwan for Calcutta at 5 miles per hour. When will A and B meet? When will they be 20 miles apart?

Ans $9\frac{1}{2}$ hrs from A's start, $7\frac{1}{2}$ hrs and $12\frac{1}{2}$ hrs from A's start

8 A policeman sets off after a thief at 7 A.M. the thief having had ($2\frac{1}{2}$ hrs start). If the thief goes 4 miles an hour and the policeman $5\frac{1}{2}$ miles an hour, when will the policeman catch the thief?

Ans At 3 P.M.

9 A monkey climbing up a greased pole, ascends 5 ft and slips down 2 ft in alternate seconds, until he reaches the top of the pole. If the pole be 17 ft high, how long will it take him to reach the top?

Ans 9 sec

10 Two taps A and B will fill a cistern in 10 and 20 hours respectively. In what time will they fill it together?

Ans 6 2/3 hrs

11 At what times between 6 and 7 are the hands of a clock (i) together, (ii) at right angles?

Ans (i) $32\frac{8}{11}$ min past 6

(ii) $16\frac{4}{11}$ min and $49\frac{1}{11}$ min past 6

12 The expenses of a family when rice is at 20 seers for a rupee are Rs 50 a month, when rice is at 25 seers for a rupee the expenses are Rs 48 a month (other expenses remaining the same) what will they be when rice is at 30 seers for a rupee? Also find how much rice can be had for a rupee when the expenses are Rs 60

Ans Rs 46 10a 8p and 10 seers per rupee

(N.B.—For solution see Graphs of Algebraical Functions, p 20)